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ORIGINAL ARTICLE

Evolutive and nonlinear vibrations of rotor on aerodynamic bearings

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Abstract The use of air as a lubricant in aerodynamic bearings is advantageous, particularly in the food industry. Aerodynamic bearings with tilting pads have complicated stiffness and damping properties and need a very detailed theoretical and experimental research. Response curves of rigid rotor supported on aerodynamic bearings are presented for a linear but evolutive mathematical model. Due to non-monotone properties of stiffness and damping matrices at variable revolutions, a new resonance appears. The mathematical model of rotor vibrations in the whole area of bearing clearance is also developed in the consideration of strongly nonlinear properties of aerodynamic bearing.

Keywords Rotordynamics · Aerodynamic bearings · Tilting pads · Numerical solution · Large displacements · Response curves · Vibrations

1 Introduction

The new progressive type of bearing is based on the aerodynamic principle working with only atmospheric pressure without any input of compressed air. The use of air as a lubricant in bearings is very advantageous, particularly in the chemical and food industry and also for ecological reasons. Each gas bearing generates complicated stiffness and damping forces during rotation, not yet sufficiently known. Especially their nonlinear and evolutive (frequency-dependent elements of stiffness and damping matrices) properties were mostly neglected.

The properties of aerodynamic tilting pads bearings were ascertained by numerical solution in the form of evolutive stiffness and damping matrices calculated in TECHLAB Ltd. and used in IT ASCR for analyses of dynamic behavior of the experimental rigid rotor. The numerically gained response curves of such linearized system show that some new resonance peaks can occur.

The behavior of the system at larger displacements of the rotor journal, where the nonlinearity of aerodynamic forces are considerable, was investigated as well.

The presented paper is a contribution to the theoretical investigation of one type of aerodynamic bearing developed in the Institute of Thermomechanics [1, 6]. The main aim of our work is to develop and to prepare (for further application) the complex but adequate simple function describing the dynamic properties of aerodynamic tilting pad bearing, which enables to study by means of asymptotic and/or numerical methods the rotor motion at great amplitudes.

These amplitudes are limited by the bearing clearance. Journal motion in the entire clearance is strongly influenced by the nonlinearity of the bearing characteristic, as is shown in the following survey.

The initial impulse for the theoretical analysis of rotor vibrations in large was inspired by the results of

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measurements on the experimental stand in IT ASCR, where the vibrations at some conditions differed very strongly from steady harmonic form.

For analytical and/or numerical solution of general types of motion of nonlinear rotor system containing aerodynamic self-acting tilting pad bearings, we need to express the nonlinear stiffness and damping properties for the whole range of clearance in analytical form.

The literature survey done as a first stage of our research work shows that the majority of articles on dynamic properties of fluid bearings describe these properties usually by means of linearized matrix of stiffness and damping coefficients.

Because it is expected that the dependence of matrix coefficients on radial displacements at oil and gas bearings are very similar, we have both types in view. From the same reason, the bearings with and without tilting pads were included into the survey as well.

References [1, 3, 5–7] describe the previous research results reached at the development of the prototypes of aerodynamic (self-acting) tilting pad bearings in Czech Republic. The presented paper extends these publications. Some parts of [2] were used in this paper for solution of linear and nonlinear dynamic problems.

Reference [4] is the basic source for study of dynamic properties of rotors supported on oil-film bearings, while [8] is the fundamental work for linear theory of tilting pad gas bearings, but it is not suitable for non-steady motion, where the nonlinearity plays a significant role.

Nicholas [9] presents a very nice state-of-art in tilting pads bearings in the last 50 years (more than 30 citations), where in addition to dynamics also, lot of other aspects were discussed (thermo effects, inputs of oil, etc.).

Paper [10] oriented on high stiffness pressuring gas bearing used nonlinear model, but the solution is based on iterative method, which is not quite suitable for our application. Strzelecki [11] studied the dynamic characteristics of five-pad journal bearing operating at turbulent oil film. We use the presented diagrams, Sommerfeld's number versus eccentricity ϵ, as a base for the choice of correction function f_{cor} of our gas bearing. Papers [12, 13] present similar dynamic characteristics of five-pad and of plain circular journal bearing but all with respect to temperature increase.

Dynamic properties were also studied in [14], where the nonlinear properties of gas bearing, but without tilting pads, were ascertained. Presented method seems rather time-consuming, but for the verification of selected cases, it would be convenient.

The dynamic behavior of real turbo-expander supported on pressurized methane gas radial bearings is presented in [15]. It is seen that the vibrations can reach very high amplitudes and complicated forms. The research of these motions is therefore important.

One of the contributions to the nonlinear behavior of rotor is [16], where the theoretical analysis of the simplest mathematical model of rotor system (mass supported by cylindrical oil bearing) is solved by numerical integration in the entire clearance field and Hopf bifurcations are indicated. Chaotic motion of simple model of rotor is derived in [17].

More complicated system is studied in [18], where the dynamic behavior in the large of a rotor on plain short hydrodynamic bearings is investigated and oil whip, whirl and other forms of oscillations including chaotic are recorded at large displacements.

The similar system – Jeffcott rotor on tilting pad oil bearings – is studied experimentally in [19] from the point of view of stability and occurrence of self-excited vibration. The linear model of stiffness and damping bearing properties are applied in the added theoretical analysis, so that the stability threshold speeds can be ascertained.

Results of theoretical investigation of rotor supported in flexibly mounted self-acting gas journal bearings [20], where it is shown that the self-excited vibration can be removed by appropriate choice of stiffness and damping of flexibly mounted bush, are very interesting.

A new procedure for diagnostic of nonlinear effects using dimensionless diagnostic indexes is proposed in [21]. It is oriented on plain oil bearing, but this idea might also be useful for gas tilting pad bearing.

Three-sleeve cylindrical oil bearing is experimentally and by numerical computation studied in [22]. Stiffness and damping coefficients show strongly nonlinear increase with increasing eccentricity. The influences of inward and outward oil flow on hydrodynamic characteristics were theoretically and experimentally investigated in [23] on the similar type of sleeve journal bearing. The strong nonlinearity causes even the dual equilibrium positions at certain rotor speed.

