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Generation of Chaotic Beats in a Modified Chua's Circuit Part I: Dynamic Behaviour

DONATO CAFAGNA* and GIUSEPPE GRASSI
Dipartimento di Ingegneria dell'Innovazione, Università degli Studi di Lecce, via Monteroni, 73100 Lecce, Italy;
**Author for correspondence (e-mail: donato.cafagna@unile.it)*

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Abstract. This paper and the companion one [Cafagna, D. and Grassi, G., *Nonlinear Dynamics*, this issue] present the new phenomenon of chaotic beats in non-autonomous Chua's circuit, driven by two sinusoidal inputs with slightly different frequencies. In particular, in this paper the behaviour of the proposed circuit is analyzed both in time-domain and state-space, confirming the chaotic nature of the phenomenon and the effectiveness of the design.

Key words: chaotic beats, non-autonomous Chua's circuit, nonlinear dynamics

1. Introduction

In the field of chaotic systems, different complicated behaviors such as period-adding sequences, generation of multi-scroll attractors, synchronization phenomena and intermittency properties have been widely studied [1–3].

In recent times, an attractive phenomenon has been investigated in [4], where the behaviors of Kerr and Duffing nonlinear oscillators driven by two sinusoidal inputs with slightly different frequencies have been analyzed. In particular, the authors of reference [4] started by considering that in linear systems the interaction of two sinusoidal signals generates the well-known phenomenon called *beats* [5]. Namely, when two waves with slightly different frequencies interfere, the frequency of the resulting waveform is the average of the frequencies of the two waves, whereas its amplitude is modulated by an envelope, the frequency of which is the difference between the frequencies of the two waves. This concept has been generalized in [4], where the generation of chaotic beats in coupled nonlinear systems with very small nonlinearities has been studied.

Based on these considerations and on the results reported in [6], this paper and the companion one [7] aim to investigate the generation of chaotic beats in a novel modified version of the Chua's circuit. The paper is organized as follows. In Section 2, the equations of the proposed non-autonomous circuit are reported. It consists in a modified version of the autonomous Chua's circuit, where two sinusoidal inputs characterized by slightly different frequencies have been added. In Section 3, numerical integrations of dimensionless equations get a first insight into the generation of chaotic beats. Finally, by exploiting PSpice simulator, Section 4 illustrates in detail the beats phenomenon in the implemented circuit (see the companion paper [7]).

2. The Proposed Circuit

The state equations of the proposed non-autonomous circuit (Figure 1), constituted by two capacitors, an inductor, a linear resistor, the Chua's diode and two external periodic excitations,

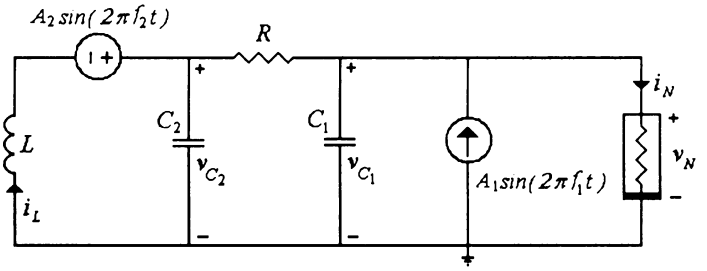

Figure 1. The proposed modified Chua's circuit.

are:

$$\begin{pmatrix} \frac{dv_{C_1}}{dt} \\ \frac{dv_{C_2}}{dt} \\ \frac{di_L}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{1}{RC_1} & \frac{1}{RC_1} & 0 \\ \frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{pmatrix} \begin{pmatrix} v_{C_1} \\ v_{C_2} \\ i_L \end{pmatrix} + \frac{1}{C_1} \begin{pmatrix} -g(v_{C_1}) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{A_1}{C_1}\sin(2\pi f_1 t) \\ 0 \\ \frac{A_2}{L}\sin(2\pi f_2 t) \end{pmatrix} \tag{1}$$

where A_1 and A_2 are the amplitudes of the periodic excitations, f_1 and f_2 are their frequencies whereas

$$g(v_{C_1}) = G_b v_{C_1} + 0.5(G_a - G_b)(|v_{C_1} + B_p| - |v_{C_1} - B_p|) \tag{2}$$

is the characteristic of the Chua's diode [2].

