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ORIGINAL PAPER

Full- and reduced-order synchronization of a class of time-varying systems containing uncertainties

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Abstract Investigation on chaos synchronization of autonomous dynamical systems has been largely reported in the literature. However, synchronization of time-varying, or nonautonomous, uncertain dynamical systems has received less attention. The present contribution addresses full- and reduced-order synchronization of a class of nonlinear time-varying chaotic systems containing uncertain parameters. A unified framework is established for both the full-order synchronization between two completely identical time-varying uncertain systems and the reduced-order synchronization between two strictly different time -varying uncertain systems. The synchronization is successfully achieved by adjusting the determined algorithms for the estimates of unknown parameters and the linear feedback gain, which is rigorously proved by means of the Lyapunov stability theorem for nonautonomous differential equations together with Barbalat's lemma. Moreover, the synchronization result is robust against the disturbance of noise. We illustrate the applicability for full-order synchronization using two identical parametrically driven pendulum oscillators and for reduced-order synchronization using the parametrically driven second-order pendulum oscillator and an additionally driven third-order Rossler oscillator.

Keywords Synchronization · Time-varying · Uncertain

1 Introduction

Over the past decade or so, a large number of researchers have been devoted to chaos synchronization not only for its importance in theory but also for its potential applications in various areas such as mechanics, biology, neural networks, and secure communications (e.g., see review [1], recent books [2, 3] and references therein). Meanwhile, several kinds of chaos synchronization have been developed, such as complete synchronization [4], generalized synchronization [5], phase synchronization [6], lag synchronization [7] and anticipating synchronization [8], along with corresponding techniques to steer synchronization. Moreover, it is expected that novel theories and further applications associated with chaos synchronization will arise.

Notice that there inevitably exist uncertainties in nature and artificial models, thus, robust synchronization of uncertain dynamical systems is a subject of great interest. Complete synchronization (CS) has been shown to occur in structurally equivalent dynamical systems, i.e., either identical systems [9] or

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systems in which the nonidentity results in a rather slight parameter mismatch [10], as well as in strictly different dynamical systems, i.e., systems with different model structures [11] involving different order [12].

CS was initially confined to coupled identical systems [4]. Yet in many models, including experimental systems and complex systems, every component cannot be assumed to identical. Hence, synchronization of oscillators with different structures is desired, as this may play an essential role in the communication process of systems of laser arrays [13], biological systems [14], and so on. CS has also been analyzed in high-dimension systems with different time delays [15] and in space-extended systems with parameter misfit [16]. In addition, synchronization of strictly different chaotic oscillators whose orders are not equal has currently received much attention for the reason that in some fields (e.g., biology and neurology [17, 18]) the synchronization is carried out even though the oscillators have different order. Synchronization of chaotic oscillators not having same order is usually termed as reduced-order synchronization [12]. It is a problem of synchronizing a slave system with the projection of a master system. Reduced-order synchronization relies upon the premise that the order of the slave system is less than that of the master system. The application for reduced-order synchronization of systems with uncertainty to secure communication is discussed in Ref. [19].

Most of what has been said describes the synchronization of structurally equivalent or strictly different, yet autonomous, dynamical systems which may be uncertain in parameter or structure. However, less has been done in time-varying uncertain, strictly different, nonlinear dynamical systems. This is of great relevance in many areas such as chaotic secure communication, engineering and other nonlinear disciplines. Therefore, our aim in this work is to discuss full- and reduced-order synchronization of a class of time-varying uncertain systems with identical or different models. To this end, in Sect. 2, we describe a unified framework for both the full-order and reduced-order synchronization. Section 3 exemplifies the application of this procedure by the parametrically driven pendulum and additionally forced Rossler oscillators. Finally, we present our conclusion in Sect. 4.