Nonlinear stiffness and damping coefficients of a hydrodynamic cylindrical bearing supporting a vertical rigid rotor is derived in [24]. The procedure for determining the characteristic nonlinearity is based on the application of Krylov–Bogoljubov's averaging method.

Nonlinear effects due to the oil film in cylindrical bearing on the response curves of horizontal rotor were studied in [25] for different unbalance values, by means of changing the rate of lubricant flow and for different oil types.

The analytical methods for solution of nonlinear problems and stability of rotor motion with regard to the nonlinear properties of gas bearings are based on the procedures elaborated in [2, 26–28].

In spite of the very large extend of publications on nonlinear dynamic properties of fluid bearings, the appropriate methods of solution of general rotor motion in three-pad aerodynamic bearings in the entire field of bearing clearance was not found. However, in each of the above-mentioned articles, there is partial information useful for our work.

The following contribution is based on the first part of the solution of linear dynamics characteristics for small displacements around the equilibrium journal position [29], which is widespread for the whole region of bearing clearance by means of correction function. It is only approximate solution, but it enables to solve very general types of oscillations occurring in the nonlinear systems.

2 Evaluation of the bearing characteristics

The theoretical model of the studied rotor describes the properties of experimental rotor supported on aerodynamic bearings with diameter $d = 50\,\mathrm{mm}$. The x-axis is oriented downwards, and y-axis is oriented to the right at the left-hand rotation.

The basic computational model of the aerodynamic bearing was elaborated in TECHLAB Ltd. using numerical solution (FEM) of Reynolds equations of flow in the bearing. An algorithm of calculation of laminar gas flow in bearing in Fortran is based on the work of Lund [8, 9].

The used program for calculation of dynamic characteristics of bearings at different revolutions takes into account inertia properties of tilting pads. The vertical load from the weight was $F_{\mathrm{st}} = 38\,\mathrm{N}$. This program gives discrete values of elements of full stiffness and damping matrices [3, 5]. Successful running of prototype [1] proves the correctness of calculation by the program developed for design of aerodynamic bearings. A photo of a tilting pad bearing is included in Figure 3b.

In spite of that, the method of calculation was elaborated for constant revolutions, the experiments show that bearing properties do not change essentially with the slow increase or decrease of rotor angular velocity.

The TECHLAB description of linear and frequency-dependent stiffness and damping properties was applied for solution of rotor motion. Discrete values must be replaced for numerical solution by continuous functions of angular velocities ω (s^{-1}). These values vary relatively very strong at different revolutions. There are no monotone functions for these variations because the inertia of tilting pads causes resonance phenomena, superimposed on the monotone increase or decrease during variation of revolutions. The substitutive functions for the following solution must be therefore selected as a combination of monotone polynomials $c + d\omega$ or $c + d\omega + f\omega^2$ and of functions describing real or imaginary components of 1-DOF system response:

$$
\begin{aligned}
K &= c + d\omega + a\frac{\Omega^2-\omega^2}{(\Omega^2-\omega^2)^2+b^2\omega^2},\\
K &= c + d\omega + f\omega^2 + a\frac{b\omega}{(\Omega^2-\omega^2)^2+b^2\omega^2}.
\end{aligned}
\tag{1}
$$

Good agreement of analytical continuous functions with discrete points (circles) can be reached by appropriate selection of parameters c, d, f, Ω^2, a, b. The functions of stiffness $K_{xx}(\omega)$ and damping $B_{xx}(\omega)$ are shown in Figs. 1 and 2 as examples.

3 Dynamical parameters of rotor

The experimental rotor (Fig. 3a) is symmetric, with total mass $m = 7.6\,\mathrm{kg}$. Inertia moment to the y- or x-axis is $I = 0.10024\,\mathrm{kg\,m^2}$. The distance between the centers of bearings is $l = 0.32\,\mathrm{m}$. The inertia properties defined by mass m and moment of inertia I can also be replaced by effects of three masses [2], two of them m_1, m_2 situated in the centers of bearings and the third mass m_3 in the center of gravity T. The centrifugal force $me\omega^2$ acts at the distance a to the right from the center T.

The rotor is supported on two identical three-pad aerodynamic bearings (diameter $d = 50\,\mathrm{mm}$, clearance $\delta = 0.05\,\mathrm{mm}$), which, at sufficiently high revolutions, do not need any supply of air pressure, as the surrounding air is drawn into bearing and forms a load-bearing lubricated film. The simple structure of aerodynamic bearing is shown in Fig. 3b.

