

# Effect of surface wave inversion non-uniqueness on 1D seismic ground response analysis

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**Abstract** Surface wave methods are increasingly being used for geotechnical site characterization. The methodology is based on the dispersive characteristic of Rayleigh waves in vertically heterogeneous medium. Experimental dispersion curve is inverted to obtain one-dimensional shear-wave velocity profile by inverse problem solution. Uncertainty associated with this surface wave inversion has drawn much attention. Inverse problem solution can provide different equivalent shear-wave velocity profiles, which may lead to different seismic site response analysis. In this study, a neighborhood algorithm has been used for inversion of dispersion curve to get a set of equivalent shear-wave velocity profiles. These equivalent velocity profiles are then used for 1D ground response analysis for different input motion record of the same earthquake at different epicentral distances. Results show significant variation in amplification spectrum in terms of maximum amplification as well as peak frequency. The extent of this uncertainty largely depends on the characteristics of the ground motion records at different epicentral distances. A linear variation is observed between mean coefficients of variation of amplification spectrum and epicentral distance of ground motion records. A gradual increase in mean value of peak frequency and peak amplification with the epicentral distance is also observed.

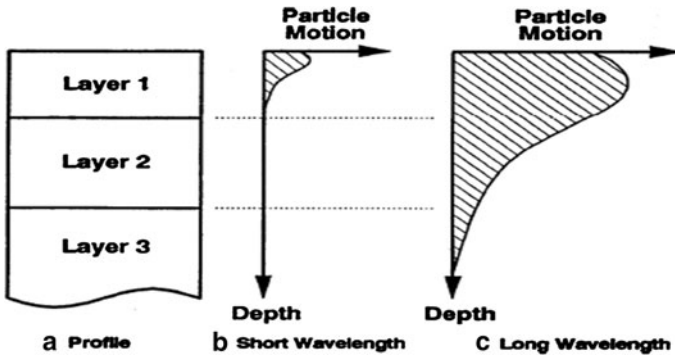
**Keywords** Inversion non-uniqueness · Dispersion curve · Coefficient of variation · Site response · Amplification spectrum

## 1 Introduction

Surface wave methods are used to characterize a site on the basis of shear-wave velocity profiles. Surface waves travel at speed governed by the shear-wave velocity-depth profile of the near-surface material. It utilizes the dispersion characteristic of Rayleigh waves. It generally takes more than two-thirds of total seismic energy generated by an impact seismic source at the ground surface. Rayleigh waves with different wave periods travel at different

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**Fig. 1** Geometrical dispersion in layered media (after Rix 1988)

velocities in a layered medium and penetrate to different soil depth due to their dispersive nature (Fig. 1). As a result of the variation of the shear stiffness of the layers, waves with different wavelengths travel with different phase velocities. The applications of surface waves in engineering field started in the 1950s with the Steady State Rayleigh Method (Jones 1958), but their revolution arrived only in the last two decades with the SASW method (Nazarian et al. 1983) and MASW method (Park et al. 1999; Xia et al. 1999; Miller et al. 1999).

Different types of surface wave methods are used for extracting the dispersion curve. Active-source tests, in which waves are generated using a seismic source (Stokoe et al. 1994; Park et al. 1999) and passive-source tests based on constant vibration of earth's surface or microtremors (Horike 1985; Tokimatsu 1995; Louie 2001; Strobbia and Cassiani 2011) are used for the analysis. Active and passive wave tests are associated with different frequency components which are directly related to the depth of investigation. In active-source tests, high-frequency components are dominating, and in passive-source tests, low-frequency components are obtained.

In surface wave tests, experimental dispersion curve is constructed from field data using different processing techniques (Strobbia and Foti 2006; Socco and Strobbia 2004). This experimental dispersion curve is then used for inverse problem solution to get shear-wave velocity variation with depth. The solution of the inverse problem is non-unique and may result in several equivalent velocity profiles, with a good fit with the experimental dispersion curve. Non-uniqueness of the solution of a dispersion curve provides only a possible solution. The surface wave data measurement uncertainty have been studied by different researchers in terms of coefficient of variation (COV) ( $COV = \sigma/\mu$ , where  $\sigma$  is the standard deviation and  $\mu$  is the mean) of phase velocity of Rayleigh wave.

