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Risk assessment and analysis of ice disaster in Ning–Meng reach of Yellow River based on a two-phased intelligent model under grey information environment

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Abstract Ice disaster is serious in Ning–Meng reach of Yellow River in China. Due to the complexity of ice disaster, it usually presents grey uncertainty. In order to evaluate the probability of ice disaster loss caused by the adverse events and reveal the development between ice regime data and ice disaster risk under grey information environment, this paper assesses and analyzes the ice disaster risk in Ning–Meng reach of Yellow River. Firstly, the index system of ice disaster risk is established based on the formation mechanism of ice disaster and the novel features of ice regime presented in recent years. Then, the two-phased intelligent model under grey information environment is proposed. The risk degree of ice disaster is assessed with grey interval relational clustering at the first phase, and the decision rules that reflect the development between ice regime information and ice disaster risk degree are extracted with grey dominance-based rough set approach at the second phase. The last empirical analysis of ice disaster risk in the year of 1996–2015 shows that the evaluated ice disaster risk degree of different years is consistent with the practical ice regime characteristics and the extracted decision rules could do as the intuitive criterion to estimate the ice disaster risk through ice regime data.

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1 Introduction

In high-latitude area, the river flow is blocked by the river ice in the period of freeze-up season. More and more ice leads to ice jam and ice dam which will raise the water level, and eventually the ice flood disaster generates when the levees break (Wang 2014). The Ning–Meng reach of Yellow River is located in the northernmost area of Yellow River basin. Due to its curving and variable river morphology, cold and complex climates, it is one of the most frequent reaches suffering from ice flood disasters in China. The ice disaster threatens the security of people's life seriously and hinders sustainable economic development in Yellow River basin. So, many practical research results have been obtained about characteristics and formation mechanism of ice disaster (Jiang et al. 2008), channel-storage increment in freeze-up period (Guo et al. 2014; Zhang et al. 2015), flow routing during ice control period (Kong et al. 2015), ice prevention with reservoir (Lu et al. 2010; Chang et al. 2016), ice regime forecasting model based on river ice dynamic (Fu et al. 2014), fuzzy reasoning (Wang et al. 2012) and neural network (Wang et al. 2008), which further improve the prevention and control of ice disaster. However, in order to analyze the probability of ice disaster loss, so as to make the countermeasure of ice prevention scientifically, the risk assessment of ice disaster, as the component of ice disaster risk management and the premise of ice disaster risk decision (Aven 2015), possesses extremely significant meanings.

At present, the classical risk assessment methods of natural disaster include probabilistic risk assessment based on mathematical statistics; comprehensive risk index method based on AHP, fuzzy mathematics and other evaluation methods; dynamic risk assessment combining remote sensing or GIS with some simulation models (Wu et al. 2014; Wang et al. 2016; Chen et al. 2014; Guo et al. 2016). The ice disaster risk in Ning–Meng reach of Yellow River not only includes the similarity of complex risk system but also possesses the individuality that is different from traditional natural disaster risk. For the complex influence and various evolution process of ice disaster in Ning–Meng reach of Yellow River, it presents a large extent of uncertainty. On the one hand, the uncertainty is connected with its special geographical location, hydrologic and meteorological characteristics and river morphology. On the other hand, the uncertainty could not separate from the imprecise methods of ice disaster risk assessment and the influence of current ice engineering. Considering the systematicness and regionalism of ice disaster in Ning–Meng reach, it still lacks a systemic risk management system of ice disaster. Whereupon, based on the comprehensive effects of the uncertain ice regime factors, Wu et al. (2015) defined the ice disaster risk as "the probability of ice disaster loss caused by the adverse events of ice jam and ice dam in the Ning–Meng reach of Yellow River" and proposed a theoretical framework of "risk identification, risk estimation, risk assessment and risk management" for ice disaster risk analysis, which opened a new chapter for the research of ice disaster risk.

In recent years, extensive economic growth pattern has destroyed the environment, leading to global warming, and the construction of various ice engineering, such as reservoir, levee, is more and more, which have had increasing influences on thermodynamic and hydraulic factors of ice regime. The ice regime in Ning–Meng reach of Yellow River has presented some novel features, which leads to a large extent of timeliness for the ice regime information in Ning–Meng reach. Namely, it is much more reasonable to use the recent ice regime information to reflect the current ice disaster risk and guide the practical ice prevention work. In addition, due to the complexity of ice disaster, the quantity of risk index is restricted and the collected ice regime data are usually grey numbers whose upper and lower bounds are known, but real values are unknown. Hence,

the ice regime information often presents grey uncertainty with "small samples and poor information" (Deng 1982). Risk assessment with grey uncertainty could not be tackled by the methods of mathematical statistics, fuzzy mathematics and numerical simulation, etc. However, the grey target (Luo and Wang 2012), grey relational evaluation (Yan et al. 2016; He and Gong 2014) and grey clustering evaluation (Li et al. 2015; Xie et al. 2014; Shao et al. 2014) based on grey system theory could effectively tackle this kind of decision problems. Luo (2014) investigated the evaluation of ice-jam disaster risk and its prediction with information of three-parameter interval grey numbers and proposed a new method based on grey idea that can be easily realized on computers. Wang and Dang (2015) put forward a dynamic multi-attribute decision-making method based on prospect theory to evaluate the risk of ice jam. Considering the hybrid existence of grey information and fuzzy information in the process of ice disaster risk evaluation, Luo and Li (2016) presented a hybrid grey multiple attribute decision-making method based on "clutch" thought.

The above researches have achieved prominent effect on the risk assessment of ice disaster and other natural disasters, but they mostly are locked into assessing the disaster risk and dividing the risk grade of different objects, which lack the further risk development analysis between ice regime information and ice disaster risk. Actually, the historical ice regime data contain much valuable information which could also reflect the ice disaster risk intuitively. If the valuable information could be extracted effectively, it can further improve the risk analysis and management of ice disaster. Furthermore, the majority of previous researches have focused on risk assessment or risk development analysis separately, and this paper focuses on an integrated efficient decision-making model for ice disaster risk identification, assessment and development analysis.

In order to tackle the grey uncertainty of ice disaster risk and analyze the development between ice regime information and ice disaster risk, this paper proposes a two-phased intelligent model to evaluate and analyze the ice disaster risk in Ning–Meng reach of Yellow River. Considering the grey uncertainty of ice disaster, this paper proposes grey interval relational clustering to evaluate the risk grade of ice disaster. Different from other risk assessment methods, grey interval relational clustering not only inherits the advantage of classical grey clustering, in which the whitenization weight function is used to tackle the situation that the clusters are overlapped and some of objects belong partially to several clusters, but also copes the clustering problem that the index values are three-parameter interval grey numbers. And then, the decision table of ice disaster risk is established through risk grade and ice regime information of each year. The grey dominance-based rough set approach is proposed as a method of data mining to extract the simplest decision rules which could reflect the development between ice regime information and ice disaster risk degree. The research results not only enrich the risk identification, risk assessment and risk development analysis of ice disaster, but provide scientifically decision basis for ice prevention preparedness in Ning–Meng reach of Yellow River.