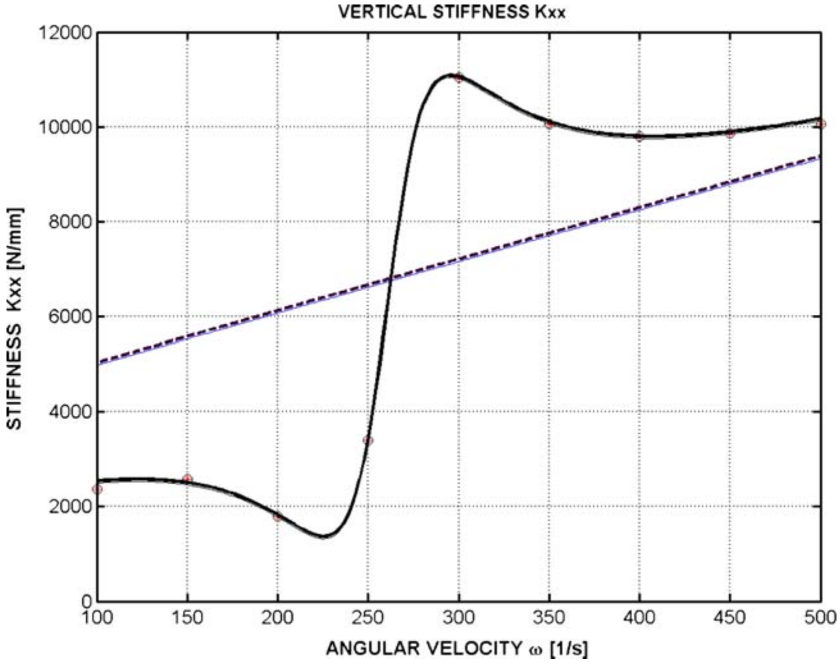

Fig. 1 Stiffness K_{xx} versus frequency ω

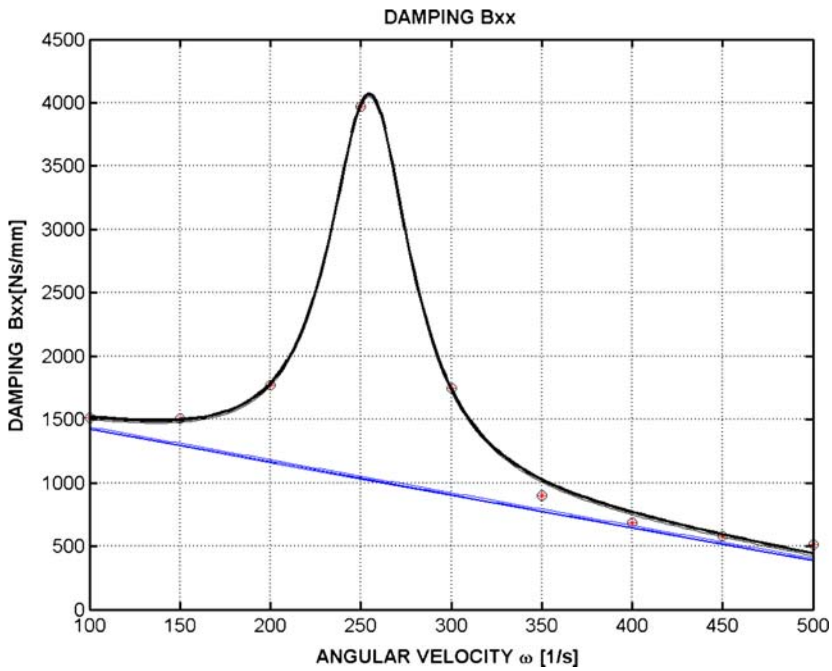

Fig. 2 Damping B_{xx} versus frequency ω

4 Differential equation of motion including the inertia effects of tilting pads

Matrices K and B are used for calculations of responses of rotor at unbalance excitation, ascertained by the following equations:

$$(m_1 + m_3/4)\ddot{x}_1 + m_3/4\ddot{x}_2 + K_{xx}(\omega)x_1 + B_{xx}(\omega)\dot{x}_1 + K_{xy}(\omega)y_1 + B_{xy}(\omega)\dot{y}_1 = (1/2 - a/l)me\omega^2 \cos \omega t,$$

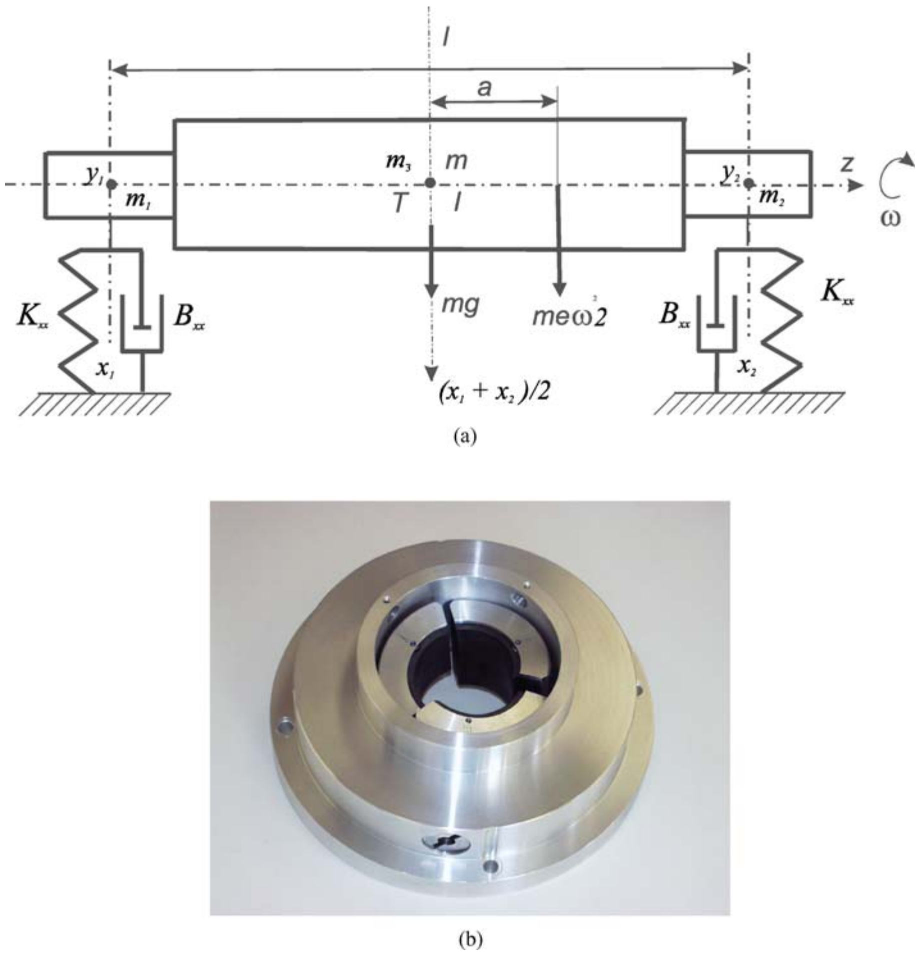

Fig. 3 (**a**) Symmetric rotor. (**b**) Aerodynamic three tilting pads bearing

$$
\begin{aligned}
&(m_2 + m_3/4)\ddot{x}_2 + m_3/4\ddot{x}_1 + K_{xx}(\omega)x_2 + B_{xx}(\omega)\dot{x}_2 \\
&\quad + K_{xy}(\omega)y_2 + B_{xy}(\omega)\dot{y}_2 = (1/2 + a/l)me\omega^2 \cos \omega t, \\
&(m_1 + m_3/4)\ddot{y}_1 + m_3/4\ddot{y}_2 + K_{yy}(\omega)y_1 + B_{yy}(\omega)\dot{y}_1 \\
&\quad + K_{yx}(\omega)x_1 + B_{yx}(\omega)\dot{x}_1 = (1/2 - a/l)me\omega^2 \sin \omega t \\
&(m_2 + m_3/4)\ddot{y}_2 + m_3/4\ddot{y}_1 + K_{yy}(\omega)y_2 + B_{yy}(\omega)\dot{y}_2 \\
&\quad + K_{yx}(\omega)x_2 + B_{yx}(\omega)\dot{x}_2 = (1/2 + a/l)me\omega^2 \sin \omega t,
\end{aligned}
\tag{2}
$$

where x_1 and x_2 are measured from the equilibrium positions, given by constant load mg.