Marosi and Hiltunen (2004a) found out the uncertainty in terms of COV of phase angle and phase velocity from the repetition of the test. They reported that there was low measurement uncertainty ( $COV \sim 2\%$ ) in the phase angle and phase velocity data. Again, Marosi and Hiltunen (2004b) evaluated the measurement uncertainty of shear-wave velocity with a COV of 5–10%. So, the inversion of phase velocity is magnifying the uncertainty. Study also reported that shear-wave velocity samples are normally distributed at a particular depth, and uncertainty in the resulting profile is increasing with depth. Lai et al. (2005) studied how the uncertainty of the experimental dispersion curves is mapped into shear-wave velocity profiles obtained by inversion process. The study shows that in experimental dispersion curve, low-frequency region is associated with higher values of

uncertainty and high-frequency region is associated with lower values of COV. Strobbia and Foti (2006) used a method MOPA (Multi-Offset Phase Analysis) for the identification of both modeling error and data uncertainty. They found out that the resulting shear-wave velocity profile may show up to 18 % uncertainty at depth as results of inversion.

Foti et al. (2009) studied the effect of the surface wave inversion uncertainty on seismic ground response analysis for profile having high impedance contrast. They found that effect of inversion uncertainty on seismic site response is insignificant. Later this study was extended by Boaga et al. (2011) for different impedance contrasts and observed that different equivalent profiles resulted from surface wave inversion are not equivalent in terms of seismic ground response analysis. For low impedance contrast, the effect is very much pronounced and for high impedance contrast, the equivalent solutions have a very little effect. Their seismic site response study is limited on soil models having different impedance contrast using a single earthquake record. However, the response of a soil column (ground response analysis) is strongly influenced by the frequency content of the input ground motion. This effect has been studied and presented in this paper.

So, the solution of the inverse problem is the most critical step of the surface wave techniques due to its non-uniqueness. For the most engineering applications, the velocity profiles are obtained blindly from the surface wave tests and are used for the site characterization or seismic site response analysis for design ground motion. Very few studies have been carried out to assess the consequences of this inversion uncertainty in the surface wave method on the seismic ground response analysis. Inversion uncertainty yields different equivalent shear-wave velocity profiles for the same measured dispersion curve. These equivalent profiles may not be exactly equivalent in terms of their seismic site response. In this paper, a study has been carried out to find out the impact of non-uniqueness of surface wave inversion on seismic response of soil column using near-source and far-source earthquake records. A statistical study also carried out to quantify the uncertainty and ultimately variation of uncertainty with epicentral distance is mapped.

## 2 Inversion using neighborhood algorithm

The neighborhood algorithm is a stochastic direct-search method for finding models of acceptable data fit inside a multi-dimensional parameter space (Sambridge 1999). A set of pseudo-random samples is generated after defining the variation of each model parameters (thickness and shear-wave velocity of each layer) in the parameter space. This set of samples is then processed to get the dispersion curves by using the forward problem algorithm which was originally developed by Thomson (1950) and Haskell (1953) and later modified by Dunkin (1965) and Knopoff (1964). The forward algorithm considers the fundamental mode of Rayleigh wave propagation by modeling soil column as a stack of horizontal and homogeneous layers. Once the theoretical dispersion curve is developed from the random samples given by the neighborhood algorithm, the misfit value is calculated. If the experimental dispersion curves are associated with an uncertainty estimate, the misfit is given by

$$\text{Misfit} = \sqrt{\sum_{i=0}^n \frac{(X_{ti} - X_{ei})^2}{\sigma_i^2 n}}$$

where  $X_{ti}$  is the theoretical and  $X_{ei}$  is the experimental phase velocity of the calculated curve at frequency  $f_i$ ,  $\sigma_i$  is the uncertainty of the frequency samples and  $n$  is the number of

frequency samples considered in the dispersion curve. If uncertainty is not provided,  $\sigma_i$  is replaced by  $X_{ii}$  in the equation. The details about the procedure are described in the literature (e.g., Wathelet et al. 2004).

