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ORIGINAL PAPER

A logistic regression classifier for long-term probabilistic prediction of rock burst hazard

Ning Li[1] · R. Jimenez[1]

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Abstract Rock burst is a complex dynamic process can lead to casualties, to failure and deformation of the supporting structures, and to damage of the equipment on site; hence, its prediction is of great importance in underground construction. We present a novel empirical method to predict rock burst based on the theory of logistic regression classifiers. An extensive database collected from the literature, which includes observations about rock burst occurrence (or not) in underground excavations in projects from all over the world, is used to train and validate the model. The proposed approach allows us to compute new class separation lines (or planes) to estimate the probability of rock burst, using different combinations of five possible input parameters—tunnel depth, H; maximum tangential stress, MTS; elastic energy index, W_{et}; uniaxial compressive strength of rock, UCS; uniaxial tensile strength of rock, UTS—among which it was found that the preferable model could be developed in H–W_{et}–UCS space. The proposed model is validated with goodness-of-fit tests and nine-fold cross-validation; results show that its predictive capability compares well with previously proposed empirical methods and confirm that, as expected, the probability of rock burst increases with excavation depth, and that both W_{et} and UCS have a similarly significant influence on rock burst occurrence. Finally, expressions are proposed for identification of conditions associated with several reference values of rock burst probability, which can be employed in preliminary risk analyses.

Keywords Rock burst · Probability theory · Logistic regression · Class separation · Cross-validation

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✉ R. Jimenez
rafael.jimenez@upm.es

[1] Universidad Politécnica de Madrid, ETSI Caminos, C. y P, C/Profesor Aranguren s/n, 28040 Madrid, Spain

1 Introduction

Rock burst is a dynamic process caused by a sudden and violent release of elastic energy accumulated in rock and coal masses. Given its "explosive" nature, rock burst can lead to significant risks (see, e.g., Brauner 1994; Ortlepp and Stacey 1994; Kaiser et al. 1996; Huang and Wang 1999; Dou et al. 2012). For instance, it can threaten human life, as demonstrated by more than 13,000 accidents, with more than 16,000 casualties, reported in Chinese metal mines between 2001 and 2007 (Zhou et al. 2012). And it can also affect the schedule and budget of projects, as illustrated by the two Jinping-II hydropower tunnels, in which the cumulative lengths of sections with rock burst problems added to (approx.) 18.5 and 16.3% of the tunnel length (Shan and Yan 2010).

Significant rock burst problems are also common in coal mining. For instance, a rock burst accident associated with a large gas emission occurred in Sunjiawan coal mine (Fuxin, Liaoning province; February 14, 2005), leading to a seismic event with Richter magnitude of 2.5 (Dou et al. 2009), and another accident occurred in Qianqiu coal mine (Henan province; November 3, 2011), with 10 people killed and 75 trapped underground. It is expected that, with the increasing complexity and depth of future underground projects, new challenges due to rock burst must be addressed.

Predicting the (likely) occurrence of rock burst in one specific project using limited information is still one of the main challenges in relation to rock burst. The reason is that, although many advances in the understanding of rock burst have been achieved since the seminal work of Cook (1966)—see, e.g., Ortlepp and Stacey (1994), Zubelewicz and Mróz (1983), Xie and Pariseau (1993), or Program (1996)—, the complexity of the phenomena involved makes it difficult to predict rock burst in real cases. In other words, there is a need of new methods to predict and control rock burst hazards during underground activities (Dou et al. 2012).

The prediction of rock burst can be divided into two categories: long-term and short-term predictions (Peng et al. 2010). Long-term prediction uses simple information, that is commonly available during the initial stages of a project, to assess the likelihood of rock burst during project development, so that it can guide decision makers in relation to excavation and control methods, whereas short-term prediction aims to predict the location and time of rock burst using data—such as information about drilling bits, microseismic monitoring, and acoustic emission—collected during construction of the actual underground facility (see, e.g., Cai et al. 2001, 2015; Lu et al. 2012; Ma et al. 2015).

Here, we focus on long-term rock burst prediction. Data mining methods and artificial intelligence have often been applied for long-term prediction of rock burst since the seminal work of Feng and Wang (1994). For instance, Zhang et al. (2011) employed a particle swarm optimization–BP neural network; Zhou et al. (2012) and Peng et al. (2014) proposed a rock burst classification based on Support vector machines; Li and Liu (2015) employed the Random forests approach; and Liu et al. (2013) employed cloud models with attribution weights. Although such data mining methods can estimate rock burst hazards, they are often complex and difficult to use on site for immediate prediction work.

In this paper, we propose a novel empirical method for long-term rock burst prediction that is based on the statistical theory of logistic regression classifier. The classifier is trained and tested using a database of case histories compiled from the literature and from technical reports on underground projects, and it allows us to estimate the likelihood of rock burst occurrence based on simple input data that is commonly available at early stages of project development.

2 Empirical methods for rock burst prediction

To be able to anticipate rock burst occurrence, we should incorporate (at least) most of the important aspects that influence rock burst. Some of such factors are discussed below, together with empirical methods proposed to predict rock burst occurrence and to evaluate the associated risk.

The rock stress within a rock mass increases with depth (Hoek and Brown 1980); therefore, the energy accumulated within the rock mass also increases with depth, so that rock burst likelihood is expected to increase as the depth of excavation increases (Dou et al. 2006). That observation motivated Hou and Wang (1989) and Pan and Li (2002) to propose empirical equations to anticipate rock burst based on a "critical depth" computed as a function of other rock mass parameters. See Table 1.