2 The statement of the problem

Consider a nonlinear time-varying dynamical system

$$\dot{x} = f(x,t) + F(x,t)p, \tag{1}$$

where $x \in \Omega_1 \subset R^n$ denotes the state variables, $p \in R^k$ denotes the uncertain parameters, and $f : R^n \times R^+ \to R^n$ and $F : R^n \times R^+ \to R^{n\times k}$ ($f, F \in L_\infty$ for $(x,t) \in \Omega_1 \times R^+$) represent nonlinear vector functions. We view model (1) as the master system and introduce a controlled slave system

$$\dot{y} = g(y,t) + G(y,t)q + u, \tag{2}$$

in which $y \in \Omega_2 \subset R^{n_1}$ denotes the state variables, $q \in R^l$ denotes the uncertain parameters, $g : R^{n_1} \times R^+ \to R^{n_1}$ and $G : R^{n_1} \times R^+ \to R^{n_1\times l}$ ($g, G \in L_\infty$ for $(y,t) \in \Omega_2 \times R^+$) are nonlinear vector functions, and u is an input or a controller. Suppose the master and slave systems are linearly dependent on their respective parameters. Clearly, many kinds of time-varying systems consisting of externally (parametrically) periodically forced ones can be described by the form of (1) or (2). Determined by the order and the structure of the considered models, systems (1) and (2) can be loosely understood to be completely identical ($n = n_1$, $k = l$, $f = g$, $F = G$, $p = q$) or strictly different ($n \neq n_1$). In the following, we plan to introduce an adaptive feedback strategy, a powerful tool in coping with control and synchronization of uncertain dynamical systems, to synchronize systems (1) and (2) in the case of completely identical and strictly different model structures.

2.1 Full-order synchronization of two identical time-varying uncertain systems

For the former case, systems (1) and (2) are completely identical yet with unknown parameters. Now, the main purpose is to design a suitable controller u to synchronize the two identical systems in spite of the differences in their initial conditions. Denote the synchronization error between the two systems as $e = y - x \in R^n$. To determine the controller, we subtract system (1) from system (2) and thus obtain the error dynamical system

$$\dot{e} = f(y,t) - f(x,t) + F(y,t)p - F(x,t)p + u. \tag{3}$$

Note that synchronizing two chaotic dynamical systems is essentially equivalent to stabilizing their corresponding error dynamical system at the origin, that is to say, two chaotic systems synchronize if the zero solution of their error system is asymptotically stable. So we can introduce the control function

$$u = f(x,t) - f(y,t) + \big[F(x,t) - F(y,t)\big]p_e - k_e e, \tag{4}$$

where p_e is the estimate of p, and $k_e e = (k_{e_1}e_1, k_{e_2}e_2, \ldots, k_{e_n}e_n)^T \in R^n$ is the linear feedback control with the updated gain $k_e = (k_{e_1}, k_{e_2}, \ldots, k_{e_n})^T \in R^n$. Thus, the synchronization error system is reduced

$$\dot{e} = \big[F(y,t) - F(x,t)\big]\tilde{p} - k_e e, \tag{5}$$

in which $\tilde{p} = p - p_e$ is the parameter estimation mismatch between the real value of the unknown parameter and its corresponding estimated value. Then the above discussion can be summarized as the following result.

Theorem 1 *If the estimates of the unknown parameters and the feedback gain contained in the adaptive controller* (4) *are on-line adjusted by the algorithms*

$$\begin{cases} \dot{p}_e = [F(y,t) - F(x,t)]^T e, \\ \dot{k}_e = \sigma e^T e. \end{cases} \tag{6}$$

in which $\sigma e^T e = (\sigma_1 e_1^2, \sigma_2 e_2^2, \ldots, \sigma_n e_n^2)^T$ *with* $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T$ *representing an arbitrary positive constant vector, then the synchronization error dynamical system* (5) *is asymptotically stable at the origin.*