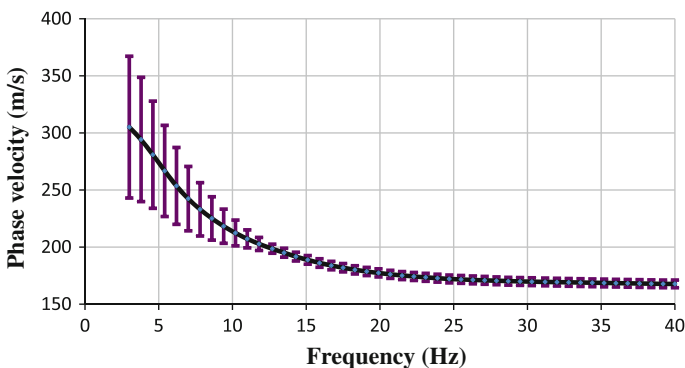
### 3 Methodology

A reference velocity profile consisting of three soil layers plus half space has been considered. Table 1 provides the details of the shear-wave velocities of different layers. Low impedance contrast is considered between half space and bottom layer because the effect of inversion uncertainty is prominent in this type of profile (Boaga et al. 2011). Next theoretical dispersion curve is generated for the considered profile using the forward problem developed by Dunkin (1965) and Knopoff (1964). In forward modeling, the value of Poisson ratio and unit weight is kept the same for all the layers (Poisson ratio 0.33 and density 1,950 kg/m<sup>3</sup>) as these parameters have a very little influence on Rayleigh wave dispersion (Socco and Strobbia 2004).

To consider errors in the development of dispersion curve arising due to measurement uncertainties in the surface wave method, the developed dispersion curve is assumed to be associated with some errors. This error in the phase velocity determination (dispersion curve) is defined based on the earlier study by Lai et al. (2005). They found out two distinct regions in dispersion curve. The threshold value of frequency between these two regions is near about 12 Hz. They found out very low COV ( $\sim 2\%$ ) at a frequency of 12 Hz and above and  $\sim 14\%$  of COV at lowest frequency of 5.6 Hz. Based on this study, we have taken a constant value of COV of 2% up to 12 Hz frequency and below this we linearly increased the COV between 2 and 14%. Figure 2 shows the dispersion curve with the calculated standard deviation of phase velocity (error) based on above considered COV values. Now the main dispersion curve associated with standard deviation is the target of our inversion.