The elastic energy index, W_{et}, defined as the ratio of retained to dissipated strain energy during one loading–unloading cycle under uniaxial compression (see Fig. 1), is another parameter with a strong influence on rock burst (Kidybiński 1981; Singh 1988). As it is also one of the most common rock tests conducted in coal mines in China (Cai et al. 2016), several criteria have been proposed to predict rock burst based on W_{et}. Among them, the criterion proposed by Kidybiński (1981), who suggested using $W_{et} \geq 2.0$ as a threshold to identify rock burst conditions, is probably the most commonly employed (see Table 1). As additional advantages, W_{et} can be measured using rock samples collected on site, with a relatively simple servo-controlled laboratory test (Wang and Park 2001) whose procedures are "normalized" (see, e.g., Ulusay and Hudson 2007); or with direct or indirect in situ evaluations (e.g., the "double hole" or "rebound" methods, respectively) (Kidybiński 1981).

But there are other mechanical parameters that could influence rock burst, and which can be assessed at the early stages of a project. The maximum tangential stress around the excavation (MTS or σ_θ), the uniaxial tensile strength of rock (UTS or σ_t), and the uniaxial compressive strength of rock (UCS or σ_c) are other mechanical parameters that have been commonly employed to predict rock burst. Examples of the associated predictive models, with their corresponding references, are listed in Table 1 (The reader should note that existing predictive models are deterministic, so that, for instance, they cannot be easily employed in the context of risk analyses. Similarly, although there are other factors—such

Table 1 Summary of previous empirical criteria for rock burst prediction

Criteria	References
$H_{cr} = 0.318\sigma_c(1 - \mu)/(3 - 4\mu)\gamma$	Hou and Wang (1989)
$H_{cr} = \sigma_c(1 - \sin\varphi)\lambda[-1 + (1 + E/\lambda)^{1/(1 - \sin\varphi)} - E/\lambda]/2E\lambda\sin\varphi$	Pan and Li (2002, 2005)
$W_{et} \geq 2.0$	Kidybiński (1981), Wang et al. (1998)
$\sigma_\theta/\sigma_c \geq 0.3$	Wang et al. (1998)
$\sigma_c/\sigma_t \leq 40$	Wang et al. (1998)
$\sigma_\theta/\sigma_c > 0.2$	Russenes (1974)

σ_θ is the maximum tangential stress of surrounding rock, MPa; σ_c is the uniaxial compressive strength of rock, MPa; σ_t is the uniaxial tensile strength of rock, MPa; μ is the Poisson's ratio; γ is the weight of the rock mass; φ is the internal friction angle of rock; λ is the softening modulus (value of elastic modulus after the peak value of stress in the stress–strain curve), MPa; E is the elastic modulus, MPa; W_{et} is elastic energy index

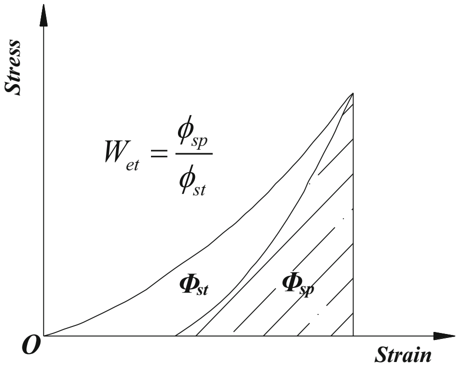

Fig. 1 Single loading–unloading cycle of uniaxial compression and definition of W_{et}

as water conditions or rock mass integrity—that could affect rock burst, they are not considered herein due to the absence of a sufficient number of reliable case histories that provide information about them in the context of rock burst occurrence. When more information about them is available, they could probably be included in future models for a better prediction work.)

We propose an alternative approach, developed using the theory of statistical classifiers, to estimate the likelihood of rock burst occurrence. To that end, we start compiling a database of 135 case histories from different types of underground excavations from all over the world, in which the occurrence (or not) of rock burst has been recorded. (83 case histories correspond to rock burst, and 52 are non-rock burst cases out of the 135 case histories within the database.) Each record (i.e., case history) in the database contains fields that correspond to the five main aspects influencing rock burst discussed above: H, MTS, UCS, UTS, and W_{et}. Figure 2 shows the boxplot of distributions for the five parameters collected in the database and Table 2 reports their main statistics. Black solid points in Fig. 2 indicate the "outliers" (or extreme cases) of each distribution, the horizontal bold lines inside the boxes represent its median values, and the bottom and top lines of each box indicate the first and third quartiles. Table 3 lists the correlations coefficients (and the corresponding p values) among the parameters; it can be observed that only the UCS–UTS and UCS–W_{et} relationships (in bold) have correlation coefficients higher than 0.6, indicating strong relationships between those two parameters respectively. The database is developed based on observations reported by Zhou et al. (2012), Zhao et al. (2007), Guo et al. (2008) and other unpublished technical reports. The complete database of case histories employed in this paper is presented in Appendix A of ESM.

The reader should note that information about some fields might be incomplete in some records from the database, so that not all case histories contain information about all parameters. This is the reason why, in the discussion below, the number of case histories might be different for the different models considered (see Table 4). The details of the formulation of the statistical classifier, which builds on the work of Jordan (2003), are discussed below; additional details about the implementation of similar classifiers to probabilistically anticipate the occurrence of complex phenomena in underground construction—respectively, the squeezing of tunnels and the stability of underground coal mine pillars—can be found in Jimenez and Recio (2011) and Recio-Gordo and Jimenez (2012).