The dynamics of (1) depend on a set of eleven circuit parameters: $C_1, C_2, L, R, G_a, G_b, B_p, A_1, A_2, f_1, f_2$. The number of parameters is reduced by normalizing the equation of the nonlinear resistor, so that its breakpoints are at ± 1 instead of $\pm B_p$. By introducing the dimensionless variables x_1, x_2, x_3, and τ:

$$v_{C_1} = x_1 B_p, \quad v_{C_2} = x_2 B_p, \quad i_L = x_3 \frac{B_p}{R}, \quad t = \tau R C_2,$$

and redefining τ as t, the following equations are obtained:

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{pmatrix} = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \alpha \begin{pmatrix} -g(x_1) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha A_1^d \sin\left(2\pi f_1^d t\right) \\ 0 \\ A_2^d \sin\left(2\pi f_2^d t\right) \end{pmatrix} \tag{3}$$

with

$$g(x_1) = b x_1 + 0.5(a - b)(|x_1 + 1| - |x_1 - 1|) \tag{4}$$

where $\alpha = \frac{C_2}{C_1}$, $\beta = \frac{R^2 C_2}{L}$, $a = RG_a$, $b = RG_b$, $A_1^d = \frac{A_1 R}{B_p}$, $A_2^d = \frac{A_2 \beta}{B_p}$, $f_1^d = RC_2 f_1$ and $f_2^d = RC_2 f_2$ (the superscript d means "dimensionless"). Thus, the set of eleven circuit parameters turns into the set of eight dimensionless parameters $\{\alpha, \beta, a, b, A_1^d, A_2^d, f_1^d, f_2^d\}$.

3. Numerical Integrations

In this section the phenomenon of beats in the dimensionless system (3) is investigated. By varying the parameters $\{\alpha, \beta, a, b, A_1^d, A_2^d, f_1^d, f_2^d\}$, several numerical integrations of Equations (3) and (4) have been carried out. After a first round of integrations, the complexity has been reduced by selecting the values $a = -1.1429$, $b = -0.7143$ and $\beta = 14.283$. Successively, extensive integrations have been carried out for several values of the parameters α, A_1^d, A_2^d and slightly different values of the frequencies f_1^d and f_2^d. In particular, it is interesting to analyze the system behaviour for $\alpha = 6.799$, $A_1^d = A_2^d = 25$, $\omega_1^d = 2\pi f_1^d = 3.0$ and $\omega_2^d = 2\pi f_2^d = 3.1$. To this purpose, Figure 2 shows the time behaviors of the