**Table 1** Reference velocity profile

Thickness (m)	Shear-wave velocity (m/s)
5	180
7	240
12	300
Half space	360

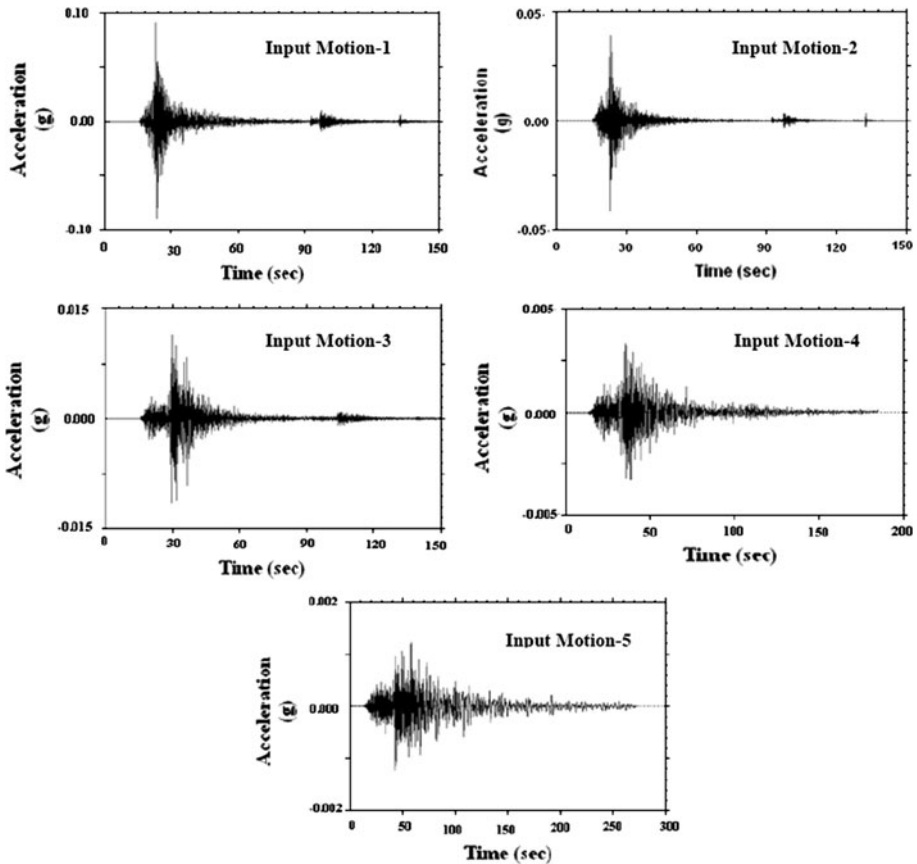


**Fig. 2** Target dispersion curve with the range of uncertainty expected in the phase velocity

After defining the target, equivalent velocity profiles are obtained by inversion using neighborhood algorithm. First, best fitting 15 profiles are selected and equivalent one-dimensional ground response study has been carried out by using SHAKE2000 software based on the SHAKE developed by Schnabel et al. (1972). Modulus reduction and damping ratio curves are selected from SHAKE2000 database. For shaking analysis, five records of the same earthquake at epicentral distance 37, 50, 103, 150 and 202 km are used as an input motion in the analysis. Recorded data of an earthquake of magnitude 6.6

**Table 2** Details of input motion records

Input motion	Epicentral distance (km)	Peak acceleration (g)	Sampling frequency (Hz)	Station name
1	37	0.092	100	Tarou
2	50	0.041	100	Yamada
3	103	0.011	100	Touwa
4	150	0.003	100	Nakasen
5	202	0.001	100	Honjoh



**Fig. 3** Acceleration time histories of input motion records at different epicentral distance used in the study

occurred on 2012/03/27 (Latitude 39.80°N, Longitude 142.33°E) in Japan are collected from K-Net database and used for the analysis. The sampling frequency of all the time histories is 100 Hz. All the time history data have been corrected for base-line correction before using it as an input in SHAKE2000. Details about the ground motion records are provided in the Table 2. Figure 3 shows the plot of acceleration time history of the recorded earthquakes for the seismic site response analysis.

## 4 Results

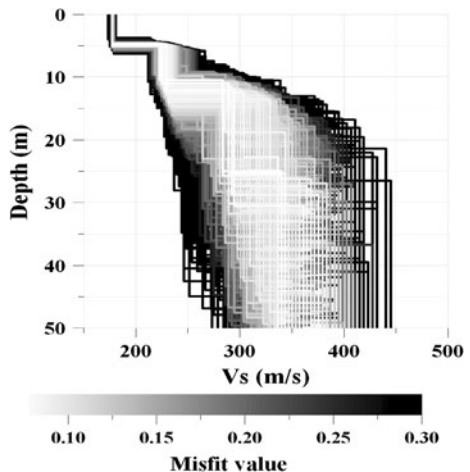
The results obtained from the analysis are presented as the equivalent profiles and variation in the seismic response for different input motion. Seismic response variation in terms of peak amplification and peak frequency of the equivalent profiles is also presented. Then uncertainty in terms of COV is quantified and plotted with epicentral distance.

A set of equivalent profiles has been generated by inversion using neighborhood algorithm to the target dispersion curve as shown earlier. Figure 4 presents generated 1D shear-wave velocity profiles with the associated misfit value after the inversion. With the increase of the misfit value, no of profiles also increase. The profiles with bright color correspond lower misfit and better approximation to the reference dispersion curve. Within these equivalent profiles, 15 best fitting profiles (maximum misfit less than 0.1) are selected for one-dimensional ground response analysis. The 1D ground response analysis provides reasonably accurate estimates of seismic responses for the level grounds. Here, our study is applicable to level grounds, where lateral heterogeneity is ignored. Figure 5 shows the profiles considered for the 1D seismic response analysis.