These equations must be transformed into such a form, where each of them contain second derivative of only one variable. Due to the linearity of expressions (2) and of the validity of the superposition principle, the influence of weight mg can be neglected and analyzed separately from the influence of periodic forces.

Response curves of vertical oscillations $x_{1\,\max}(\omega)$ and $x_{2\,\max}(\omega)$ for eccentricity $e = 5\,\mu$m and for unbalance shifted out of the center by $a/l = 0.25$ are plotted in the upper half of Fig. 4. Response curves of horizontal oscillations $y_{1\,\max}(\omega)$ and $y_{2\,\max}(\omega)$ are in the bottom half. All response curves are very flat due to the high damping and each of the curves has only one marked resonance peak.

For estimation of eigenfrequencies, the solution was repeated once more with the damping eight times lower. It is notable in Fig. 5 that five resonance frequencies are recorded, which can be explained by the influence of additional tilting pad resonances and in consequence by the strongly evolutive properties of the entire bearing.

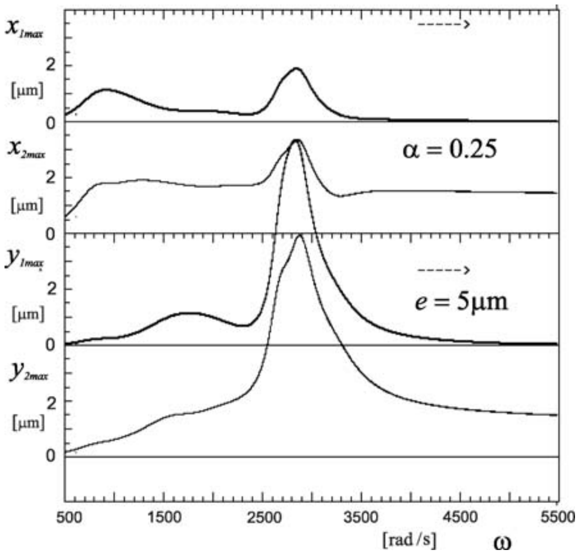

Fig. 4 Unbalance excitation in $a/l = 0.25$

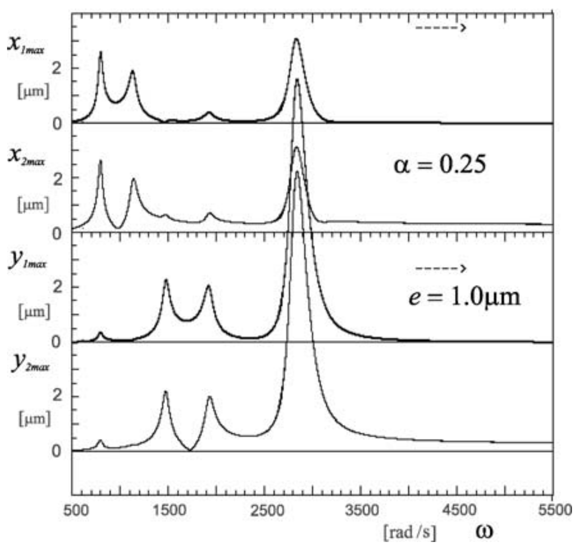

Fig. 5 Response curves for eight times lower damping

5 Extending of linear characteristics for the large displacements

Characteristics calculated in TECHLAB are suitable due to their linearity only for small displacements from the equilibrium position. This equilibrium point is defined by the constant shift of the center of journal from the center of bearing bush due to the rotor weight at certain angular velocity in the range $\omega \in (1000, 5500)$ rad/s.

The nonlinear form has to be used for larger displacement. Exact calculation of this more accurate form is very difficult and no numerical algorithm is at disposal at the present time. Therefore, a simple approximate method has to be applied for extended range of the validity. The presented method of extending the linear characteristics to large displacements is only approximate and based on the knowledge of characteristics of other fluid bearings (e.g., [4, 11]). After getting some adequately reliable data from measurements on our prototypes, this method will be treated more precisely.

The logical presumption is that the force field in bearing is central symmetric, which implies that it depends only on radius r, but not on the angle φ. The extension of force properties on the entire bearing field is then realized by multiplying the force characteristics by a correction function $f_{\text{cor}}(r)$, which must ensure the identity of the force field characteristic in the equilibrium point $r = r_0$ with the values calculated by TECHLAB Ltd.

First step is the transformation of linear stiffness and damping characteristics from Cartesian coordinates x, y into polar coordinates r, φ.

After this transformation, we can extend the validity of characteristic on the other positions of journal in the limits of bearing $r \le \delta$, $\varphi \in (-\pi, \pi)$. Based on the knowledge of other types of fluid bearings (e.g. [4]), let us apply the hyperbolic correction function

$$f_{\text{cor}} = \frac{s}{p + \delta - r} - h \tag{3}$$

for the bearing with clearance δ and with three free parameters s, p, h. These parameters can be ascertained from the conditions:

1) Zero values f_{cor} in the center of bearing $r = 0$: $\frac{s}{p+\delta} = h$.
2) Value $f_{\text{cor}} = 1$ at the shift r_0 (e.g. $r_0 = 0.4\delta$), i.e. in the point of stationary state at selected angular velocity ω: $h + 1 = \frac{s}{p+\delta-r_0}$.
3) Vertical force $F_{r0} = G(r_0) = mg/2$ acting against weight $mg/2$ and producing the deviation r_0 of journal axes is given by integration of increments of force $K_{rr} \cdot f_{\text{cor}}\, dr$ from 0 to r_0 at constant angular frequency ω.

$$G(r_0) = \int_0^{r_0} K_{rr} f_{\text{cor}}\, dr = K_{rr} \int_0^{r_0} \left(\frac{s}{p + \delta - r} - h\right) dr = K_{rr}\left[s \ln\left(\frac{p+\delta}{p+\delta-r_0}\right) - h r_0\right] = mg/2, \tag{4}$$

