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Impacts of Public Transport Policy on City Size and Welfare

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Abstract
This paper introduces a clear distinction between interregional and intraregional transportation cost in a mixed New Economic Geography and Urban Economics model (Krugman and Livas Elizondo 1996; Martin and Rogers 1995). With the assumptions that public spending on transport infrastructure has some different effects on city size and welfare, and that it is financed by a proportional tax on regional income, an absence of regulation enables productive activities to agglomerate in the most favored region. Considering the presence of urban costs (e.g. commuting costs and land rents), public transport policy for developed countries can be used as a strategic instrument for regional planning, leading to a decrease in the spatial size of cities.

Keywords Economic geography · Urban costs · Transportation · Public policy · Welfare

1 Introduction

Interregional and intraregional transportation costs have direct and indirect effects on industrial location and regional integration. Regions with better access to domestic transport networks are usually the most attractive for economic activity (Stepniak and Rosik 2018); consequently, urbanization processes usually develop around the city center, which leads to higher urban costs, entailing numerous socio-economic and environmental changes over the long term, including dynamic demographic, residential, industrial and commercial zoning (Cavailhès et al. 2007). Public transport policies are essential to accompany the relocation of both firms and workers and guide sustainable, responsible and efficient development, taking into consideration the combination of the endogenous characteristics of cities and transport networks to enhance spatial equity (Bertolini and Spit 1998; Holl and Mariotti 2018). This paper uses two

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kinds of transportation costs to analyze the impacts of public infrastructure spending on the city size and welfare: the intra and interregional transportation costs.

In the literature there are different theoretical models as those proposed by Helpman and Krugman (1985) and Krugman (1991),[1] which studied the relationship between interregional trade costs and industrial location. These models had two points in common: firstly, they demonstrated that the reduction in the interregional transportation cost induces more agglomeration in fewer regional centers, and therefore increases spatial disparities between regions. Secondly, by assuming the neutrality of space, they considered regions as simple dots without spatial dimensions, and consequently ignored intraregional transportation costs (Behrens et al. 2009).

Martin and Rogers (1995) considered domestic and international transportation infrastructure in their analysis of how public spending on transport infrastructure affects the location of economic activity and welfare. They showed that any improvement in local transport networks in the home country attracts firms to this country, if the increase in the demand for manufacturing goods is larger than the decrease in demand due to the associated increase in taxes. On the other hand, improving international transportation infrastructure in a country which has a poor quality of domestic infrastructure will imply a relocation of firms outside this country.

In reality, the intraregional transportation costs are significant and positively affected by higher urban costs (poor quality of transportation networks, land rents, commuting cost, delays, accidents, pollution, etc.); these important dimensions form the basis for urban economics. Several research contributions have been made, but most of them are neglected by interregional trade economists. Tabuchi (1998) unified the disparate disciplines that form the basis of urban economic, particularly proposing a synthesis of Alonso (1964) and Krugman (1991) by developing a general equilibrium model, with the presence of congestion costs (land rents and commuting costs). Ottaviano et al. (2002) analyzed variations in transportation cost and travel using a system of two cities (with agriculture sector and fixed housing consumption), where the commuting cost is exogenous. According to a quasi-linear utility function, they succeeded in providing an analytical solution that demonstrated the possibility of asymmetric equilibrium (when the commuting costs are different between cities), and found the inverted U-shape equilibrium when transport costs decrease. Other papers contributed to the linkage of these two growing fields. For instance, Krugman and Elizondo (Krugman and Livas Elizondo 1996) studied the effects of trade liberalization and congestion costs on the spatial sizes of cities in a developing country (Mexico).

Land rents and commuting costs[2] along with numerous other factors can define urban costs. In most developed and developing countries they represent a large and growing share of the household budget. The performance of firms is negatively affected by the increase of urban costs in large cities (Cavailhès et al. 2007). In other words, local firms will bear higher production costs due to higher land rents, and wages that

[1] In 2008, the Nobel Prize in Economics was awarded to Paul Krugman for his major contribution to new theories of trade and location of activities, highlighting the importance of urban economics in analyzing contemporary societies.

[2] Henderson et al. (2001) estimated that property prices and commuting times are 100% higher for a metropolitan area of 5 million inhabitants than for a city of 100,000. Bairoch (1985) has noted that in a city of 100,000 inhabitants (assuming 35,000 inhabitants per km^2), it is possible to travel from anywhere in the city to the center in less than 15 min, while in a city of one million inhabitants, this same trip can take one hour.

should be paid to workers to compensate urban costs. Consequently, these costs act as barriers to entry and result in higher prices, rendering firms less competitive in both local and foreign markets.

Despite the advantages of the economic agglomeration (Duranton and Puga 2004; Duranton and Turner 2012), the presence of high urban costs may negatively affect the location of firms within large cities (Brakman et al. 1996; Krugman and Livas Elizondo 1996; Tabuchi 1998; Brueckner 2000; Duranton and Puga 2001; Cavailhès et al. 2004; Cavailhès et al. 2007; Goryunov and Kokuvin 2014; Jedwab et al. 2017). This incentivizes firms to leave the central urban areas, forming secondary employment centers or clusters (Henderson and Mitra 1996; Lucas Robert and Rossi Hansberg 2002). Workers may profit from a less localization cost by choosing to live in suburban or rural areas (Glaeser and Khan 2004; Holmes and Stevens 2004; Zhang 2016). Firms may be able to pay lower wages than in the city center, while relocating inside the metropolitan area (i.e. proximal to the traditional center), thus benefiting from agglomeration economics. Also, urban costs may slow down (intensify) rural-to-urban (urban-to-rural) migration, thereby decreasing urban expansion in the long run (Jedwab et al. 2017). The formation of small clusters or cities within a wider metropolitan area, often radiating from a traditional center, is known as a *polycentric city*, which appears to be a natural response to increasing urban costs (Cavailhès et al. 2007).

We also propose a mixed general equilibrium model of New Economic Geography (NEG) and New Urban Economics (NUE), including a public sector. In this model, we assume that public expenditure may positively affect the labor supply and thus the productivity of workers when increasing the quality of local transportation infrastructure (Duranton et al. 2014; Mayer and Trevien 2017). For this, we introduce two kinds of transport costs: intraregional and interregional transport costs. Also, we consider the existence of urban cost inside each region in terms of commuting cost and resulting land rent, then we can analyze the different impacts of public spending on transport infrastructure on city size and welfare. Our main contribution is the explicit consideration of impacts the two types of transportation costs (and their associated infrastructures) in a general equilibrium model that features both NEG and NUE attributes. Unlike previous studies (Krugman 1991; Krugman and Livas Elizondo 1996; Tabuchi 1998; Ottaviano et al. 2002), our model integrates the public sector by introducing taxes on regional incomes, with the assumption of the immobility of labor between regions.