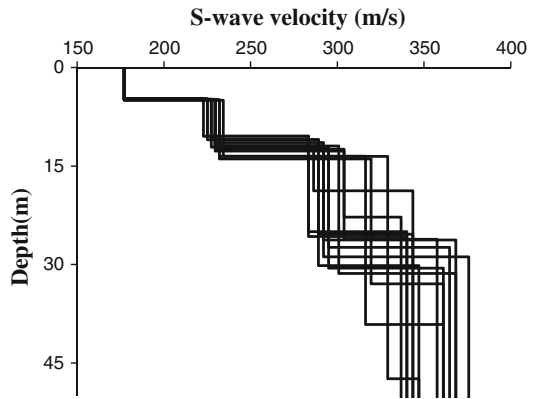
### 4.1 Seismic site response of equivalent profiles

Seismic response analysis results by using input motion-1 recorded at an epicentral distance of 37 km, and the result of ground response analysis of the equivalent profiles shows significant differences in the amplification spectrum. Here, amplification represents the amplification ratio, but not the absolute amplification of the surface motion. Amplification

**Fig. 4** Equivalent shear-wave velocity profiles with associated misfit value generated after inversion



**Fig. 5** First 15 best fitting shear-wave velocity profiles considered



ratio is the ratio between the surface motion and the input/base motion. The spectrums show variation in terms of peak amplification as well as in peak frequency (Fig. 6). Peak amplification of equivalent profiles varies from 7.2 to 9.2 and peak frequency varies 1.4–2.6 Hz. For statistical analysis, we have calculated the COV of the amplification ratio at each frequency of the equivalent profiles. Figure 7 shows the plot, the COV of amplification with respect to frequency mean value of COV is found as high as 20 % (solid line in Fig. 7). High value of COV and data scatter is observed up to frequency 7 Hz.

For input motion-2 (epicentral distance 50 km), the result (Fig. 6) shows similar kind of variation from the point of view of peak frequency (1.4–2.75 Hz) and as well as in peak amplification (7.6–8.9) in comparison with previous case. From Fig. 7, it is observed that mean COV of amplification is slightly high (22 %) in comparison with the former case.

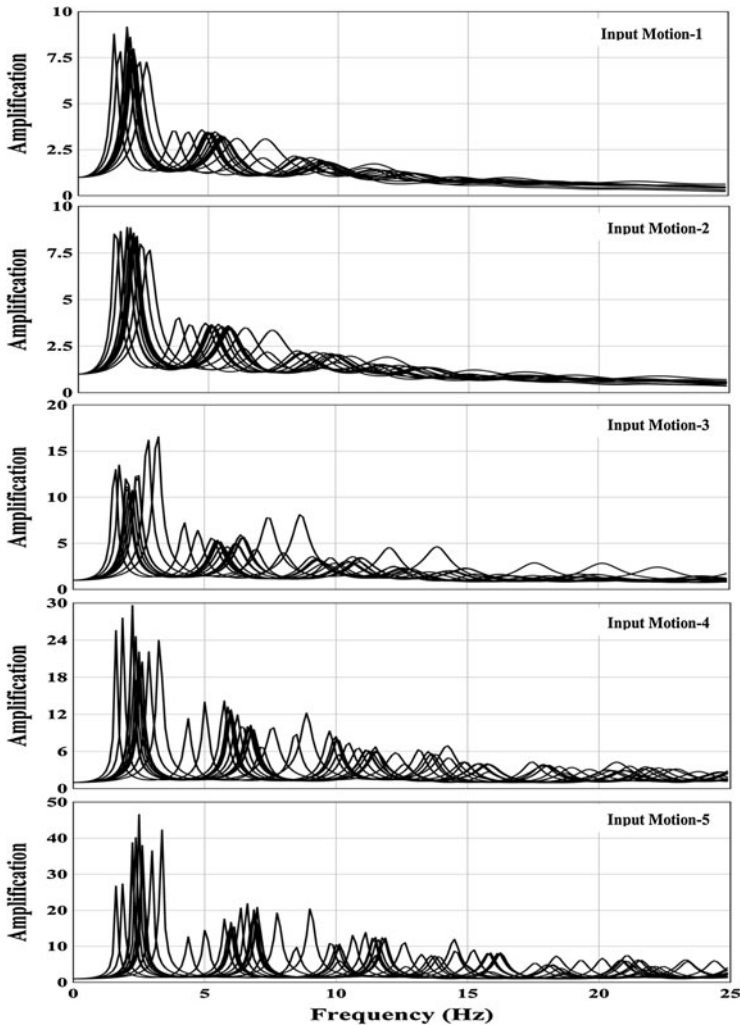
Large difference in the amplification spectrum is observed (Fig. 6) for input motion-3. Variation in amplification spectra in terms of amplitude (7.4–16.6) and peak frequency (1.62–3.25 Hz) is found out. Very high value of mean COV (mean 38 %) is observed for the amplification spectrum (Fig. 7) derived from best fitting 15 equivalent profiles, and high data scatter is observed up to 10 Hz frequency.

