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ORIGINAL ARTICLE

Dynamic stability of passive magnetic bearings

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Abstract The instability of kinematic pairs made with permanent magnets (passive magnetism) significantly restricts their technical use. On the other hand, they show very good tribological properties: null friction and wear. In previous works, it has been verified that by using Mathieu Functions any instability of the levitated member may be removed if one of the two members is subjected to a parametric excitation. In this paper, the problem is discussed directly utilizing the nonlinear equation of motion of the levitated member, and it is confirmed that it may be stable.

Keywords Bearings · Magnetism · Stability · Dynamics

1 Introduction

The advantages of passive magnetic bearings, i.e. almost null friction and wear, are partially nullified because of their instability [1]; for instance for the levitated ring of the axial bearing in Fig. 1 is unstable in radial direction. Actually, the unstable radial forces acting on the ring may be much smaller than the stable axial forces [2]. Moreover if one of the rings, the lower [3] or the upper [4], is submitted to an axial excitation, it would seem that instability may be removed. In [3, 4], the parametric equation of Mathieu [5] has been adopted, and small [3] or very small [4] stability spaces have been identified.

In order to examine these results thoroughly, in this work the behavior of the levitated ring is studied by solving its equations of motion with analytic and numerical methods, without the aid of the Mathieu method.

This work refers to the dynamic stability of permanent magnets. For stationary stability of permanent magnets and static stability of diamagnetic or superconducting materials see, for example, [6, 7].

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2 Magnetic fields and forces

2.1 Magnetic fields

Well-known relations of passive magnetism are

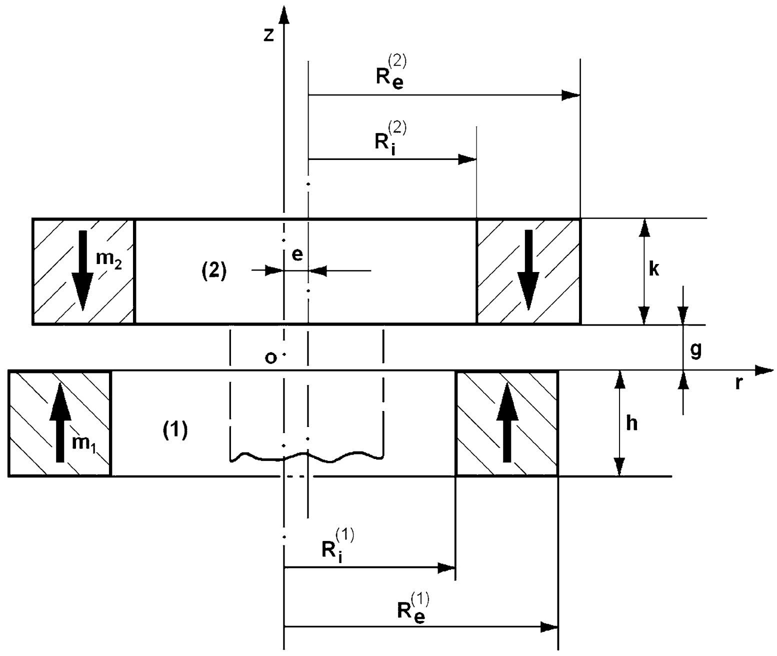
$$\bar{B} = \mu_0(\bar{M} + \bar{H}) \tag{1}$$

$$\bar{M} = \bar{M}_r + \chi \bar{H} \tag{2}$$

$$\nabla \cdot \bar{B} = 0 \tag{3}$$

where B is the flux density of the magnetic field which generates the magnetic force F, M the magnetization of the material and M_r the residual magnetization, H the intensity of magnetic field, μ_0 the vacuum permeability, μ the permeability of the material with $\mu_r = \mu/\mu_0$ its relative permeability, and $\chi = \mu_r - 1$ its susceptibility.

Fig. 1 Passive magnetic axial ring bearing



2.2 Magnetic forces

Forces F interacting between permanent magnets may be suitably computed with the theory of the volume and surface equivalent currents j, which cross the material along elementary circuits s. The calculation, starting from relations such as $d\bar{F} = j d\bar{s} \times \bar{B}$, have to be made by numerical methods [6, 8].