This paper is in four parts. Section 2 presents the general equilibrium model of economic geography and urban economics and derives the short-run equilibrium conditions. Section 3 shows the impact of public transport policies on the city size. It analyses the stability conditions of the spatial geographic equilibriums. Section 4 derives policy implications on welfare. Section 5 presents conclusions.

2 The Model

According to Krugman and Livas Elizondo (1996), models of economic geography should incorporate a tension between different spatial forces (agglomeration and dispersion forces). For example, agglomeration forces can take into consideration both external economies and market size effects, (i.e., the forward and backward linkages).

Dispersion forces can include some external diseconomies, i.e., pollution, congestion, commuting costs and urban land rents, and the delocalization from large cities to small ones. A general description of the model is presented in Fig. 1.

2.1 Assumptions

In this model we include only the centripetal forces that arise from the interaction among economics of scale, intra and interregional transport costs and market size, i.e., backward and forward linkages. The only dispersion force that we take into account is commuting cost/land rent. Despite the fact that there are some other external diseconomies in real urban areas, we adopt this choice in order to keep this model as simple as possible. As usual, the agglomeration force finds its origin in the need to reduce the interregional transportation costs of manufacturing goods, but the principal dispersion force comes from the land consumption and the resulting need for urban workers to commute between their homes and workplaces. While considering the general equilibrium framework of monopolistic competition, we introduce the commuting cost explicitly (Krugman and Livas Elizondo 1996; Tabuchi 1998), associated with the iceberg transportation costs of economic geography.

Consider an economy with two symmetric regions, 1 and 2. In each region there are two zones, one urban and one rural; and two factors of production, labor and land. The labor force is perfectly mobile between sectors *within* each region, but impossible *between* regions. This assumption reflects the lower mobility of workers between regions, especially in Europe; in fact, less than 2% of workers change their origin region to another (Faini 1999). However, workers can change their sector of activity if they get a higher wage rate. In every region inter-sectorial competition exists aiming to attract workers. The inter-sectorial repartition of labor is endogenous in this model. In order to consider the existence of the agriculture sector, we suppose that there is an activity tied to the land.

The total population L is presented below, while L_1 and L_2 represent the composite populations in regions 1 and 2, respectively. The repartition of the total population between both regions is fixed exogenously and equally, to take into account the perfect

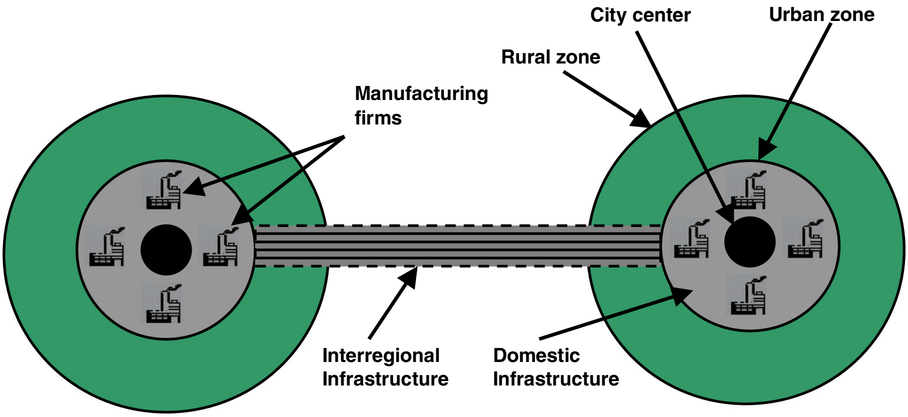

Fig. 1 General description of the model

symmetry of the model. It includes all categories of workers that can be employed in the agricultural sector, the manufacturing sector, and the public sector.

$$L = L_1 + L_2 \quad (1)$$

In each region, there is a share of workers λ_k that works and lives in the urban zone, employed exclusively in the industrial sector. The other share $1 - \lambda_k$ lives and works in the rural zone, employed exclusively in the agricultural sector. We assume that each urban worker needs a fixed living space, one unit of land.[3] The size of the urban zone is $\lambda_k L_k$; with a linear city, the maximum distance to the city center is given by the following equation. Since workers are concentrated in each region, the most remote manufacturing workers must commute a distance$\lambda_k L_k/2$, and all urban workers who live closer to the center must pay a land rent that absorbs any saving in commuting cost.

$$x = \lambda_k L_k/2 \quad (2)$$

Regarding the rural zone, the wage is normalized to the unity and the land, as abundant and having an opportunity cost equal to zero. Contrary to the manufacturing workers, the share of workers employed in the agricultural sector pays almost no land rent and have no commuting cost. The total net income that is necessary for the consumption of homogenous and differentiated industrial goods is:

$$E_{k,0} = (1-t)(1 + F_k/L_k) \quad (3)$$

where F_k is the land rent in the urban zone k, and t is a proportional tax on the regional income.

In the urban zone, the net income for a worker localized at the distance x from the city center is given by the following expression, where $(1 - \gamma_k x)w_k$ is the net wage of both commuting cost (γ_k) and land rents for all workers. It means that workers who live outside the city center will not pay any land rent, but receive a net wage due to the time spent in commuting. Workers who live in the urban zone will receive more money, but also pay for the land rent. Commuting costs and the resulting land rent are obviously diseconomies of city size (Krugman and Livas Elizondo 1996).

$$E_k(x) = (1-t)((1-\gamma_k x)w_k + F_k/L_k) - F_k(x) \quad (4)$$

The land rent $F_k(0)$ is trivially a decreasing function of x, minimum for $x = \lambda_k L_k/2$. Land being allocated to the highest bidder, this minimum is equal to the opportunity cost of land, which is equal to zero, $F_k(\lambda_k L_k/2) = 0$. From this, we can deduce the final expression for the industrial wage.

$$w_k = \frac{2}{2-\gamma_k \lambda_k L_k} \quad (5)$$

[3] The assumption of consumption of an identical and exogenous "unit of land" between consumers is very restrictive, and it is particularly important to note that the consumption increases with proximity from the city center (Fujita 1989). Considering an endogenous consumption of housing complicates the task, thus it is not considered in this model. However, the numerical simulations carried out by Tabuchi (1998) showed that the main conclusions will be the same.