Figure 6 shows very high and irregular amplification spectrum for input motion-4. Results exhibit large variation in peak amplification (19–29.6), and large variation is also observed in case of peak frequency (1.62–3.25 Hz). Statistical distribution shows very high data scatter up to frequency 15 Hz, and mean value of COV is 51 % (Fig. 7). Mean value of COV is increasing with the epicentral distance.

For input motion-5, amplification peaks are very high in amplitude (Fig. 6) than previous case and variation is very much irregular (26.7–46.5). Variation in maximum amplification frequencies also shows a significant variation (1.62–3.37 Hz). Mean value of COV of amplification spectra is very high (67 %), and high data scatter is observed up to a frequency of 20 Hz (Fig. 7).

#### 4.2 Mapping of inversion uncertainty with epicentral distance

From the above analysis, it can be seen that equivalent profiles in terms of surface wave inversion can lead to a very significantly different seismic response in terms of peak amplification as well as peak frequency and the input ground motion also plays an important role. From the results, it is observed that mean COV of amplification spectrum is increasing with the epicentral distance of recorded ground motion. An attempt has been made to show the variation of the seismic response uncertainty with the epicentral distance.

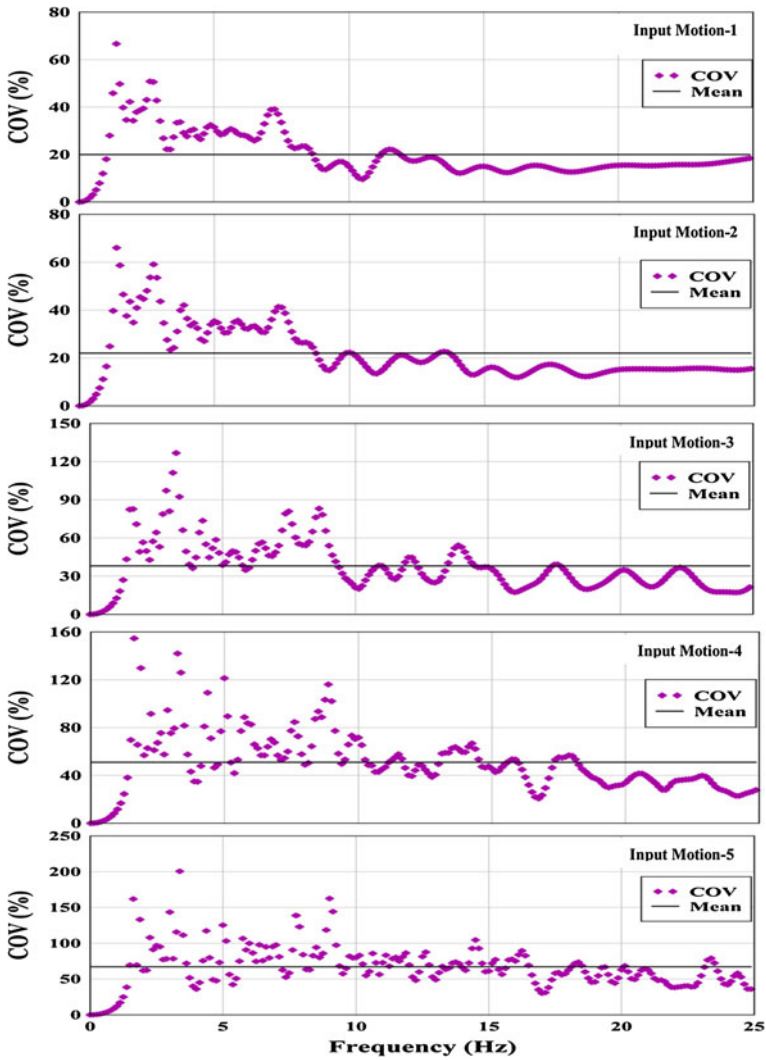


**Fig. 6** Variation of amplification spectrums of equivalent profiles for different input motions

The mean COV of amplification spectrums of the equivalent profiles has been plotted for five input motions with the epicentral distance. Figure 8 shows the plot between mean COV with the epicentral distance for the five input motion cases (dots). The calculated points exhibit a linear variation (Fig. 8), defined as  $y = 0.2877x + 8.4128$ , where  $x$  is epicentral distance and  $y$  is the mean COV.