2.3 Instability

Equation (3) shows that B is solenoidal, thus the components F_x, F_y, F_z of F and their stiffnesses K_x, K_y, K_z are ruled by Equation (5) which becomes (6) for the ring in Fig. 1

$$\nabla F = \frac{\partial F_x}{\partial_x} + \frac{\partial F_y}{\partial_y} + \frac{\partial F_z}{\partial_z} = K_x + K_y + K_z = 0, \quad (4)$$

$$\nabla F = \frac{\partial F_z}{\partial z} + \frac{\partial F_r}{\partial r} = K_z + 2K_r = 0 \quad (5)$$

(4) and (5) highlight the instability of F almost along one direction [9].

2.4 Magnetic materials

Today magnetic bearings are generally made with Rare-Earths (REs, alloys with Lantanides, especially Nd) which show to a large degree the properties that ideal magnets are required to have: homogeneity of the material with homogeneous M, complete magneto-crystalline and shape anisotropy and high coercitivity [10]. RE have so $\mu_r \cong 1$, then $\chi \approx 0$ and $M = M_r$; $B_r \cong \mu_0 M_r$, $M \cong j$.

Consequently, F may be expressed as a function B only, where

$$\frac{\mu_0}{4\pi} j_1 j_2 \Rightarrow \frac{\mu_0}{4\pi} M_{r1} M_{r2} \Rightarrow \frac{1}{4\pi\mu_0} B_{r1} B_{r2} \quad (6)$$

B_r is given by manufactures.

In this work, dimensionless forces F' and stiffnesses K' (7) between the rings of the axial bearing in Fig. 1 have been calculated.

$$F'_z = \frac{F_z}{\frac{B_{r1}B_{r2}(R_e^{(1)})^2}{4\pi\mu_0}}, \quad F'_r = \frac{F_r}{\frac{B_{r1}B_{r2}(R_e^{(1)})^2}{4\pi\mu_0}};$$
$$K'_z = \frac{K_z}{\frac{B_{r1}B_{r2}R_e^{(1)}}{4\pi\mu_0}}, \quad K'_r = \frac{K_r}{\frac{B_{r1}B_{r2}R_e^{(1)}}{4\pi\mu_0}}, \quad (7)$$

with $g' = \frac{g}{R_e^{(1)}}$, $e' = \frac{e}{R_e^{(1)}}$
Figure 2a shows the axial force F'_z and stiffness $|K'_z|$, as a function of the gap g' for some values of eccentricity e'. Figure 2b shows the radial force F'_r and stiffness K_r as a function of e', for some values of g'.