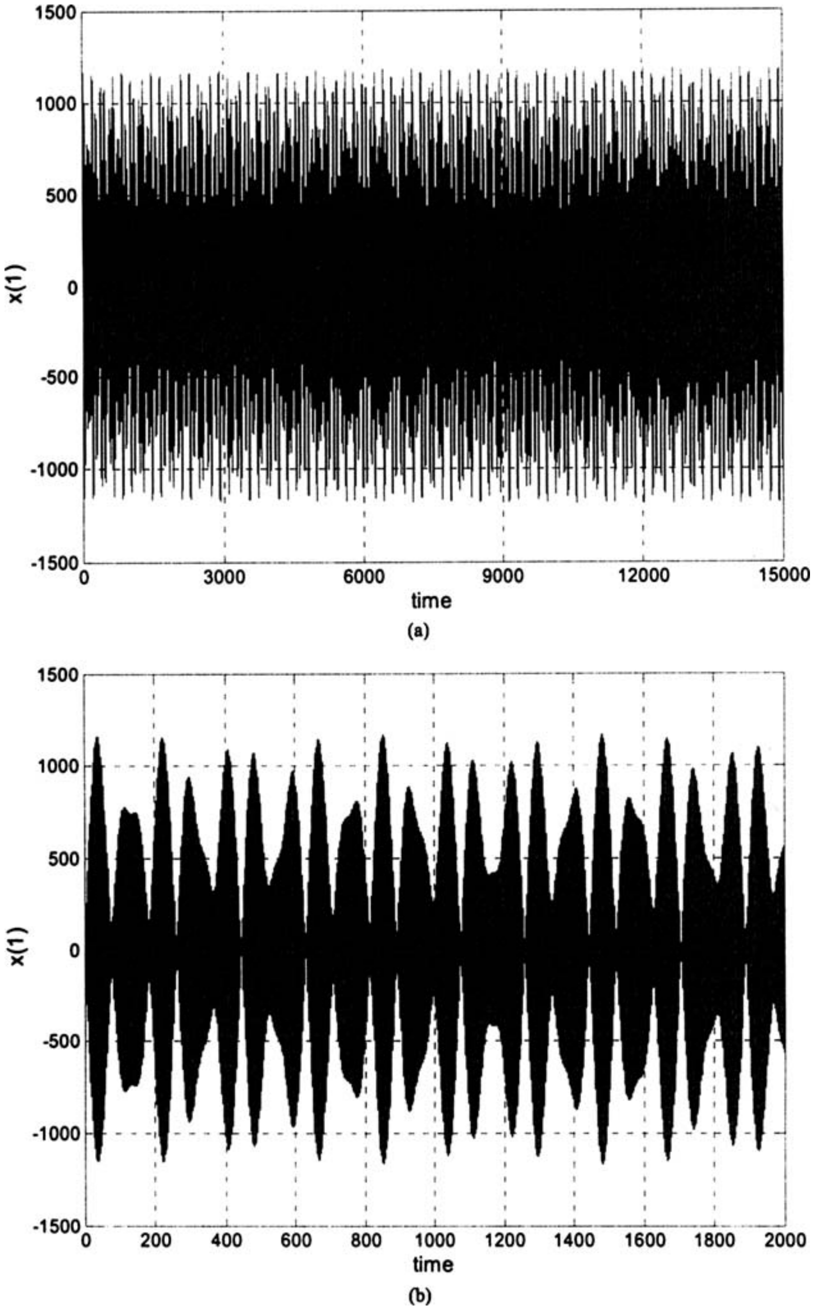

Figure 2. Behaviors of the variable x_1 for different time-scales; (a): $t \in [0, 15{,}000]$; (b): $t \in [0, 2000]$; (c): envelope of x_1 for $t \in [0, 2000]$; (d): $t \in [0, 170]$.

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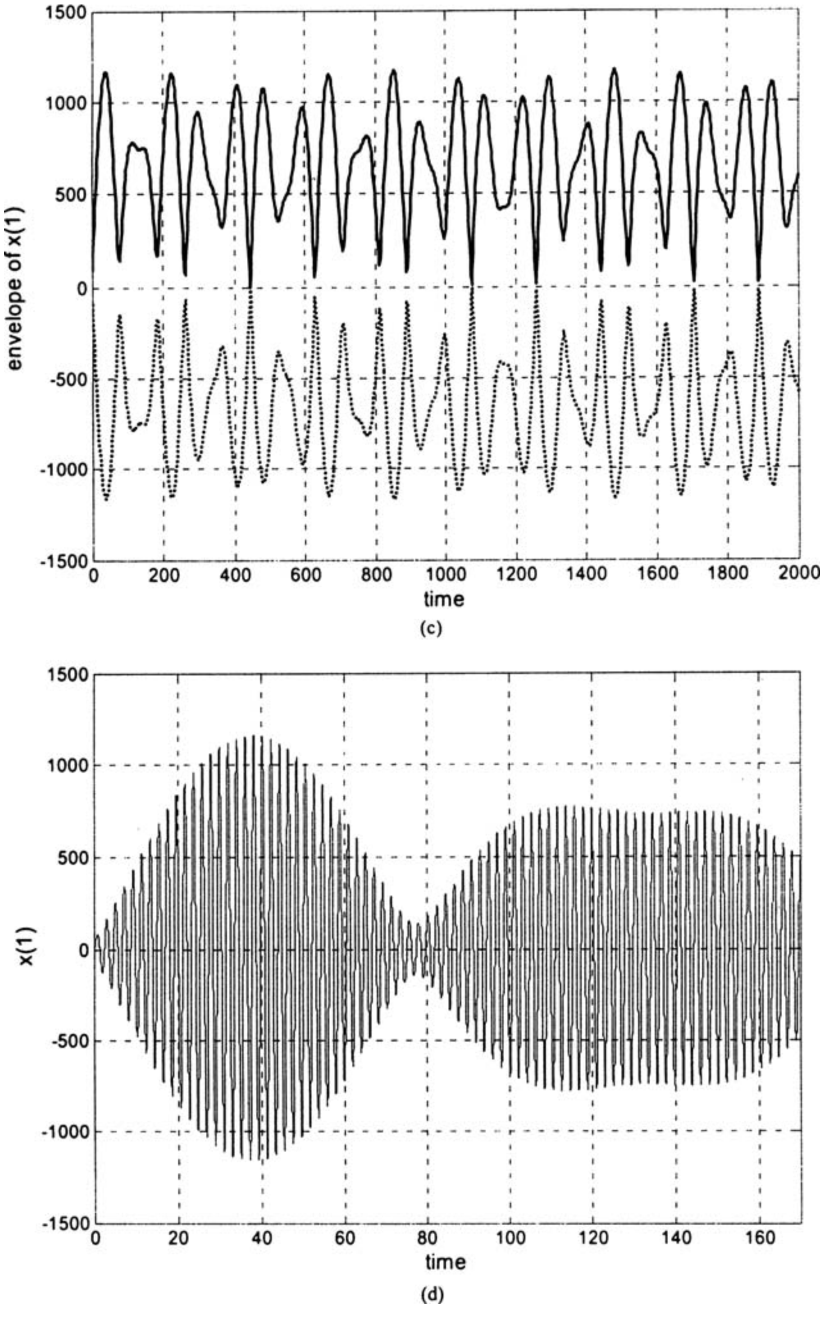


