



Application of Complex Networks Theory in Urban Traffic Network Researches

Rui Ding¹ · Norsidah Ujang² · Hussain Bin Hamid³ ·
Mohd Shahrudin Abd Manan² · Rong Li⁴ · Safwan Subhi Mousa Albadareen³ ·
Ashkan Nochian² · Jianjun Wu⁴

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Abstract

Complex network theory is a multidisciplinary research direction of complexity science which has experienced a rapid surge of interest over the last two decades. Its applications in land-based urban traffic network studies have been fruitful, but have suffered from the lack of a systematic cognitive and integration framework. This paper reviews complex network theory related knowledge and discusses its applications in urban traffic network studies in several directions. This includes network representation methods, topological and geographical related studies, network communities mining, network robustness and vulnerability, big-data-based research, network optimization, co-evolution research and multilayer network theory related studies. Finally, new research directions are pointed out. With these efforts, this physics-based concept will be more easily and widely accepted by urban traffic network planners, designers, and other related scholars.

Keywords Complex network theory · Urban traffic network · Spatial network · Multilayer network

✉ Jianjun Wu
jjwu1@bjtu.edu.cn

¹ School of Big Data Application and Economics, Guizhou University of Finance and Economics, Guiyang 550025, China

² Faculty of Design and Architecture, Universiti Putra Malaysia, 43400 Seri Kembangan, Selangor, Malaysia

³ Faculty of Engineering, Universiti Putra Malaysia, 43400 Seri Kembangan, Selangor, Malaysia

⁴ State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

1 Introduction

The importance of urban traffic network studies to urban economic and social development is self-evident. The most far-reaching impact on the modern urban form has been the development of traffic technologies such as the automobile, highway, metro and subway. The expansion of these urban traffic networks have shaped the morphology of modern cities, while the change of urban forms will in turn affect urban traffic network structures. Here we define an “urban traffic network” as an urban land-based traffic network, with emphasis on road and rail networks, and it both related with the traffic flow and infrastructure. Air transport and waterway transport are not included. In the urban traffic planning process, normally following this basic process: traffic survey, background prediction, traffic forecasting, layout scheme design, and project evaluation. Moreover, urban traffic network design is the most crucial point of layout scheme design (Liu 2001), as it fundamentally determines the basis of the future urban planning, economic development direction, and operational efficiency, while their functional layout conspicuously affects the urban form (Fig. 1). The development of traffic networks provides a strong guarantee for steady growth in the urban economy, and the rational distribution of a traffic network can effectively promote the flow of urban economic activities (Rodrigue *et al.* 2013). Due to the stability of urban development and the difficulty of changing land-use patterns, the research and prove of how the traffic network topology structure to affect urban traffic distribution has earned much attention (Ducruet and Lugo 2013; Lammer *et al.* 2006; Wu *et al.* 2006a, 2006b). Hence, the designation and choice of new networks layout call for particular concern.

Complex network theory is a multidisciplinary part of complexity science, which has seen a surge of interest since Watts and Strogatz (1998) and Barabasi and Albert (1999) described the collective dynamics of small-world networks, and the emergence of scaling in random scale-free networks. A small-world network is structured with a

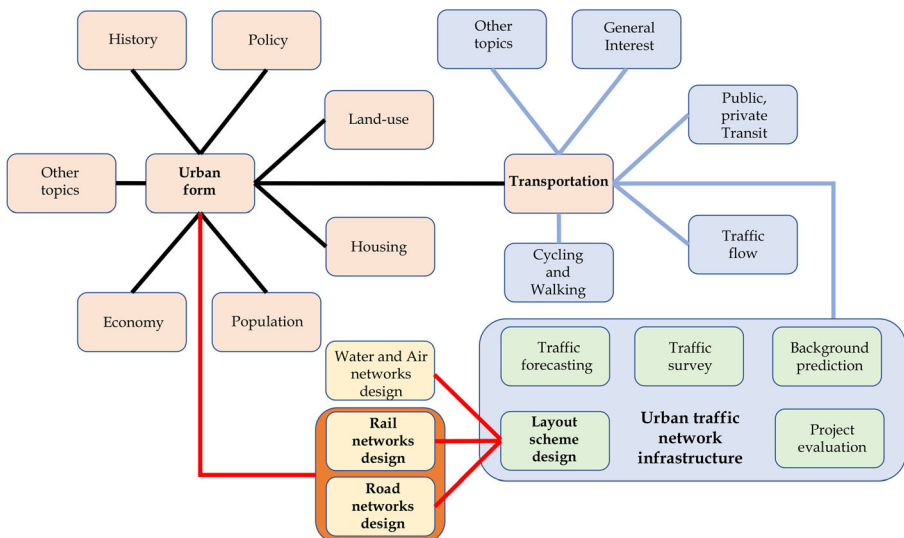


Fig. 1 The linkages between urban traffic networks and urban form

high clustering coefficient and small average shortest distance, while a scale-free network is a type of network in which the degree distribution of nodes obeys the Power-law distribution. Complex networks theory has been widely used and led to astonishing achievements in Empirical Science and Basic Science, being regarded as a paradigm representative of complexity system science. It mainly concentrates on the following aspects: the empirical research of the networks characteristics; network hub nodes detection; dynamical changing and spreading processes; the seeking and detection of communities and groups; robustness and vulnerability; and multilayer network theory and applications. We denote that the terms graph and network, node and vertex, and link and edge are respectively synonymous in this context.

Research into complex networks provides an affordable and solvable method and novel insight into analyzing a complex urban system, while urban traffic networks are continually evolving. The paradigms of the small-world and scale-free networks changed the stereotyped thinking of urban traffic networks. Before these two models were proposed, urban traffic networks were normally addressed as either regular networks or ER random networks (Erdos and Renyi 1960), as inherited from the definitions of graph theory. However, recently scholars have recognized the importance of complex network science, including Masud *et al.* (2008), who treated this theory as an independent and key chapter in the book *Operations Research and Management Science Handbook*. In urban studies, Neal (2012) described in *The Connected City: How Networks are Shaping the Modern Metropolis* the application of network science related indices in this specific direction. Additionally, Batty (2013), in *The New Science of Cities*, related his vast experience in urban models and complexity research. Most recently, Barthelemy (2018) announced his new book *Morphogenesis of Spatial Networks*, which not only introduces some basic concepts of complex network theory, but also introduces the Operations Research based idea “optimization” to the complex network theory. These works have made significant contributions to the art of the complex network research in modern urban science. Hence, we believe that the complex network theory offers huge potential for urban studies.

As a favourable practice, many network analyses based studies have been conducted to help urban and transportation planners to examine the structures and functions of the urban traffic networks (Chan 2007; Domenech 2009; Erath *et al.* 2009; Gao *et al.* 2006; Holme 2003; Scheurer *et al.* 2008; Sen *et al.* 2003). With an understanding of urban traffic networks in the context of network science, we can better understand the reasons for methodical urban form variation and then determine the potential parts of the future development (Ding *et al.* 2015), while determining static and dynamic structural characteristics can provide relevant references to the urban planning, design, optimization, and sustainable development and maintenance (Barthelemy and Flammini 2008, 2009; Batty 2007, 2012, 2013; Boccaletti *et al.* 2014; Boccaletti *et al.* 2006; Morris and Barthelemy 2012).

The history and basic indicators and functions of such research have been reviewed by many researchers (Barthelemy 2011; Derrible and Kennedy 2011; Ducruet and Beauguitte 2014; Gao *et al.* 2006; Xie and Levinson 2009a). Also discussed was the origin of graph theory, illustrating that after Euler opened the door of network science, network science inherited and comprehensively combined with the characteristics of the Scientific Management, System Science, Cybernetics, Information Technology, Behavioural Science, Economics, Operations Research, and other disciplines. Based

on these accomplishments, researchers have begun work on the ultimate goals of this subject, which are the optimization of the network structure and maximizing network performance. However, certain novel applications and new directions have been generated, and some of these reviews did not consider the network growth and evolution processes, which are critical in urban traffic network planning research; further, the network optimization methods are not considered. So, on the basis of these reviews, we aimed to fill these gaps, complement the above-mentioned contents and complex network knowledge and classifications, and cover their applications in urban traffic network studies in detail. With the perfection of our efforts, this novel concept in urban planning may be more easily and widely accepted and used by urban planners, designers, and other related scholars.