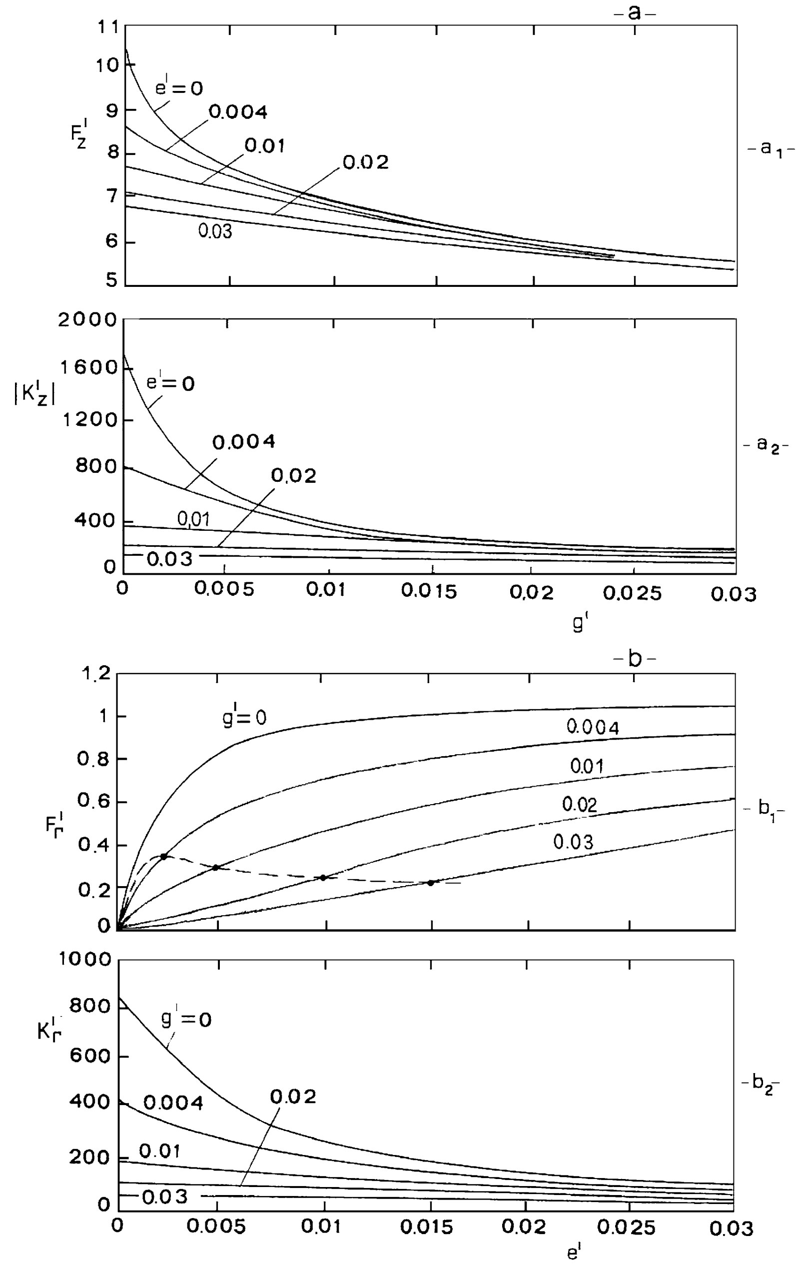


Fig. 2 Axial force (a_1) and stiffness (a_2) versus gap, for constant values of the eccentricity. Radial forces (b_1) and stiffness (b_2) versus eccentricity for constant values of the gap

M has not been calculated, because if the forces of the levitated ring are stable, the same must be true for the moments [11], especially for the loaded bearing shown in Fig. 4b.

In (7), $R_e^{(1)}$ is the highest of the external radii of the rings of the bearing; B_{r1} and B_{r2} are their residual inductions. The results refer to a bearing with $R_e^{(1)} = R_e^{(2)} = R_e = 0.03$ m, $R_i^{(1)} = R_i^{(2)} = R_i = 0.018$ m, $h = k = 0.012$ m, $B_{r1} = B_{r2} = B_r = 1.1$ T, (RE).

3 Dynamic behavior

The canonical equations of motion of the levitated ring of mass $m_2 = m$ in axial and radial direction are

$$m\ddot{g} + F_z = 0 \tag{8}$$

$$m\ddot{e} + F_r = 0 \tag{9}$$

Fig. 3 Radial forces versus eccentricity, for some values of the ratio eccentricity/gap

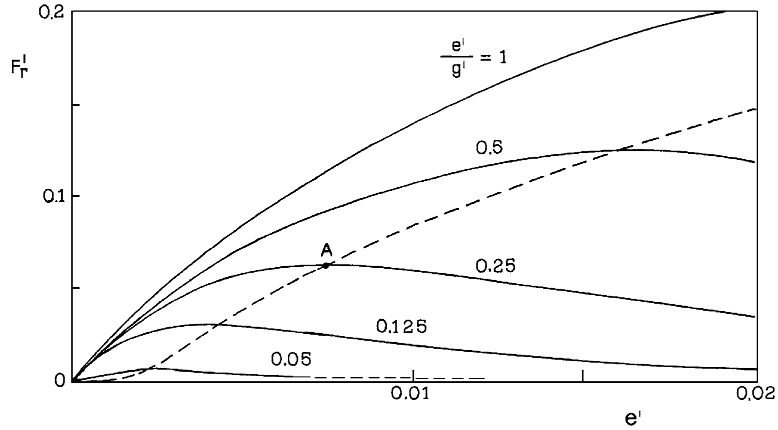


Figure 2a$_1$ shows that, starting from very small values of g ($g' > 0, 02$, then $g = g'R_e = 6 \times 10^{-4}$ m) F_z is nearly independent of e, thus it is nearly independent of F_r; Fig. 2a$_2$ shows in addition that K_z is almost constant. So it is possible to put

$$F_z = F_z \cos \Omega_z t, \quad g = g_2 \cos \Omega_z t,$$

$$\text{with} \quad \Omega_z = \sqrt{\frac{K_z}{m}}, \quad g_2 = \frac{\Delta g}{2} \text{ (Fig. 4b)} \tag{10}$$

On the other hand, Fig. 2b$_1$ shows that F_r depends significantly on g, thus it is not independent of F_z. Moreover, Fig. 2b$_2$ shows that K_r is almost constant.