The statistical distribution of peak amplification and peak frequency is presented in terms of mean and standard deviation with epicentral distance. There is a rapid increase of mean value of peak amplification with epicentral distance. In case of standard deviation, also we can observe there is a gradual increase with epicentral distance (Table 3). The value of standard deviation is very high at 150 and 202 km epicentral distances. Figure 9 presents the plot of mean of peak amplification value of equivalent profiles with different epicentral distances. The vertical bar associated with each point represents the standard deviation.



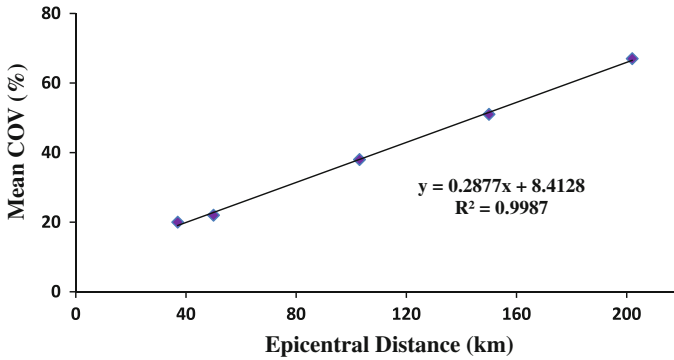


**Fig. 7** Variation of COV of amplification spectrums for equivalent profiles with respect to frequency

Figure 10 shows the plot between average peak frequency and epicentral distance with standard deviation associated with each point. Standard deviation of peak frequency does not vary much, but a gradual increase in average peak frequency with epicentral distance is observed.

### 5 Discussion

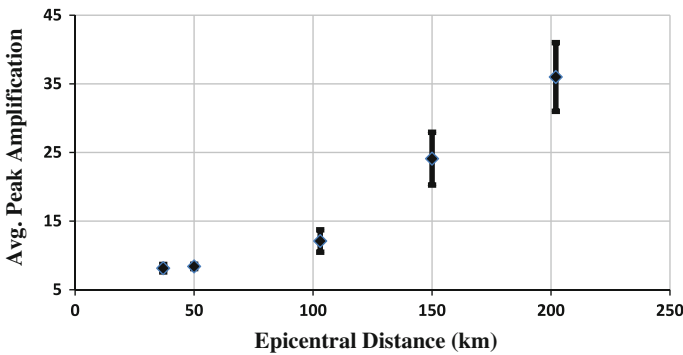
The above analysis shows that the characteristic of the ground motion records plays an important role in seismic site response analysis. We know that our earth acts as a low pass filter, that is, motion with low-frequency content can travel a long distance, but the higher



**Fig. 8** Variation of mean COV of amplification spectrums of equivalent profiles with different epicentral distances

**Table 3** Variation of peak amplification and peak frequency at different epicentral distances

Epicentral distance (km)	Mean of peak amplification	SD of peak amplification	Mean of peak frequency (Hz)	SD of peak frequency (Hz)
37	8.14	0.54	2.00	0.28
50	8.40	0.31	2.05	0.32
103	12.10	1.63	2.28	0.40
150	24.10	3.84	2.42	0.37
202	36.00	5.00	2.45	0.41



**Fig. 9** Distribution of peak amplification (mean with standard deviation) with epicentral distance

frequency motion dies out early. When an earthquake motion propagates through earth, its high-frequency component attenuates very fast and at greater epicentral distance low frequency dominates. Figure 11 clearly shows the distribution of frequency contents of the input motions. Input motion-1 (recorded at 37 km epicentral distance) and input motion-2 (recorded at 50 km epicentral distance) are having a nearly uniform frequency contents. From the results, also we can observe that mean COV of amplification spectrums for this two cases is nearly similar (20 and 22 %). For the input motion-4 (150 km epicentral