Figure 2. (*Continued*)

state variable x_1 for different resolutions of the time scale. More precisely, Figure 2(a) makes perceive the chaotic nature of the signal x_1, whereas Figure 2(b) highlights the occurrence of chaotic beats generated by its envelope (Figure 2(c)). Moreover, Figure 2(d) reveals in the signal x_1 both an amplitude modulation due to the chaotic envelope and the presence of a fundamental frequency. Further analysis, related to the power spectral density (Figure 3), confirms the chaotic nature of signal x_1 as well as the presence of a fundamental frequency $f^* = 0.48828$ (i.e., $\omega^* = 3.0679$).

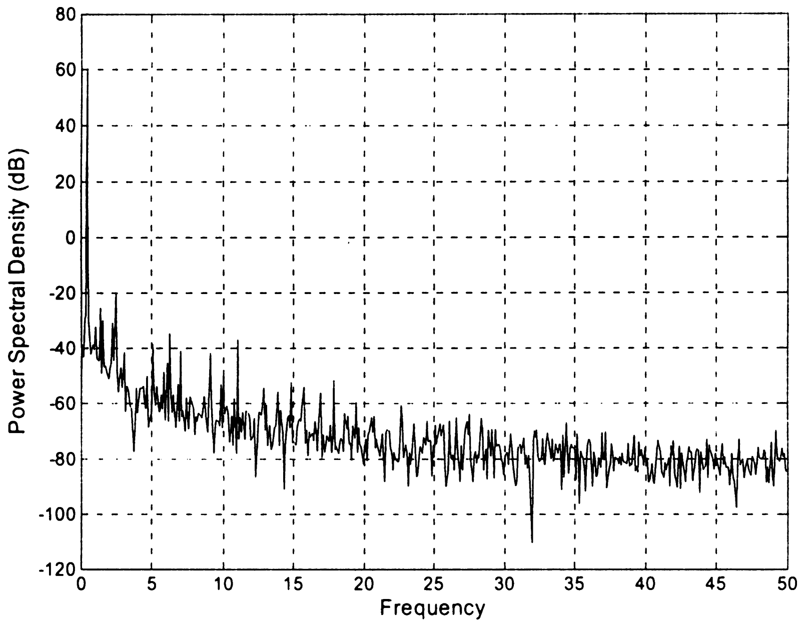


Figure 3. Broadband spectral density of the signal x_1 (fundamental frequency at $f^* = 0.48828$).

Finally, referring to the chaotic nature of the signal x_1, the Lyapunov exponents of system (3) are calculated. By considering each sinusoidal forcing term as parameter, a null exponent is obtained. Namely, the Lyapunov exponents different from zero are:

$$\lambda_1 = 0.00043, \quad \lambda_2 = -0.03805, \quad \lambda_3 = -4322.4.$$

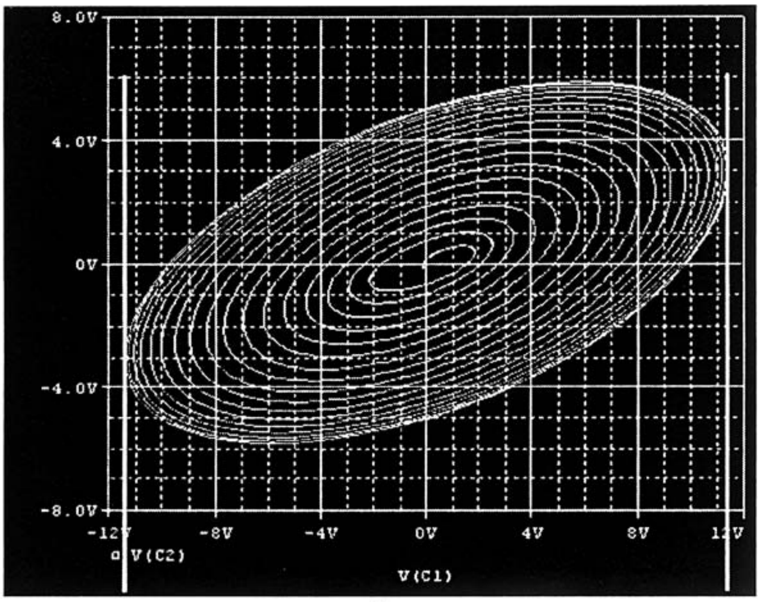
Notice the presence of one positive Lyapunov exponent, which confirms the chaotic dynamics of system (3).

4. PSpice Simulations

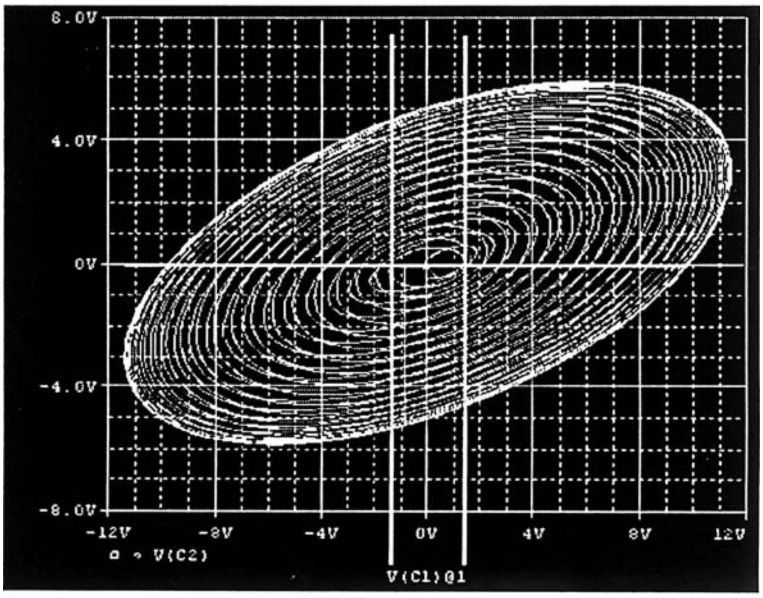
Since numerical integrations of the dimensionless system (3) have shown the occurrence of chaotic beats, the aim of this section is to investigate the beats phenomenon in the circuit, which has been designed in the companion paper [7]. In particular, based on the dimensionless parameter values reported in the previous section, the design procedure has led to the following dimensional parameter values (see details in [7]):

$$\begin{aligned}
&R = 1000\,\Omega, \quad C_1 = 10\,\text{nF}, \quad C_2 = 67.99\,\text{nF}, \quad L = 4.76\,\text{mH}, \\
&B_p = 0.01\,\text{V}, \quad G_a = -1.1428\,\text{m}\Omega^{-1}, \quad G_b = -0.7142\,\text{m}\Omega^{-1} \\
&A_1 = 0.25\,\text{A}, \quad A_2 = 17.49\,\text{V}, \quad f_1 = 7022.57\,\text{Hz}, \quad f_2 = 7256.66\,\text{Hz}.
\end{aligned} \tag{5}$$

By exploiting (5), PSpice simulator is used to describe the chaotic amplitude modulation of the beats in the proposed circuit. In particular, several phase portraits are carried out in the (v_{C_1}, v_{C_2})-state space, at different time units. The results are reported in Figures 4(a)–(d), where in each figure the evolution of the trajectory with respect to the previous illustration has been highlighted. In particular, Figure 4(a) shows that the circuit dynamics start from the origin and expand until v_{C_1} approximately reaches the values ± 11.4 V. Then, Figure 4(b) illustrates that the trajectory of variable v_{C_1} shrinks back until the values ± 1.4 V are approximately reached. In Figure 4(c) the dynamics expand again until they reach the values ± 8.15 V. Successively, in Figure 4(d) the values ± 11.25 V are approximately reached. These



(a)



(b)

Figure 4. Phase portraits at different time units using Pspice. (a): $t = 2.5$ ms; (b): $t = 5.2$ ms; (c): $t = 7.5$ ms; (d): $t = 15$ ms.