The behaviors of $F_z'(g')$ indicate the axial stability of the levitated ring; those of $F_r'(e')$ indicate its radial instability; this fact confirms (3).

Note that Fig. 2 shows F_z' and K_z', which change with g' while e' remains constant, and F_r' and K_r', which change with e' while g' remains constant. But neither g' nor e' remain really constant, in fact F imposes a curved trajectory on the levitated ring. There is a small shift (see below), which may however be regarded as rectilinear, where e' and g' varies proportionally. So, as an example, for $e' = 0.002, 0.005, 0.01, 0.015$ it may be that $g' = 0.004, 0.01, 0.02, 0.03$, i.e. $e'/g' = 0.5$. In this case F_r' in Fig. 2b$_1$ assumes the values 0, 0.034, 0.29, 0.25, 0.22, which are connected by a dashed line. Fig. 3 shows $F_r'(e')$, for some values of ratio e'/g'.

To the right of the dashed line, F_r' decreases, a necessary condition for radial stability. The diagrams in Fig. 3 refer to the initial condition $g' = 0.03$, $e' = 0$. $e'/g = 1$ refers to displacements $0 \le e' \le 0.02$ and $0.03 \le g' \le 0.05$, thus $\Delta g' = \Delta e' = 0.02$; $e'/g' < 1$ refer to $\Delta e' < \Delta g'$.

Figure 4a, shows $F_r(e)$ of a bearing with an equal ring, with $R_e = 0.015$ m, $R_i = 0.0095$ m, $h = 0.002$ m and $B_r = 1.44$ T [12]; the levitated ring bears a load $W = 18$ N ($m = 1.84$ kg), thus $g_w = 0.003$ m. The diagram refers to starting conditions $g = 0.0015$ m, $e = 0$ and to $e/g = e'/g' = 0.25$.

Radial force $F_r(e)$ may be stated in the polynomial form (11), which is suitable for the treatment outlined in Section 4.

$$F_r = K_r e - 1.63 \times 10^8 e^2 + 2.05 \times 10^{11} e^3 \\ -0.903 \times 10^{14} e^4 \tag{11}$$

with $K_r = 4.75 \times 10^4$ N m^{-2}; thus (9) becomes

$$\ddot{e} + \Omega_{r0}^2 e - \frac{1.63 \times 10^8 e^2}{m} + \frac{2.05 \times 10^{11} e^3}{m} \\ -\frac{0.903 \times 10^{14} e^4}{m} = 0 \tag{12}$$

with $\Omega_{r0}^2 = \frac{K_r}{m} = 2.58 \times 10^4$ rad^{-2} s^{-2}

Note that the term $K_r e$ in (11) represents the behavior of F_r for $g = 0$, Fig. 2b$_1$; the others are corrective terms due to the fact that g varies simultaneously with e. Further details will be given in Section 4.

4 Solutions

4.1 Analytical solution

4.1.1 Radial motion

As Equation (12) is highly nonlinear, the Ritz-Galerkin method [13] may be used, which does not require small