In section 2, inheriting and carrying forward these review papers, we reviewed these novel applications of complex network theory in urban traffic network studies in several directions, including network representation methods, topological and geographical indicators and their applications, the mining of the urban traffic network communities, the network robustness and vulnerability and their applications, big-data-based research, the optimization (both structural and flow related optimization), the co-evolution research and the multilayer network theory with their applications. In the conclusion, we illustrate the weak points and potential paths for the further research.

2 The Application of Complex Network Theory In Urban Traffic Network Studies

This paper utilised several channels to identify relevant literature regarding urban traffic network and complex network theory. Academic databases such as Science Direct Journals, Scopus, Web of Science, and Google Scholar were utilised. Reference and citation lists from key papers in the field were checked. Combinations of the following search terms were used to find the relevant literature: complex network, urban traffic network, rail network, road or street network. Theoretical studies that did not examine the urban traffic networks or highlight the major limitations and those complex network research more emphasise on Physics were excluded from search results. A total of 89 related articles were analysed, with references in Table 3.

Due to the influence and stimulation of the new network science, a new generation of urban traffic network analysis expanded and developed from the main spirit of the older ones. Truthfully, whether from a philosophical point of view or the perspective of sociology, the cognition of the complex network science is based on such a process. This includes the excavation of foundation structures, the cognition of basic properties, attribute partition, evolutionary method and performance optimization, and ultimately object control. Here, based on the six stages of division of Haggett and Chorley (1969), the two levels of research of Gattuso and Miriello (2005), and the five models of Barthelemy (2011), according to the characteristics of the urban traffic network and the corresponding requirements as well as cognitive logic, we reviewed most of the research directions of complex network theory in urban traffic network research. Actually, these research directions are always associated and it is difficult to divide them. However, they all place particular emphasis on different points; hence, we should discuss them in detail and

understand these different techniques. Although this makes this paper appear segmented, the sections are strongly connected and complementary.

Considering the research topics, in Fig. 2a, we use R, S, B to describe objectives including rail network, street network, and bus network respectively. We can see that around forty typical papers analysed street networks. Another hot topic is rail network-based research, with more than twenty papers having focused on the network properties of the rail network and its combination with bus network and street network.

After that regarding the representation methods, in Fig. 2b we use P, D, M, and C for Primal, Dual, Multilayer, and Community approaches respectively. With the simplicity and authenticity of primal approach, the vast majority of research papers used this technique to represent spatial networks.

For the basic network models used in references, in Fig. 2c we use R, E, S, W to represent the regular, ER random network, scale-free, and small world models, respectively. Analysis of these research papers indicates that many works about spatial scale-free network have been done.

Then in Fig. 2d, we divide the complex network-related urban traffic network research into several levels in terms of topological and geographic features, network communities mining, network robustness and vulnerability, network optimization, co-evolution research, multilayer network theory related studies, weighted research, hub nodes or important links, network growth, and integrated studies (population, land-use

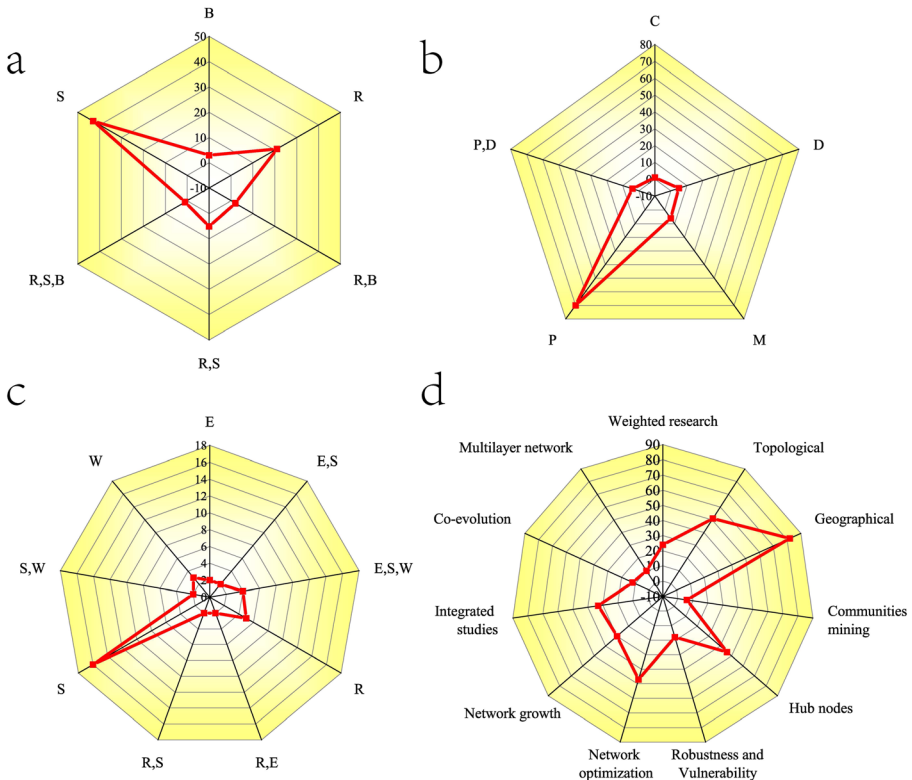


Fig. 2 Analysis of these related references

integrated). More than 50% and 90% of these researches focus on the network topological and geographic features, respectively. Extra efforts should give to the network communities mining, network robustness and vulnerability, co-evolution research, and multilayer network theory related studies.

2.1 The Networks Representation Methods

In network science, there are a large number of network representation methods. Considering spatial networks, there are four basic network representation methods, which are based on the network topological structure and simplicity of expression. These methods, known as the primal approach (Space L) and the dual approach (Space P), are more widely used compared with the multilayer approach (Space M) and the community approach (Space C).

The primal approach, or Space L method, was recommended by Porta *et al.* (2006b) and is the simplest and perceptual intuition method. Distance is measured, while simply treating intersections as nodes, and streets as edges in the graph. It is easy to connect with and directly used by the geographic coordinate system-based software like ArcGIS, some further advanced spatial analysis can be directly applied, but the transfer information and 3D based connection relationships can hardly represented. This approach is used by nearly all urban traffic network related analysis, especially when considering distance (Dolev *et al.* 2010; Erath *et al.* 2009; Porta *et al.* 2009; Wang *et al.* 2011; Barthélemy and Flammini 2009).

The dual approach, or Space P method, see streets turned into nodes and intersections as edges. This definition was also proposed by Porta *et al.* (2006b), who also discussed the relation between Space Syntax (Hillier 2007; Hillier *et al.* 1976) and dual approach, the transfer information is shown intuitively, but when applying to real projects, it needs additional elaboration and description, it deliberately ignores the impact of distance. It is widely used when considering decision-making (Curtis and Scheurer 2010), network transfer or turning ability (Hu *et al.* 2008), the connectivity graph of urban streets (Jiang and Claramunt 2004a, 2004b), and network topological structure-related research (Masucci *et al.* 2014).

The multilayer approach, or Space M method, was proposed by Buldyrev *et al.* (2010) and treats different modes of traffic network as different layers, connected by the cooperation strength (Ding *et al.* 2017). It is the closest approach to the real situation, as the complex relationships between upper-layer and lower-layer can be clearly shown by one graph; however, the measurement methods are still under development and not as hot as planar network because of its representation and calculation complexity. Sometimes, it is used for the measure of traffic dynamics on multilayer networks (Du *et al.* 2014; Gu *et al.* 2011; Morris and Barthelemy 2012) and its optimization (Du *et al.* 2016).

The Space C approach is a complementary projection method in which vertices denote some communities and an edge exists if two communities have at least one overlapping points. Here, the community may represent assumed communities (like bus services networks) or real communities, or treat a traffic or activity route as a node or an edge, it can simplify the measurement from the macro perspectives but oversimplified from the micro perspective. It is used for bus service networks (Sun *et al.* 2015b; Xu *et al.* 2007) and clustering method simplification (Gleyze 2013).