2 Study area

The Ning–Meng reach of Yellow River is located in the northernmost area of Yellow River basin in China. The river channel of here seems like the symbol "∏," as is shown in Fig. 1. It starts at Heishanxia of Ningxia Province and ends at Zhungeer Banner in Inner Mongolia. The total length of Ning–Meng reach is 1217 km, including 397 km in Ningxia and 820 km in Inner Mongolia. Because of its long and cold winter, the minimum temperature can reach −35 °C, and the river frozen days of Ning–Meng reach can last 4–5 months. Generally, the
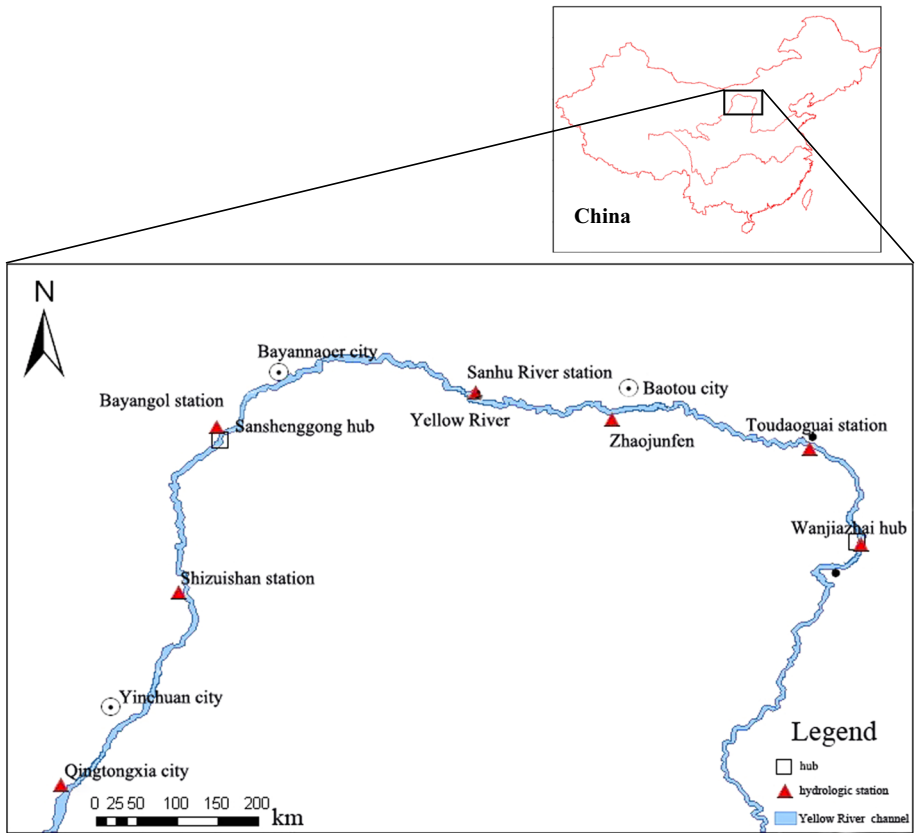
Fig. 1 Ning–Meng reach of Yellow River

ice flow starts in November and the river unfreezes in March of the next year. The average frozen days last 100 days, sometimes 150 days in an extreme climate. In the period of freeze-up, the ice thickness reaches 0.5–1.0 m. Most of reaches are stable frozen in winter, and the average length of frozen is about 800 km. Due to its curving and variable river morphology, cold and complex climates, the river freezes from lower reach to upper reach in winter and unfreezes from upper reach to lower reach in spring of next year, which generates ice jam and ice dam easily and leads to ice flood disaster eventually. Therefore, the Ning–Meng reach is one of the most serious reaches suffering from ice flood disasters. According to the statistics in Ning–Meng reach from 1969 to 2010, there have been a total of 132 times for ice jam and ice dam, in which 56 times lead to ice flood disaster. The ice disaster with various severities occurs almost every year, and the extensive disaster occurs on average every 2 years.

3 Novel features of ice regime in recent years and the index system of ice disaster risk

Risk identification is the basis of risk analysis. In order to analyze the evolution process of ice disaster leading to loss, identifying the formation mechanism and main influencing factors of ice disaster, Jiang et al. (2008) quantitatively described the changes in the

characteristics of ice phenology including the flow rate and freeze-up/break-up dates of the Yellow River based on observations from 1950s to 2000s; Wang (2014) researched the evolution of ice in the period of ice flow, freeze-up and break-up, and analyzed the formulation mechanism, evolution and hazard of ice jam and ice dam; Wu et al. (2015) defined the risk of ice disaster and analyzed the influencing factors of ice disaster risk from four aspects: river morphology; temperature uncertainty; impact analysis of reservoir flow and the other ice engineering. However, in recent years, with the global warming and the increase of ice engineering, some novel features of ice regime have been presented in Ning–Meng reach of Yellow River.

3.1 Novel features of ice regime in recent years

1. The dates of ice flow and freeze-up are deferred and the break-up date comes early. The freeze-up and break-up of river are unstable.
   As is shown in Table 1, in the year of 2001–2015, the average dates of ice flow, freeze-up and break-up are November 24, December 4 and March 25, respectively. Comparing with the years of 1950–2015, the ice flow date is deferred 6 days, the freeze-up date is deferred 3 days, and the break-up date is earlier 2 days. In addition, in the year of 2001–2002 and 2013–2014, the Ning–Meng reach appeared the phenomenon of twice freeze-up and twice break-up. Especially in the year of 2009–2010, the alternate of freeze-up and break-up happened thrice. The freeze-up and break-up of river are much more unstable, which complicates the evolution process of ice regime. The variation of temperature is the main reason. Since 1990s, influenced by the global warming, the average temperatures of Ning–Meng reach raised during ice freezing season, and the temperature fluctuated wildly. As the statistics in Chang et al. (2016), in the 1990s, the average temperature was higher than in the 1980s. After 2000, the average temperature increased about 3.6 °C than before, and the rate of increase was about 0.5 °C for every 10 years.

2. The under-ice river flow is reduced, and channel-storage increment in freeze-up is increased.
   As is shown in Table 2, in the year of 2001–2015, the average level of max channel-storage increment in freeze-up was 1.485 billion m^3, and it increased 0.433 billion m^3 than the average level before the year of 2000. In the years before 2000, the channel-storage increment usually increased to maximum in the late January, and lasted to the late February. But, after the years of 2000, the channel-storage increment usually increased to maximum in the early or middle February and lasted to the early March. Especially in the year of 2010–2011, the channel-storage increment increased to maximum in early March and lasted to the eve of break-up, which extremely increased the ice disaster risk and the difficulty of ice prevention.

Table 1 Statistics of freeze-up and break-up in Ning–Meng reach

Index	Average date of 1950–2015	Average date of 2001–2015	Earliest date	Latest date
Date of ice flow	Nov.18	Nov.24	Nov.4, 1969	Nov.30, 2006
Date of freeze-up	Dec.1	Dec.4	Nov.7, 1969	Dec.30, 1989
Date of break-up	Mar.27	Mar.25	Mar.12, 1998	Apr.5, 1970

Table 2 Average level of max channel-storage increment in different years (billion m^3)

Index	Years of 1952–1968	Years of 1969–1985	Years of 1986–2000	Years of 2001–2015
Max channel-storage increment	0.875	1.01	1.27	1.485

Because the flood control standard, ice diversion project and river building in Ning–Meng reach are increasing, the river morphology presents the characteristics of multiple shallows and bends. The amount of sediment entering Yellow River has increased sharply, and the under-ice river flow is reduced. In the period of freeze-up, the water from upper river gathers in the shallows, which increases the channel-storage increment simultaneously. In addition, with the development of agricultural economy, large-scale agricultural reclamation occurs near the embankment of Ning–Meng reach, and the production in the river-beach area is intertwined, which causes the a large amount of ice water to stay in the river-beach and increases the channel-storage increment.