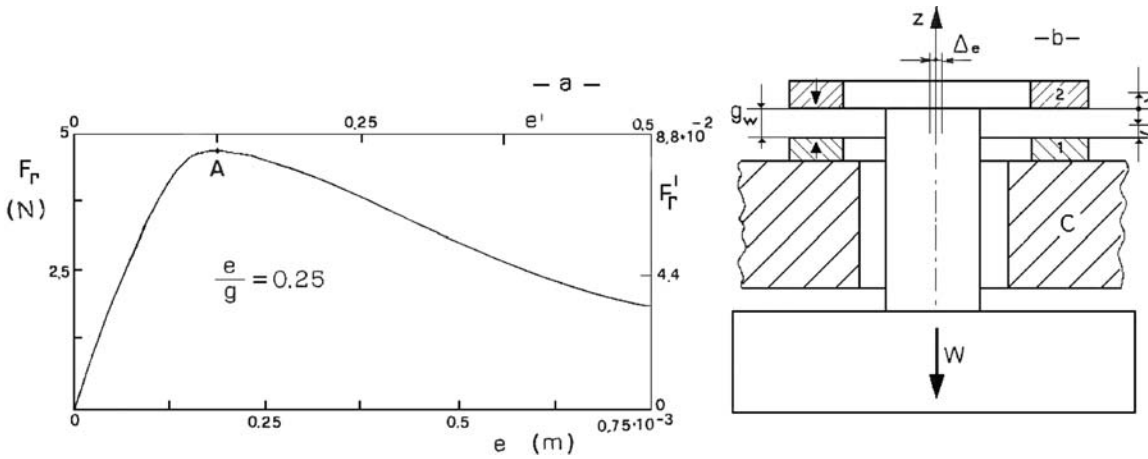


Fig. 4 (a) Radial force versus eccentricity for a selected value of the ratio eccentricity/gap. (b) Loaded bearing

nonlinear terms, but imposes a priori an approximate time solution. Once (12) has been rewritten in generalized coordinates

$$\ddot{q} + \omega_n^2 q - q^2 + q^3 - q^4 = 0 \tag{13}$$

the method allows us to state that in the absence of even terms, (13) has the harmonic solution

$$g(t) = a_{1,3}\ \cos\left(\Omega_r^2 t + \Phi_{1,3}\right) \equiv a_{1,3}\ \cos\theta,$$

$$\text{with}\quad \Omega_r^2 = \Omega_{r0}^2 + \frac{3}{4}a_{1,3}^2 \tag{14}$$

The effect of the even terms may be evaluated by the principle of "harmonic balance" [14] which, along with the hypothesis that the solution of (13) may be almost periodic, poses (14) in the form

$$q(t) = (a_2 + a_{1,3}\ \cos\Omega_r t + a_4),$$

$$\text{with}\quad \Omega_r = \Omega_{r0} + \bar{\omega}(a_2, a_{1,3}, a_4) \tag{15}$$

where the values of a_2, a_4 and $\bar{\omega}$ are obtained by the system

$$\begin{aligned} &\Omega_{r0}^2 a_2 + p_2(a_2, a_{1,3}) = 0 \\ &\Omega_{r0}^2 a_4 + p_4(a_4, a_{1,3}) = 0 \\ &\left(\Omega_{r0}^2 - \Omega_r^2\right)a_{1,3} + p_{1,3}(a_2, a_{1,3}, a_4) = 0 \end{aligned} \tag{16}$$

with

$$\begin{aligned} p_2(a_{2,4}, a_{1,3}) &= \frac{1}{2\pi}\int_0^{2\pi} f(a_2 + a_{1,3}\cos\theta)\cos\theta\, d\theta \\ p_{2,4}(a_{2,4}, a_{1,3}) &= \frac{1}{2\pi}\int_0^{2\pi} f(a_4 + a_{1,3}\cos\theta)\cos\ \theta\, d\theta \\ p_{1,3}(a_{2,4}, a_{1,3}) &= \frac{1}{2\pi}\int_0^{2\pi} f(a_2 + a_4 + a_{1,3}\cos\theta) \\ &\quad \times \cos\theta\, d\theta \end{aligned} \tag{17}$$

The first two of (17) show that $a_2 = a_4 = a_{2,4}$, thus (16) are reduced to

$$\begin{aligned} &\Omega_{r0}^2 a_{2,4} + p_{2,4}(a_{2,4}, a_{1,3}) = 0 \\ &\left(\Omega_{r0}^2 - \Omega_r^2\right)a_{1,3} + p_{1,3}(a_{2,4}, a_{1,3}) = 0 \end{aligned} \tag{18}$$

with

$$\begin{aligned} p_{2,4}(a_{2,4}, a_{1,3}) &= \frac{1}{2\pi}\int_0^{2\pi} f(a_{2,4} + a_{1,3}\cos\theta)\cos\theta\, d\theta \\ p_{1,3}(a_{2,4}, a_{1,3}) &= \frac{1}{\pi}\int_0^{2\pi} f(a_{2,4} + a_{1,3}\cos\theta)\cos\theta\, d\theta \end{aligned} \tag{19}$$

where [14],

$$\begin{aligned} f(a_{2,4}, a_{1,3}\cos\theta) &= (a_{2,4} + a_{1,3}\cos\theta)^3 \\ &= \frac{3}{4}a_{1,3}^3\cos\theta + \frac{3}{2}a_{2,4}a_{1,3}^2 \end{aligned} \tag{20}$$