2.2 The Topological Indicators and Applications

Within the inevitable limitations and subject to the lower computing ability, limited data accessibility, and the poorly-developed theories of the time, the pioneers of traffic network science suffered a limited and heuristic point of view to explore their inner characteristics. However, some brilliant works have emerged, such as the work of Garrison and Marble (1961), Kansky (1963), Chorley (1967), Haggett *et al.* (1977), and Sheffi (1985), their theories, methods and ideas have continually are still informing research tendencies. After decades of improvement, scholars now know the particular factors that affect the traffic hub nodes and network coverage and have identified evaluation methods to consider both the topology and geography of networks (Garrison and Marble 1961, 1964; Kansky 1963). This research is grounded on the definition of network topological indicators (Table 1), which are mainly used to describe the basic characterises of urban traffic networks.

Certain classical graph theory measure indices remain common and are still widely used for some particular investigations (Ducruet & Beauguitte 2014). Without considering the real length and edge weights, Berge (1962), based on the equivalent properties and characterises of a tree, demonstrated the number of loops, which can be measured by the function of relations of nodes and edges, Ore (1963) gave a similar definition. Based on this, Garrison and Marble made notable progress on the basis of network design. For the description of the potential number of loops and edges, the ratio of edges versus vertices, labelled as α , γ and β , respectively (Garrison 1960; Garrison and Marble 1961, 1964). These three widely used indicators in the transport literatures enable us to better understand different network characteristics. Other indexes like overlap are also considered in some research (Aleta *et al.* 2016).

Most urban traffic network studies have illustrated these basic characteristics of traffic networks. The basic network characteristics of more than 40 worldwide street networks was measured by Buhl *et al.* (2006), the 20 largest German urban road networks were examined by Lammer *et al.* (2006) and 40 urban street networks in the USA were topologically measured and tested by Jiang (2007), there are many

Table 1 The topological indicators and applications

Indicators	Functions	References	Related applications
The number of loops	$\mu = M - N + s$	Berge (1962); Ore (1963)	Buhl <i>et al.</i> (2006); Lammer <i>et al.</i> (2006); Jiang (2007); Derrible and Kennedy (2010a, 2010b); Quintero-Cano (2011); Quintero <i>et al.</i> (2013)
The potential number of loops	$\alpha = \frac{M-N+1}{2N-5}$	Garrison (1960); Garrison and Marble (1961); Garrison and Marble (1964)	
The potential number of edges	$\gamma = \frac{M}{3(N-2)}$		
The ratio of edges versus vertices	$\beta = \frac{M}{N}$		

N is the number of nodes, the number of edges is denoted as M, s is the subgraph number

related works not listed here. Derrible and Kennedy also made a noteworthy role in network traffic analysis by using a new methodology to redraw metro networks into graphs using two indicators: directness and structural connectivity (Derrible and Kennedy 2010a, 2010b), they characterised 33 metro networks around the world, reviewed the impact of network size, and discussed the implications for topology. Quintero *et al.* (2013) focused on pre-existing network indicators, whereas Kansky summarised the definitions of transit network properties (Kansky 1963), and in addition presented several macro-level prediction models for transit infrastructure, traffic network topology, transit route design, and transit performance and operations (Quintero-Cano 2011).

2.3 The Geographical Indicators and Applications

Haggett and Chorley (1969) offered an early and extensive treatment of networks and graph-based analysis of geographic networks. Based on the development progress of graph theory, Peter Haggett defined the six stages in the analysis of nodal regional systems (Fig. 3) in his book *Locational Analysis in Human Geography* (Haggett *et al.* 1977). He believed that regional analysis is the good approach to geography, in which the results of the spatial and ecological analysis are combined. In other words, that means networks structure and the human behaviour can be measured in one system. Typically, these six meaningful processes are part of the fresh context of network science with a hinge and compared perfectly, the interaction process versus human behavioural interactions. The networks and nodes processes can be treated as static network research and its characteristics, while the hierarchies process is most studies are about network density and centrality. The surface process includes interactions between urban land-use patterns and network structures. The diffusion process is the part of network dynamics that considers the co-evolution procedures. Nevertheless, in a review of those contemporary studies, they also found that network growth research has been somewhat fragmentary, due to limited computing ability.

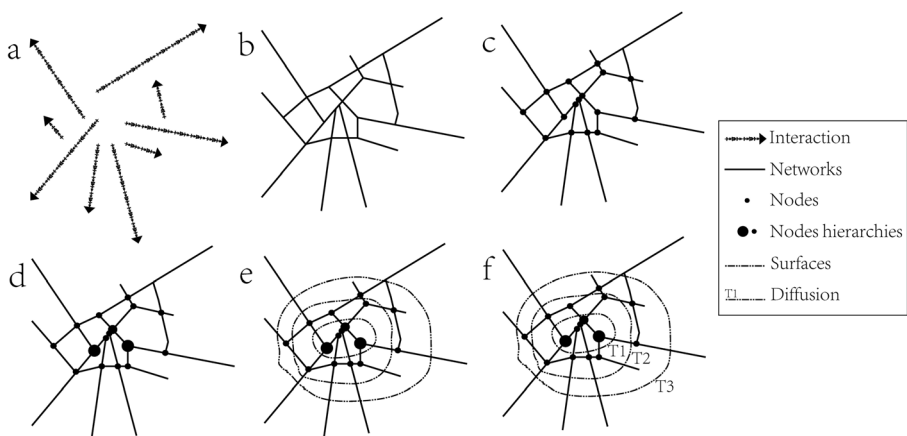


Fig. 3 The six stages in the analysis of nodal regional systems. **a** Interaction. **b** Networks. **c** Nodes. **d** Hierarchies. **e** Surfaces. **f** Diffusion. (adjusted from Haggett *et al.* 1977)

Based on traffic flow research, Kansky (1963) recommended the index of, as this indicator can be used in urban rail transit to measure average passenger flow per station (see Table 2). He inherited the concepts suggested by Garrison & Marble, while some relevant indicators are proposed, like the average edge length. The next indicator is, also known as network extension. In the description of Derrible and Kennedy (2011), the reason this indicator is named and relates to the calculation of the circumference of a circle. It is always used to calculate the network density. The next is which is related with the weight of nodes. We can also expand it to the edges, while can be used when we only know the weights of the network rather than flows. There are many other indicators, like the network covering degree and so on. When we design a new or renew network plan of a specific area, we can compare these indicators in a more direct way. Gattuso and Miriello (2005) computed most of these indicators of 13 megacities, then compared and classified them. Similarly, Zhang *et al.* (2013) discovered the universal characteristics of urban rail transit networks (URTNs) from six indicators and calculated the topological efficiency of the network. Boeing analyzed 27,000 urban street networks in the USA from multi-scale perspectives, with most of these being geographical indicators (Boeing 2017).

Importantly, the growth of a traffic network not only involves local traffic demand but also derives from optimising traffic capacity across an entire network. As a result, concern for the basic structure of urban traffic networks will yield tremendous benefits in the long run for urban forms and land-use. This process, for now, has been mainly based on the assessment and analysis of network centralities.