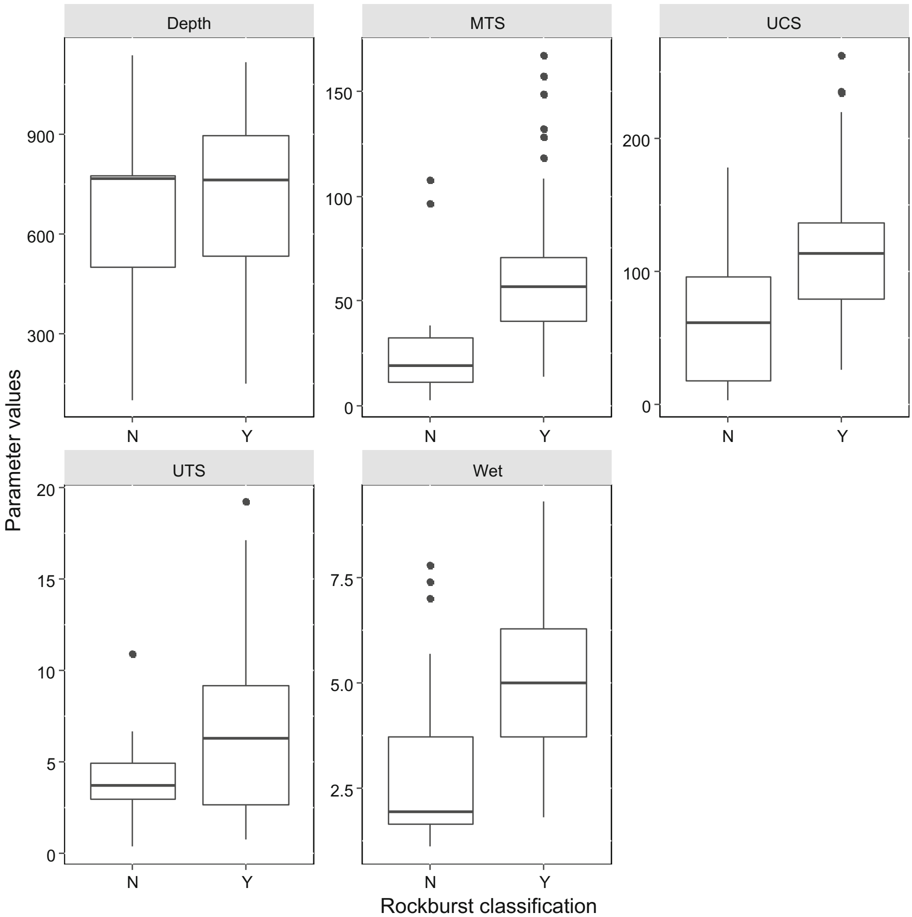

Fig. 2 Boxplot of rock burst parameters ("N" represents non-rock burst and "Y" represents rock burst)

Table 2 Descriptive statistics of the input parameters for case histories within the database

Parameter	Available	Missing	Min	Max	Mean	SD
H (m)	119	16	100	1140	705.97	274.53
MTS (MPa)	100	35	2.6	167.2	56.28	33.21
UCS (MPa)	134	1	2.9	263	97.32	54.69
UTS (MPa)	123	12	0.38	19.2	5.68	3.58
W_{et}	117	18	1.1	9.3	4.41	2.05

3 A logistic regression classifier for rock burst prediction

Statistical classifiers aim to assign "labels" to observations based on their characteristic features; in other words, they aim to identify the "class" or "group" to which observations belong. In the context of rock burst prediction, our statistical classifiers are trained to assign a discrete-valued random variable, Y, that represents the possible rock burst

Table 3 Pearson's correlation coefficients (*p* values) between the parameters collected in the database (The *p* value is expressed in probability levels so that the smaller *p* level shows the significant relationship)

	H	MTS	UCS	UTS	W_{et}
H	1	0.161 (0.140)	−0.245 (0.008)	−0.193 (0.046)	−0.169 (0.091)
MTS	0.161 (0.140)	1	0.216 (0.031)	0.207 (0.039)	0.293 (0.003)
UCS	−0.245 (0.008)	0.216 (0.031)	1	**0.601 (0.000)**	**0.685 (0.000)**
UTS	−0.193 (0.046)	0.207 (0.039)	**0.601 (0.000)**	1	0.299(0.002)
W_{et}	−0.169 (0.091)	0.293 (0.003)	**0.685 (0.000)**	0.299(0.002)	1

Table 4 Possible models using parameters collected

Types of parameters		Error cases	Case histories number	Error rate (%)
Environmental	Rock intrinsic			
H	$\boldsymbol{W_{et}}$	**10 + 10**	**101**	**19.8**
H	UCS	20 + 8	118	23.7
H	UTS	25 + 5	107	28.0
MTS	W_{et}	10 + 2	100	12.0
MTS	UTS	10 + 2	100	12.0
MTS	UCS	11 + 2	100	13.0
H, MTS	W_{et}	4 + 4	85	9.4
H, MTS	UCS	7 + 4	85	12.9
H, MTS	UTS	8 + 2	85	11.8
H	$\boldsymbol{W_{et}}$**, UCS**	**2 + 4**	**99**	**6.1**
MTS	UCS, W_{et}	10 + 2	100	12.0
H	W_{et}, UTS	4 + 4	89	9.0
H	UCS, UTS	23 + 7	107	28.0
MTS	W_{et}, UTS	10 + 3	100	13.0
MTS	UCS, UTS	9 + 2	100	11.0

For the catalogue of error cases, the first figure indicates that observations assigned a rock burst label when they actually presented non-rock burst conditions (false positive), whereas the second indicates observations for which a non-rock burst label was assigned even though the rock burst occurred in reality (false negative)

occurrence—"rock burst" or "non-rock burst"—, based on the observation of a set of variables, X, defined in terms of the input parameters discussed above. The main difference with many of the previous efforts to address rock burst is that the assignments can be conducted probabilistically, using the computed conditional probability $P(Y|X)$, as explained below.