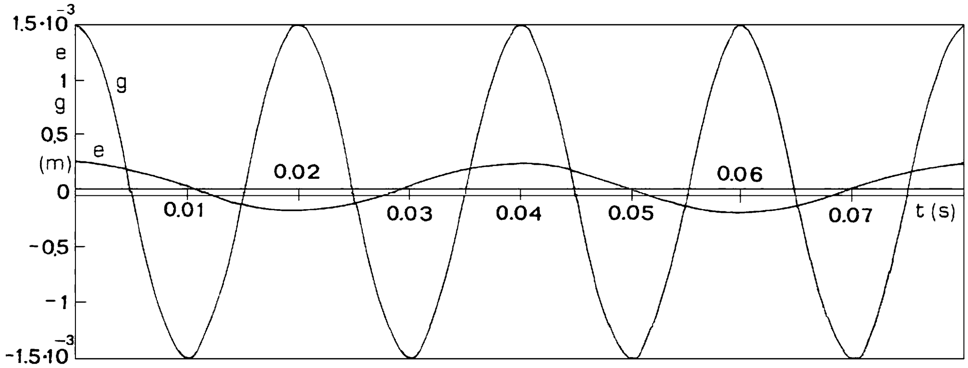


Fig. 5 Axial and radial vibrations of the levitated ring of a magnetic axial ring bearing

The third member of (20) is obtained from the second by neglecting terms with $\cos n\theta$ for $n > 1$, [15]. Thus $p_{2,4}$ and $p_{1,3}$ are reduced to

$$p_{2,4} = \frac{3}{8}a_{1,3}^3, \quad p_{1,3} = \frac{3}{4}a_{1,3}^3 \tag{21}$$

and (19) become

$$\begin{aligned} &\Omega_{r0}^2 a_{2,4} + \frac{3}{8}a_{1,3}^3 = 0 \\ &\left(\Omega_{r0}^2 - \Omega_r^2\right)a_{1,3} + \frac{3}{4}a_{1,3}^3 = 0 \end{aligned} \tag{22}$$

from which

$$a_{2,4} = -\frac{3}{8}\frac{a_{1,3}^3}{\Omega_{r0}^2}, \quad \Omega_r^2 = \Omega_{r0}^2 + \frac{3}{4}a_{1,3}^2 \tag{23}$$

In the end the even terms of (13) modify the amplitude of the vibration, and the cubic term modifies the frequency.

If the foregoing process is applied to the bearing under examination, we obtain $\Delta e/2 = 3.75 \times 10^{-4}$ m, thus $(3/4)a_{1,3}^2 = 1.05 \times 10^{-7}\ \mathrm{s}^{-2}$. Moreover $\Omega_{r0}^2 = 2.58 \times 10^4\ \mathrm{rad}^2\,\mathrm{s}^{-2}$, then $\Omega_{r0} = 160\ \mathrm{rad\ s}^{-1}$. So, if $e_{1,2} = a_{1,3}$ and $e_{2,4} = a_{2,4}$, we obtain

$$\begin{aligned} e &= e_{2,4} + e_{1,3}\cos\Omega_r t = -1.01 \times 10^{-4} + 3.75 \\ &\quad \times 10^{-4}\cos[(2.58 \times 10^4 + 1.05 \times 10^{-7})^{0.5}]t \\ &\quad \cong -1.01 \times 10^{-4} + 3.75 \times 10^{-4}\cos 160t\ m \\ \Omega_r &= \left(\Omega_{r0}^2 + 3/4a_{1,3}^2\right)^{0.5} = (2.58 \times 10^4 + 1.05 \\ &\quad \times 10^{-7})^{0.5} \cong 160\,\mathrm{rad\,s}^{-1} \end{aligned}$$

Consequently, the even terms of (13) significantly modify the amplitude of the vibration, while the cubic term leaves the frequency almost unchanged, as predicted in [14].