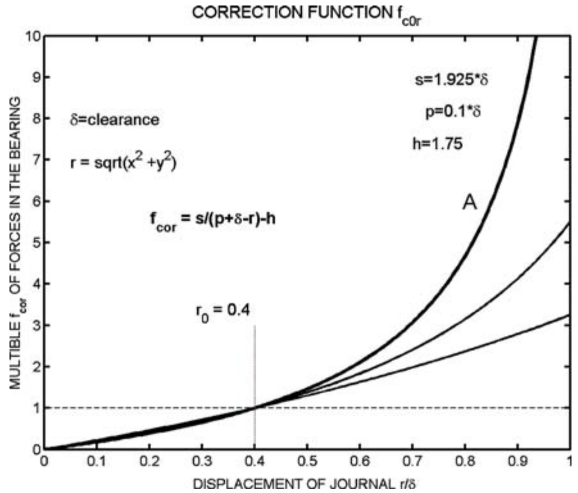

Fig. 6 Correction function $f_{\mathrm{cor}}(r)$

where $K_{rr} = K_{xx}$. By appropriate selection of constants, we get correction function $f_{\mathrm{cor}}(r)$ and its integral $G(r)$ for given revolutions. Three examples of such functions are shown in Fig. 6. The parameters of curve A are given in Fig. 6. The differential equations of rotor motion expressed in polar coordinates we get from (2) after introducing the expressions for displacements, velocities, and accelerations are as follows:

$$\begin{aligned}
x_i &= r_i \cos\varphi_i, \quad y_i = r_i \sin\varphi_i,\\
\dot{x}_i &= \dot{r}_i \cos\varphi_i - r_i\dot{\varphi}_i \sin\varphi_i,\\
\dot{y}_i &= \dot{r}_i \sin\varphi_i + r_i\dot{\varphi}_i \cos\varphi_i \quad i = 1, 2\\
\ddot{x}_i &= \ddot{r}_i \cos\varphi_i - 2\dot{r}_i\dot{\varphi}_i \sin\varphi_i - r_i\dot{\varphi}_i^2 \cos\varphi_i\\
&\quad - r_i\ddot{\varphi}_i \sin\varphi_i,\\
\ddot{y}_i &= \ddot{r}_i \sin\varphi_i + 2\dot{r}_i\dot{\varphi}_i \cos\varphi_i - r_i\dot{\varphi}_i^2 \sin\varphi_i\\
&\quad + r_i\ddot{\varphi}_i \cos\varphi_i,
\end{aligned} \tag{5}$$

and after such a transformation assuring that each of the equations contains the second derivative of only one variable. So we get

$$\begin{aligned}
\ddot{x}_1 &= F_{1x}\alpha_2 - F_{2x}\alpha_3 = \ddot{r}_1 \cos\varphi_1 - r_1\ddot{\varphi}_1 \sin\varphi_1\\
&\quad - 2\dot{r}_1\dot{\varphi}_1 \sin\varphi_1 - r_1\dot{\varphi}_1^2 \cos\varphi_1,\\
\ddot{y}_1 &= F_{1y}\alpha_2 - F_{2y}\alpha_3 = \ddot{r}_1 \sin\varphi_1 + r_1\ddot{\varphi}_1 \cos\varphi_1\\
&\quad + 2\dot{r}_1\dot{\varphi}_1 \cos\varphi_1 - r_1\dot{\varphi}_1^2 \sin\varphi_1,\\
\ddot{x}_2 &= F_{2x}\alpha_1 - F_{1x}\alpha_3 = \ddot{r}_2 \cos\varphi_2 - r_2\ddot{\varphi}_2 \sin\varphi_2\\
&\quad - 2\dot{r}_2\dot{\varphi}_2 \sin\varphi_2 - r_2\dot{\varphi}^2 \cos\varphi_2,\\
\ddot{y}_2 &= F_{2y}\alpha_1 - F_{1y}\alpha_3 = \ddot{r}_2 \sin\varphi_2 + r_2\ddot{\varphi}_2 \cos\varphi_2\\
&\quad + 2\dot{r}_2\dot{\varphi}_2 \cos\varphi_2 - r_2\dot{\varphi}_2^2 \sin\varphi_2,
\end{aligned} \tag{6}$$

where α_1, α_2, and α_3 are the functions expressing the mass distribution in rotor and the functions F_{1x}, F_{2x}, F_{1y}, and F_{2y} contain all members written in Equation (2), excluding those with second derivatives. Further transformation gives

$$\begin{aligned}
&(F_{1x}\alpha_2 - F_{2x}\alpha_3)\cos\varphi_1 + (F_{1y}\alpha_2 - F_{2y}\alpha_3)\sin\varphi_1\\
&\quad = \ddot{r}_1 - r_1\dot{\varphi}_1^2,\\
&(F_{1x}\alpha_2 - F_{2x}\alpha_3)\sin\varphi_1 + (F_{1y}\alpha_2 - F_{2y}\alpha_3)\cos\varphi_1\\
&\quad = r_1\ddot{\varphi}_1 + 2\dot{r}_1\dot{\varphi}_1,\\
&(F_{2x}\alpha_1 - F_{1x}\alpha_3)\cos\varphi_2 + (F_{2y_1}\alpha_1 - F_{2y}\alpha_3)\sin\varphi_1\\
&\quad = \ddot{r}_2 - r_2\dot{\varphi}^2,\\
&(F_{2x}\alpha_1 - F_{1x}\alpha_3)\sin\varphi_2 + (F_{2y_1}\alpha_1 - F_{1y}\alpha_3)\cos\varphi_2\\
&\quad = r_2\ddot{\varphi}_2 + 2\dot{r}_2\dot{\varphi}_2.
\end{aligned} \tag{7}$$

The final equations of journals motion can be written in the short form:

$$\begin{aligned}
\ddot{r}_1 &= r_1\dot{\varphi}_1^2 + F_{r1} \quad r_1\ddot{\varphi}_1 = -2\dot{r}_1\dot{\varphi}_1 + F_{\varphi 1},\\
\ddot{r}_2 &= r_2\dot{\varphi}_2^2 + F_{r2} \quad r_2\ddot{\varphi}_2 = -2\dot{r}_2\dot{\varphi}_2 + F_{\varphi 2}.
\end{aligned} \tag{8}$$