Network centrality assessment is a fundamental concept in geographical network analysis. It was introduced earlier in the context of social network research (Bracey 1956; Freeman 1977; Smailes 1946). Numerous measurement methods have been proposed since then, and more recently, the Multiple Centrality Assessments (MCA) model and Kernel Density Estimation (KDE) methods were well-advised as introduced by the research group of Sergio Porta, Paolo Crucitti and Vito Latora. They assigned a family set of measure and quantified the centralities of urban spatial networks, compared with the method of space syntax, as a new alternative technical perspective of urban planning and design (Crucitti *et al.* 2006; Latora and Marchiori 2003; Porta *et al.* 2006a, 2006b, 2008; Porta and Latora 2007; Porta *et al.* 2012; Porta *et al.* 2009; Wang *et al.* 2011). Over the last decade, the group of Sergio Porta, Paolo Crucitti and Vito Latora carried out some practical projects to illustrate the MCA and KDE methods, like in Bologna (Porta *et al.* 2009) and Barcelona (Porta *et al.* 2012) within a similar research paradigm. The related explanation and demonstration of KDE method can be found in the in-depth reference of Porta *et al.* (2009). First, plot real urban situations, then use the KDE method to figure out the bumps of these activities. Subsequently, with analysis of the properties of network centrality, mixed maps of urban economic activities and network centralities are obtained. Finally, their interrelations emerge to be assessed by correlation analysis. This paradigm is quite simple for the better understanding of the relationship between human activities such as land-use types (Rui 2013) and landscape patterns, as proposed by Liu *et al.* (2015), and the formation and constitution of network structures. A variation model with a greedy algorithm considering integrated MCA index has been proposed to analyse urban green space accessibility (Chen and Chang 2015). Spatial autocorrelation indicators like Spearman, normally used to assesses the relationship between two variables, can be better described

Table 2 The geographical indicators

Indicators	Functions	References	Description of Functions
The average passenger flow per station	$\theta = \frac{\sum F}{N}$	Kansky (1963); Derrible and Kennedy (2011)	F is the traffic flow on each edge, the sum $\sum F$ means the total traffic flow.
The average edge length	$\eta = \frac{L}{M}$		L is the total edge length of the network.
The network extension	$\pi = \frac{L}{D}$		Here D is the network diameter, a fixed value.
The average weight of nodes	$\iota = \frac{L}{w}$		Here w is the weight.
The network covering degree	$Ncd = \frac{S_{network}}{S_{territory}}$		It is the surface ratio between the surface served by the network and territory surface.
The degree centrality	$DC(v_i) = \frac{k_i}{N-1}$	Freeman (1977)	The number of links incident on a node that can reflect the importance of the node v_i in relation to spatial geography, which indicates that a node with more neighbors is more important in a network.
The closeness centrality	$C_{closeness}(v_i) = \frac{1}{D_{ij}} = \frac{N-1}{\sum_{i \neq j} d_{ij}}$	Marchiori and Latora (2000)	It is denoted as the reciprocal of the average distance between each node pairs $D_{ij} = \frac{\sum_{i \neq j} d_{ij}}{N-1}$. This index means that if a node is closer to other rest nodes, it is more important in the network; it describes the relative location of a node.
The clustering coefficient	$Cc(v_i) = \frac{e_i}{m_i(m_i-1)}$	Watts and Strogatz (1998)	This coefficient is a measure of the extent to which a node, v_i , shares neighbors with other nodes, where e_i is the number of edges shared with local neighbors of node v_i , and m_i is the connection degree of local neighbors of node v_i . $Cc(v_i)$ is derived from the unit interval 0 to 1, which is called the local clustering coefficient. The global clustering coefficient is denoted as $Cc(G) = \frac{\sum_{i=1}^N Cc(v_i)}{N}$.
The betweenness centrality	$BC(v_i) = \frac{\sum_{j \neq s \neq t \in V} d_{ij}^{min, st} d_{im, st}^{min, st}}{(N-1)(N-2)}$	Freeman (1977)	It is defined as the total number of shortest paths between two separate nodes $d_{min, st}$ and passing through node v_i ; it reflects the load on node v_i and can alternately understood as the controllability of the node. On this basis, centrality can be clarified as $C_{betweenness}(v_i) = \sum_{i \neq s \neq t \in V} \frac{d_{ij}^{min, st}}{d_{min, st}}$, and the normalization of

Table 2 (continued)

Indicators	Functions	References	Description of Functions
The straightness centrality	$C_{straightness} = \frac{1}{N-1} \sum_{i=1}^N \sum_{i \neq j} \frac{d_{ij}^{Euclidean}}{d_{ij}}$	Vragović <i>et al.</i> (2005)	<p>betweenness $BC(v_i)$ is described as function shows, where $(N-1)(N-2)$ is the maximum possible value.</p> <p>It is used to measure the Euclidean distance versus the shortest path between nodes v_i and v_j.</p>
The assortativity coefficient	$r = \frac{\sum_{k_i k_j} k_i k_j [e(k_i k_j) - q(k_i)q(k_j)]}{\sigma_q^2}$	Newman (2002)	<p>It is normally used to measure the growth mechanism of urban traffic network. Joint probability distribution $P(k, j)$ means the probability of randomly choose one edge which the ends of this edge have degree k and j. $P(k, j) = \frac{m(k, j)\mu(k, j)}{2M}$, where $m(k, j)$ stands for the number of edges between nodes which have degree k and j, for $\mu(k, j) = 2$ if $k = j$, $\mu(k, j) = 1$ if $k \neq j$. Let $e(k_i, k_j)$ be the fraction of links that connect a node of degree k_i to a node of degree k_j. Let $P(k_j)$ be the probability that a node selected randomly in the network has degree k_j. The distribution of the excess degree of a node at the end of a randomly chosen link is $q(k_j) = \frac{(k_{j+1})p(k_{j+1})}{\sum_k k_j p(k_j)}$, σ_q^2 is the standard deviation of the distribution $q(k_j)$. In a network if the link between two nodes not related with their degrees value, then we can say the network has no degree correlations. Otherwise, the network has degree correlations. If the nodes have a bigger degree value preferred to connect with nodes also have a bigger degree value, then the network is assortative. If the nodes have bigger degree preferred to connect with nodes have smaller degree value then the network is disassortative.</p>
The eigenvector centrality	$x_a = \frac{1}{\lambda} \sum_{t \in G} a_{v,t} x_t$	Newman (2008)	<p>It is used to measure the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than same connections to low-scoring nodes. Normally we use the adjacency</p>

Table 2 (continued)

Indicators	Functions	References	Description of Functions
			matrix to find eigenvector centrality: Let $A = (a_{v,t})$ be the adjacency matrix $a_{v,t} = 1$, if vertex v is linked with vertex t , equal 0 otherwise. The centrality score of vertex v can be defined as: $x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G(v)} a_{v,t} x_t$ Where $M(v)$ is a set of the neighbours of v and λ is a constant. With a small rearrangement this can be rewritten in vector notation as the eigenvector equation: $Ax = \lambda x$.
The average shortest path length	$APL = \frac{1}{N(N-1)} \sum_{i \neq j} d_{min}^{ij}$	Albert and Barabási (2002)	Defined as the average number of steps along the shortest paths for all possible pairs of network nodes.
The network efficiency	$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{min}^{ij}}$	Latora and Marchiori (2002); Latora and Marchiori (2003)	It is the inverse of the APL between each pair of nodes v_i and v_j , it shows the average efficiency of transit flow or information between nodes in the network, and also passable to extended to nodes or edges.
The information centrality	$IC = \frac{\Delta E(G)}{E(G)} = \frac{\Delta E(G) - E(G')}{E(G)}$		It is defined by the change of network efficiency caused by removing some nodes or edges, where G' is the new network after the removing.

by a monotonic function. They can be used to examine the relationship between road network centralities and land-use categories (Rui 2013).

Notably, a comprehensive example has been provided by Scheurer *et al.* (2008). Their research transplanted the key assessment method of MCA to the integrated land-use and traffic planning in the Perth metropolitan area within five different scenarios, the betweenness centrality variation of scenario frequency boost, scenario light rail corridors, scenario middle ring centres, scenario fringe expansion and scenario composite wishbone. However, limitations still exist, as this method is based on the sapiential discussion of workshops and the experiences of some experts. Related suggestions only provide local optimum solutions. Taking many factors into consideration and observing the potential of the network centrality research being widely used in further exploration, an open-source urban traffic network analysis toolbox for ArcGIS was programmed by Sevtsuk and Mekonnen (2012).

2.4 The Mining of the Urban Traffic Network Communities

In the mathematics and computer science literature, this topic is usually termed ‘graph clustering’ or ‘partition’. It is derived from the ideas of sociologists to identify the

subgroups within different delineations. That is, in many complex networks, nodes group together to form clusters characterised by properties which are more or less independent of the properties of individual nodes and the network as a whole. A large number of algorithmic approaches to the problem have been considered (Barthelemy 2011; Curtis and Scheurer 2009; Donetti 2004; Newman 2012; Newman and Girvan 2004; Porter *et al.* 2009; Zhong *et al.* 2014), but interest in recent years has focused particularly on statistical inference methods. Network communities can be identified through the definition and optimization of modularity by the research of Newman and Girvan (2004). After that, Expert *et al.* (2011) expanded it with the interplay between distant locations, in which spatial interactions are ruled by a Power-law distance decay effect and the communities detected from the interaction network (Liu *et al.* 2014).