3. The peak flow is reduced, and the ice regime trends to complexity.

In the break-up period of Ning–Meng reach, the river unfreezes from upper river to lower river, and the peak flow increases gradually all over the process. Especially in the reaches of violent break, the peak flow increases rapidly with the release of channel-storage increment. Considering the variation of temperature in break-up period, river morphology and reservoir engineering in recent years, the peak flow is reduced and the ice regime trends to complexity. The average peak flow in the years of 2001–2015 was 1811 m^3/s, which decreased 17.6% than the years of 1968–2000. Especially in the year of 2014–2015, the peak flow was 920 m^3/s, and it was the lowest level in record.

This is because the release of water in the shallow and river-beach is slow, which affects the flood peak process and increases the difficulty of flood forecasting. What is more, reservoir operation changes the natural flow of river. In the period of ice flow, the reservoir could be used to increase the discharge flow. The hydrodynamic force in the lower river is increased by the additional increment of flow, and the water temperature is raised correspondingly. Then, the frozen date could be postponed artificially. Similarly in the period of break-up, the peak flow could be restricted through decrease the discharge flow. Especially in the year of 2014, the Haibowan Reservoir, located 87 km from the upper river of Sanshenggong hydro-junction, was completed, and it is the first reservoir whose primary mission is ice prevention in Yellow River. Through the joint operation of reservoirs, the peak flow is decreased to some extent.

3.2 Index system of ice disaster risk

According to four principles: comprehensively considering the formation mechanism of ice disaster and the novel features of ice regime in recent years; the index data should be accessible and completed; reflecting the probability of ice disaster loss caused by the adverse events of ice jam and ice dam; consulting relative experts and references, the index system of ice disaster risk is established, as is shown in Table 3.

Table 3 Index system of ice disaster risk

Factor	Index	Explanation about the index
Thermodynamic factors	T1: frozen days(FD)	The period of ice flow, freeze-up and break-up
	T2: average temperature (AT)	The average temperature of freezing season for the important hydrological monitoring stations in Ning–Meng reach
	T3: cumulative negative temperature (CNT)	Cumulative negative temperature of freezing season for the important hydrological monitoring stations in Ning–Meng reach
Hydraulic and human factors	H1: average flow of freeze-up (AFF)	Average flow of freeze-up for the important hydrological monitoring stations in Ning–Meng reach
	H2: peak flow(PF)	The peak flow in the period of break-up
	H3: largest flow in 10 days(LF10)	Largest flow in 10 days
	H4: max channel-storage increment (MCSI)	Max channel-storage increment from Shizhuishan station to Toudaoguai station

Generally, thermodynamic factors include solar radiation heat, temperature, water temperature, rainfall and snowfall, etc. In winter and spring, the Ning–Meng reach of Yellow River is affected by cold air in Siberia and Mongolia. The climate is dry and cold, and the snowfall is scarce. Therefore, temperature is the focus of the thermodynamic factors that affect the change of ice regime. In freeze-up season, the river will freeze when the cumulative negative temperature reaches a certain level. Simultaneously, taking into account that the freeze-up and break-up appear alternately, the frozen days, the average temperature and the cumulative negative temperature are taken as the indexes reflecting the thermodynamic factors.

Hydraulic factors include flow, flow velocity, water level and wind speed. The role of wind speed is very small, which is usually ignored. The influence of flow on ice regime is mainly reflected in flow velocity and fluctuation of water level. So, the flow can be used to characterize the impact of hydraulic factors. Considering the novel features of ice regime in recent years and the formation mechanism of ice disaster, the average flow of freeze-up, peak flow, largest flow in 10 days and max channel-storage increment are taken as the indexes reflecting the hydraulic factors.

River morphology factors change small in a short time and the relative data lack integrity, so they are not considered. The impact of human factors presents mainly in the reservoir project, and the action of reservoir operation is reflected in the change of flow. So, the human factors are combined with hydraulic factors.

4 Data and model

4.1 Data source and preprocessing

Yellow River Web site (http://www.yellowriver.gov.cn/) and Yellow River Conservancy Commission set up the special theme for ice prevention of Yellow River in November of each year. The ice regime of Yellow River is recorded by the daily dynamic of ice prevention. Hence, the ice regime information in the years of 1996–2015 is collected from the important hydrological monitoring stations in Ning–Meng reach, and the outlier test

and missing value substitution are used as the methods of data preprocessing. Generally, the ice regime data collected from each year are not single real numbers, but a series of numbers distributed in some intervals. In this paper, the minimum, most probable and maximum values of each index in multiple time points or multiple measurements of ice period are collected, and the multiple values are represented by three-parameter interval grey numbers. Grey numbers information is taken as the basic representation of ice regime data, which is more reasonable than the averaging method used in previous literatures. The years are as the object set, and the risk indices are as the attribute set; then, the information system of ice disaster risk with three-parameter interval grey number is constructed, which can more comprehensively reflect the practical ice regime.

4.2 The two-phased intelligent model under grey information environment

In order to evaluate the probability of ice disaster loss caused by the adverse events ice disaster risk and extract the development between ice regime information and ice disaster risk, the two-phased intelligent model under grey information environment is proposed to evaluate and analyze the ice disaster risk in Ning–Meng reach of Yellow River.

In the first phase, for the information system of ice disaster risk whose index values are three-parameter interval grey numbers, the grey interval relational clustering under grey information environment is proposed to assess the risk grade of ice disaster. At first, the grey interval relational coefficient matrix is obtained through computing the grey interval relational coefficients of each sub-factor to ideal sequence factors. Then, the comprehensive clustering coefficients are achieved based on whitenization weight function, through which the objects could not only be assigned to corresponding grey class but also be ranked in its own class.

In the second phase, the decision table of ice disaster risk could be established through combining the clustering results obtained in the first phase with information system of ice disaster risk. Then, the grey dominance-based rough set approach is proposed to extract decision rules that could reflect the development between ice regime and ice disaster risk.

4.2.1 Grey interval relational clustering under grey information environment

Let the object set be $U = \{x_1, x_2, \ldots, x_n\}$, and the clustering index set (attribute set) be $AT = \{a_1, a_2, \ldots, a_m\}$. Set s grey classes. The index value of object x_i ($i = 1, 2, \ldots, n$) to index a_j ($j = 1, 2, \ldots, m$) is denoted as $f(x_i, a_j)$, and $f(x_i, a_j)$ is three-parameter interval grey number, denoted as $f\left(x_i, a_j\right) \in \left[x_{ij}^l, x_{ij}^m, x_{ij}^u\right]$, in which x_{ij}^l, x_{ij}^m and x_{ij}^u refer to the lower bound, center and upper bound of three-parameter interval grey number (Luo and Wang 2012). We can get the evaluation matrix $U = (f(x_i, a_j))_{n \times m}$. Put the object x_i into grey class k ($k = 1, 2, \ldots, s$) according to the index value $f(x_i, a_j)$ of object x_i about index a_j.

Firstly, perform the grey extreme difference transform on evaluation matrix to make it dimensionless and for better comparison.