Proof For convenience, we first group the two systems consisting of the synchronization error system (5) and parameter estimation error and feedback gain system (6) as an augmented system. The point $O = (e = 0, \tilde{p} = 0, k_e = k^*)$ in which $k^* = (k_1^*, k_2^*, \ldots, k_n^*)^T \in R^n$ denotes a certain positive constant vector, is a stationary point of the augmented system. Construct a function in some neighborhood of this stationary point

$$V = \frac{1}{2}\left(e^T e + \tilde{p}^T\tilde{p} + \sum_{i=1}^{n}\frac{1}{\sigma_i}\big(k_{e_i} - k_i^*\big)^2\right). \tag{7}$$

The time derivative of V along the trajectories of the time-varying augmented system is

$$\begin{aligned} \dot{V} &= \dot{e}^T e - \tilde{p}^T\dot{p}_e + \sum_{i=1}^{n}\frac{1}{\sigma_i}\big(k_{e_i} - k_i^*\big)\dot{k}_{e_i} \\ &= \big[\big(F(y,t) - F(x,t)\big)\tilde{p} - k_e e\big]^T e \\ &\quad - \tilde{p}^T\big[F(y,t) - F(x,t)\big]^T e + \sum_{i=1}^{n}\big(k_{e_i} - k_i^*\big)e_i^2 \\ &= -\sum_{i=1}^{n}k_i^* e_i^2 \le 0. \end{aligned} \tag{8}$$

It is easy to check that the auxiliary function V is a Lyapunov function. According to the Lyapunov stability theorem for nonautonomous differential equations [20], the equilibrium point O of the augmented system is Lyapunov stable, i.e., $e \in L_\infty$, $\tilde{p} \in L_\infty$, and $k_e \in L_\infty$. From (8) one can see that $e \in L_2$. Then, by Barbalat's lemma [21], for any initial condition, (5) implies $\dot{e} \in L_\infty$, which in turn indicates $e \to 0$ as $t \to \infty$. Then the error system (5) is asymptotically stable at the origin. This illustrates that the two identical time-varying uncertain systems (1) and (2) can attain full-order synchronization by the adaptive controller (4) with the updated laws (6) for parameter estimation and feedback gain. This completes the proof. □

2.2 Reduced-order synchronization of two different time-varying uncertain systems

For the latter case of $n_1 < n$, synchronization between the two systems in the form of (1) and (2) would be obtained only in reduced order. The reduced-order synchronization, first proposed in Ref. [12], is actually a problem of controlling a slave system to the projection of a master system. So, the master system should be divided into two parts, with the size of one part equal to that of the slave system. Then the first part of the master system (1), the projection, is assumed in the form

$$\dot{x}_p = f_p(x,t) + F_p(x,t)p, \tag{9}$$

in which $x_p \in R^{n_1}$, $f_p : R^n \times R^+ \to R^{n_1}$ and $F_p : R^n \times R^+ \to R^{n_1 \times k}$.

The other part of the master system (1), is given as

$$\dot{x}_r = f_r(x,t) + F_r(x,t)p, \tag{10}$$

where $x_r \in R^{n-n_1}$, $f_r : R^n \times R^+ \to R^{n-n_1}$ and $F_r : R^n \times R^+ \to R^{(n-n_1)\times k}$.

Let $e = y - x_p \in R^{n_1}$ denote the synchronization error. Then the error dynamical system between systems (2) and (9) is obtained:

$$\dot{e} = g(y,t) + G(y,t)q - f_p(x,t) - F_p(x,t)p + u. \tag{11}$$

Now, the aim is to design an adaptive controller u to synchronize system (2) with system (9). Viewing the error system (11), the controller can be chosen

$$u = f_p(x,t) - g(y,t) + F_p(x,t)p_e - G(y,t)q_e - k_e e, \tag{12}$$

where q_e is the estimate of q and $k_e e = (k_{e_1}e_1, k_{e_2}e_2, \ldots, k_{e_{n_1}}e_{n_1})^T \in R^{n_1}$ is the linear feedback control with the updated gain $k_e = (k_{e_1}, k_{e_2}, \ldots, k_{e_{n_1}})^T \in R^{n_1}$.