In some sense, the mining of urban traffic network communities is imperative for the planning and design process, as design the partition of OD zones or divide of urban areas into small or local communities, for human mobility patterns research, and benefit our scientific management. Traditional approaches in the mining community include spectral graph partitioning and hierarchical clustering. The spectral graph partitioning method finds the best division to cut the complete graph into two groups, then further subdivides those two until reaching the required number of groups. Michael Batty provided a method stemming from the algorithm from Newman and Girvan (2004), a divisive method in which edges are progressively removed from a network chosen with inverted order by computing their betweenness scores. Additionally, de Arruda *et al.* (2016) introduced the fast greedy algorithm to detect the communities of geographical networks. For uncovering urban socioeconomic clusters, Zhong *et al.* (2014) used the modularity detecting method, based on the movement data and its changing trends to identify essential elements of the urban spatial structure of Singapore, and figured out the city hubs and centres. Our recent work also focuses on the modularity detecting method but more emphasis on the growth impact of urban traffic multi-layer networks (Ding *et al.* 2018). Then, the spatial distribution of vehicles was analysed, and some giant vehicle clusters and distribution of vehicle cluster sizes are observed by Gong *et al.* (2016). Other related studies are shown in Table 3.

2.5 The Network Robustness and Vulnerability

Investigations have clearly shown that the statuses of the real network nodes are not equal. There is an obvious inhomogeneous network structure, and the nodes in different positions have different importance to the general network, also have different reactions to the network anti-destroying and anti-jamming ability. In a scale-free network, if 5% of important nodes were attacked, the whole network will be soon be paralysed. When a rail network is affected by internal or external factors, the capacity of some of the nodes or edges are exceeded, when this occurs, failure or error emerged.

Because of the combinations and connections with surrounding nodes and edges, failure and error are amplified and spread. This ripple effect eventually leads to the jam of part of a network or even the collapse of the entire network (Albert *et al.* 2000; Motter and Lai 2002). This phenomenon is called cascading failure. In other words, a network may be vulnerable, and research about robustness and vulnerability of urban traffic network is critical for our future planning of the network layouts, can make the

new network became functionally steady when facing traffic jams or natural disasters, and protect existing networks (Crucitti *et al.* 2004b).

Recently, most academics have focused on the connectivity and reliability of networks and tested their robustness and vulnerability in terms of network efficiency changes (Cao *et al.* 2013; Huang and Chow 2010), the ratio of giant component and the variation of general travel cost (Balijepalli and Oppong 2014), the accessibility alteration, and then in turn, to optimize the network functional layout (Crucitti *et al.* 2004a; Crucitti *et al.* 2003; Crucitti *et al.* 2004b; Sun *et al.* 2008b; Watts 2002; Wu *et al.* 2006a, 2007a, 2008a, 2008b; 2007b), and understand the diffusing rates (Sun and Wu 2005; Wu *et al.* 2004a; Yang *et al.* 2012; Zheng *et al.* 2007; Zhou *et al.* 2015). Among them the greatest and most widely used traffic related cascading failure model is the load and capacity model (Wu *et al.* 2006a, 2007a, 2007b), as proposed by Motter and Lai (2002).

Fundamentally, the cascading failure method in urban traffic networks is based on the removal of nodes or edges, to character what kind of changing will happen. There is another method exactly in contrast, adding nodes or edges, which is quite common in nature especially in the traffic networks. Although we can treat them as the same process in some sense, adding a road into the already existing road system is more common than abandoning a road in our daily life. Researchers have tested various kinds of attack strategies, normally based on the descending order of node degrees based, load-based, and other centrality measures mentioned previously in this paper (Albert *et al.* 2004; Crucitti *et al.* 2004b; Holme *et al.* 2002; Tang *et al.* 2014). Then, they determined the most harmful attack strategies and the criticality level of network elements, which depend on the role played in the network structure and the flow passthrough (Rodriguez-Nunez and Garcia-Palomares 2014). Computing the degree distribution, average path length, and clustering coefficient of a public transportation system in Shanghai, Li *et al.* (2007) assessed the complex characteristics of that system while simultaneously exploring the error tolerance and attack vulnerability of the network. Other researchers who examined a rail transit system have analysed the statistical characteristics and assessed the connectivity and reliability of the network using robustness theory (Taylor 2008; Taylor *et al.* 2006; Yin and Xu 2010). Among the earliest attempts to validate this idea, Buhl *et al.* (2006) not only considered the topological patterns and network efficiency, but also analysed fragility against random failures both of the Minimal Spanning Tree and Greedy Triangulation networks. Based on the network robustness analysis. Further results were provided by Ding *et al.* (2015), which found that the rail network protection should emphasise those nodes with the largest degrees and the highest betweenness values, and that a rail network's growth is likely based on the nodes with the biggest lengths of the shortest path. With these great efforts, research on complex network reliability, robustness and vulnerability made significant progress and gradually became one of the development trends in the traffic research field. More inspirations about network vulnerability and resilience can refer to Mattsson and Jenelius (2015).

2.6 The Big-Data-Based Complex Networks Research

To meets the current needs of traffic system while addressing the social and economic needs of future generations (Eladaway 2014), big-data-based research has now become

an important paradigm amongst researchers into urban traffic management. The social network data like smartphone data, social network data, taxi data, smart card data are widely used for the measure of Origin-Destination distribution, the traffic flow Assignment, the traffic congestion detection, the mining of travel behaviours and the urban computing (Chen and Yang 2013; Othman *et al.* 2014; Sun and Jin 2015; Widhalm *et al.* 2015; Yang *et al.* 2014; Zhao *et al.* 2016b), more importantly the movement of these social network users can represent the mobility of urban traffic users and the evolution trends of urban traffic network structure (Singha and Kalita 2013; Zhong *et al.* 2014).

The application of complex networks theory in big data are more focusing on the following parts: First is create the weighted network based on the big data, treat the user's movement data as traffic flows on networks (Soh *et al.* 2010), then some basic and traditional user behaviours can be detected, such as network congestion and centrality (Gao *et al.* 2013). The second direction is to evaluate the city's polycentricity and the location of Central Business District, to clarify the influence of people's travel behavior to the network structure and vice versa (Zhong *et al.* 2014). The third is exploration and mining of complex user traffic behaviour, based on records at the micro level of each cardholder's detailed travel behaviour, and in the medium and macroscopic level also reflects the characteristics of the users and groups (Frumin and Zhao 2012; Manley *et al.* 2015), to optimize the space layout, the traffic networkability, and to achieve sophisticated traffic management requirements (Wang *et al.* 2012; Zeydan *et al.* 2016), such as finding a real passenger path in a complex transit network using big data (Min *et al.* 2013; Sun and Xu 2012).

2.7 The Optimization and Related Applications

Some previous studies we mentioned above actually already contain a part of the content of network optimization methods, such as the research of network robustness and vulnerability and the application of big-data-based analysis, which give us a clear vision for the future development. The concepts of network optimization have been inherited and developed from the operational research, and have a long history and relatively dependable background. Ordinarily, most traditional network design problems belong to this class (Ahuja *et al.* 1993), based on the classification of Motter and Toroczkai (2007), for the research aims which are more focusing on the urban spatial network structures, we define the optimization methods within two classes, structural optimization and the flow related optimization. Although we distinguished these two methods, nevertheless, they always mixed together.

2.7.1 Structural Optimization

Structural optimization is based on the purely graph-theoretic methods and problems to shape the urban traffic network structures, and fulfill some specified properties. In fact, the methods of removing nodes or edges (Jiang *et al.* 2013; Liu *et al.* 2007; Zhang *et al.* 2007; Zhang *et al.* 2014b), adding nodes or edges (Huang and Chow 2010; Ma *et al.* 2016b; Zhang *et al.* 2014a), changing route directions (Hu *et al.* 2010), and network rewiring (Jiang 2014; Zhang *et al.* 2015) to enhance the network connectivity, capacity

and efficiency all belong to this class. For now, most optimization research without spatial constraints uses similar contexts to those we proposed above.