The urban land rent is given by the following expression, which acknowledges that the level of land rent depends on the city size and the commuting cost. If the city size becomes very large by attracting more workers from the agricultural sector in the rural zone, then the land rent will increase.

$$F_k = 2\int_0^{\lambda_k L_k/2} F_k(x)dx = (1-t)\gamma_k w_k \frac{(\lambda_k L_k)^2}{4} \tag{6}$$

After some substitutions and simplifications, we find the expression of the net income for urban workers.

$$E_k(x) = E_{k,0} = (1-t)\left(1 + \frac{\gamma_k \lambda_k^2 L_k}{2(2-\gamma_k \lambda_k L_k)}\right) \tag{7}$$

Like most models in economic geography (Krugman 1991; Krugman and Livas Elizondo 1996; Puga 1998), we consider an economy with two regions and two private sectors: an agricultural sector with constant returns to scale (tied to the land); and a manufacturing sector with increasing returns to scale. The latter is imperfectly competitive and produces differentiated manufactures. There are two production factors, each of which is assumed to be specific to one sector. Consumers have a preference for manufacturing varieties, and manufactured goods can be exported from one region to the other one with an "iceberg" transportation cost: for each unit of goods shipped from one region to the other, only a fraction arrives. τ is an inverse index of transportation cost. In this model, we introduce a public sector which produces public infrastructure, and is paid by a proportional tax on regional incomes. The government exogenously allocates public investments among the two symmetric regions.

The easiest way to do this is with the Dixit-Stiglitz monopolistic competition model (Dixit and Stiglitz 1977). We suppose that there are a large number of symmetric products. Each producer acts as a profit-maximizing monopolist, but free-entry drives profits to zero. Consumers have the same preferences in both regions. They have a Cobb-Douglas preference function over homogenous product and a CES aggregate of the N manufactured goods.

$$U = \frac{1}{\mu^\mu (1-\mu)^{1-\mu}} M^\mu A^{1-\mu} \quad \text{with} \quad M = \left[\sum_{i=1}^{N} q_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \; 0 < \mu < 1 \,\text{and}\, \sigma > 1 \tag{8}$$

where A is the quantity of homogenous product consumed, M is the global quantity of manufactured goods consumed, q_i is the quantity of each manufactured good consumed, μ is the substitution elasticity between manufactured goods and the product tied to the land, and σ is the elasticity of substitution between different manufactured goods.

E is the income, p_i is the price of the variety i, the price of the homogenous product tied to land equal to one. Consumers maximize their utility under income constraints:

$$\begin{array}{l} A = (1-\mu)E \\ M = \mu E/I \quad \text{with} I = \left[\sum_i p_i^{1-\sigma}\right]^{1/(1-\sigma)} \\ q_i = \mu E I^{\sigma-1} p_i^{-\sigma} \end{array} \tag{9}$$

In the agricultural sector, one unit of labor produces one unit of product and wage is the same in both regions. Therefore, the price of homogeneous product provided by this sector is also equal to one.

$$P_k^A = w_k^A = 1 \tag{10}$$

The production of a quantity Q_i of any variety i requires a fixed and a variable quantity of a specific labor input. The cost function in the manufactured sector is:

$$l_i = \alpha + \beta Q_i \tag{11}$$

l_i is the quantity of labor necessary to produce Q_i units of product i. α and β are respectively the fixed and the variable cost in each region.

2.2 The Short-Run Equilibrium Conditions

Each producer faces an elasticity of demand equal to the elasticity of substitution and therefore will charge a price that is a constant markup over marginal cost.

$$p_k = \frac{\sigma}{(\sigma-1)} \beta w_k \tag{12}$$

Given the assumption that free entry will drive profit to zero, there is a unique zero-profit for each product.

$$Q_i = Q = \frac{\alpha}{\beta}(\sigma-1) \tag{13}$$

The Dixit-Stiglitz index price is:

$$I_k = \left[n_k p_k^{1-\sigma} + n_l (\tau p_l)^{1-\sigma}\right]^{1/(1-\sigma)} \tag{14}$$

and the quantities consumed for each good produced in k are:

$$q_{k,k} = \mu E_{k,0} I_k^{\sigma-1} p_k^{-\sigma} = \frac{\mu E_{k,0} p_k^{-\sigma}}{n_k p_k^{1-\sigma} + n_l (\tau p_l)^{1-\sigma}} \tag{15}$$

and for the good produced in region l,

$$q_{k,l} = \mu E_{k,0} I_k^{\sigma-1} (\tau p_l)^{-\sigma} = \frac{\mu E_{k,0} (\tau p_l)^{-\sigma}}{n_k p_k^{1-\sigma} + n_l (\tau p_l)^{1-\sigma}} \tag{16}$$

This total demand should equal the total supply given by eq. (13)

$$\frac{\mu L_k E_{k,0} p_k^{-\sigma}}{n_k p_k^{1-\sigma} + n_l (\tau p_l)^{1-\sigma}} + \frac{\mu L_l E_{l,0} (\tau p_k)^{-\sigma}}{n_k (\tau p_k)^{1-\sigma} + n_l p_l^{1-\sigma}} = \frac{\alpha}{\beta}(\sigma - 1) \tag{17}$$

The demand of one manufacturing firm is given by the following equation:

$$l = \alpha + \beta Q = \alpha + \beta \frac{\alpha}{\beta}(\sigma - 1) = \alpha\sigma \tag{18}$$

The total demand for n_k manufacturing firms in region k is:

$$n_k l = n_k \alpha\sigma \tag{19}$$

Regarding the labor market, we assume that in each urban zone we have two kinds of workers: the first one is employed in the provision of domestic transport infrastructures g_{γ_k}/w_k and the other in the manufacturing sector. The labor demand for all industrial firms is:

$$n_k l = n_k \alpha\sigma + \frac{g_{\gamma_k}}{w_k} \tag{20}$$

The labor supply is equal to the quantity of time net of commuting time spent between homes and workplaces. For a worker localized in x, this time is equal to $1 - \gamma_k x$, so we can easily deduce the total supply:

$$F_k = 2\int_0^{\lambda_k L_k/2} (1 - \gamma_k x) dx = \lambda_k L_k \left(1 - \frac{\gamma_k \lambda_k L_k}{4}\right) \tag{21}$$

The global income in region k equals to the sum of all revenues

$$R_k = L_k + (1 - t)\frac{(w_k - 1)^2}{\gamma_k w_k} \tag{22}$$

3 Public Transportation Policies and City Size

3.1 Public Transport Policies

In this model, we introduce two kinds of transport costs: (i) an interregional transport cost on the manufacturing product, and (ii) an intraregional transport cost (commuting cost). Like Martin and Rogers (1995), we interpret these transport costs as being directly related to the quality of transport infrastructure and public services in each region: τ, γ_1 and γ_2 are the infrastructure costs of interregional trade, commuting cost in region 1 and commuting cost in region 2. Every change in these costs represents a change in transport infrastructure. For example, a reduction in $\tau$is considered as an improvement in the quality of interregional transport infrastructure, and vice versa. In this case, we consider the building of an international airport or of a harbor to be an improvement of the interregional infrastructure. The assumption of a public infrastructure impact on the regional manufacturing technology can be introduced in this model through two ways. When the public transport policy affects the quality of domestic transport infrastructure, it can reduce the commuting cost, consequently exerting a positive impact on the total labor supply. When the public intervention is focused on the improvement of the interregional transport infrastructure, it may facilitate the transportation of goods, posing some implications for the distribution of workers and firms.