The expressions F_{r1}, F_{r2}, $F_{\varphi 1}$, and $F_{\varphi 2}$ are very complicated. Their structures can be shown in the simple case, when both journals move independently. This can be reached by appropriate mass distribution along the rotor length. Let us suppose that distribution of rotor mass is changed so that $m_3 = 0$, i.e. $\alpha_3 = 0$ and $\alpha_1 = \alpha_2 = 1/m_0 = 2/m$. Then the motions in both bearings are independent. Forced motion of journal in one of the bearings is covered by the first two equations of (7) or the first row in (8). Neglecting the index and considering that the radial and tangential components of force are

$$\begin{aligned}
F_r &= F_x \cos\varphi + F_y \sin\varphi,\\
F_\varphi &= F_x \sin\varphi + F_y \cos\varphi,
\end{aligned} \tag{9}$$

we get the differential equations of journal motion in one of the bearings

$$\begin{aligned}
\ddot{r} &= r\dot{\varphi}^2 - F_r/m_0 + e\omega^2 \cos\omega t + g,\\
r\ddot{\varphi} &= -2\dot{r}\dot{\varphi} - F_\varphi/m_0 + e\omega^2 \sin\omega t,
\end{aligned} \tag{10}$$

where increments of forces in radial F_r and circumferential F_φ directions are ($\dot{r} = v, \dot{\varphi} = \omega$)

$$\begin{bmatrix} dF_r \\ dF_r \end{bmatrix} = f_{\mathrm{cor}} \begin{bmatrix} K_{rr} & K_{r\varphi} \\ K_{\varphi r} & K_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} dr \\ r\,d\varphi \end{bmatrix} +$$

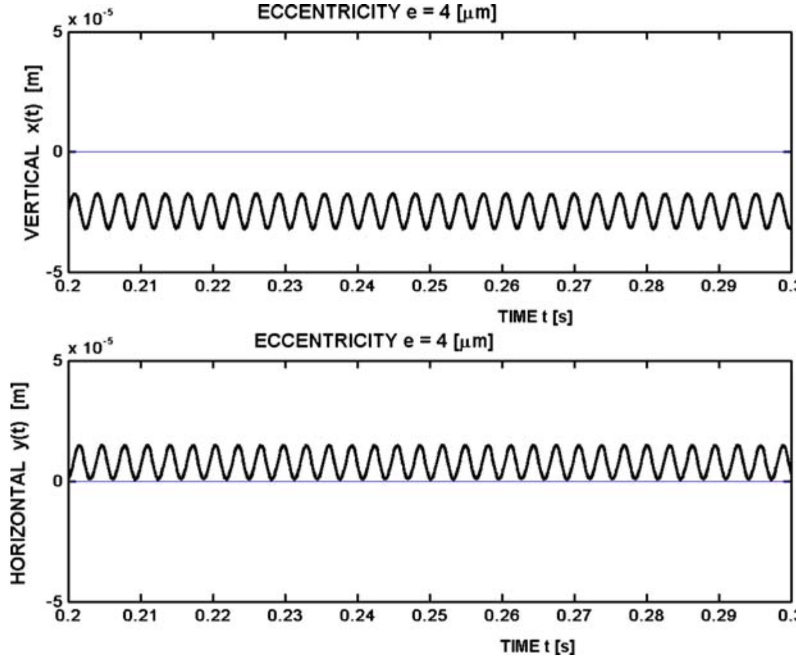

Fig. 7 Vibrations of well-balanced rotor with eccentricity $e = 4\,\mu$m

$$+ f_{\mathrm{cor}} \begin{bmatrix} B_{rr} & B_{r\varphi} \\ B_{\varphi r} & B_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} dv \\ r\, d\omega \end{bmatrix}. \tag{11}$$

We know the *K, B* properties in the equilibrium point ($r = r_o, \varphi = 0$). Due to the central symmetry in all points on the circle of radius r_o, the stiffness and damping matrices are centrally oriented and have the same elements as those in the point $r_o, \varphi = 0$:

$$\begin{bmatrix} K_{rr} & K_{r\varphi} \\ K_{\varphi r} & K_{\varphi\varphi} \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}$$
$$\begin{bmatrix} B_{rr} & B_{r\varphi} \\ B_{\varphi r} & B_{\varphi\varphi} \end{bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}. \tag{12}$$

The rheological properties can then be defined in all points $0 < r < r_0, -\pi \le \varphi \le \pi$ by means of the correction function $f_{\mathrm{cor}}(r)$, and its integral $G(r)$. So we get forces F_r and F_φ and the motion in all points is described by

$$\begin{aligned} \ddot{r} &= r\dot{\varphi}^2 - (K_{rr} G(r) + K_{r\varphi} f_{\mathrm{cor}}(r) r\varphi + B_{rr} f_{\mathrm{cor}}(r)\dot{r} \\ &\quad + B_{r\varphi} f_{\mathrm{cor}}(r) r\dot{\varphi})/m_0 + g \cos\varphi \\ &\quad + e\omega^2 \cos(\omega t - \varphi) \\ r\ddot{\varphi} &= -2\dot{r}\dot{\varphi} - (K_{\varphi r} G(r) + K_{\varphi\varphi} f_{\mathrm{cor}}(r) r\varphi + B_{\varphi r} f_{\mathrm{cor}}(r)\dot{r} \\ &\quad + B_{\varphi\varphi} f_{\mathrm{cor}}(r) r\dot{\varphi})/m_0 + g \sin\varphi + e\omega^2 \sin(\omega t - \varphi). \end{aligned} \tag{13}$$

The records of time history and phase plane trajectories of various motions are shown in the following section.

6 Example

The influence of nonlinearity on the stationary vibrations of rotor is demonstrated in Figs. 7–11, all calculated at the angular velocity $\omega = 2000$ rad/s (318.3 Hz). The first record (Fig. 7) gives the response of the well-balanced rotor with small eccentricity $e = 4\,\mu$m. Centrifugal force at this eccentricity and at given revolutions is approximately 60 N.