Our classifier is a discriminative model (Mitchell 2015), so that it aims to maximize the quality of its predictions for the training set, using the logistic function to construct a "map" between physical features and rock burst probabilities. In particular, if we use $X = (X_1,\ldots, X_n)$ to denote the input vector that includes information about (some of) the parameters that influence rock burst described above—i.e., H, W_{et}, MTS, UTS, and/or UCS—, then the conditional probability of the class label, Y, can be computed based on a set of observations on X, as $P(Y = y|X = x)$. And, since Y is a Bernoulli-type random variable ($Y = 1$ indicates "rock burst", whereas $Y = 0$ indicates "non-rock burst"), the conditional probability can be expressed as

$$p(y|x) = f(x)^y(1 - f(x))^{1-y} \quad (1)$$

where $f(x)$ is the parameter of the Bernoulli distribution—note that it is a function of input data—, with $f(x) = p(y = 1|x) = E(y|x)$.

Furthermore, we assume that $f(x)$ can be computed using the logistic function and a linear transformation of the input features, as

$$f(x) = \frac{1}{1 + \exp(-\theta^T x)} \quad (2)$$

where θ is the parameter vector of the logistic regression classifier.

To "learn" the parameter vector, a maximum likelihood approach—that aims to maximize the probability that the model produces the observations available within the training dataset—is common. In our case, assuming independence between observations in the training data set, we can define the likelihood as

$$p(y_1, \ldots, y_N|x_1, \ldots, x_N, \theta) = \prod_{i=1}^{N} f(x_i)^{y_i}(1 - f(x_i))^{1-y_i} \quad (3)$$

and, taking logarithms to facilitate the optimization, we obtain the log-likelihood as

$$\log p = \log(y_1, \ldots, y_N|x_1, \ldots, x_N, \theta) = \sum_{i=1}^{N} \{y_i \log f(x_i) + (1 - y_i)\log(1 - f(x_i))\} \quad (4)$$

The log-likelihood in Eq. (4) could be optimized using a wide variety of methods, but Newton–Raphson methods—which are iterative methods that employ first and second order derivatives—and, in particular, the iteratively reweighted least squares (IRLS) algorithm, are commonly employed due to their good performance. The final expression is (see Jimenez and Recio 2011 for details of the derivation):

$$\theta^{i+1} = \theta^i + (X^T W_i X)^{-1} X^T (y - f(x_i)) \quad (5)$$

where X is the matrix of observations (with an additional column of ones to provide an independent item); $W = \mathrm{diag}\{f(x_1)(1 - f(x_1)), \ldots, f(x_N)(1 - f(x_N))\}$ is a diagonal weight matrix that changes from iteration to iteration—hence the algorithm's name—; y is a $N \times 1$-dimensional vector with the observed outcomes of "rock burst" or "non-rock burst" occurrence for each input; and $f(x_i)$ is another $N \times 1$ vector with the Bernoulli's model parameters computed with Eq. (2) for each input vector with the current value of the parameter vector, θ^i.

4 Results and discussion

4.1 Model training and probability calculation

The algorithm described in Sect. 3 has been implemented in MATLAB, and several models with different combinations of "environmental" (or "external") and "rock intrinsic" parameters have been tested to assess their performance and predictive capabilities.

For simplicity, we only construct models with two or three input parameters (out of the five parameters discussed above), so that at least one parameter is chosen from the "environmental" parameters (H or MTS) and at least one is chosen from the "rock intrinsic" parameters (W_{et}, UCS or UTS). The possible models constructed using such combinations of parameters, and their training results, are listed in Table 4. Results show that, as expected, models with three parameters perform better than models with two. Results also show that the models with two input parameters that use MTS as "environmental" parameter obtain lower error rates, but their prediction are biased, which makes us to suggest that other non-biased models be used instead. (Although they are biased toward the conservative side, with non-rock burst cases being predicted as rock burst, unbiased models are preferable in the context of risk analyses). Furthermore, results show that models that do not consider W_{et} and MTS have consistently higher error rates (above 20%).

Next, we discuss two of the models in Table 4 that, due to their simplicity and performance, were considered preferable to predict rock burst. The first model (referred to as Model A) considers only two input (predictive) variables: W_{et} and H. Model B contains three input variables: W_{et}, UCS, and H.

Based on the discussion above, the classier for Model A is developed using the following input observation matrix:

$$X = [1, X_1, X_2] = [1, W_{et}, H] \tag{6}$$

where X is a $N \times 3$-dimensional matrix that includes a first column with a vector of ones to supply an independent term in the regression, and N is the number of observations in the collected case histories. The rock burst observations form a $N \times 1$-dimensional vector, Y, in which a value of 1 represents rock burst occurrence (using 0 otherwise). Similarly, the observation matrix for Model B is a $N \times 4$-dimensional matrix constructed as:

$$X = [1, X_1, X_2, X_3] = [1, \mathrm{UCS}, W_{et}, H] \tag{7}$$

The convergence of the IRLS algorithm is fast, and the parameter vector, θ, can be trained in just a few iterations. (Results presented herein are computed with a convergence tolerance of 1E−9.) As an example of the results obtained, Table 5 lists the iterative estimates of the parameter vector for Model B obtained during its training with the IRLS algorithm. (For Model A, the algorithm converges in just six steps, with a final result of $\theta_1 = -3.5092$, $\theta_2 = 0.9920$, and $\theta_3 = 0.0013$.)

Table 5 Iterative parameter estimates for the logistic regression classifier of Model B

Iteration	θ_1	θ_2	θ_3	θ_4
0	0	0	0	0
1	−2.8830	0.0166	0.2271	0.0019
2	−4.8288	0.0332	0.3437	0.0029
3	−6.7756	0.0492	0.4899	0.0038
4	−8.2241	0.0600	0.6191	0.0046
5	−8.7536	0.0638	0.6747	0.0048
6	−8.8060	0.0641	0.6811	0.0048
7	−8.8064	0.0641	0.6812	0.0048
8	−8.8064	0.0641	0.6812	0.0048

Once the classifier has been trained, Eq. (2) is employed to develop the logistic regression classifier for different values of probability (indicated by $f(x)$), as:

$$\theta^T x = \ln\left(\frac{f(x)}{1 - f(x)}\right) \tag{8}$$

And, particularly for Model A, we obtain:

$$\theta_1 \cdot 1 + \theta_2 W_{\mathrm{et}} + \theta_3 H = \ln\left(\frac{f(x)}{1 - f(x)}\right) \tag{9}$$

where the θ_i values are the computed values listed above.