Formally, we assume like Barro (1990) that the government applies a tax rate on regional incomes, which it allocates between regions. We suppose also that public transport infrastructure can only be supplied by government (Martin and Rogers 1995). The total budget is equal to G:

$$G = t(R_1 + R_2) \tag{23}$$

where t indicates the tax rate that is applied by government, and R_1,$R_2$are the global incomes in region 1 and region 2 (respectively).

This total budget should finance public spending in interregional infrastructure g_τ and intraregional infrastructure g_{γ_k}. The repartition function of expenditure is:

$$G = g_\tau + g_{\gamma_1} + g_{\gamma_2} \tag{24}$$

We assume also that both transport infrastructures are produced using only the labor factor. Thus, the quantity of labor used for the intraregional infrastructure is provided by the residents of the urban zone. The quantity of labor used for the interregional transport infrastructure is provided by the residents of the rural zone. Since the wage in rural zone is equal to one, the total quantity of labor used for the construction of the interregional infrastructure will be equal to g_τ. Given that the wage in the urban zone k is equal to w_k, the necessary quantity of labor to use for the provision of the intraregional infrastructure will be equal to g_{γ_k}/w_k.

Transport costs are decreasing functions of spending on interregional and intraregional infrastructure. This allows us to reformulate these costs as follows:

$$\tau = \tau(g_\tau)\ \gamma_k = \gamma_k\left(g_{\gamma_k}/w_k\right) \text{and} \, \partial\tau/\partial g_\tau < 0 \, \text{with} \, \partial\gamma_k/\partial g_{\gamma_k} < 0 \tag{25}$$

3.2 The Equilibrium Location of Firms

Our main focus lies in studying the impact of the intra and interregional transportation costs on the spatial distribution of firms and workers. Despite the fact that we have all necessary equations that define the short-run equilibrium, there is no analytical solution for this equilibrium model. Due to the complexity of the equation system, analytical results are limited to cases of high transportation costs τ=4 and zero transportation costs τ=1. In particular, we examine in this paper the equilibrium stability of urban concentration and dispersion. Like Tabuchi (1998), long-run stable equilibrium means that relocation from region 1 to region 2 is not profitable for any single firm. In this case, if the profitability does not exceed one, the stability equilibrium can be qualified as stable (Appendix 1).

3.2.1 The Asymmetric Equilibrium

We suppose that all economic activities are concentrated in region 1. In other words, both manufacturing and agriculture sectors exist in region 1, while only the agricultural sector exists in region 2. Noting that the urban zone will disappear completely from region 2, the land rent will be equal to zero. This is the same case for the share of urban labor, which must be equal to zero. The total labor can be employed only in the agricultural sector, to produce the homogeneous product. In region 2, since a higher variety of manufacturing goods should be imported from region 1 with high transportation costs, the price index must be high. Consequently, farmers will enjoy consuming more agricultural products and housing space.

$$\lambda_2 = 0, F_2 = 0 \tag{26}$$

Substituting eq. (26) in the short-run equilibrium equations, we find the following equation system. The first equation describes the goods market equilibrium and the second one defines the public budget equilibrium.[4]

$$\begin{cases} \mu(1-t)[\tau(L_1+\psi)+L_2] = \tau\left(\dfrac{\lambda_1 L_1\left(4-\gamma_1\lambda_1 L_1+2g_{\gamma_1}\gamma_1\right)-4g_{\gamma_1}}{4}\right)\left(\dfrac{2}{2-\gamma_1\lambda_1 L_1}\right) \\ g_\tau + g_{\gamma_1} = g(L_1+L_2+\psi) \end{cases} \tag{27}$$

where $\psi = (1-t)\frac{\gamma_1(\lambda_1 L_1)^2}{2(2-\gamma_1\lambda_1 L_1)}$.

In Fig. 2, when the interregional transportation cost is null ($\tau = 1$), with the presence of low intraregional transportation cost ($\gamma_1 = 0.1$), sudden agglomeration takes place (i.e., manufacturing firms and all workers agglomerate). In this case, only the industrial sector continues to exist in region 1, but the agricultural sector will disappear completely. A decrease in shipping costs encourages firms in the large city to export their manufacturing goods to the periphery, which tends to diminish price differentials and hence wage differentials. In the large city, variety in manufacturing goods

[4] In figures 2-3 and 6-7, we assume $L_1 = 1$, $L_2 = 1$, $\mu = 0.6$, $\sigma = 5$

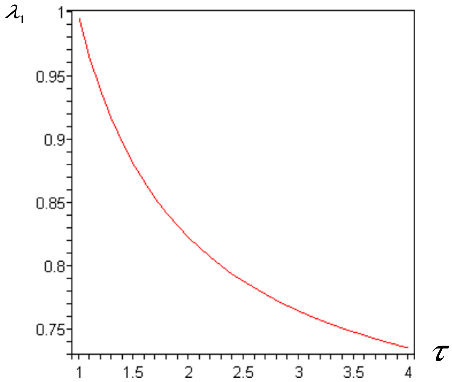

Fig. 2 Asymmetric equilibrium with lower intraregional transportation cost

(agglomeration force) becomes more important than urban costs (dispersion force). In addition, when the manufacturing share increases ($\mu > 0.5$), residents can benefit from a decrease in the price index, leading to an increase in the demand for manufacturing goods. Similarly, manufacturing firms in region 1 will profit from a larger market size. These effects lead to a full agglomeration of manufacturing activity. This concentration takes the spectacular form of catastrophic agglomeration. However, with the presence of higher intraregional transportation cost (Fig. 3), this spatial configuration seems impossible to be realized in actuality. Despite the tendency toward agglomeration in region 1, it should be noted that both sectors continue to exist in the same region. In other words, the urban costs supported by a share of workers within the agglomeration become too high to be compensated by improved access to manufacturing goods. Therefore, they will choose to relocate in the rural zone to enjoy consuming more space.