Other researchers like Schweitzer *et al.* (1997) investigated the evolution of road networks during the optimization process based on minimised travel bypass and focused on minimised construction cost. Additionally, Barthelemy and Flammini (2006) suggested the optimization principles of traffic networks and proposed a spanning tree simulation model, in which tree structures change with their defined numerical value parameters. The maximum centrality spanning trees method is also proposed to optimise the network structure (Scellato *et al.* 2006).

Whether or not studies consider the spatial constraints, some critical qualitative features of these networks can be imitated using the simulated optimised network structures. These results showed that a traffic network with a fixed structure could be distinguished as a random or a uniform state, and they revealed that roads and hierarchies, which are often thought to be the products of conscious design, can also be self-organized, or obey the Power law. A Monte Carlo optimization model that minimised the construction and maintenance costs of a planar network, bearing in mind the entire Euclidean distance, is presented by Gastner and Newman (2006), named the “GN model”. Moreover, they created and applied a population-driven model to the USA case, which is more related to the real situations. A local optimization process was given by Barthelemy and Flammini (2008), for the purpose of maximising the reduction of the cumulative distance, the network growing process was simulated. Most critically and realistically, Kleinberg (2000) and Li *et al.* (2010) proved that with the suggested optimal controlling parameter, an optimised network structure can be observed. After that, Rui and Ban (2011) and Rui (2013) tested the relationship between projected and existing nodes and segments based on the local constraints and legality tests in some real situations when the local network is growing, including angles (between projected segment and existing segments) and distance (between projected nodes and existing nodes).

2.7.2 Flow Related Optimization

Flow-related optimization is founded on a different perspective, with concerns about some discrete and continuous problems for the purpose of reducing traffic jams or minimise the travel time (cost) or maximise the system utility, and more specifically, as the traffic network design models did, considering about traffic flows effects. A great general network design model was proposed by Magnanti and Wong (1984), in which almost all problems encountered in transportation planning can be viewed as its specialisations or variations. In fact, DNDP and CNDP are both considered bi-level nonlinear optimization problems with and without the budget constraint situation (Liu 2001, 2003), with the user equilibrium assignment (UE), the system optimum assignment (SO) and other system equilibrium situations all in mind. The group of Gao, Sun, Wu investigated and analysed an optimal traffic network structure for resisting traffic congestion featuring different volumes of traffic. They introduced a cost function based on UE assignment that ensured the flow balance of traffic systems and expanded both the bi-level programming model and its solution algorithms (Gao *et al.* 2005; Sun *et al.* 2008a; Wu *et al.* 2008a). Oliveira *et al.* (2014) developed flow conditions based on the research of Li *et al.* (2010) in small world networks and obtained the best flow

conditions using decentralised algorithms. A centrality index based on the maximum power flow through the edges was introduced, and the Max-Flow Min-Cut Theorem (Ford–Fulkerson theorem) is used to estimate the capacity of the edge (Dwivedi and Yu 2013; Dwivedi *et al.* 2010). Based on this, optimal solutions of the maximum flow problem on complex networks with capacity constraints on nodes were given. Another interesting model, the Microscopic Congestion Model (MCM), was introduced to identify the urban traffic congestion hotspots in the context of the complex network (Solé-Ribalta *et al.* 2016b, 2016c).

Most network optimization methods are based on the single-objective and multi-objective programming (Gutierrez-Jarpa *et al.* 2017; Gutierrez-Jarpa *et al.* 2013; Jozefowicz *et al.* 2008; Ulungu and Teghem 1994). Some research uses the multi-objective programming method directly to optimize urban traffic network (Gutierrez-Jarpa *et al.* 2017; Gutierrez-Jarpa *et al.* 2013), this kind of method is figurative, speedy and handy. Thus, the basic function of the single-objective bi-level programming can be easily expanded as a multi-objective bi-level programming model (Chen *et al.* 2010), when the systems have more than one aim.

2.8 The Co-Evolution Research

The study of co-evolution of urban traffic networks must be combined with the study of the growth of urban traffic networks. General urban traffic network growth can refer to work by Xie and Levinson (2009a), which fundamentally demonstrates the network growth models and the inner relationships between the traffic networks, urban forms and some macro and microeconomic policies and other aspects of urban subsystems. Urban land-use and urban traffic network co-evolution processes and some general models have been proposed by Levinson, Yerra and Xie (Iacono *et al.* 2008; Levinson *et al.* 2007; Levinson and Yerra 2005, 2006; Xie and Levinson 2007, 2009b; Yerra and Levinson 2005). These models consider the network structure, travel demand model, revenue model, cost model, investment model and accessibility and land-use model in a comprehensively way. A paradigm thus appears, which is use these various of models to simulate the changing trends and interrelations between the land-use and traffic system, followed a sensitivity analysis with considering the influences of changing variables, then some intuitional and visualized results earned. Furthermore, many varieties and further research were proposed recently with general bi-level programming design (Li *et al.* 2015) and applied with CA models (Wu *et al.* 2016), and much further co-evolution among land development, population, roads and vehicle ownership were considered.

Focusing on the co-evolution model of the 19th and twentieth century London, Levinson and Xie found that population distribution and network density are positively correlated. They validated a simulation model to fit the empirical evidence better and noted that evolution is an iterative process of interaction, investment and divestment. Moreover, they illustrated how surface traffic networks can grow and decline spontaneously over time, providing further evidence for the property of self-organization (Levinson 2007; Xie and Levinson 2007, 2009a, 2009b). In a similar vein, a new dynamics model based on the logistic equation to capture the dynamic characteristics of the co-evolution process between the road surface and urban traffic structure was developed (Wu *et al.* 2013). After that, the co-evolution of road network expansion

and urban traffic growth grounded in the case of Beijing is described (Wu *et al.* 2014), and a co-evolution model is suggested with a stability analysis and numerical simulation. Zhao *et al.* furthered the research of Levinson *et al.* and revealed the interactions between population distribution and urban road network evolution based on the simulation models. Using a relative neighborhood graph (RNG) and a Fermat-Weber location problem as the connection mechanism, these authors revealed that the population distribution in a city is the leading cause of heterogeneity in a network topology structure (Zhao *et al.* 2014, 2016a, 2015). They analysed and simulated the relationship between two types of economic agents – government investment decisions versus household and company locations – to show that average accessibility for employment and population increases in the evolutionary process.

Although those previous investigations are important and necessary for further research of urban planning and design, they all treat networks as purely planar and do not consider the influence of multilayer networks (similar meaning with independent networks or multiplex networks), which illustrate and represent different appropriate urban traffic modes and lines.

2.9 Multilayer Network Theory and its Applications

More recently, multilayer network-based studies have attracted attention, illustrating and representing various appropriate urban traffic modes. Development of rail and road networks is inseparable from the development of a prosperous urban area; thus, research on multilayer networks has scientific potential and fulfills a real need. To bridge the gap of the multilayer network representation of real-world networks, Kurant and Thiran (2006) first proposed a general multilayer model that facilitated the description and analysis of multilayer networks. The authors examined three transportation networks and found that a tiny error on a multilayer network could cause cascading failures. They also investigated the relationships among degree, betweenness and real loads and found that, as opposed to the commonly acknowledged view, the correlations in their dataset between the three factors were not that apparent. Later, Buldyrev *et al.* (2010) demonstrated that electrical blackouts resulted in the cascading failure of the Internet communication network and power stations in Italy. Their research is a breakthrough milestone in the measurement of interactions in layered networks. They deciphered the critical percolation threshold, which is larger than the equivalent threshold of a single-layer network of the same size. Albert *et al.* (2016) have introduced a standardised model to simulate the elements navigating those networks and analysed congestion in multilayer transportation networks.

Most outstandingly, a model of traffic dynamics revealing a transition at the onset of cooperation between layered networks is proposed by Gu *et al.* (2011); they introduced the notion of the cooperation strength of different layers to illuminate the relation of coupling networks. Furthermore, the ratio of speeds of coupling different modes, network accessibility, mobility and the behaviour of different layers were considered (Aleta *et al.* 2016; Gallotti *et al.* 2015; Ma *et al.* 2016a, 2015; Morris and Barthelemy 2012; Solé-Ribalta *et al.* 2016a; Strano *et al.* 2015), and later Ding *et al.* (2017) proposed a heuristic urban transportation network design method showing that transit users' route choices change dramatically through the co-evolution process.