For the benefit indices

$$u_{ij}^l = \frac{x_{ij}^l - x_j^{l\nabla}}{x_j^{u*} - x_j^{l\nabla}}, u_{ij}^m = \frac{x_{ij}^m - x_j^{l\nabla}}{x_j^{u*} - x_j^{l\nabla}}, u_{ij}^u = \frac{x_{ij}^u - x_j^{l\nabla}}{x_j^{u*} - x_j^{l\nabla}} \tag{1}$$

For the cost indices

$$u_{ij}^l = \frac{x_j^{u*} - x_{ij}^u}{x_j^{u*} - x_j^{l\nabla}}, u_{ij}^m = \frac{x_j^{u*} - x_{ij}^m}{x_j^{u*} - x_j^{l\nabla}}, u_{ij}^u = \frac{x_j^{u*} - x_{ij}^l}{x_j^{u*} - x_j^{l\nabla}} \tag{2}$$

where $x_j^{u*} = \max_{1 \le i \le n}\left\{x_{ij}^u\right\}$, $x_j^{l\nabla} = \min_{1 \le i \le n}\left\{x_{ij}^l\right\}$. Then, we get the standardized evaluation matrix.

$$U' = (u_{ij})_{n \times m} \tag{3}$$

The ideal sequence is defined as

$$u^+ = (u_1^+, u_2^+, \ldots u_m^+) \tag{4}$$

where $u_j^+ \in \left[u_j^{l+}, u_j^{m+}, u_j^{u+}\right]$, and $u_j^{l+} = \max_{1 \le i \le n}\left\{u_{ij}^l\right\}$, $u_j^{m+} = \max_{1 \le i \le n}\left\{u_{ij}^m\right\}$, $u_j^{u+} = \max_{1 \le i \le n}\left\{u_{ij}^u\right\}$.

Denote

$$\Delta_{ij}^{l+} = \left|u_j^{l+} - u_{ij}^l\right|, \Delta_{ij}^{m+} = \left|u_j^{m+} - u_{ij}^m\right|, \Delta_{ij}^{u+} = \left|u_j^{u+} - u_{ij}^u\right|;$$

$$m^{l+} = \min_{1 \le i \le n} \min_{1 \le j \le m} \Delta_{ij}^{l+}, M^{l+} = \max_{1 \le i \le n} \max_{1 \le j \le m} \Delta_{ij}^{l+};$$

$$m^{m+} = \min_{1 \le i \le n} \min_{1 \le j \le m} \Delta_{ij}^{m+}, M^{m+} = \max_{1 \le i \le n} \max_{1 \le j \le m} \Delta_{ij}^{m+};$$

$$m^{u+} = \min_{1 \le i \le n} \min_{1 \le j \le m} \Delta_{ij}^{u+}, M^{u+} = \max_{1 \le i \le n} \max_{1 \le j \le m} \Delta_{ij}^{u+};$$

The grey interval relational coefficient of sub-factor u_{ij} to ideal sequence factors u_j^+ is defined as

$$r_{ij}^+ = \frac{1}{2}\left[(1 - \lambda)\frac{m^{l+} + \beta M^{l+}}{\Delta_{ij}^{l+} + \beta M^{l+}} + \frac{m^{m+} + \beta M^{m+}}{\Delta_{ij}^{m+} + \beta M^{m+}} + \lambda\frac{m^{u+} + \beta M^{u+}}{\Delta_{ij}^{u+} + \beta M^{u+}}\right] \tag{5}$$

where $\beta \in (0, 1)$ is recognition coefficient and generally $\beta = 0.5$. $\lambda \in [0, 1]$ is the adjustment coefficient given by decision maker according to the preference of the lower bound and upper bound.

The grey interval relational coefficient matrix of each sub-factor to ideal sequence factors is denoted as

$$r_{ij} = \begin{bmatrix} r_{11}^+ & r_{12}^+ & \cdots & r_{1m}^+ \\ r_{21}^+ & r_{22}^+ & \cdots & r_{2m}^+ \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1}^+ & r_{n2}^+ & \cdots & r_{nm}^+ \end{bmatrix}_{n \times m} \tag{6}$$

In matrix (6), $r_j^+ = (r_{1j}^+, r_{2j}^+, \ldots, r_{nj}^+)'(j = 1, 2, \ldots, m)$ denotes the grey interval relational coefficient vector of each object to ideal object under index j. According to the value range of r_j^+, the value range is divided into s grey intervals correspondingly. Set the whitenization weight function $f_j^k(x)(k = 1, 2, \ldots, s)$ of each grey interval. In this condition, the whitenization weight function of each index in the same grey interval is equal. The whitenization weight function includes lower, moderate and upper whitenization weight

functions, denoted by $\left[-, -, x_j^k(3), x_j^k(4)\right]$, $\left[x_j^k(1), x_j^k(2), -, x_j^k(4)\right]$ and $[x_j^k(1), x_j^k(2), -, -]$, respectively (Xie et al. 2014).

The comprehensive clustering coefficient of object x_i $(i = 1, 2,\ldots, n)$ to grey class k $(k = 1, 2,\ldots, s)$ is defined as

$$\sigma_i^k = \sum_{j=1}^{m} f_j^k(r_{ij}^+) \cdot \eta_j \tag{7}$$

where the η_j $(j = 1, 2,\ldots, m)$ is the index weight. The comprehensive clustering coefficient matrix is denoted as

$$\sigma_i^k = \begin{bmatrix} \sigma_1^1 & \sigma_1^2 & \cdots & \sigma_1^s \\ \sigma_2^1 & \sigma_2^2 & \cdots & \sigma_2^s \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_n^1 & \sigma_n^2 & \cdots & \sigma_n^s \end{bmatrix}_{n \times s} \tag{8}$$

Determine the class that certain object belongs to according to the comprehensive clustering coefficient matrix. Object x_i belongs to grey class k^* if

$$\max_{1 \leq k \leq s} \{\sigma_i^k\} = \sigma_i^{k^*} \tag{9}$$

Then, all the objects could be assigned to corresponding grey class.

4.2.2 Grey dominance-based rough set approach

Rough set theory is a mathematical tool which is proposed to take advantage of information inherent in a given data without requiring any auxiliary information or subjective judgments (Pawlak 1982). Pawlak's classical rough set could not tackle the information system whose attributes present preference-ordered domains, and then Greco et al. proposed dominance-based rough set approach (Greco et al. 2001, 2002). Due to the uncertainty of ice disaster risk, the collected ice regime data are grey numbers and the indices are preference-ordered. In order to reveal the development between ice regime information and ice disaster risk, this paper proposes grey dominance-based rough set approach. Based on the grey dominance relation proposed in our previous research at Luo et al. (2016), this paper put forward an approach to extract decision rules from decision table whose index values are three-parameter interval grey numbers.

4.2.2.1 Grey dominance relation (Luo et al. 2016) An information system with three-parameter interval grey number is a quadruple $S = (U, AT, V, f)$, where U is a finite non-empty set of objects, and AT is a finite non-empty set of attributes; $\forall a \in AT$, V_a is the domain of attribute a, $V = \cup_{a \in AT} V_a$, and $f : U \times AT \to V$ is a total function such that $f(x, a) \to V_a$ for every $a \in AT$, $x \in U$, called an information function, where V_a is a set of three-parameter interval grey numbers.

A decision table with three-parameter interval grey number is a special information system $S = (U, AT \cup d, V, f)$, where d is decision attribute, and AT is called condition attribution correspondingly, $d \cap AT = \varnothing$. V_d is the domain of d, and V_d is single-value set.

Definition 1 Let $a(\otimes) \in [a^l, a^m, a^u]$ and $b(\otimes) \in [b^l, b^m, b^u]$ be three-parameter interval grey numbers, and the possibility degree of $a(\otimes) \geq b(\otimes)$ is defined as

$$p(a(\otimes) \geq b(\otimes))$$

$$= \lambda \frac{\min\{(a^m - a^l) + (b^m - b^l), \max(a^m - b^l, 0)\}}{(a^m - a^l) + (b^m - b^l)}$$

$$+ (1 - \lambda) \frac{\min\{(a^u - a^m) + (b^u - b^m), \max(a^u - b^m, 0)\}}{(a^u - a^m) + (b^u - b^m)}$$

where $\lambda \in [0, 1]$ is the adjustment coefficient given by decision maker according to the preference of the lower bound and upper bound.