As $f(x)$ is a probability, we can obtain lines in the W_{et}–H space with constant rock burst hazard. For example, to develop a line with 50% of rock burst probability (with $f = 50\%$), we obtain:

$$H(m) = 2699.38 - 763.08 W_{\mathrm{et}} \tag{10}$$

Similarly, Model B can be transformed to:

$$\theta_1 \cdot 1 + \theta_2 \mathrm{UCS} + \theta_3 W_{\mathrm{et}} + \theta_4 H = \ln\left(\frac{f(x)}{1 - f(x)}\right) \tag{11}$$

where the θ_i values correspond to the converged results listed in Table 5.

As a particular example, we can obtain the class separation plane with a 50% likelihood of rock burst occurrence under the W_{et}–UCS–H three-dimensional space, as

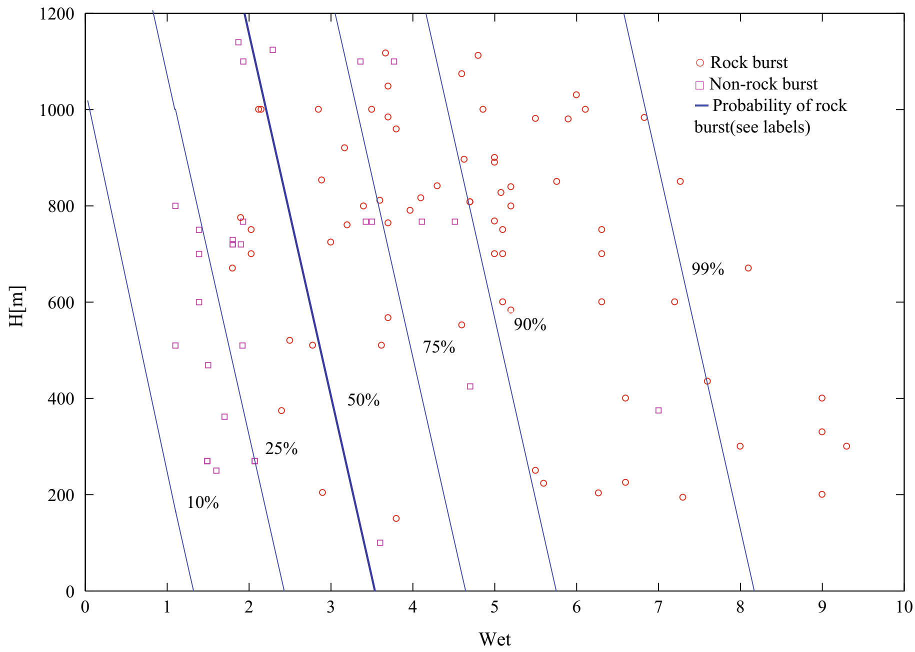

Fig. 3 Equi-probability lines for rock burst prediction with Model A

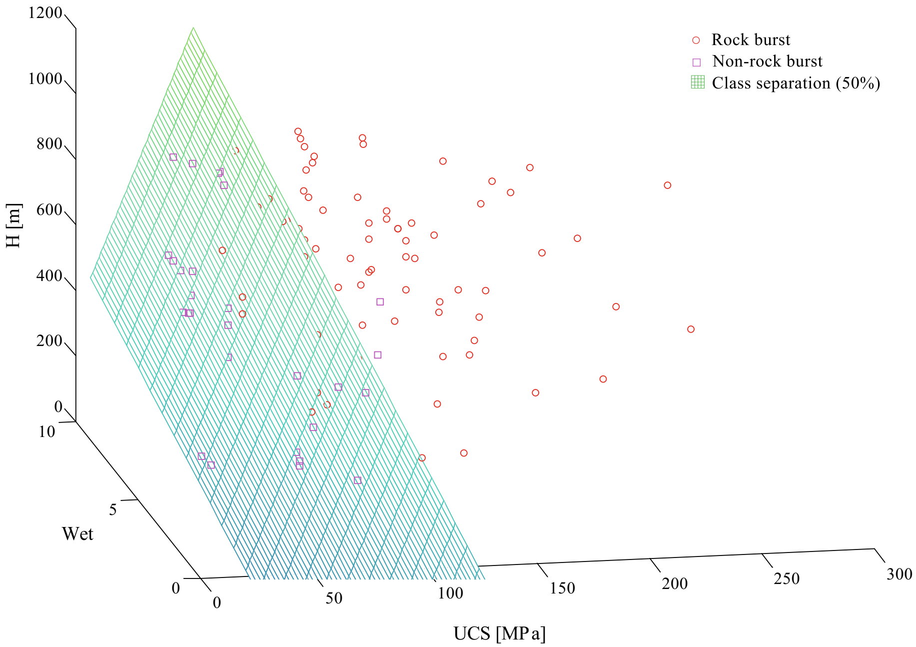

Fig. 4 50% of probability plane for rock burst prediction with Model B

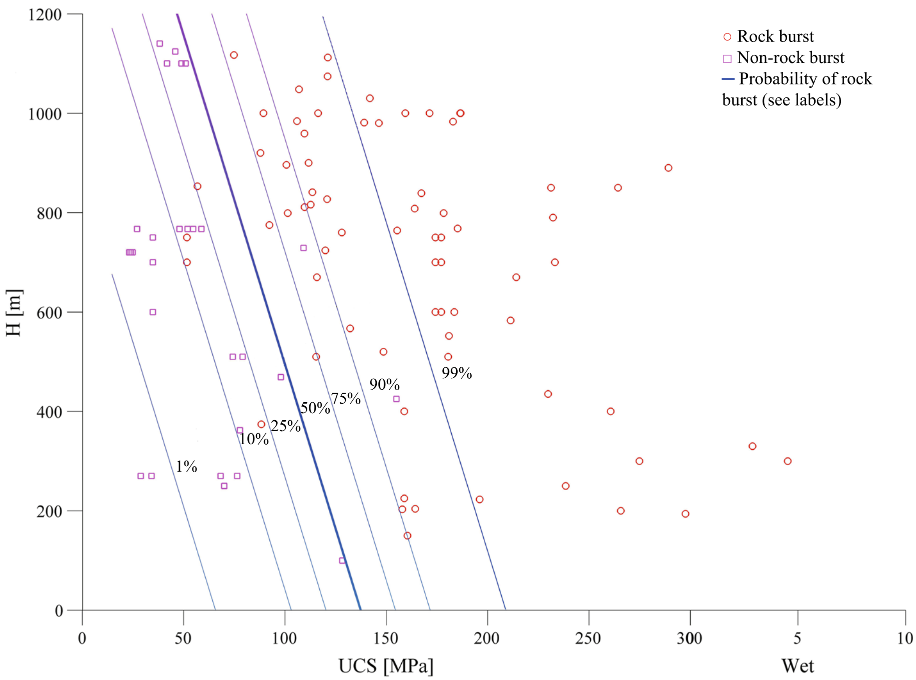

Fig. 5 Equi-probability planes (viewed under a certain angle) for rock burst prediction with Model B

$$H(m) = 1834.67 - 13.35\text{UCS} - 141.92W_{\text{et}} \tag{12}$$

Figure 3 shows the resulting probability lines [for $f(x) = (1\%, 10\%, 25\%, 50\%, 75\%, 90\%, 99\%)$] for Model A. Figure 4 shows the class separation plane (corresponding to a 50% probability of rock burst) of Model B, which divides the input space into two regions: one in which "rock burst" is more likely than "non-rock burst" (the upper half-space) and another in which it is otherwise. Figure 5 shows another view of the results of Model B shown in Fig. 4, in which additional probability planes have been added (for rock burst probabilities of 1%, 10%, 25%, 75%, 90%, and 99%), and in which the angle of view have been selected so that such planes appear as lines in the plot.

4.2 Goodness of fit and predictive performance

Goodness-of-fit tests aim to understand whether the model predictions accurately reflect the observed data outcomes, so that the performance of a classifier can be evaluated comparing its predictions to available observations (Dreiseitl and Ohno-Machado 2002).

4.2.1 Deviance and Akaike's information criterion

Deviance is a goodness-of-fit test for logistic regression models, based on the likelihood ratio between our fitted model and the saturated one (one in which each observation gets its own parameter). The null deviance shows how well the output is predicted by a model with an intercept only, and the residual deviance shows how well the output is predicted by the model when the predictors are included.