These research works offer a broad range of new perspectives for investigating the aspects of multilayer networks, considering the interrelationship and cooperation of different traffic layers and modes, and much further, optimising network layout (Morris and Barthelemy 2012; Strano *et al.* 2015).

3 Conclusion and Future Research Directions

This paper comprehensively reviewed recent applications of complex network theory in urban traffic network studies, especially road and rail networks, in terms of urban planning-related points.

In section 2 we analysed recent research and found strong potential for the further application of complex network theory in urban traffic network studies. For these studies in Table 3 and Fig. 2, even though some parts of them can be highly-connected, mixed or position exchanged, we can still easily comprehend research trends when considering the background of urban traffic networks. These topics are innovative and remain open to urban traffic network planners and have significant and obvious research value. The network topology and related centralities studies can illustrate some of the network characteristics, but in urban studies, a comprehensive zonal research is appreciated and needs to be further discussed. The numerous research findings of the topological and geographical characteristics provide new holistic concepts of urban traffic networks for planners beyond their basic images and functions, not only from micro but also from macro-perspective. Many explorations of this concept focus on network dynamics and the network optimization, but integrated co-evolution studies are still relatively rare. Findings of network operations and dynamics offer new visions of urban traffic network management and optimization, such as how to figure out hub nodes or potential areas for the future investment and the change and impact of user's behaviours. Some novel concepts of physics have never been imported to urban traffic network planning, like the network robustness and vulnerability, but obviously, they have great research importance and possibility. Recently, the infrastructure network vulnerability related analysis already used by some researches to fight against with natural disasters (Kermanshah *et al.* 2017), more and more related and extended research can be seen soon. Also, as the acquisition of big data is difficult, most data concerning network evolution and growth come from these simulation models, actual data is relatively rare (these data include demography data, the land-use changing data, the economic centre shifting data, and most importantly, network structure changing data). Actually, in the context of urban traffic networks studies, the co-evolution process is obviously crucial to our understanding of the relationships between traffic networks and urban land-use. As the hot point of network science, multilayer research has recently become more attractive compared with single-layer, because it is closer to the real situations (the complex system of rail and road networks). These discussions concerning big data sets between different layers are thought-provoking, as they offer infinitely varied contents to link the traffic studies and network studies, and might lead to a new paradigm of study urban traffic networks in a comprehensive manner.

Sadly, those physical models in recent urban planning paradigm are not evident or popular, predominantly because of the requirement of knowledge of Modern Physics, Mathematics and Statistics. Urban planners and designers considering these social problems rarely have this particular skill. The education of quantitative geography should include the complex network theory as modern operations research did.

For now, an application framework and methodological perspective for applying complex network theory in urban planning are still not established. A more integrated real urban and metropolitan co-evolution model is needed. Actually, most of these models focus on network centralities, the relation and integration between networks and surrounding contents also need to pay more attention, not only from the perspectives of single layer but also multilayer networks. Like the integrated researches of land-use growth, traffic network growth, population growth, human behaviour, industrial distribution and economic development related studies need to be conducted further. The most important effort of these evolution models is to discover the actual co-evolution laws matching different urban scales.

Tremendous interest has been aroused in other research into synchronous and coherent behaviour, such as the timetable optimization (Tao and Ceder 2015; Wu *et al.* 2015), traveller behaviours and vehicle operation (Meignan *et al.* 2007) and network controllability (Liu *et al.* 2011). These new research directions have strong research possibilities which are especially obvious for their use in urban traffic studies.

Research into network weights and directions in urban traffic planning should be more stressed, especially in areas that use weights to represent the transportation ability or capacity. Most complex network related research treats urban traffic network as unweighted and undirectional network, so the process becomes highly simplified.

Effort should put into clearly identifying the exploration of important nodes. Although many node importance indicators have been proposed, most of them do not comprehensively take into account complex urban properties.

The optimization method still cannot perfectly fulfill the requirement of real situations. Specifically, multi-objective, multilayer methods and modularity design technology need more attention. Particularly, a user-friendly GUI design in the GIS design environment can attract urban designers' broader attention. As Ducruet and Lugo (2013) (p. 15) illustrated that "...no software proposes yet an integrated package combining spatial statistics, GIS, graph visualization, and network analysis 'all in one'...", this situation is not changed, hence for the computer scientists and geography researchers, with further consider the data acquisition and network generation, there exist a great opportunity.

Ultimately, our efforts have led to a new understanding of recent research and future directions which deserve further investigation. Hopefully, this review can benefit to urban and transport planners for their integrated researches.

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Appendix

Table 3 The complex network theory related applications in urban traffic network research

References	Representation methods	Basic network model used	Network	Weighted research	Topological related studies	Geographical related studies	Network communities mining related studies
Latora and Marchiori (2002)	P	S	R,B		•	•	
Sen <i>et al.</i> (2003)	P	W	R		•	•	
Jiang and Claramunt (2004a)	S		S		•	•	
Jiang and Claramunt (2004b)	S		S		•	•	
Wu <i>et al.</i> (2004b)	P	S	R		•	•	
Gattuso and Mirrello (2005)	P		R		•	•	
Hillier and Iida (2005)	P		S		•	•	
Levinson and Yerra (2005)	P	R	S	•	•	•	
Andersson <i>et al.</i> (2006)	P	S	S		•	•	
Angeloudis and Fisk (2006)	P		R		•	•	
Barthélemy and Flammini (2006)	P	E	S	•	•	•	•
Buhl <i>et al.</i> (2006)	P		S		•	•	
Crucitti <i>et al.</i> (2006)	P		S		•	•	
Gastner and Newman (2006)	P		S	•	•	•	
Lammer <i>et al.</i> (2006)	P	S	S		•	•	
Levinson and Yerra (2006)	P	R	S		•	•	
Porta <i>et al.</i> (2006a)	D	S,W	S	•	•	•	
Porta <i>et al.</i> (2006b)	P		S		•	•	
Scellato <i>et al.</i> (2006)	P		S		•	•	

Table 3 (continued)

References	Representaiton methods	Basic network model used	Network	Weighted research	Topological related studies	Geographical related studies	Network communities mining related studies
Taylor <i>et al.</i> (2006)	P		S			•	
Wu <i>et al.</i> (2006b)	P	E,S,W				•	
Jiang (2007)	P	S	S		•		
Liu <i>et al.</i> (2007)	P	S		•			
Wu <i>et al.</i> (2007a)	P	S					
Wu <i>et al.</i> (2007b)	P	E,S,W	S	•			
Xu <i>et al.</i> (2007)	P,D		B		•		
Zheng <i>et al.</i> (2007)	P	S			•		
Barthélemy and Flammini (2008)	P		S		•		
Hu <i>et al.</i> (2008)	D	R,S	S		•		
Porta <i>et al.</i> (2008)	P		S	•			
Scheurer <i>et al.</i> (2008)	P		R	•			
Taylor (2008)	P		S		•		
Domenech (2009)	P	W	R		•		
Masucci <i>et al.</i> (2009)	P,D	R,E	S		•		
Xie and Levinson (2009b)	P	R	R	•			
Curtis and Scheurer (2010)	P		R	•			
Daganzo (2010)	P		R,B	•	•		
Hu <i>et al.</i> (2010)		S			•		
Derrible and Kennedy (2010a)	P		R		•		
Derrible and Kennedy (2010b)	P		R		•		
Huang and Chow (2010)	P	S			•		

Table 3 (continued)