In Figs. 2 and 3, the simple existence of higher interregional transportation costs implies more dispersion of activities, leading to a decrease in the city size. In this case, goods shipping should become more costly, inducing a higher regional price index and therefore a lower demand for home goods in region 1. In addition, with the presence of a higher intraregional transportation cost in region 1 ($\gamma_1 = 0.6$), manufacturing firms

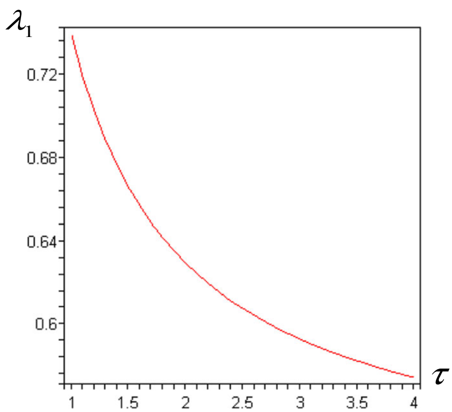

Fig. 3 Asymmetric equilibrium with higher intraregional transportation cost

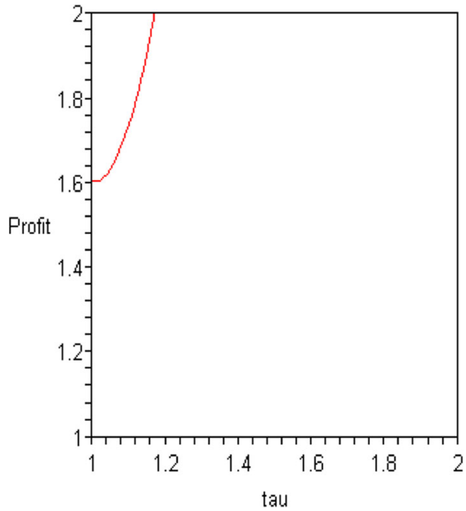

Fig. 4 Asymmetric stability with higher intraregional transportation cost

will bear higher production costs due to higher wages that should be paid to compensate workers for urban costs. Consequently, workers will consume more agricultural products and more housing space in the rural zone; because prices and the wage rates rise proportionally, workers must be better off (Tabuchi 1998).

The simulation results show that the relocation of firms from the region 1 to the region 2 is possible, thus the agglomeration equilibrium is unstable (Fig. 4). However, it becomes stable when the interregional transportation cost and the commuting cost are sufficiently low (Fig. 5). Similarly, it becomes stable with intermediate values of interregional transportation costs, where the commuting cost γ_1 is null and the elasticity of substitution σ is close to one. In Fig. 4, we assume$L_1 = 1$,$L_2 = 1$,$\sigma = 5$,$t = 0.3$, $\gamma_1 =$

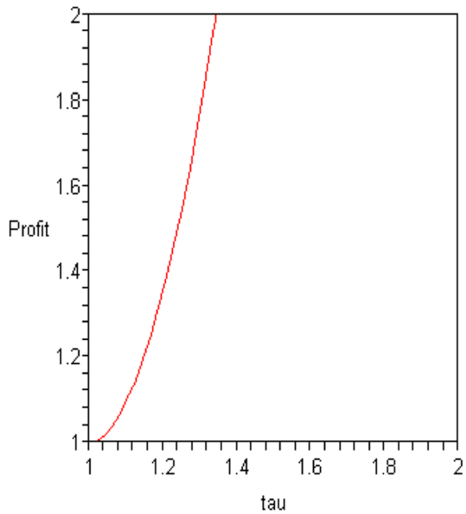

Fig. 5 Asymmetric stability with lower intraregional transportation cost

0.3, $\sigma = 5$ and $\lambda_1 = 0.6$. In Fig. 5, we assume$L_1 = 1$,$L_2 = 1$,$\sigma = 5$,$t = 0.2$, $\gamma_1 = 0.000001$, $\sigma = 5$ and $\lambda_1 = 0.6$.

3.2.2 The Symmetric Equilibrium

In this case we suppose that all economic activities are distributed equally between both regions. In other words, the agricultural and manufacturing sectors are present equally in each region without any discrimination. The symmetric equilibrium is given by the following equation system, which shows that the total population, government expenditure on local infrastructure, and the industrial wage are distributed equally across regions.

$$\begin{array}{l} L_1 = L_2 = L \\ g_{\gamma_1} = g_{\gamma_2} = g_\gamma \\ w_1 = w_2 = w \end{array} \tag{28}$$

By substituting the eq. (28) in the short-run equilibrium equations, we finally arrive at the following equation system.

$$\begin{cases} (1-t)\left(2L(2-\gamma\lambda L) + (1-t)\gamma(\lambda L)^2\right) = \psi\left(\lambda L\left(4-\gamma\lambda L + 2g_\gamma\gamma\right)-4g_\gamma\right) \\ g_\tau + 2g_\gamma = 2t\left(L + (1-t)\dfrac{\gamma(\lambda L)^2}{2(2-\gamma\lambda L)}\right) \end{cases} \tag{29}$$

where: $\psi = \frac{(\tau^\sigma+\tau)}{\mu(\tau^\sigma+1)}$.

In Figs. 6 and 7, when the cost of transporting goods is sufficiently high, all manufacturing goods that should be imported from each other become expensive, leading to a higher regional price index and thus to a lower demand for home goods. In addition, with the presence of higher urban costs, firms will bear a higher production cost due to higher wages they should pay to compensate workers for their urban costs. This cost is a barriers to the entry of new firms, leading to an increase in the regional prices and therefore to a decrease in the demand for manufacturing goods. The opposite effect is due to the presence of higher manufacturing share, which enhances the demand for manufacturing goods and thus the agglomeration force. These effects may cause an increase in the spatial size of the cities. Here, it is very important to note that the economy tends to agglomerate when interregional transportation costs are sufficiently high. This spatial development scheme is exactly the opposite of that obtained by Krugman (1991) in the center-periphery model.

When firms are located within the urban zone in each region, workers distribute themselves around the city center and commute between their homes and workplaces daily. Competition for land generates a land rent whose value decreases as the distance to the employment center rises. This implies that both the land rent and the average commuting cost increase when more workers from the rural zone relocate to the urban zone (i.e., rural-urban migration). Indeed, as the population keeps rising, the urban

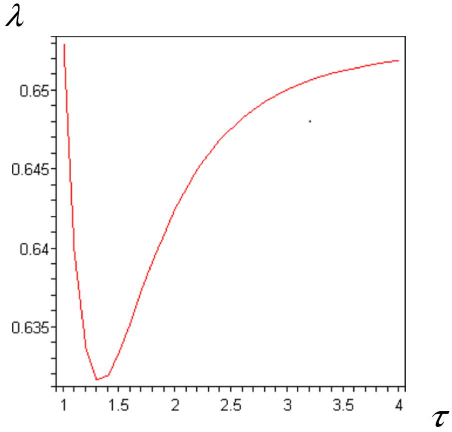

Fig. 6 Symmetric equilibrium with lower intraregional transportation cost

costs supported by workers within the agglomeration become too high to be compensated by a better access to manufacturing goods. Thus, dispersion arises when the interregional transportation cost is intermediate ($\tau = 1.3$) compared to urban costs (Tabuchi 1998). In other words, with the presence of higher intraregional transportation costs (Fig. 7), the industrial location in each urban zone becomes less attractive. The urban cost discourages the free entry of new firms, inducing higher prices and lower demand for manufacturing goods. Thus, urban residents become more sensitive to higher levels of urban costs, but less sensitive to the availability of manufacturing goods produced locally and imported at lower cost from the other region. Thus workers will be interested in the consumption of more agricultural products and housing space by relocating in the rural zone, implying a reduction in the spatial size of the city. This is the same result with the presence of lower intraregional transportation cost as shown in Fig. 6. Thus, it seems that the agglomeration force is always dominated by the negative effect of the presence of urban costs.