Definition 2 Let $S = (U, \mathrm{AT}, V, f)$ be information system with three-parameter interval grey number, $\forall a \in \mathrm{AT}$, $\forall x, y \in U$, and the grey dominance degree of object x to y on attribute a is denoted by $D_a(x, y)$ such that

$$D_a(x, y) = \begin{cases} 0, & x^u \leq y^l \\ 1, & x^l \geq y^u \, or \, x = y \\ p(f(x, a) \geq f(y, a)), & \text{otherwise} \end{cases}$$

Let $\theta \in [0, 1]$ be the dominance threshold which is given aforehand, and the θ-grey dominance relation with respect to attribute a is defined as

$$S_a^\theta = \{(x, y) \in U^2 | D_a(x, y) \geq \theta\}$$

$\forall A \subseteq \mathrm{AT}$, the θ-grey dominance relation with respect to attribute set A is defined as

$$S_A^\theta = \{(x, y) \in U^2 | \forall a \in A, (x, y) \in S_a^\theta\}$$

For a given θ, if $(x, y) \in S_a^\theta$, then x is considered as dominating y on attribute a with the at least grey dominance degree of θ. We denote it by $f(x, a) \geq {}^\theta f(y, a)$ or $f(y, a) \leq {}^\theta f(x, a)$.

4.2.2.2 Lower and upper approximations and decision rules In decision table $S = (U, \mathrm{AT} \cup \{d\}, V, f)$, the decision attribute value $f(x, d)$ is single-valued. The decision attribute d makes a partition of U into a finite number of classes. Let $\mathrm{Cl} = \{\mathrm{Cl}_t, t \in T\}$, $T = \{1, 2, \ldots, n\}$ be a set of these classes that are ordered, that is, $\forall t, s \in T$, if $t \leq s$, then the objects from Cl_s are preferred to the objects from Cl_t. The sets to be approximated are upward unions and downward unions of classes, which are defined, respectively, as $\mathrm{Cl}_t^\geq = \cup_{s \geq t} \mathrm{Cl}_s$, $\mathrm{Cl}_t^\leq = \cup_{s \geq t} \mathrm{Cl}_s$, $\forall t \in T$. The statement $x \in \mathrm{Cl}_t^\geq$ means "x belongs to at least class Cl_t", whereas $x \in \mathrm{Cl}_t^\leq$ means "x belongs to at most class Cl_t."

Let $S = (U, \mathrm{AT} \cup \{d\}, V, f)$ be a decision table with three-parameter interval grey number, $A \subseteq \mathrm{AT}$, $\forall x \in U$, and the θ-dominating set and θ-dominated set with respect to x on A are denoted by

$$S_A^{\theta+}(x) = \{y \in U | (y, x) \in S_A^\theta\}, S_A^{\theta-}(x) = \{y \in U | (x, y) \in S_A^\theta\}$$

Therefore, the set of all objects belonging to $\mathrm{Cl}_t^\geq$ without any ambiguity constitutes the lower approximation of $\mathrm{Cl}_t^\geq$, denoted by $\underline{A^\theta}(\mathrm{Cl}_t^\geq)$, and the set of all objects that could belong to $\mathrm{Cl}_t^\geq$ constitutes the upper approximation of $\mathrm{Cl}_t^\geq$, denoted by $\overline{A^\theta}(\mathrm{Cl}_t^\geq)$:

$$\underline{A^\theta}(\mathrm{Cl}_t^\geq) = \{x \in U | S_A^{\theta+}(x) \subseteq \mathrm{Cl}_t^\geq\},$$

$$\overline{A^\theta}(\mathrm{Cl}_t^\geq) = \{x \in U | S_A^{\theta-}(x) \cap \mathrm{Cl}_t^\geq \neq \varnothing\}.$$

Similarity, the lower and upper approximations of $\mathrm{Cl}_t^{\leq}$ are defined as

$$\underline{A^\theta}(\mathrm{Cl}_t^{\leq}) = \{x \in U | S_A^{\theta-}(x) \subseteq \mathrm{Cl}_t^{\leq}\},$$

$$\overline{A^\theta}(\mathrm{Cl}_t^{\leq}) = \{x \in U | S_A^{\theta+}(x) \cap \mathrm{Cl}_t^{\leq} \neq \varnothing\}.$$

Like decision rules in Qian et al. (2008), from lower and upper approximations of $\mathrm{Cl}_t^{\geq}$ and $\mathrm{Cl}_t^{\leq}$, the there are four types of decision rules to be considered as follows:

1. Certain $\geq^\theta$ decision rules with following syntax: If $(f(x,a_1) \geq^\theta v_{a_1}) \wedge (f(x,a_2) \geq^\theta v_{a_2}) \wedge \ldots \wedge (f(x,a_k) \geq^\theta v_{a_k}) \wedge (f(x,a_{k+1}) \leq^\theta v_{a_{k+1}}) \wedge \ldots \wedge (f(x,a_m) \leq^\theta v_{a_m})$, then $x \in \mathrm{Cl}_t^{\geq}$. //These rules are supported only by objects from the lower approximations of the upward unions of classes $\mathrm{Cl}_t^{\geq}$.

2. Possible $\geq^\theta$ decision rules with the following syntax: If $(f(x,a_1) \geq^\theta v_{a_1}) \wedge (f(x,a_2) \geq^\theta v_{a_2}) \wedge \ldots \wedge (f(x,a_k) \geq^\theta v_{a_k}) \wedge (f(x,a_{k+1}) \leq^\theta v_{a_{k+1}}) \wedge \ldots \wedge (f(x,a_m) \leq^\theta v_{a_m})$, then x could belong to $\mathrm{Cl}_t^{\geq}$. //These rules are supported only by objects from the upper approximations of the upward unions of classes $\mathrm{Cl}_t^{\geq}$.

3. Certain $\leq^\theta$ decision rules with the following syntax: If $(f(x,a_1) \leq^\theta v_{a_1}) \wedge (f(x,a_2) \leq^\theta v_{a_2}) \wedge \cdots \wedge (f(x,a_k) \leq^\theta v_{a_k}) \wedge (f(x,a_{k+1}) \geq^\theta v_{a_{k+1}}) \wedge \cdots \wedge (f(x,a_m) \geq^\theta v_{a_m})$,, then $x \in \mathrm{Cl}_t^{\leq}$. //These rules are supported only by objects from the lower approximations of the downward unions of classes $\mathrm{Cl}_t^{\leq}$.

4. Possible $\leq^\theta$ decision rules with the following syntax: If $(f(x,a_1) \leq^\theta v_{a_1}) \wedge (f(x,a_2) \leq^\theta v_{a_2}) \wedge \cdots \wedge (f(x,a_k) \leq^\theta v_{a_k}) \wedge (f(x,a_{k+1}) \geq^\theta v_{a_{k+1}}) \wedge \cdots \wedge (f(x,a_m) \geq^\theta v_{a_m})$,, then x could belong to $\mathrm{Cl}_t^{\leq}$. //These rules are supported only by objects from the upper approximations of the downward unions of classes $\mathrm{Cl}_t^{\leq}$.

where $A_1 = \{a_1, a_2, \ldots, a_k\}$, $A_2 = \{a_{k+1}, a_{k+2}, \ldots, a_m\}$, $A_1 \cup A_2 = \mathrm{AT}$ where A_1 is increasing preference and A_2 is decreasing preference.

4.2.2.3 Extract simplest decision rules through attribute reduction To extract briefer decision rules, it is necessary to reduce the dispensable attributes in the condition part of a given decision table. The discernibility matrix has been proved to be as an effective method for attribute reduction.