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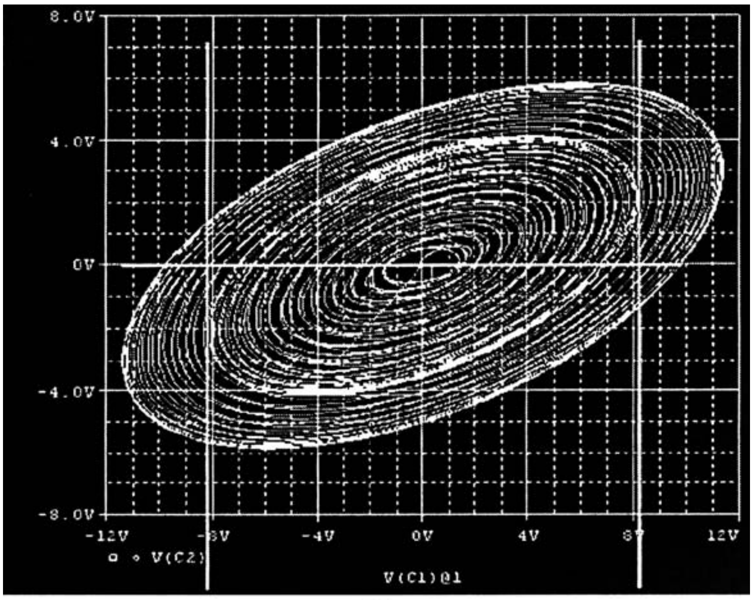
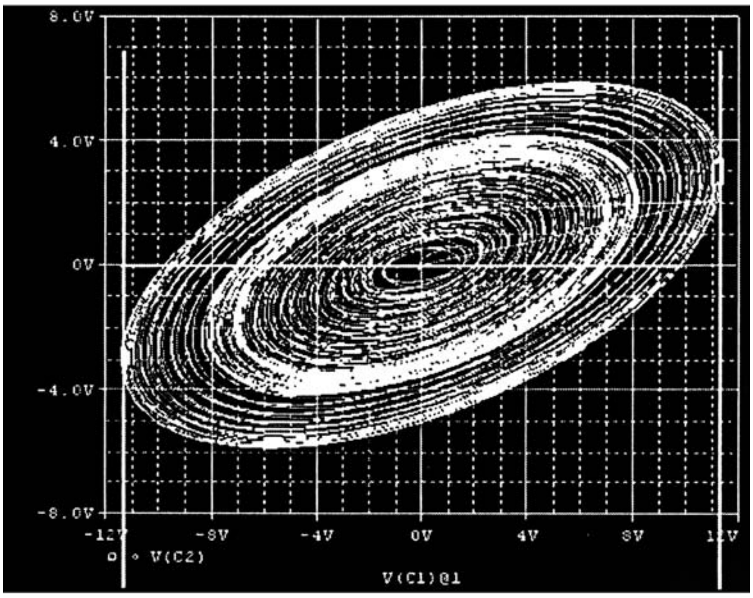


(c)



(d)

Figure 4. (*Continued*)

expanding and contracting behaviors go on *chaotically* for increasing times until the "final" attractor is obtained (see also [7]).

5. Discussion

Now, a brief discussion is reported with the aim of better understanding the formation of the chaotic beats. To this purpose, the circuit dynamics are analyzed by taking the circuit parameters (5) but