References	Representaiton methods	Basic network model used	Network	Weighted research	Topological related studies	Geographical related studies	Network communities mining related studies
Li <i>et al.</i> (2010)	P	R,E	R		•	•	
Yin and Xu (2010)	P		S	•		•	
Eisenstat (2011)	P		S		•	•	
Gu <i>et al.</i> (2011)	M	E,S,W	R	•	•	•	
Rui and Ban (2011)	P	R	S		•	•	
Wang <i>et al.</i> (2011)	P		S		•	•	
Derrible (2012)	P		R	•	•	•	
Morris and Barthelemy (2012)	P	E			•	•	
Porta <i>et al.</i> (2012)	P		S		•	•	
Sevtsuk and Mekonnen (2012)	P		S		•	•	
Batty (2013)	P,D	E,S,W	R,S,B	•	•	•	•
Gutierrez-Jappa <i>et al.</i> (2013)	P		R		•	•	
Jiang <i>et al.</i> (2013)	P	S			•	•	
Quintero <i>et al.</i> (2013)	P		S		•	•	
Rui (2013)	P	R,S	S		•	•	
Balijepalli and Oppong (2014)	P		S	•		•	
Chen <i>et al.</i> (2014)	P,D		B			•	
Du <i>et al.</i> (2014)	M	S			•	•	
Rodriguez-Nunez and Garcia-Palomares (2014)	P		R		•	•	
Jiang (2014)		E,S				•	
Masucci <i>et al.</i> (2014)	D	W	S		•	•	

Table 3 (continued)

References	Representaiton methods	Basic network model used	Network	Weighted research	Topological related studies	Geographical related studies	Network communities mining related studies
Zhang et al. (2014b)	P	E,S			•		
Zhao et al. (2014b)	P		S			•	
Zhong et al. (2014)	P		S	•		•	•
Chen and Chang (2015)	P		S			•	•
Ding et al. (2015)	P		R		•	•	
Li et al. (2015)	P	R	S	•		•	
Liu et al. (2015)	P		S		•	•	
Yang et al. (2015)	P		R	•		•	
Mattsson and Jenelius (2015)			R,S			•	
Strano et al. (2015)	P		R,S	•		•	
Sun et al. (2015a)	P		R			•	
Sun et al. (2015b)	C		B		•	•	•
Zhao et al. (2015)	P		S			•	
Aleita et al. (2016)	M		R,S	•		•	
de Aruda et al. (2016)	P		S		•	•	•
Xing et al. (2016)	P	S,W	R	•	•	•	
Du et al. (2016)	M				•		
Solé-Ribalta, Gómez et al. (2016b)	P		S			•	
Wu et al. (2016)	P		S			•	
Zhao et al. (2016a)	P		S			•	
Ding et al. (2017)	M		R,S			•	

Table 3 (continued)

References	Representation methods	Basic network model used	Network	Weighted research	Topological related studies	Geographical related studies	Network communities mining related studies
Wang <i>et al.</i> (2017)	P	S	S	•		•	
Gutierrez-Jarpa <i>et al.</i> (2017)	P		R		•	•	
Ding <i>et al.</i> (2018)	M		R,S		•	•	
Lin <i>et al.</i> (2018)	P		S			•	
Zhang <i>et al.</i> (2018)	P		R		•	•	
Wang <i>et al.</i> (2018)	P	S	S	•		•	

References	Hub nodes or important links	Network robustness and vulnerability related studies	Network optimization related studies	Network growth	Integrated studies	Co-evolution studies	Multilayer network related studies
Latora and Marchiori (2002)							
Sen <i>et al.</i> (2003)							
Jiang and Claramunt (2004a)	•						
Jiang and Claramunt (2004b)	•						
Wu <i>et al.</i> (2004b)	•						
Gattuso and Mirriello (2005)							
Hillier and Iida (2005)	•						
Levinson and Yerra (2005)	•		•			•	
Andersson <i>et al.</i> (2006)				•	•		
Angeloudis and Fisk (2006)		•					

Table 3 (continued)

References	Hub nodes or important links	Network robustness and vulnerability related studies	Network optimization related studies	Network growth	Integrated studies	Co-evolution studies	Multilayer network related studies
Barthélemy and Flammini (2006)			•				
Buhl et al. (2006)		•	•				
Crucitti et al. (2006)			•				
Gastner and Newman (2006)	•		•				
Lammer et al. (2006)	•		•				
Levinson and Yerra (2006)	•		•			•	
Porta et al. (2006a)	•						
Porta et al. (2006b)	•						
Scellato et al. (2006)							
Taylor et al. (2006)	•	•				•	
Wu et al. (2006b)							
Jiang (2007)	•						
Liu et al. (2007)	•	•	•				
Wu et al. (2007a)		•					
Wu et al. (2007b)		•					
Xu et al. (2007)							
Zheng et al. (2007)							
Barthélemy and Flammini (2008)	•		•				
Hu et al. (2008)			•				
Porta et al. (2008)	•		•				
Scheurer et al. (2008)			•			•	
Taylor (2008)		•	•			•	

Table 3 (continued)

References	Hub nodes or important links	Network robustness and vulnerability related studies	Network optimization related studies	Network growth	Integrated studies	Co-evolution studies	Multilayer network related studies
Domenech (2009)				•		•	
Masucci <i>et al.</i> (2009)			•	•			
Xie and Levinson (2009b)	•		•	•			
Curtis and Scheurer (2010)	•		•		•		
Daganzo (2010)			•				
Hu <i>et al.</i> (2010)	•			•			
Derrible and Kennedy (2010a)	•						
Derrible and Kennedy (2010b)			•	•		•	
Huang and Chow (2010)	•	•		•			
Li <i>et al.</i> (2010)			•	•			
Yin and Xu (2010)	•	•	•	•			
Eisenstat (2011)			•	•			
Gu <i>et al.</i> (2011)			•	•			
Rui and Ban (2011)			•	•		•	•
Wang <i>et al.</i> (2011)	•		•	•			
Derrible (2012)	•						
Morris and Barthélemy (2012)			•	•			•
Porta <i>et al.</i> (2012)							
Sevtsuk and Mekonnen (2012)	•				•		
Batty (2013)	•		•	•			
Gutiérrez-Jaña <i>et al.</i> (2013)	•		•	•			
Jiang <i>et al.</i> (2013)			•	•			
Quintero <i>et al.</i> (2013)			•	•			

Table 3 (continued)

References	Hub nodes or important links	Network robustness and vulnerability related studies	Network optimization related studies	Network growth	Integrated studies	Co-evolution studies	Multi-layer network related studies
Rui (2013)			•	•	•	•	
Balijepalli and Oppong (2014)	•	•	•				
Chen et al. (2014)	•		•				
Du et al. (2014)			•	•			•
Rodriguez-Nunez and Garcia-Palomares (2014)	•	•	•				
Jiang (2014)	•		•	•			
Masucci et al. (2014)			•	•			
Zhang et al. (2014b)		•	•				•
Zhao et al. (2014b)	•		•	•	•	•	
Zhong et al. (2014)	•				•		
Chen and Chang (2015)			•		•		
Ding et al. (2015)	•	•	•	•	•	•	
Li et al. (2015)	•		•	•	•	•	
Liu et al. (2015)	•				•		
Yang et al. (2015)	•	•			•		
Mattsson and Jenelius (2015)		•					
Strano et al. (2015)	•		•		•		•
Sun et al. (2015a)	•	•			•		
Sun et al. (2015b)							
Zhao et al. (2015)	•		•	•	•	•	
Aleta et al. (2016)			•				•

Table 3 (continued)

References	Hub nodes or important links	Network robustness and vulnerability related studies	Network optimization related studies	Network growth	Integrated studies	Co-evolution studies	Multilayer network related studies
de Arnuda <i>et al.</i> (2016)							
Xing <i>et al.</i> (2016)	•	•					
Du <i>et al.</i> (2016)			•	•			•
Solé-Ribalta, Gómez <i>et al.</i> (2016b)	•		•		•		
Wu <i>et al.</i> (2016)				•	•	•	
Zhao <i>et al.</i> (2016a)	•		•	•	•	•	
Ding <i>et al.</i> (2017)			•	•	•	•	
Wang <i>et al.</i> (2017)	•				•		
Gutierrez-Jarpa <i>et al.</i> (2017)	•		•		•		
Ding <i>et al.</i> (2018)			•	•	•	•	•
Lin <i>et al.</i> (2018)	•						
Zhang <i>et al.</i> (2018)	•	•					
Wang <i>et al.</i> (2018)	•				•		

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