Let $S = (U, \mathrm{AT} \cup \{d\}, V, f)$ be a decision table with three-parameter interval grey number, and $D_{\{d\}}^+ = \{(x,y) \in U^2 | f(x,d) \geq f(y,d)\}$ is a dominance relation of decision attribute. S is referred to as θ-consistent if and only if $S_{AT}^\theta \subseteq D_{\{d\}}^+$, i.e., $\forall x,y \in U, (x,y) \in S_{AT}^\theta \Rightarrow (x,y) \in D_{\{d\}}^+$. Otherwise, it is inconsistent. This paper only discusses the consistent decision table. Hence, for $\forall A \subseteq \mathrm{AT}$, if $S_A^\theta \subseteq D_{\{d\}}^+$ and $S_B^\theta \nsubseteq D_{\{d\}}^+$ for any $B \subset A$, then A is a θ-attribute reduction of S.

Let $D^* = \{(x,y) | f(x,d) < f(y,d)\}$, the discernibility set is denoted by

$$C_{\mathrm{AT}}^\theta(x,y) = \begin{cases} \{a \in \mathrm{AT} | (x,y) \notin S_a^\theta\}, & (x,y) \in D^* \\ \varnothing, & (x,y) \notin D^* \end{cases}$$

and the discernibility matrix is denoted by $C_{\mathrm{AT}}^\theta = \{C_{\mathrm{AT}}^\theta(x,y) | x,y \in U\}$. Correspondingly, the discernibility function is denoted by $L = \wedge\{\vee\{a | a \in C_{\mathrm{AT}}^\theta(x,y)\} | x,y \in U\}$.

Similar to the proof of attribute reduction in Qian et al. (2008) and Yang et al. (2015), we conclude that attribute set A is a θ-reduction of AT if and only if A is a conjunctive form of the minimal disjunctive normal form of discernibility function L.

Through the attribute reduction results, the dispensable attributes in the condition part could be eliminated, and the decision rules extracted in section 4.2.2.2 could be simplified.

4.2.3 Risk assessment and analysis of ice disaster based on the two-phased intelligent model under grey information environment

The steps of the proposed method are as follows, and the basic framework of this paper is reflected in Fig. 2.

Step 1 Let the years of 1996–2015 be the object set, and let the index system of ice disaster risk be the attribute set. Construct information system of ice disaster risk according to the collected ice regime data of 1996–2015 on different index. Determine grey class s according to the requirement of risk grade;

Step 2 Standardize the information system of ice disaster risk with formula (1)–(2), and get standardized evaluation matrix (3);

Step 3 Compute ideal sequence with formula (4) and grey interval relational coefficient with formula (5). Construct grey interval relational coefficient matrix (6);

Step 4 Set the whitenization weight function $f_j^k(x)$ of each grey interval. Determine the clustering weight $\eta_j (j = 1, 2, \ldots, m)$ of each index;

Step 5 Calculate the comprehensive clustering coefficient with formula (7), and get the comprehensive clustering coefficient matrix (8); judge the grey class that each object belongs to with formula (9). Determine the risk grade of each object;

Step 6 Establish the decision table of ice disaster risk through combining risk grade with information system of ice disaster risk, and standardize the attribute indexes values into increasing preference;
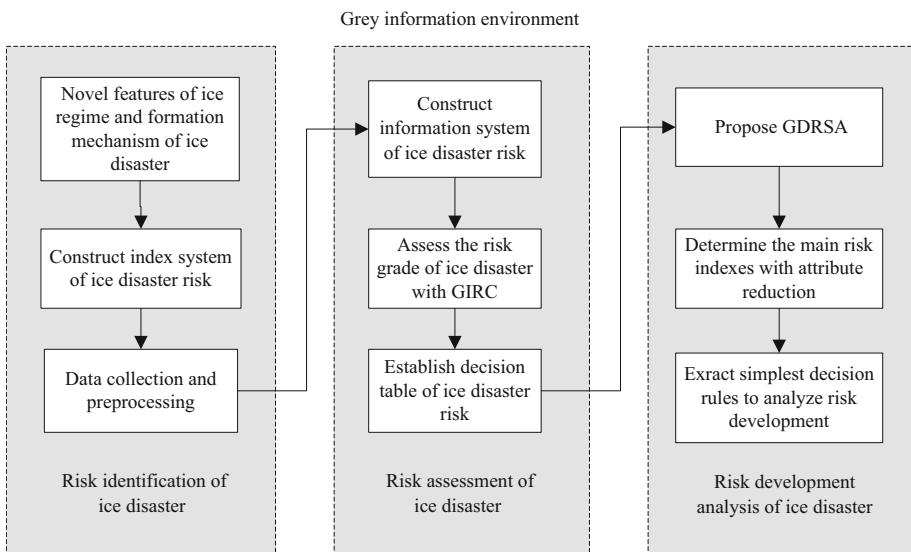

Fig. 2 Basic framework of this paper

Step 7 Construct grey dominance relation through section 4.2.2.1 in 4.2.2;

Step 8 According to section 4.2.2.2 in 4.2.2, define the lower approximation set and upper approximation set of upward unions $Cl_t^{\geq}$ and downward unions $Cl_t^{\leq}$;

Step 9 Extract the simplest decision rules through attribute reduction according to section 4.2.2.3 in 4.2.2. Analyze the development between ice regime information and ice disaster risk.

5 Result and analysis

The ice regime data of each index during 1996–2015, represented with three-parameter interval grey numbers, are shown in Fig. 3. The information system of ice disaster risk whose index values are three-parameter interval grey numbers could be constructed. According to the actual requirement, three grey classes are set, in which the low-risk class, middle-risk class and high-risk class correspond to the three risk grades 1, 2 and 3, respectively.

Let $\beta = 0.5$, $\lambda = 0.5$. Then, the grey interval relational coefficient matrix could be achieved. Considering the uncertainty of the index weight, the equal weight is adopted to reduce the loss caused by uncertain factors. The whitenization weight functions about three grey classes are $f_j^1[-, -, 0.4, 0.55]$, $f_j^2[0.4, 0.6, -, 0.75]$ and $f_j^3[0.65, 0.85, -, -]$. The clustering results are shown in Table 4.

As is shown in Table 4, according to the ice disaster risk grade for the years of 1996–2015, the year of grade 1 (low risk) is 7, with the proportion 35%; the year of grade 2 (middle risk) is 8, with the proportion 40%; the year of grade 3 (high risk) is 5, with the proportion 25%.