In Fig. 7, when the interregional transport cost is sufficiently low ($1 \leq \tau < 1.3$), associated with a higher intraregional transport cost, workers tend to agglomerate in

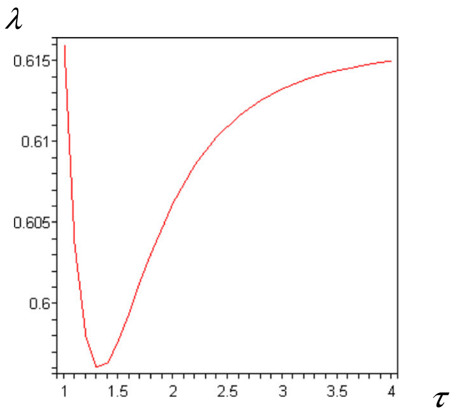

Fig. 7 Symmetric equilibrium with higher intraregional transportation cost

each urban zone, leading to an increase in the city size. Contrary to the findings of Tabuchi (1998) and Ottaviano et al. (2002), workers become less sensitive to the higher levels of urban costs (Fig. 7), but more sensitive to the availability of manufacturing goods at lower cost. Therefore, agglomeration arises once shipping costs have reached a sufficiently low level relative to commuting costs. It is important to note that manufacturing share plays a significant role in the intensity of this agglomeration. In the same vein, once shipping goods from each region is becoming very low, this can induce a lower regional price index and a higher demand for home manufacturing goods. Moreover, with a higher manufacturing share, residents can benefit from a decrease in the price index, leading to an increase in the demand for manufacturing goods. Similarly, firms locating in each region will profit from a larger market size. These economic impacts will encourage the concentration of manufacturing activity. Here, it should be noted that the intensity of agglomeration can vary with the levels of urban costs (see Fig. 6).

To study the question of stability for the symmetric equilibrium (Appendix 2), we apply a method that consists in linearizing at an equilibrium point of a system of differential equations (profit and number of firms). When differentiating n_1 by taking into account the symmetric configuration and the free-entry condition, we obtain these following expressions:

$$\frac{dq_1/q_1}{dn_1/n_1} = \frac{A_{22}B_1 + A_{12}B_2}{A_{11}A_{22} - A_{12}A_{21}}, \frac{dq_2/q_2}{dn_1/n_1} = \frac{A_{21}B_1 + A_{11}B_2}{A_{11}A_{22} - A_{12}A_{21}} \tag{30}$$

Since $\frac{\beta w_k q_k}{\sigma - 1} > 0$, $d\pi_k/dn_1$ has the same sign as dq_k/dn_1. To be stable, it must be that $dq_1/dn_1 < 0$ and $dq_2/dn_1 > 0$, then,

$$\frac{A_{22}B_1 + A_{12}B_2}{A_{11}A_{22} - A_{12}A_{21}} < 0 \text{ and } \frac{A_{21}B_1 + A_{11}B_2}{A_{11}A_{22} - A_{12}A_{21}} > 0 \tag{30a}$$

Due to the complexity of these two expressions, we conduct some numerical simulations in order to determine the equilibrium stability. When the cost of transporting goods takes an intermediate value, the symmetric equilibrium is stable. However, it becomes unstable when the interregional transport cost is sufficiently low. Two additional cases whereby symmetric equilibrium can be stable are: (i) when the intraregional transportation cost is sufficiently low ($\gamma = 0, 01$), and (ii) when the intraregional and interregional transportation costs are sufficiently high.

4 Transportation Policies and Welfare Implications

The objective consists to examine the welfare implications of the regional transportation policies for the region receiving public funds. To do so, we suppose that all activities are concentrated in region 1. As in previous models (Helpman 1995; Martin and Rogers 1995; Gallego and Zofio 2018), welfare can be derived from the indirect utility level.

$$V_1 = (1-t)\left(1 + \frac{\gamma_1\lambda_1^2 L_1}{2(2-\gamma_1\lambda_1 L_1)}\right) I^{-\mu} \quad (31)$$

When we substitute the eqs. (5), (7), (12), (14) and (22) in the eq. (31), the indirect utility level is:

$$V_1 = (1-t)\left(1 + \frac{\gamma_1\lambda_1^2}{2(2-\gamma_1\lambda_1)}\right)\left[\left(\frac{\lambda_1\left(4-\gamma_1\lambda_1 + 2g_{\gamma_1}\gamma_1\right)-4g_{\gamma_1}}{4\alpha\sigma}\right)\left(\frac{2\sigma\beta}{(\sigma-1)(2-\gamma_1\lambda_1)}\right)^{1-\sigma}\right]^{-\mu/(1-\sigma)} \quad (32)$$

In the *asymmetric equilibrium*, the welfare level depends on the spatial distribution of urban workers, the public spending in intraregional infrastructure g_{γ_1} and the tax rate t. It is important to note that only the sector tied to land continues to exist in region 2. Thus, the welfare level will not depend on the quality of the interregional transport infrastructure.

$$\partial V_1 / \partial g_{\gamma_1} < 0$$

When the regional policy improves the quality of domestic infrastructure, which consists to decrease the commuting cost, the welfare level will decrease in region 1 (Fig. 8 and Fig. 9). This result can be explained by two main factors. Firstly, in region 1 all activities are supposed to be concentrated (i.e., industry and agriculture). Thus, agglomeration may imply a higher level of urban costs due to the large size of the urban zone, leading to an increase in the regional price index. Although government expenditure is focused on the improvement of local infrastructure, the real effect on household welfare is limited to a certain extent. Secondly, workers must pay a higher tax rate to finance the spending on local transportation infrastructure, leading to a decrease in their real incomes. In addition, since the agglomeration state is unstable most of the time, the relocation of firms from the large region to the small one becomes plausible in the long run, thereby implying a higher cost for the consumers of the region that loses firms. All these factors may negatively decrease household welfare. In terms of public policy, we may say that government intervention is not desirable in the process of agglomeration. Therefore, Martin and Rogers (1995) proposed that if the improvement of domestic infrastructure could be financed by a third party (e.g. another region or the private sector), the negative effect on regional incomes would be null; thus, long-term household welfare would be improved without incurring direct costs.