The error dynamical system (11) can be rewritten

$$\dot{e} = G(y,t)\tilde{q} - F_p(x,t)\tilde{p} - k_e e, \tag{13}$$

where $\tilde{q} = q - q_e$ is the parameter estimation misfit.

Similar to the discussion in Sect. 2.1, the error dynamical system (13) can be asymptotically stable at the origin if some adapted laws concerning parameter estimation and linear feedback gain are appropriately selected. Omitting the detailed proof, we directly list the main result in Theorem 2.

Theorem 2 *If the updated algorithms for the estimates of unknown parameters and the feedback strength included in the adaptive controller (12) are selected*

$$\begin{cases} \dot{p}_e = -[F_p(x,t)]^T e, \\ \dot{q}_e = [G(y,t)]^T e, \\ \dot{k}_e = \sigma e^T e, \end{cases} \tag{14}$$

where $\sigma e^T e = (\sigma_1 e_1^2, \sigma_2 e_2^2, \ldots, \sigma_{n_1} e_{n_1}^2)^T$ *with* $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{n_1})^T$ *denoting an arbitrary positive constant vector, then the adaptive controller (12) can successfully steer the error system (13) to the origin. That is to say, synchronization between systems (1) and (2) is obtained in reduced order.*

3 Applications of synchronization

To show how the proposed procedure applies in chaos synchronization, two typical chaotic systems of a parametrically driven second-order nonlinear pendulum oscillator and an additionally forced third-order nonlinear Rossler oscillator are used. The chaotic behaviors in the considered pendulum [22, 23] and Rossler [24, 25] oscillators as well as their control and synchronization have been widely discussed by theoretical derivation, numerical simulation and experimental analysis. However, investigation of synchronization of the uncertain case has received less attention. Thus, in the following, we demonstrate the full-order synchronization of the two uncertain pendulum oscillators in spite of the differences in initial conditions in Example 1. Then we demonstrate the reduced-order synchronization of the uncertain pendulum and Rossler oscillators in Example 2.

Example 1 We first apply Theorem 1 to synchronize the parametrically driven uncertain Pendulum oscillator [22] whose states are evolved by

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\beta x_2 - (1 + \rho\cos 2t)\sin x_1. \end{cases} \tag{15}$$

This system, containing the unknown parameters β and ρ, represents the master. The controlled slave system, having the same form as the master except for the addition of a controller, is constructed as

$$\begin{cases} \dot{y}_1 = y_2 + u_1, \\ \dot{y}_2 = -\beta y_2 - (1 + \rho\cos 2t)\sin y_1 + u_2. \end{cases} \tag{16}$$

According to the general framework addressed in Sect. 2.1, the adaptive controller is designed

$$\begin{cases} u_1 = -e_2 - k_{e_1}e_1, \\ u_2 = \beta_e e_2 + 2(1 + \rho_e\cos 2t)\sin\frac{e_1}{2} \\ \qquad \times \cos\left(x_1 + \frac{e_1}{2}\right) - k_{e_2}e_2. \end{cases} \tag{17}$$

The updated laws for the parameter estimate (β_e, ρ_e) and the feedback gain (k_{e_1}, k_{e_2}) in the above controller are determined

$$\begin{cases} \dot{\beta}_e = -e_2^2, \\ \dot{\rho}_e = -2\cos 2t \sin\frac{e_1}{2}\cos\left(\frac{e_1}{2} + x_1\right)e_2, \\ \dot{k}_{e_1} = \sigma_1 e_1^2, \quad \dot{k}_{e_2} = \sigma_2 e_2^2. \end{cases} \tag{18}$$

We point out that the sixth-order Runge–Kutta method with a time step size 0.001 is employed throughout the numerical simulation. The simulated values of the unknown parameters of the pendulum