The years of low risk are 2001–2002, 2002–2003, 2003–2004, 2006–2007, 2008–2009, 2013–2014 and 2014–2015, respectively. The main characteristics of these years were the higher temperature than other years, which shortened the frozen days to some extent. In these years, the ice jam and ice dam were also appeared, but they did not cause serious ice disaster. This is because the ice disaster risk is the comprehensive action of multiple risk factors. With the year of 2008–2009 as an example, the max channel-storage increment in Ning–Meng reach was 1.8 billion m^3 high, but the changing process of the temperature in break-up period was reposeful. The channel-storage increment released slowly, and the flow in break-up period was smooth accordingly. Therefore, the ice disaster risk caused by violent break was decreased. Especially in the year of 2014–2015, the ice disaster risk was obviously low. In the break-up period, the peak flow was 920 m^3/s, and the largest water in 10 days was 0.56 billion m^3, both of them at the historical minimum level. On the one hand, because the cold air was relatively weak, resulting in the delayed ice-occurring and freeze-up date, the frozen days were short. On January 13, 2015, the Mahuanggoukou Reach, located in the upper of Ning–Meng reaches, unfroze firstly, which was more than 30 days in advance of the historical average level. The whole reaches unfroze on March 18, and the break-up period lasted 65 days. With the extension of break-up period, the channel-storage increment released slowly, and the ice disaster risk was decreased. On the other hand, the Haibowan Reservoir was completed in the year of 2014. Through the operation of Haibowan Reservoir and other reservoir in Ning–Meng reach, the ice disaster risk was decreased most in that year, which also proved the opinion that the changes induced by reservoirs joint operation may vastly exceed than by single reservoir operation (Chang et al. 2016).
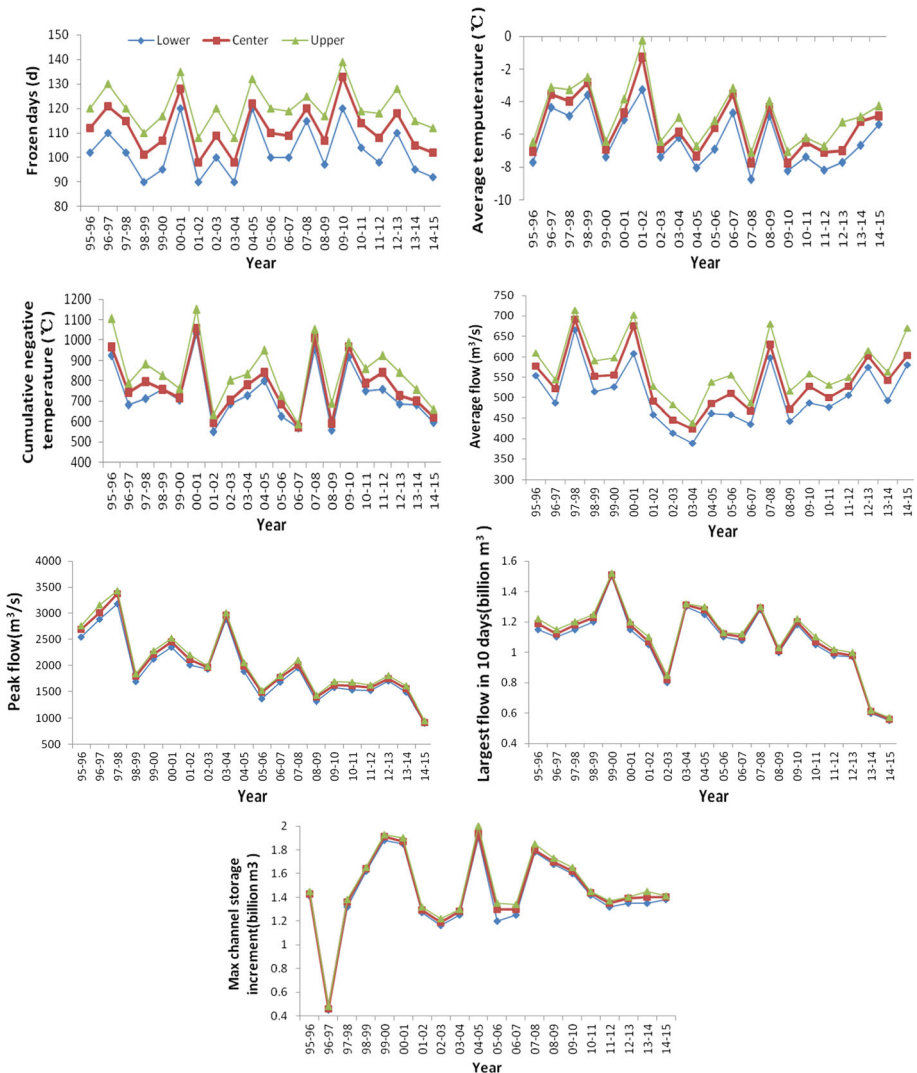
Fig. 3 Ice regime data of each index in the year of 1996–2015

The years of middle risk are 1995–1996, 1996–1997, 1997–1998, 1998–1999, 2005–2006, 2010–2011, 2011–2012 and 2012–2013. In these years, the ice disaster had occurred to some extent, and the ice disaster prevention and control should not be ignored.

The years of high risk are 1999–2000, 2000–2001, 2004–2005, 2007–2008 and 2009–2010. In these years, the practical ice disaster was serious, which caused great loss. From the comprehensive clustering coefficients of the years of 2000–2001 and 2007–2008, we know the ice disaster risk is higher than other years, which corresponds with the practical ice disaster loss in these years completely. In the year of 2000–2001, the cold air came prematurely. The ice flow date was November 5, which was 15 days earlier than the historical average level. The early and long ice flow period aggravated the hazard of ice jam. What is more, the serious disaster loss in this year not only related to the max channel-

Table 4 Assessment results of ice disaster risk for Ning–Meng reach of 1996–2015

Order	Year	Clustering coefficients of grey classes			Risk grade
		Low	Middle	High	
1	1995–1996	0.0009	0.6168	0.2179	2
2	1996–1997	0.3346	0.341	0.0907	2
3	1997–1998	0.0598	0.5404	0.2857	2
4	1998–1999	0.3623	0.4169	0.0116	2
5	1999–2000	0.1692	0.3017	0.3976	3
6	2000–2001	0	0.3782	0.5714	3
7	2001–2002	0.5911	0.3067	0	1
8	2002–2003	0.506	0.2634	0.1096	1
9	2003–2004	0.3255	0.3121	0.0657	1
10	2004–2005	0.1689	0.289	0.368	3
11	2005–2006	0.3694	0.442	0	2
12	2006–2007	0.4723	0.3958	0	1
13	2007–2008	0.0634	0.1866	0.4433	3
14	2008–2009	0.5147	0.2568	0.0554	1
15	2009–2010	0.1813	0.286	0.3111	3
16	2010–2011	0.2749	0.4832	0.0724	2
17	2011–2012	0.2876	0.4421	0.1429	2
18	2012–2013	0.2522	0.4564	0.0913	2
19	2013–2014	0.4841	0.4076	0	1
20	2014–2015	0.5226	0.3602	0	1

storage increment that reached 1.87 billion m^3 but also connected with the abrupt temperature change in break-up period. Especially in the year of 2007–2008, the Ning–Meng reach was suffered the most serious ice disaster in last 40 years. The average temperature in ice season was nearby −8.5, 3.5 °C lower than the historical average level, which is the lowest average temperature since the year of 1952. In the break-up period, the temperature rose rapidly, and the break-up only lasted 15 days. It is the year that kept the double records of the shortest break-up and the largest daily water release.

For the above, the results of risk assessment in this paper is almost consistent with the average occurrence probability of ice disaster in Ning–Meng reach of Yellow River, and the ice disaster risk grade of each year is nearly according with the practical ice regime features.

The decision table of ice disaster risk whose index values are three-parameter interval grey numbers is constructed with the years as the object set, the risk indexed as the condition attribute set, and the risk grade as the decision attribute set. Let $\theta = 0.6$, namely a three-parameter interval grey number dominates another one with not less than 0.6 possibility degree and then they satisfy 0.6-grey dominance relation. According to the grey dominance-based rough set approach, the lower approximation of upward unions and downward unions is presented in Table 5. (Because the decision table is 0.6-consistent, the upper approximation is equal to the lower approximation. We only present the low approximation.)