In the *symmetric equilibrium*, the indirect utility level is:

$$V = (1-t)\left(1 + \frac{\gamma\lambda^2}{2(2-\gamma\lambda)}\right)\left[\left(\frac{\lambda\left(4-\gamma\lambda + 2g_{\gamma}\gamma\right)-4g_{\gamma}}{4\alpha\sigma}\right)\left(\frac{2\beta\sigma}{(\sigma-1)(2-\gamma\lambda)}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\right)\right]^{-\mu/(1-\sigma)} \quad (33)$$

$$\partial V / \partial g_{\gamma} < 0$$

In this case, we consider that technological progress decreases the intraregional cost of commuters. When the spending in the intraregional transportation infrastructure is high, leading to a decrease in the commuting cost, the indirect utility level becomes low (Fig.

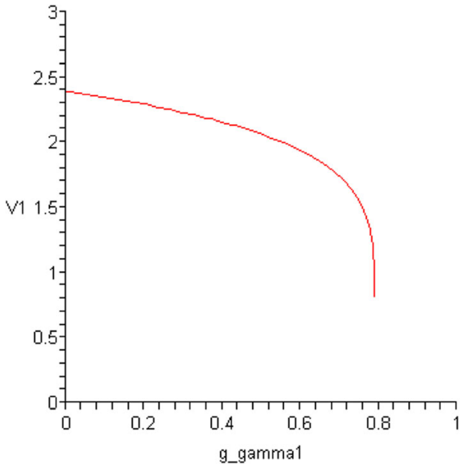

Fig. 8 Welfare implication with higher intrarregional transportation cost

10 and Fig. 11). There are two factors that can explain this result. Firstly, workers will pay a higher tax rate in each urban zone to finance the provision of the intraregional infrastructure, leading to a decrease in their incomes and therefore in their demand for goods. Secondly, firms will pay lower wages as a result of the decreasing commuting cost. These two effects should negatively affect the welfare level of workers. Moreover, technological progress is able to decrease the intraregional transportation cost (Trefil 1994), but the reduction of the latter is limited to a certain extent, since it includes the commuting cost between homes and workplaces. Indeed, technological progress would decrease the interregional transportation cost of transporting goods, but it would reduce the commuting cost little (Tabuchi 1998). This can be explained by the existence of the physical constraints of rush-hour congestion of local transport networks, which cannot be reduced without the development of an ultra-rapid mass transit generating little congestion within urban zones. In other words, compared to the interregional

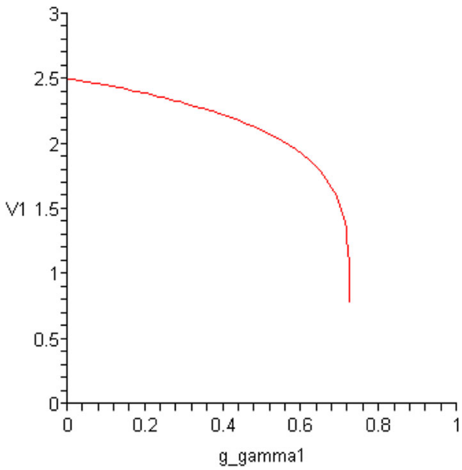

Fig. 9 Welfare implication with lower intrarregional transportation cost

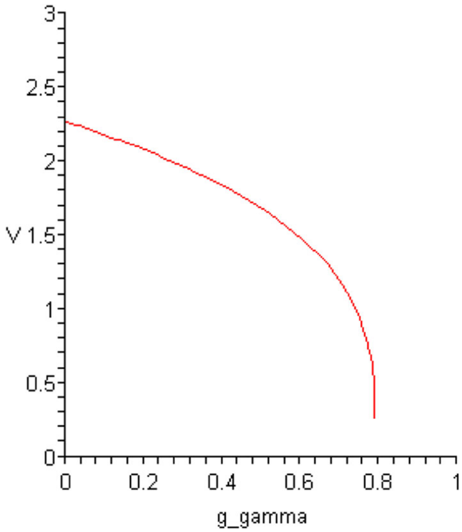

Fig. 10 Welfare implication with higher intrarregional transportation cost and higher interregional transportation cost

transportation costs of goods, commuting costs seem difficult to overcome. Hence, the result would still be valid and important.

$$\partial V/\partial_{\tau} < 0 \Leftrightarrow \partial V/\partial_{g_{\tau}} > 0$$

When the regional policy is focused on the improvement of the interregional transportation infrastructure, leading to a decrease in the cost of transporting goods, the welfare

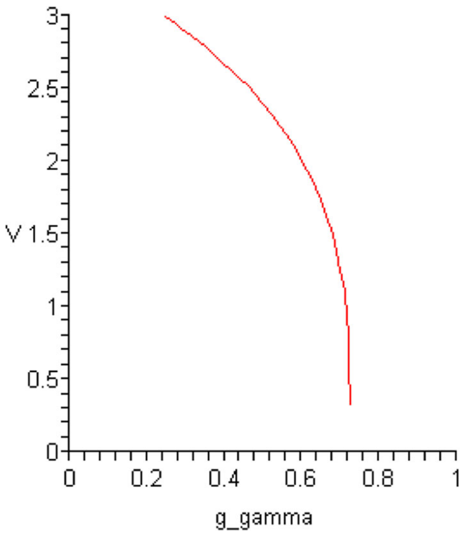

Fig. 11 Welfare implication with lower intrarregional transportation cost and lower interregional transportation cost

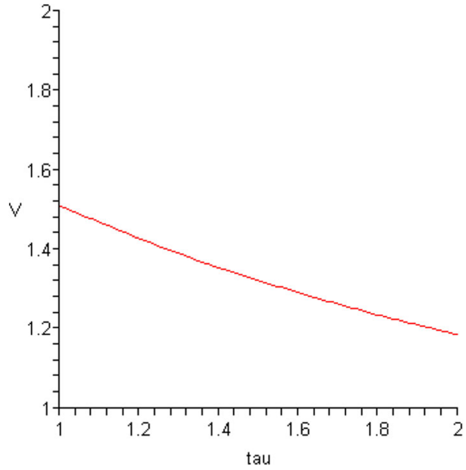

Fig. 12 Welfare implication with higher intrarregional transportation cost

level will increase continuously (Fig. 12 and Fig. 13). In fact, there are two opposite effects of this regional policy. Firstly, an improvement in the interregional infrastructure induces lower prices of manufacturing goods imported from the other region. This implies an increase in the real wage rate of workers and therefore in the demand for goods. Secondly, workers will bear higher taxes on their wages, thereby inducing a decrease in demand for manufacturing goods. It appears in this case that the net impact is positive on the welfare level of workers, since the first effect is larger than the second one (Martin and Rogers 1995). Contrary to the findings of Tabuchi (1998), it appears that the welfare level in the dispersion state is usually better than that in the agglomeration state when interregional transportation costs become sufficiently low. Since the even dispersion state is stable with an intermediate level of interregional transportation costs, it can be facilitated by government intervention.