Akaike's information criterion (AIC; Akaike 1974) can also be used for model selection. It represents an objective methodology to select the "best" model, given some observations, among a set of candidate models; ranking the models considering both their fit and complexity (Simply comparing goodness of fit is a poor model selection technique, as it is always possible to improve it increasing the number of model parameters, but too complex models should be penalized to avoid overfitting.)

Table 6 shows the results computed for both models. The null deviances of models A and B are slightly different because their total number of case histories is different, and both models decrease their deviances when predictors are added. It is also clear that Model B has a lower residual deviance and AIC than Model A, illustrating the effects of including UCS as a predictive parameter.

4.2.2 Hosmer–Lemeshow (H–L) test

The Hosmer–Lemeshow (H–L) test is another statistical test for goodness of fit for logistic regression models. It is often employed in risk prediction models. It evaluates whether the observed proportions of events within subgroups of the population match the expected

Table 6 Summary of deviances and AIC for Model A and Model B

Model	Null deviance	Residual deviance	AIC
A	122.882	83.141	81.474
B	117.930	41.095	49.095

Table 7 H–L test for Model A

Group	Probability zone	Observed no rock burst cases	Observed rock burst cases	Expected no rock burst cases	Expected rock burst cases
1	[0.149,0.25]	12	0	9.58	2.42
2	(0.25,0.357]	6	3	6.07	2.93
3	(0.357,0.548]	3	7	5.52	4.48
4	(0.548,0.713]	2	8	3.49	6.51
5	(0.713,0.824]	2	8	2.21	7.79
6	(0.824,0.902]	4	6	1.41	8.59
7	(0.902,0.927]	0	10	0.83	9.16
8	(0.927,0.965]	0	10	0.58	9.42
9	(0.965,0.982]	1	9	0.24	9.76
10	(0.982,0.998]	0	10	0.06	9.94

Table 8 H–L test for Model B

Group	Probability zone	Observed no rock burst cases	Observed rock burst cases	Expected no rock burst cases	Expected rock burst cases
1	[0.003492,0.04859]	10	0	9.74	0.26
2	(0.04859,0.2093]	7	3	8.71	1.29
3	(0.2093,0.5667]	9	1	6.52	3.48
4	(0.5667,0.8969]	1	9	1.80	8.20
5	(0.8969,0.94]	0	10	0.78	9.22
6	(0.94,0.9867]	1	8	0.35	8.65
7	(0.9867,0.9961]	0	10	0.06	9.94
8	(0.9961,0.9986]	0	10	0.03	9.97
9	(0.9986,0.9999]	0	10	0.005	9.996
10	(0.9999,1]	0	10	0.0003	9.9997

proportions (Hosmer Jr et al. 2013). In our proposed model, we divide the case histories into 10 groups for evaluation (see Tables 7 and 8). The p values for the H–L test are 0.04 for Model A and 0.41 for Model B. This suggests that Model A has a worse performance (as its p value is the less than 0.05), whereas Model B provides better results for more reliable future predictions.