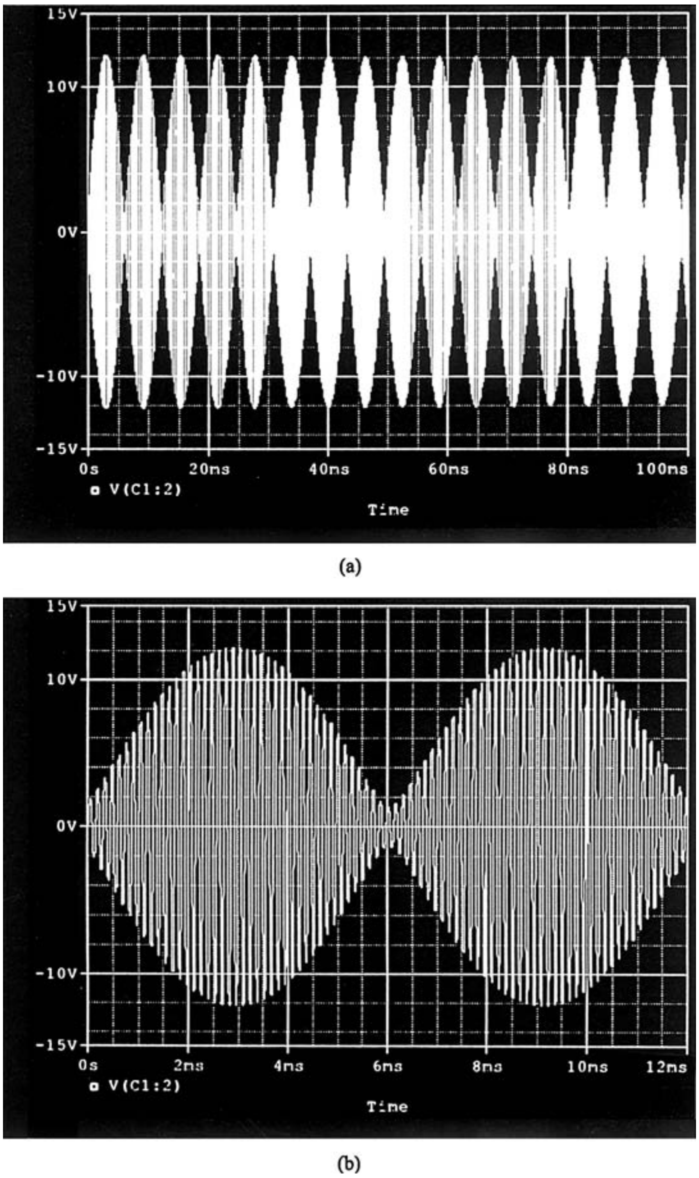


Figure 5. Behaviors of v_{C_1} for different time-scales using PSpice; (a): $t \in [0, 100\,\text{ms}]$; (b): $t \in [0, 12\,\text{ms}]$.

equal frequencies $f_1 = f_2 = 7022.57\,\text{Hz}$. The resulting time waveforms of the variable v_{C_1} are reported in Figures 5(a)–(b) for two different resolutions of the time scale. Figure 5 clearly highlights that in this case the expanding and contracting behaviour goes on *periodically* for increasing times. More precisely, Figure 5(a) shows the presence of beats due to a *periodic* envelope, whereas Figure 5(b) reveals the presence of a fundamental frequency. It can be concluded that for equal frequencies $f_1 = f_2 = 7022.57\,\text{Hz}$ the proposed non-autonomous circuit is not able to generate chaotic beats. However, notice that in this case the circuit exhibits *periodic* amplitude modulated signals, which are very similar to the beats obtained in *linear* systems.

6. Conclusions

This paper and the companion one [7] have focused on a modified version of the Chua's circuit, characterized by two sinusoidal inputs with slightly different frequencies. In particular, in this paper the new phenomenon of chaotic beats has been analyzed both in time-domain and state-space, confirming the effectiveness of the design approach developed in [7].

References

1. Chen, G. and Ueta, T., *Chaos in Circuits and Systems*, World Scientific Series on Nonlinear Science, Singapore, 2002.
2. Chua, L., 'Chua's circuit 10 years later', *International Journal of Circuit Theory and Applications* **22**, 1994, 279–305.
3. Grassi, G. and Miller, D. A., 'Theory and experimental realization of observer-based discrete-time hyperchaos synchronization', *IEEE Transactions on Circuits and Systems – I* **49**, 2002, 373–378.
4. Grygiel, K. and Szlachetka, P., 'Generation of chaotic beats', *International Journal of Bifurcations and Chaos* **12**, 2002, 635–644.
5. Wylie, C. R., *Differential Equations*, McGraw-Hill, New York, 1979.
6. Cafagna, D. and Grassi, G., 'Generating chaotic beats in coupled non-autonomous Chua's circuits', in *Proceedings of 16th European Conference on Circuit Theory and Design (ECCTD 2003)*, Krakow, Poland, September 1–4, 2003.
7. Cafagna, D. and Grassi, G., 'Generation of chaotic beats in a modified Chua's circuit – Part II: Circuit Design', *Nonlinear Dynamics*, 44, 101–108.