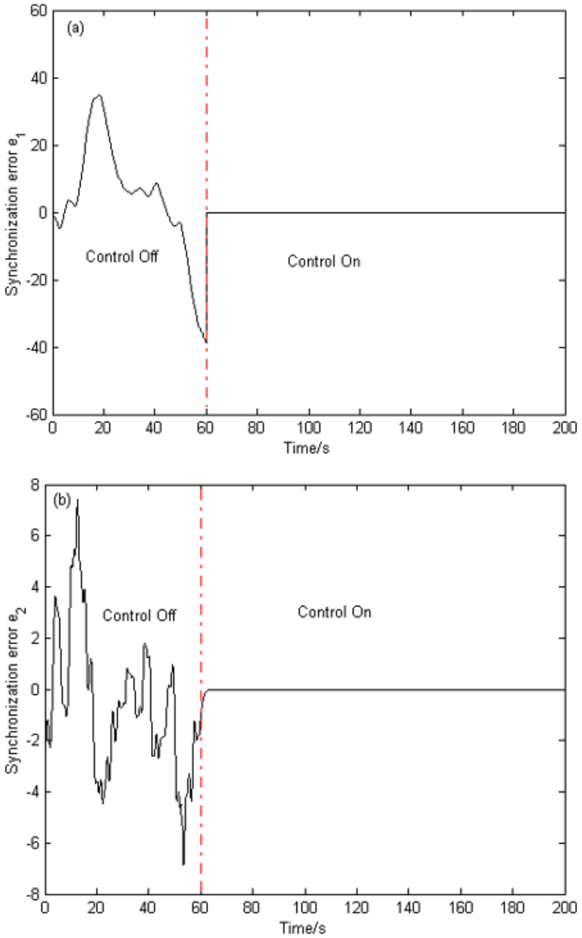

Fig. 1 **a** and **b** depict the full-order synchronous behavior of two identical uncertain pendulum oscillators. The adaptive controller (17) is activated at $t \geq 60$

oscillator are adopted $\beta = 0.1$ and $p = 2.0$ such that it can display chaotic dynamics. Without loss of generality, the initial values for the master and slave are located at $(x_1, x_2) = (0.1, 1.5)$ and $(y_1, y_2) = (0.3, –1.0)$, respectively. The parameter estimations and feedback gains all start from zero. When selecting $\sigma_1 = \sigma_2 = 1.0$ and activating control at $t \geq 60$, the synchronization results of the two identical uncertain pendulum oscillators are presented in Fig. 1, from which one can see that the synchronization errors between systems (15) and (16) display an erratic behavior without control actions, while they tend to zero promptly once the controller (17) switches at $t \geq 60$.

Example 2 We apply Theorem 2 to synchronize the above described pendulum oscillator to an additionally periodically driven Rossler oscillator. Thus, the Rossler oscillator, given by the following state equations [25], is termed as the master system

$$\begin{cases} \dot{x}_1 = -x_2 - x_3, \\ \dot{x}_2 = x_1 + a x_2, \\ \dot{x}_3 = b - c x_3 + x_1 x_3 + d \cos t. \end{cases} \tag{19}$$

The evolution of the controlled slave pendulum oscillator is depicted in (16). All of the parameters a, b, c, d, β and ρ involved in the master and slave systems are unknown or uncertain. To meet the aim of synchronizing the pendulum oscillator with a projection of the Rossler oscillator, we project the Rossler oscillator in the (x_2, x_3) plane

$$\begin{cases} \dot{x}_2 = x_1 + a x_2, \\ \dot{x}_3 = b - c x_3 + x_1 x_3 + d \cos t. \end{cases} \tag{20}$$