From Table 5, the certain $\geq$ $^{0.6}$ decision rules and certain $\leq$ $^{0.6}$ decision rules could be achieved, but in order to extract simplest decision rules, the attribute reduction is necessary. Through the methods of discernibility matrix, the all 0.6-attribute reduction results of decision table are presented as follows: {$T1$, $H3$}, {$T1$, $T3$, $H4$}, {$T1$, $H1$, $H4$}, {$T2$, $H1$, $H3$}, {$T1$, $H2$, $H4$}, {$T3$, $H1$, $H3$}, {$T2$, $H2$, $H4$}, {$T2$, $T3$, $H1$, $H4$}, {$T3$, $H2$, $H3$, $H4$}. All of the nine kinds of attribute reduction results include both T factors and H factors, which reflects that ice disaster risk is the comprehensive action of thermodynamic factors, hydraulic factors and human factors. Because the impact of human factors presents mainly in the change of flow, the comprehensive influencing factors for ice disaster could be divided into thermodynamic and hydraulic factors, which is also corresponding with the analysis in Wang (2014) and Wu et al. (2015).

Through the attribute reduction result {$T1$, $H3$}, eliminate dispensable attributes and the decision rules contained by others. The simplest decision rules could be extracted as follows. (The decision rules extracted through other attribute reduction results are similar, and we do not enumerate them here.)

The certain $\geq$ $^{0.6}$ decision rules are as follows:

$R1$: If FD $\geq$ $^{0.6}$ [95, 107, 117] and LF10 $\geq$ $^{0.6}$ [1.5, 1.51, 1.52], then ice disaster risk grade is 3;
$R2$: If FD $\geq$ $^{0.6}$ [120, 128, 135] and LF10 $\geq$ $^{0.6}$ [1.15, 1.18, 1.2], then ice disaster risk grade is 3;
$R3$: If FD $\geq$ $^{0.6}$ [120, 122, 132] and LF10 $\geq$ $^{0.6}$ [1.25, 1.28, 1.3], then ice disaster risk grade is 3;
$R4$: If FD $\geq$ $^{0.6}$ [115, 120, 125] and LF10 $\geq$ $^{0.6}$ [1.28, 1.29, 1.3], then ice disaster risk grade is 3;
$R5$: If FD $\geq$ $^{0.6}$ [90, 101, 110] and LF10 $\geq$ $^{0.6}$ [1.2, 1.23, 1.25], then ice disaster risk grade is at least 2;
$R6$: If FD $\geq$ $^{0.6}$ [98, 108, 118] and LF10 $\geq$ $^{0.6}$ [0.98, 1, 1.02], then ice disaster risk grade is at least 2;
$R7$: If FD $\geq$ $^{0.6}$ [110, 118, 128] and LF10 $\geq$ $^{0.6}$ [0.97, 0.98, 1], then ice disaster risk grade is at least 2.

The certain $\leq$ $^{0.6}$ decision rules are as follows:

$R1$: If FD $\leq$ $^{0.6}$ [100, 108, 114] and LF10 $\leq$ $^{0.6}$ [0.8, 0.82, 0.85], then ice disaster risk grade is 1;
$R2$: If FD $\leq$ $^{0.6}$ [90, 98, 108] and LF10 $\leq$ $^{0.6}$ [1.3, 1.31, 1.32], then ice disaster risk grade is 1;

Table 5 Lower approximation set of $Cl_t^{\geq}$ and $Cl_t^{\leq}$

Low approximation set	The order of years
$\underline{AT}^{0.6}$ ($Cl_1^{\geq}$)	All years
$\underline{AT}^{0.6}$ ($Cl_2^{\geq}$)	1, 2, 3, 4, 5, 6, 10, 11, 13, 15, 16, 17, 18
$\underline{AT}^{0.6}$ ($Cl_3^{\geq}$)	5, 6, 10, 13, 15
$\underline{AT}^{0.6}$ ($Cl_1^{\leq}$)	7, 8, 9, 12, 14, 19, 20
$\underline{AT}^{0.6}$ ($Cl_2^{\leq}$)	1, 2, 3, 4, 7, 8, 9, 11, 12, 14, 16, 17, 18, 19, 20
$\underline{AT}^{0.6}$ ($Cl_3^{\leq}$)	All years

*R*3: If FD $\leq$ ^{0.6} [100, 109, 119] and LF10 $\leq$ ^{0.6} [1.08, 1.1, 1.12], then ice disaster risk grade is 1;

*R*4: If FD $\leq$ ^{0.6} [102, 112, 120] and LF10 $\leq$ ^{0.6} [1.15, 1.19, 1.22], then ice disaster risk grade is at most 2;

*R*5: If FD $\leq$ ^{0.6} [110, 121, 130] and LF10 $\leq$ ^{0.6} [1.1, 1.12, 1.15], then ice disaster risk grade is at most 2;

*R*6: If FD $\leq$ ^{0.6} [110, 115, 120] and LF10 $\leq$ ^{0.6} [1.15, 1.18, 1.2], then ice disaster risk grade is at most 2;

*R*7: If FD $\leq$ ^{0.6} [90, 101, 110] and LF10 $\leq$ ^{0.6} [1.2, 1.23, 1.25], then ice disaster risk grade is at most 2.

Then, the whole information contained in the decision table of ice disaster risk could be simplified as the 14 decision rules above. The development between ice regime information and ice disaster risk could be revealed by the decision rules extracted from historical ice regime data. The above decision rules could be used for us to analyze the corresponding risk grade when the risk index values reach certain extent, even for someone who lacks relevant experience about risk assessment. With the certain $\geq$ ^{0.6} decision rule *R*5 as an example to explain the meaning of the "if—then" decision rules, it means that if the frozen days dominate [90, 101, 110] with at least the grey dominance degree of 0.6 and the largest water in 10 days dominates [1.2, 1.23, 1.25] with at least the grey dominance degree of 0.6, then the ice disaster risk grade is at least 2. In addition, the decision rules with three-parameter interval grey numbers represent the more practical ice regime information, and it contains the condition that the collected ice regime information is real numbers. This is because the ice regime data in real numbers could be viewed as special three-parameter interval grey numbers whose lower, center and upper bounds are equal. Hence, the above decision rules could also reflect ice disaster risk even though the collected ice regime data are real numbers.

According to the above decision rules, the risk warning line of ice disaster can be set intuitively. With the monitoring of real-time dynamic information of ice regime, the early warning of ice disaster risk could be conducted. Some ice prevention work could be prepared beforehand when the changed ice regime data approach the early warning line.

6 Conclusions

As an important part of ice disaster risk management and the premise of ice disaster risk decision, ice disaster risk assessment has important theoretical significance and application value for ice prevention and control. Different from the previous risk assessment, considering the grey uncertainty of ice disaster risk, this paper proposes a two-phased intelligent model which not only assesses risk grade of ice disaster but also reveals the development between ice regime data and ice disaster risk.

The research results show that: (1) According to the ice disaster risk grade for the years of 1996–2015, the year of low risk is 7, with the proportion 35%; the year of middle risk is 8, with the proportion 40%; the year of high risk is 5, with the proportion 25%. The results of risk assessment in this paper are almost consistent with the average occurrence probability of ice disaster in Ning–Meng reach of Yellow River, and the ice disaster risk grade of each year is nearly according to the practical ice regime features. (2) All of the nine kinds of attribute reduction results for the decision table of ice disaster risk reflect again that ice disaster risk is the comprehensive action of thermodynamic factors, hydraulic

factors and human factors. The development between ice regime information and ice disaster risk could be presented intuitively by the decision rules extracted from historical ice regime data. (3) The research enriches the risk identification, risk assessment and risk development analysis of ice disaster in Ning–Meng reach of Yellow River.

The further research will focus on the dynamic assessment of the ice disaster risk. Based on the scientific constitution and quantitative assessment of ice disaster risk, considering the evolution and interaction of the ice disaster factors, the dynamic assessment and real-time early warning model of ice disaster risk will be constructed toward the whole ice process and multi-scale of ice regime.

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