4.1.2 Axial motion

With reference to the axial vibration, as the axial stiffness around g_0 is $K_z \approx -1.9 \times 10^5\ \mathrm{N\ m}^{-1}$, then the axial frequency is $\Omega_Z = 321\ \mathrm{rad\ s}^{-1}$, almost twice the radial frequency, as already seen in [4]. Moreover, the amplitude of the vibration is $g_2 = \Delta g/2$. For the axial vibration, we thus have $g_2 = 15 \times 10^{-4}$ m, $\Omega_Z = 321$ rad s^{-1}.

Then, with reference to Fig. 4b, if at $t = 0$ it is, for example $g = g_0 = 15 \times 10^{-4}$ m, $\dot{g} = 0$ and $e = e_0 = 2 \times 10^{-4}$ m, $\dot{e} = 0$ (point A in Fig. 4a) the levitated ring begins to vibrate in an axial direction with amplitude $g_2 = 15 \times 10^{-4}$ m and frequency $\Omega_Z = 321\ \mathrm{rad\ s}^{-1}$, and in a radial direction with $e = 2.74 \times 10^{-4}$ m and $\Omega_r = 160\ \mathrm{rad\ s}^{-1}$.

Figure 5 shows the axial g and radial e vibrations of the ring.

Note that as the ring is stable for $g = g_0$ and $e = e_0$ (coordinate of point A, where F_r has a maximum, Fig. 4a), it is certainly stable for points (g, e) to the right of A.

4.2 Numerical solution

To evaluate more precisely the behaviors of e, Equation (12) was solved numerically using the dynamic section of the software application Simulink (MATLAB library). This software allows systems to

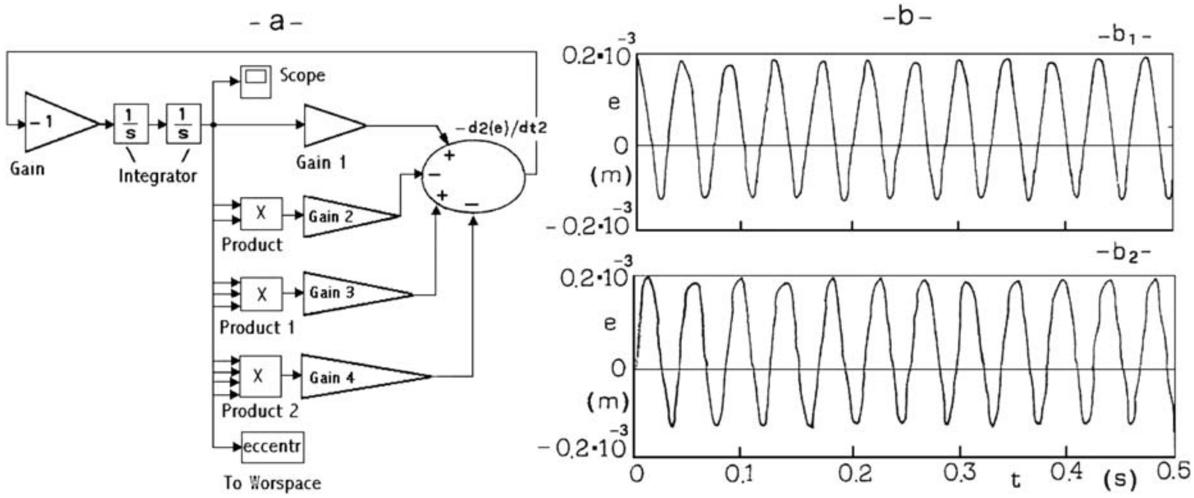


Fig. 6 (a) Block diagram of the dynamic model of the bearing. (b) Radial vibrations of the levitated ring

be modeled, simulated and analysed with changing exit. The blocks of the software (each standing for a dynamic elementary system) were linked to represent the dynamic of the bearing. Figure 6a shows the block diagram. Figure $6b_1$ shows the radial motion of the levitated ring for $e_0 = 2 \times 10^{-4}$ m and $\dot{e} = 0$. The behavior of e is periodic and almost harmonic, with total amplitude $e = 1.65 \times 10^{-4}$ m, smaller than its analytic value 2.74×10^{-4} m and with frequency $\Omega_r = 149$ rad s^{-1}, which is quite near to its analytical value 160 rad s^{-1}.