4.2.3 Confusion matrices

Rock burst prediction is a two-class prediction problem, so that there are four possible situations which can be conveniently recorded as confusion matrices (Fawcett 2006). If the actual outcome is positive (i.e., "rock burst"), and it is classified as positive, it is counted as a true positive (TP); if it is classified as negative (i.e., "non-rock burst"), it is counted as a false negative (FN). If the actual outcome is negative and the prediction is also negative, it can be defined as true negative (TN); if it is classified as positive, it is counted as a false positive (FP).

Table 9 Confusion matrix of "Rock burst" prediction with Model A

Predicted			
Yes	No		
61(TP)	10(FN)	Yes	Actual
10(FP)	20(TN)	No	

"Actual" indicates the real rock burst condition recorded in the database, and "predicted" indicates the predictions of our proposed model. "Yes" represents rock burst occurrence, and "No" represents non-rock burst

Table 10 Confusion matrix of "Rock burst" prediction with Model B

Predicted			
Yes	No		
67(TP)	4(FN)	Yes	Actual
2(FP)	26(TN)	No	

"Actual" indicates the real rock burst condition recorded in the database, and "predicted" indicates the predictions of our proposed model. "Yes" represents rock burst occurrence, and "No" represents non-rock burst

Based on these definitions, and using the class separation lines (or planes) computed with our trained classifiers in Models A and B, we obtain the performance results illustrated by the confusion matrices listed in Tables 9 and 10. The error rates for Model A and Model B are 19.8% and 6.1%, respectively. As expected (because it has only two parameters), Model A has a higher error rate, which decreases when additional "rock intrinsic" parameters are considered (see Table 4), showing that considering UCS can significantly improve the predictions. Therefore, Model B is proposed as an improved logistic regression classifier that can provide better estimations, while still using simple input information.

4.2.4 Relative operating characteristic curve

The relative operating characteristic curve (ROC; Metz 1978) is a simple empirical description that indicates all possible combinations of the relative frequencies of correct and incorrect decisions. A model with no discrimination ability will generate a ROC curve that follows the 45° line, whereas a perfect discrimination is when the ROC curve follows the left hand and top axes of the unit square with the true positive rate equals one and the false positive rate equals zero (Pearce and Ferrier 2000).

The ROC curves of our proposed models are presented in Fig. 6. The areas under the curve (AUC) are 0.873 and 0.961 for Model A and Model B, respectively. (Hosmer Jr et al. 2013 proposed that the AUC values above 0.9 represent an outstanding discrimination, whereas values between 0.8 and 0.9 represent an excellent discrimination.) Therefore, our proposed Model B is also better when its performance is assessed using ROC curves and their corresponding AUC values.

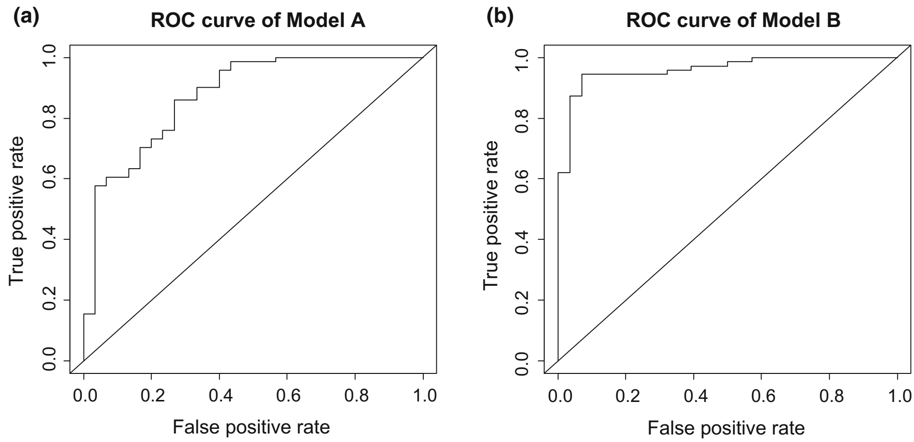

Fig. 6 ROC curves of Models A and B

4.2.5 Cross-validation

At the same time, a *k*-fold cross-validation exercise (with $k = 9$) is employed to validate Model B and to further test its performance. For such combination of input parameters (H, W_{et}, and UCS), there are 99 case histories in the initial database, so that they can be randomly divided into nine groups of data. Then, for each group, the model is trained using the other eight groups, and the originally selected group is used to predict rock burst occurrence with the trained logistic classifier model, and to compare its predictions with the observations within the group. If this process is repeated for the nine groups, a nine-fold cross-validation exercise is obtained. Results are reported in Table 11, showing that Model B maintains a low average error rate of 9.1% (Note that, although some groups have higher error rates, the key result of the validation is the average error rate.)

4.3 Use for risk analyses

We can use two additional case histories to illustrate the applicability of the developed model to estimate the probability of rock burst occurrence, given a set of input parameters corresponding to a new rock underground project: the first one has the following set of representative input parameters: $H = 700$ m, $W_{\mathrm{et}} = 2.87$, and UCS $= 70.68$ MPa; the second has $H = 630$ m, $W_{\mathrm{et}} = 0.88$, and UCS $= 59$ MPa. Then, given the set of fitted model parameters listed in Table 5 ($\theta_1 = -8.8064$, $\theta_2 = 0.0641$, $\theta_3 = 0.6812$, and $\theta_4 = 0.0048$), we can use Eq. (2) to compute the probability of rock burst occurrence, $f(x)$, resulting values of 73.9% and 19.8%, respectively. Such probability values can then be incorporated, with the corresponding estimates of costs associated to such failures (whose discussion is outside the scope of this work), into risk analyses.