The synchronization error states can be defined as $e_i = y_i - x_{i+1}$, $i = 1, 2$. According to the general framework addressed in Sect. 2.2, the adaptive controller is devised

$$\begin{cases} u_1 = -(e_2 + x_3) + a_e x_2 - k_{e_1} e_1, \\ u_2 = x_1 x_3 + \beta_e (e_2 + x_3) \\ \qquad + (1 + \rho_e \cos 2t) \sin(e_1 + x_2) + b_e \\ \qquad - c_e x_3 + d_e \cos t - k_{e_2} e_2. \end{cases} \tag{21}$$

The parameter estimation $(\beta_e, \rho_e, a_e, b_e, c_e, d_e)$ and feedback gain (k_{e_1}, k_{e_2}) should be updated by the following laws

$$\begin{cases} \dot{a}_e = -e_1 x_2, \quad \dot{b}_e = -e_2, \\ \dot{c}_e = e_2 x_3, \quad \dot{d}_e = -e_2 \cos t, \\ \dot{\beta}_e = -(e_2 + x_3) e_2, \\ \dot{\rho}_e = -e_2 \cos 2t \sin(e_1 + x_2), \\ \dot{k}_{e_1} = \sigma_1 e_1^2, \quad \dot{k}_{e_2} = \sigma_2 e_2^2. \end{cases} \tag{22}$$

The simulated values of the unknown parameters in the Rossler system are taken as $a = 0.398$, $b = 2.0$, $c = 4.0$, $d = 1.0$ to ensure the oscillator's chaotic behavior. The initial conditions for the master and slave oscillators, for the parameter estimations and feedback gains are set generally as $(x_1, x_2, x_3) = (0, 0, 0)$, $(y_1, y_2) = (0.1, 1.5)$, $(\beta_e, p_e) = (1.5, 1.1)$, $(a_e, b_e, c_e, d_e) = (0.1, 2.1, 3.0, 1.5)$ and $(k_{e_1}, k_{e_2}) = (0, 0)$, respectively. The numerical results of the reduced-order synchronization between the pendulum and Rossler oscillators are presented in Fig. 2 when selecting $\sigma_1 = \sigma_2 = 1.0$ and activating controller (21)

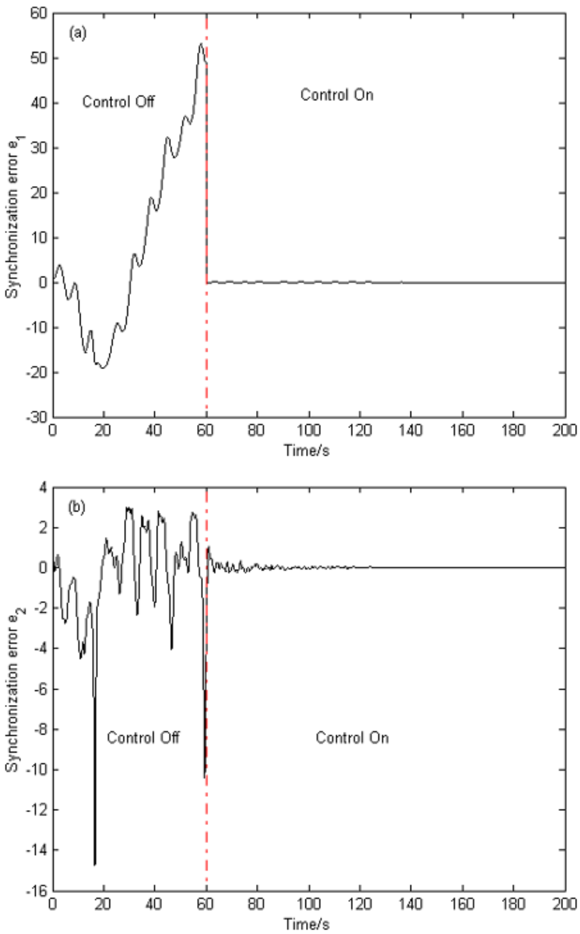

Fig. 2 **a** and **b** display the reduced-order synchronous behavior of the uncertain pendulum and Rossler oscillators. The adaptive controller (22) is activated at $t \geq 60$

at $t \geq 60$. The evolutions of the synchronization errors between systems (16) and (20), depicted in Figure 2(a) and (b), display an erratic behavior when the controller is off, while they asymptotically tend to zero as soon as the controller is activated at $t \geq 60$.