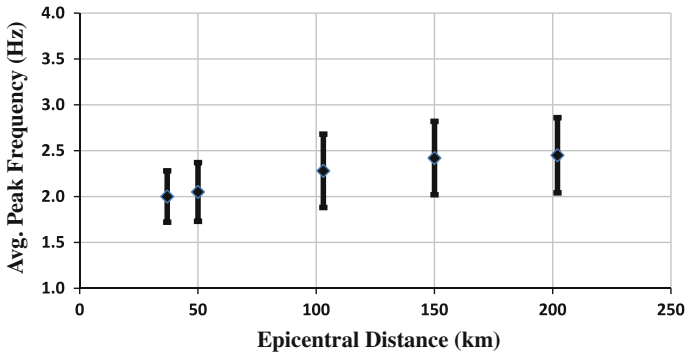


Fig. 10 Distribution of peak frequency (mean with standard deviation) with epicentral distance

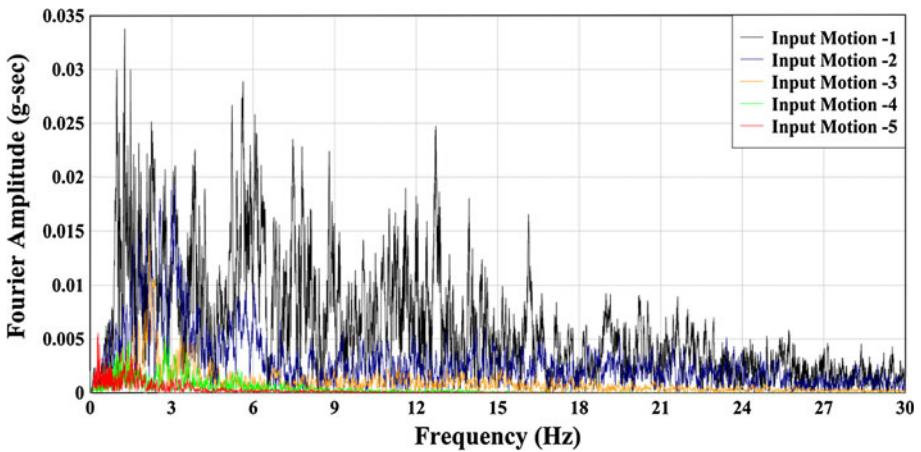


Fig. 11 Comparison of Fourier amplitude spectra of input motions

distance) and 5 (202 km epicentral distance), higher frequency content is absent, and as a result, they exhibit very high value of mean COV (51 and 67 %).

When site response is analyzed for far-field earthquakes, the influence of non-uniqueness of surface wave methods on seismic site response is very much significant. However, for near-field earthquake motions, this effect is comparatively low.

Site-specific response analysis of a site under far-field earthquake is not only non-unique but significant variations are observed both for peak amplification and peak frequency in the amplification spectra of the considered earthquake motion. This non-uniqueness and the differences arise due to uncertainty in surface wave techniques (Inversion). The analysis clearly indicates that if accurate shear-wave velocity profile is not derived, obtained response does not reflect the actual scenario of ground motion, particularly for far-field earthquake scenarios.

## 6 Conclusions

Surface wave methods have gained a great deal of popularity from geophysical and geo-technical point of view, but very few research has been made to find out the level of

confidence associated with these methods. In this article, an attempt has been made to quantify the consequences of inversion uncertainty in the surface wave methods on 1D seismic site response studies. Findings obtained from the whole study can be summarized as follows:

- a. Equivalent profiles resulting from non-uniqueness of surface wave inversion are not equivalent in terms of their seismic site response.
- b. Effect of non-uniqueness of surface wave inversion on seismic site response analysis is significantly influenced by characteristics of input ground motion. To study this effect, five records of the same earthquake motion at different epicentral distances have been considered. Higher COV of amplification spectrums is observed for input motion recorded at higher epicentral distances. Mean COV of amplification linearly varies with the input motion recorded at different epicentral distances.
- c. While Boaga et al. (2011) observed variations in the amplification spectra in terms of only peak frequency; we observed variations in amplification spectra in terms of both peak amplitude as well as peak frequency for the considered earthquake motions.
- d. A rapid increase of mean and standard deviation is observed for peak amplification with increase in the epicentral distance. In case of peak frequency, slow increase of mean and standard deviation is observed with the increase in epicentral distance.
- e. This study clearly indicates that the inversion uncertainty has a pronounced effect on the 1D ground response analysis particularly for the far-field earthquake scenarios. This can mislead the calculations for design ground motions if these uncertainties are not properly addressed.

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