Table 11 Nine-fold cross-validation results of Model B

Group	Error rates (%)	Confusion matrices			
No. 1	9.1	Predicted			
		Yes	No		
		6	0	Yes	Actual
		1	4	No	
No. 2	0	Predicted			
		Yes	No		
		9	0	Yes	Actual
		0	2	No	
No. 3	18.2	Predicted			
		Yes	No		
		7	1	Yes	Actual
		1	2	No	
No. 4	0	Predicted			
		Yes	No		
		9	0	Yes	Actual
		0	2	No	
No. 5	18.2	Predicted			
		Yes	No		
		7	1	Yes	Actual
		1	2	No	
No. 6	18.2	Predicted			
		Yes	No		
		6	1	Yes	Actual
		1	3	No	
No. 7	9.1	Predicted			
		Yes	No		
		4	0	Yes	Actual
		1	6	No	
No. 8	9.1	Predicted			
		Yes	No		
		9	1	Yes	Actual
		0	1	No	
No. 9	0	Predicted			
		Yes	No		
		10	0	Yes	Actual
		0	1	No	
Average	9.1				

5 Discussion

Based on the results presented in Figs. 3, 4, and 5, we can observe that, as expected and in agreement with previous research (see, e.g., Dou et al. 2006), the new classifier predicts higher rock burst probabilities as H, UCS, or W_{et} increase. Results also illustrate that, in this case, significantly better results are obtained when an additional input parameter is

Table 12 Comparison of error rates with four different empirical methods and with the logistic regression classifier of Model B with all data and with cross-validation

Methods	Unavailable cases	Available cases	Error rate (%)
Stress coefficient (Russenes 1974)	15	84	10.7
Stress coefficient (Wang et al. 1998)	15	84	17.9
Rock brittleness coefficient (Wang et al. 1998)	11	88	22.7
Elastic energy index (Kidybiński 1981; Wang et al. 1998)	0	99	13.1
Model B with all data	0	99	6.1
Model B with cross-validation	0	99	9.1

included in the model: the predictive performance is significantly improved in Model B—using both the original set of case histories and the ninefold cross-validation exercise—, when the UCS is added to Model A as an additional input parameter.

The validity of predictions obtained with the model proposed can be further assessed using the available observations. For instance, the 50% rock burst probability line in the W_{et}–H space of Model A produces a total of 20(10 + 10) misclassifications, as ten false positives and ten false negatives are produced. Similarly, there are 6(2 + 4) misclassifications in the W_{et}–UCS–H space of Model B (see Table 4), and such misclassifications are almost balanced with respect to the 50% probability plane. Therefore, both models produce (almost) unbiased classifiers that are beneficial in risk analyses. And the rock burst probability lines (or planes) computed for other probability values—1%, 10%, 25% , 50%, 75%, 90%, and 99%—also suggest that the rock burst misclassifications tend to stay within the expected ranges.

For comparison, Table 12 presents the prediction results of the proposed logistic regression classifier, together with the outcomes expected with some other common empirical methods for rock burst prediction presented in Table 1. Our model has the lowest error rate among those considered, while still maintaining a simple equation that only needs simple information commonly available in the initial stages of a project.

To assess the significance of the predictors and that of the model, the usual approach of hypotheses testing is employed. To that end, the Wald test statistics are computed using the R software (R Core Team 2014), obtaining that the p values for the H, UCS and W_{et} coefficients are 0.00567, 0.0000702, and 0.04441, respectively. The multivariable Wald test statistic is also employed to test the whole model significance, obtaining a final p value of 0.00007354. These results show that our proposed Model B has a significant relationship with the selected predictors (Hosmer Jr et al. 2013). Therefore, our proposed model is considered reliable in the prediction work, so that we could consider Model B as the optimal classifier for rock burst prediction, due to its simplicity of use (or its small requirements for input parameters), to its goodness of fit and to its low error rate and unbiased predictions, which provide a better predictive ability than previous empirical methods.

Finally, it is important to remind readers that the trained parameters shown in the model presented in Eq. (12) cannot be considered as the final solution to the problem of rock burst prediction. The reason is that, as more case histories become available in the future, the database should be extended and the classifier should be updated so that better models can be developed. Similarly, the collected database has limits that affect the applicability of the proposed model. For instance, the values of H in the database range between 100 m and 1140 m, and the use of the classifier for prediction should also be confined to such limits.

6 Conclusions

We present a novel logistic regression classifier for long-term prediction of rock burst. It is based on simple input data that are commonly available at the early stages of design of civil or mining underground structures. Its main difference with traditional alternative methods is that results are computed probabilistically; hence, they can be naturally incorporated into risk analyses. Five parameters are considered: two are “environmental” or “non intrinsic” parameters (the depth of the excavated underground facility, H; and the maximum tangential stress, MTS); and three are “rock intrinsic” parameters (the elastic energy index, W_{et}; the uniaxial compressive strength, UCS; and the uniaxial tensile strength, UTS).

Several combinations of two and three parameter models were trained and tested using a database compiled with 135 case histories, and they were further validated using goodness-of-fit tests and nine-fold cross-validation. Results suggest that the classifier in the H–W_{et}–UCS (Model B) space should be preferentially employed in practice, as it shows significantly better goodness of fit and predictive performance (which is probably adequate for practice). In addition, the proposed model improves previously available methods, because it produces a reduced number of misclassification cases, and because it provides probabilistic boundaries to estimate the probability of rock burst.

Finally, the reader should be reminded that the proposed method is only an empirical solution to a complex problem, and that the classifier has been trained using a limited database, so that it should not be used outside such range of data. Similarly, this method is only an additional item within a wider “toolbox” of methods employed to deal with rock burst hazards, such as numerical simulation, field testing, and seismic monitoring (see, e.g., Sun et al. 2007; Patynska and Kabiesz 2009; Fan et al. 2012).

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