Due to this small excitation, the response is harmonic and synchronous with angular revolutions $\omega(T = 2\pi/\omega)$ and the system can be considered as linear. The constant shift of the vertical mean value $x = -2.5\,\mu$m is caused by the weight *mg*. The aerodynamic forces at rotation cause the shift in horizontal direction $y = 0.8\,\mu$m. The range of displacements *x, y* (scale $-5e-5, 5e-5$) corresponds to the clearance of the aerodynamic bearings.

The enlargement of eccentricity to the value $e = 6\,\mu$m (+50%) creates vibrations which are three times greater, as well as considerable change in their form. This is shown in Fig. 8, where the very strong 1/3 sub-harmonic component originates both in the vertical *x* and in horizontal *y* directions. The oscillations

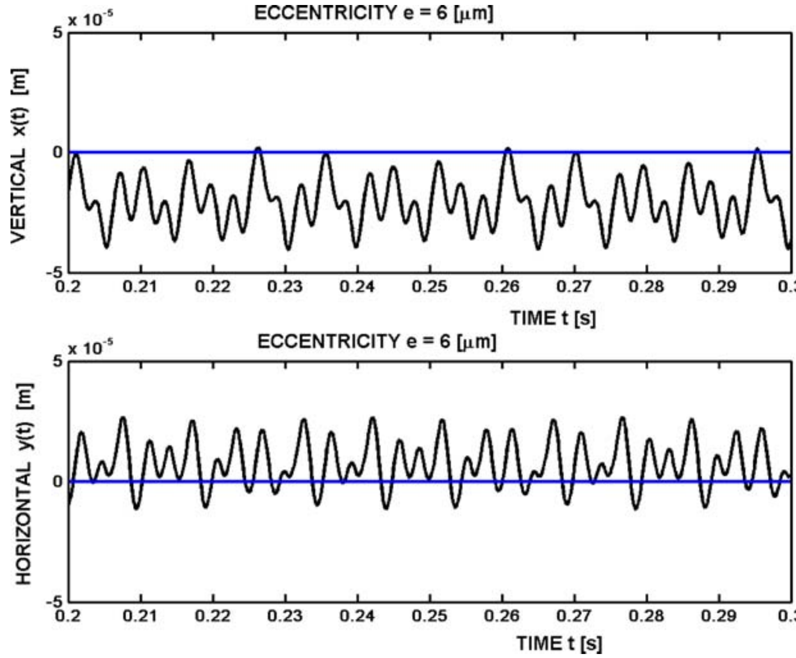

Fig. 8 Vibrations of rotor with eccentricity $e = 6\,\mu$m

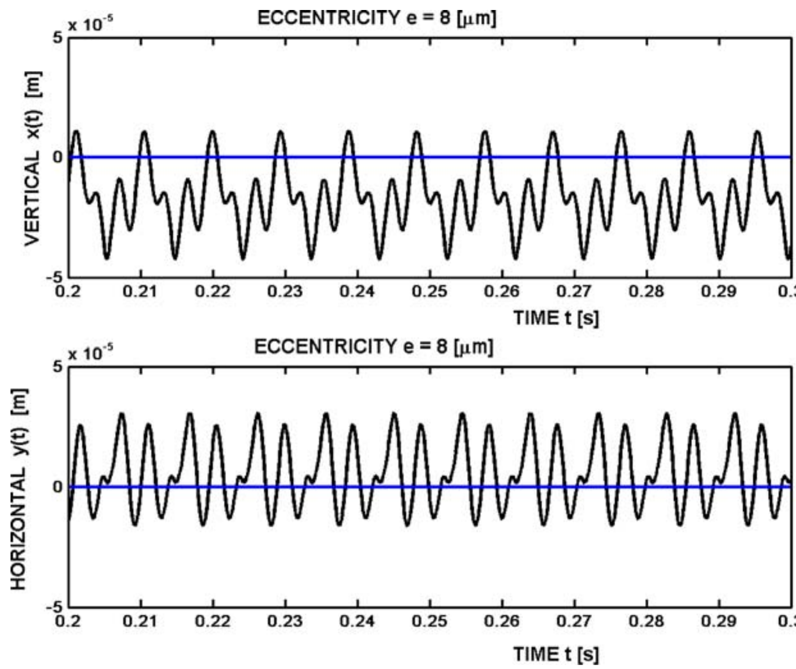

Fig. 9 Vibrations of rotor with eccentricity $e = 8\,\mu$m

are not simple stationary periodic, but have marked chaotic or quasi-periodic behavior very close to the periodic oscillations with the period 11 times greater than the revolutions of rotor ($e = 6$, see also Fig. 11, upper right).

If we enlarge the eccentricity to $e = 8\,\mu$m (centrifugal force $me\omega^2 = 120\,$N), the oscillation becomes stable and turns into periodic motion with three times greater period than the revolutions as shown in Fig. 9. It is remarkable that the number of the marked peaks in the horizontal (y) direction is twice as much as the number of those peaks in the vertical (x) direction. Amplitudes of these oscillations increase roughly proportionally to the previous case (i.e. in the ratio 8:6).