Figure $6b_2$ refers to $e_0 = 0$ and $\dot{e} = 2.4 \times 10^{-2}$ m and it is $e = 1.55 \times 10^{-4}$ and $\Omega_r = 147$ rad s^{-1}, These values almost the same as the former ones.

The behavior of g is shown again in Fig. 5a.

Apart from the difference with the analytical one, the numerical solution confirms that the radial motion of the levitated ring may be a periodic, stable vibration, under the action of a suitable axial motion.

Finally, note that the levitated ring is almost motionless as its axial vibration is small and its radial one is even smaller.

5 Experiments

The stability of the studied bearing was obtained for "ideal" permanent magnets. To verify theoretical results, experiments on permanent magnet rings were made [16]. Some difficulties have been met concerning the non homogeneous magnetisation M, which commercial permanent magnets, including RE still have today.

For example, some measured values of the flux density Ø. of a ring in $Nd_2Fe_{14}B$, with $R_e = 0.04$ m, $R_i = 0.015$ m, $h = 0.01$ m and $B_r = 1.37$ T highlighted a radial and circumferential non uniformity of ϕ. thus of M, with changes greater than 20% as well. We are now researching into ring magnets that have a better uniformity, though of smaller dimensions, and also into ring systems of elementary magnets.

6 Conclusions

Previous works have used Mathieu equations to understand how to minimize the radial instability of the levitated member of permanent magnet pairs, and small spaces, where it may be stable under suitable conditions of excitations.

This work has shown that stability may exist under more extensive conditions, and that the spaces of stability may be more extensive than the Mathieu ones. A nonlinear equation of motion of the levitated ring has been determined, and solutions have been obtained both by analytical and numerical methods. The ring, once excited axially, vibrates both axially and radially in a stable mode, with small vibration amplitudes.

References

1. Earnshaw, S.: On the nature of molecular forces which regulate the constitution on luminiferrous ether trans. Cambridge Philos. Soc. **7**, 97–112 (1842)
2. Bassani, R., Villani, S.: Passive magnetic bearing: conic-shaped bearing. Proc. Instn. Mech. Engrs. **213**, 151–161 (1999)
3. Bassani, R.: A stability space of a magneto-mechanical bearing. ASME, J. Dyn. Syst. (in press)
4. Bassani, R.: Stability of permanent magnets bearings under parametric excitations. Trib. Trans. **48**, 457–463 (2005)
5. McLachlan, N.W.: Theory and Application of Mathieu Functions, p. 401, Oxford, Claredon (1947)
6. Bassani, R.: Permanent magnetic levitation and stability. Find. Trib. Micro-Macro, Kluver 547–554 (2001)
7. Bassani, R.: Earnshow S. (1805–1888) and magnetic stability. Meccanica **41**, 375-389 (2006)
8. Bassani, R., Di Puccio, F., Ciulli, E., Mugolino, A.: Study of conic permanent magnet bearings. I° AIMETA International Tribology Conference 447–454 (2000)
9. Bassani, R.: Stabilità dei corpi in campi potenziali. DCMN, Università di Pisa, RL **524**, 4 (1991)
10. Coey, J.M.D.: Rare-Hearth Iron Permanent Magnets, p. 522, Claredon, (1996)
11. Lemarquaud, G., Yonnet, J.P.: Classification des suspensions magnètiques. RGE **6**, 31–34 (1990)
12. Tokoro, H., Uchida, K.: High energy product Nd-Fe-B sintered magnets produced by wet compacting process. IEEE Trans. Mag. **37**, 2463–3466 (2001)
13. Newland, D.E.: Mechanical Vibration Analysis and Computation, p. 583, Longman, Harbow and Wiley, New York (1989)
14. Szemplińska-Stupnicka, V.: The Behavior of Nonlinear Vibrating Systems, Vol. I°, p. 251, Kluver A.P., (1990)
15. Zuillinger, D.: Standard Mathematical Tables and Formulae, p. 812, CRC, (1996)
16. Bassani, R., Ciulli, E.: Passive magnetic bearings. Low instability and possibile stability. Proceedings of the World Tribology Congress 2005-63546, Washington DC, (September 2005)