Another essential element concerning chaos synchronization is to check whether the synchronization remains against the disturbances such as the action of external (internal) noise on the systems under consideration. Now, we analyze the robustness of our proposed adaptive control strategy in a noisy environment. Suppose there is small additive uniformly distributed random noise disturbing the master system and the procedure as described above is still carried out. Fig. 3 indicates that when an additively uniformly distributed random noise in the range of $[-20, 20]$ is working on the Rossler oscillator (19), the adaptive

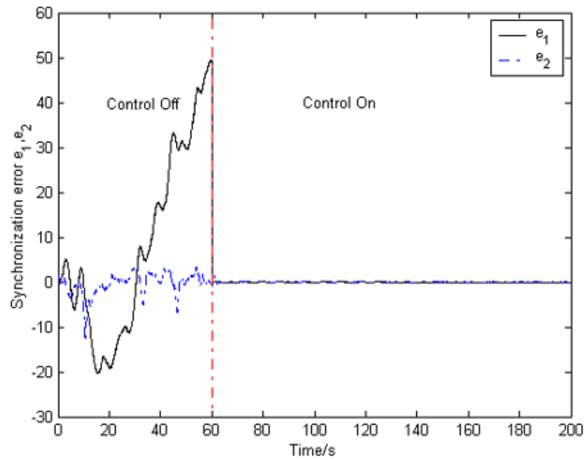

Fig. 3 The graph shows the synchronization errors of the controlled pendulum oscillator (16) and the driving Rossler oscillator (19) when the disturbance of additive uniformly distributed random noise with the range of $[-20, 20]$ is working on the Rossler oscillator

controller (21) is still sufficient to steer the synchronization error to the origin once the control action is activated, though the behavior of the error dynamical system is something different from that of the noise-free one before activating the control.

4 Conclusion

In summary, the main contribution of this paper is in describing a unified mathematical frame for global synchronization of both two completely identical and two strictly different time-varying uncertain chaotic systems. The former situation involves the synchronization of two trajectories starting from arbitrary conditions, namely, full-order synchronization; and the latter situation is the synchronization of all the state variables of the slave system with those of the projection of the master system, namely, reduced-order synchronization. A remarkable feature as well as a central distinction between the present study and the previous ones in the literature is that the dynamical systems involved are nonautonomous and uncertain in parameters. Due to this fact, an adaptive control scheme, associated with the updated laws for the estimates of unknown parameters and feedback gains, is systematically implemented. The feasibility of the principle is rigorously proved using the union of the Lyapunov stability theorem for nonautonomous differential equations and Barbalat's lemma. Applications to the full-order synchronization of two identical nonlinear pendulum oscillators and the reduced-order synchronization of the pendulum oscillator and a nonlinear Rossler oscillator are presented. The numerical simulations agree well with the theoretical analysis. Moreover, the robustness property of this synchronization strategy against the disturbance of noise is checked, and the results show that the procedure applies for the case of a noisy environment.

Finally, we stress that the adaptive strategy proposed in this study is introduced in a unified mathematical frame and it is in particular adequate for the synchronization of a wide class of time-varying and uncertain nonlinear dynamical systems. Moreover, the linear feedback strength contained in the adaptive controller is on-line adjusted according to the current state errors of the involved oscillators, which can avoid the occurrence of the too large or too small fixed gain value estimated before activating the control. Clearly, these characteristics of the present scheme are of great practical interest in synchronization theory; however, there are also some trade-offs in terms of full knowledge of the systems' states and a little complex structure of the controller. In fact, designing a simple and universal control strategy is desired in real-world situations. This effect will require far more research effort and a careful investigation is left for future work.

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