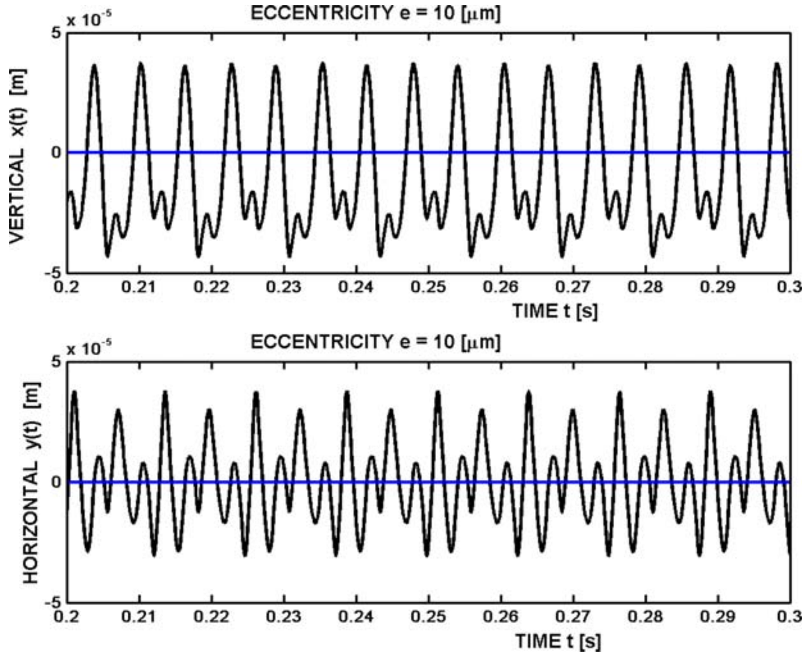

Fig. 10 Vibrations of rotor with eccentricity $e = 10\,\mu$m

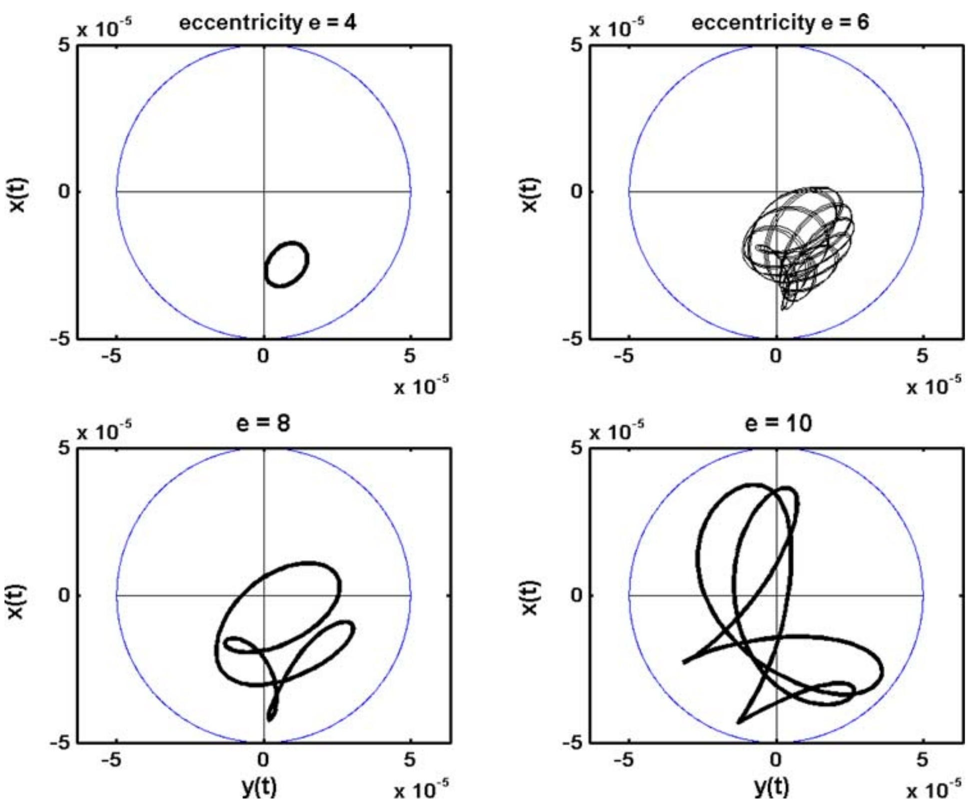

Fig. 11 Vibrations of rotor in polar coordinates

Figure 10 shows that the eccentricity $e = 10\,\mu\text{m}$ ($me\omega^2 = 160\,\text{N}$) again causes the expected increase of total amplitudes, especially of the sub-harmonics of orders 1/2 and 1/4. Period is $T_4 = 4T_0 = \frac{4\cdot 2\pi}{\omega}$. The sub-harmonic 1/2-components dominate in the vertical x-direction; in horizontal direction, the 1/4 and the basic harmonics are outstanding.

Figure 11 summarizes the view on the time histories of motions described in Figs. 7–10 into diagrams in x, y (or polar r, φ) coordinates. Also, these records were calculated for the same frequency $\omega = 2000$ 1/s and for the same short time interval 0.1 s, and therefore, it is not possible to say if the motion at $e = 6\,\mu\text{m}$ (subplot top right) is chaotic or quasi-periodic, but types of the other three motions ($e = 4$, 8, $10\,\mu\text{m}$) can be ascertained quite correctly.

Vibrations excited by the small eccentricity $e = 4\,\mu\text{m}$ (top left) is pure periodic with the excitation frequency. Trajectories in subplot left bottom $e = 8\,\mu\text{m}$ are more complex and have three maxima and three minima in both x-, y-directions. Simple line proves the periodicity of the motion.

The more complicated curve is shown in the right bottom subplot in Fig. 11, $e = 10\,\mu\text{m}$, which is again periodic, but with period T_4. It has four maxima and four minima both in vertical (x) and in horizontal (y) directions.

The common property of all presented cases is the limited displacements, smaller than the clearance of bearings. Even at a much greater excitation, the strong nonlinearity of aerodynamic forces near to the surface prevents from rub-contact of journal and stator.

7 Conclusions

Analysis of dynamic properties of rigid rotor supported on aerodynamic bearings was carried out for cases where we consider the inertia properties of tilting pads. It follows from this numerically based analysis that the inertia of pads strongly influences the characteristics of bearings. Stiffness and damping matrices are non-symmetric and their elements are non-monotonous functions of angular frequency ω.

Corresponding response curves of such linear but evolutive system are more complicated than those of system with constant parameters. Analysis of this linear model, valid only for small displacements from the equilibrium position shows that new resonance peaks can emerge due to tilting pads inertia properties. Investigation of rotor vibrations in large can be done only by means of nonlinear model.

Therefore, a mathematical approximate model of nonlinear evolutive system was derived in Cartesian and in polar coordinates by means of the correction function $f_{\text{cor}}(r)$. This correction enables to solve the rotor motion at the given angular velocity ω in the whole area of bearing clearance $0 < r < r_0, -\pi \le \varphi \le \pi$. Derived strongly nonlinear mathematical model will be used in further research of rotor vibrations.

In addition to the evolutive linear model of bearing, the real aerodynamic bearings (as all fluid bearings) have strongly nonlinear characteristics, which are here respected by introducing the correction function f_{cor} into the mathematical model. Nonlinearity produces at higher eccentricities $e > 5\,\mu\text{m}$ distortions of oscillations formed by higher and sub-harmonic components, and gives rise to complicated forms of oscillations as sub-harmonic, quasi-periodic, and chaotic. Several examples show these forms of vibrations.

This contribution is an extended work of paper [7].

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