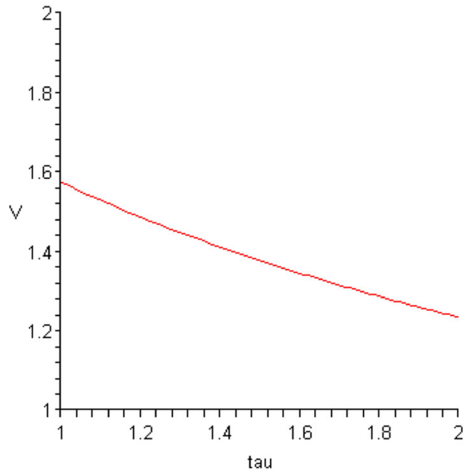

Fig. 13 Welfare implication with lower intrarregional transportation

5 Conclusions

This article introduced two kinds of transport infrastructures to study their impacts on city size and welfare. We presented a mixed new economic geography and new urban economics model in two symmetric regions, comprising an urban and rural zone. In this model, we also incorporated a negative externality related to the existence of commuting costs and lands rents in each urban zone. In the agglomeration state, we found that agglomeration occurs when the interregional transportation cost becomes sufficiently low, corroborating Krugman (1991). In the dispersion state, we illustrated the transition from agglomeration to dispersion, and then agglomeration when the interregional transport cost becomes sufficiently low, contrary to Tabuchi (1998).

In terms of welfare implications, we found that the welfare level in the dispersion state is usually higher when the regional policy is focused on the improvement of the interregional transportation cost. However, a regional policy whose objective is to improve the quality of the intraregional transportation infrastructure, which consists to reduce the time cost of commuting (opportunity cost of time), decreases the welfare level of workers. Consequently, it should be noted that the government intervention is desirable in the process of decentralization, but no policies are needed in the process of urbanization.

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Appendix 1: Asymmetric stability

In region 1, we have the following equations at the equilibrium:

$$\begin{aligned}
w_1 &= \frac{2}{2-\gamma_1\lambda_1 L_1}\\
p_1 &= \frac{\sigma}{(\sigma-1)}\beta w_1\\
E_{1,0} &= (1-t)(1+F_1/L_1)\\
F_1 &= (1-t)\frac{\gamma_1(\lambda_1 L_1)^2}{2(2-\gamma_1\lambda_1 L_1)}\\
n_1 &= \frac{\lambda_1 L_1\left(4-\gamma_1\lambda_1 L_1+2g_{\gamma_1}\gamma_1\right)-4g_{\gamma_1}}{4\alpha\sigma}\\
q_1 = q^* &= \frac{\mu L_1 E_{1,0} p_1^{-\sigma}}{n_1 p_1^{1-\sigma}} + \frac{\mu L_2 E_{2,0}(\tau p_1)^{-\sigma}}{n_1(\tau p_1)^{1-\sigma}}
\end{aligned}$$

By replacing $E_{1,0}$ and $E_{2,0}$, we find:

$$q_1 = \frac{\mu(1-t)(L_1+F_1)}{n_1 p_1} + \frac{\mu L_2(1-t)}{n_1\tau p_1}$$

By replacing F_1,p_1, n_1 and w_1, we find the total demand for manufacturing goods in region 1:

$$q_1 = \frac{\mu(1-t)\left(\tau\left(L_1 + (1-t)\frac{\gamma_1(\lambda_1 L_1)^2}{2(2-\gamma_1\lambda_1 L_1)}\right) + L_2\right)}{\frac{2\beta\sigma\tau}{(\sigma-1)(2-\gamma_1\lambda_1 L_1)}\left(\frac{\lambda_1 L_1\left(4-\gamma_1\lambda_1 L_1 + 2g_{\gamma_1}\gamma_1\right)-4g_{\gamma_1}}{4\alpha\sigma}\right)}$$

In region 2, we have the following equations at the equilibrium:

$$\begin{aligned} w_2 &= 1 \\ p_2 &= \frac{\sigma}{(\sigma-1)}\beta \\ E_{2,0} &= (1-t) \\ q_2 &= \frac{\mu L_2 E_{2,0} p_2^{-\sigma}}{n_1(\tau p_1)^{1-\sigma}} + \frac{\mu L_1 E_{1,0}(\tau p_2)^{-\sigma}}{n_1 p_1^{1-\sigma}} \end{aligned}$$

By replacing $E_{1,\,0}$ and $E_{2,\,0}$, we get:

$$q_2 = \frac{\mu L_2(1-t)p_2^{-\sigma}}{n_1(\tau p_1)^{1-\sigma}} + \frac{\mu(1-t)(L_1 + F_1)(\tau p_2)^{-\sigma}}{n_1 p_1^{1-\sigma}}$$

By replacing F_1, n_1, p_1, p_2 and w_2, we find this final expression for total manufacturing demand:

$$q_2 = \frac{\mu(1-t)\left(L_2 + \left(L_1 + (1-t)\frac{\gamma_1(\lambda_1 L_1)^2}{2(2-\gamma_1\lambda_1 L_1)}\right)\tau^{1-2\sigma}\right)}{\left(\frac{\lambda_1 L_1\left(4-\gamma_1\lambda_1 L_1 + 2g_{\gamma_1}\gamma_1\right)-4g_{\gamma_1}}{4\alpha\sigma}\right)\left(\frac{2}{2-\gamma_1\lambda_1 L_1}\right)^{1-\sigma}\left(\frac{\sigma\beta}{\sigma-1}\right)\tau^{1-\sigma}}$$

Finally we find the profitability expression:

$$\frac{q_2}{q_1^*} = \frac{\left(L_2 + (L_1 + \Omega)\tau^{1-2\sigma}\right)\left(\frac{2\tau}{2-\gamma_1\lambda_1 L_1}\right)^{\sigma}}{\tau(L_1 + \Omega) + L_2} \text{ with } \Omega = (1-t)\frac{\gamma_1(\lambda_1 L_1)^2}{2(2-\gamma_1\lambda_1 L_1)}$$

Appendix 2: The symmetric equilibrium stability

Differentiation with respect to n_1

$$\frac{dE_{k,0}}{E_{k,0}} = \frac{dF_k}{L_k + F_k} - \frac{dt}{1-t}, k = 1,2$$

$$\frac{dw_k}{w_k} = \frac{\gamma_k \lambda_k L_k}{2-\gamma_k \lambda_k L_k} \frac{d\lambda_k}{\lambda_k}, k = 1,2$$

$$\frac{dF_k}{F_k} = \left(\frac{w_k + 1}{w_k - 1}\right) \frac{dw_k}{w_k} - \frac{dt}{1-t}, k = 1,2$$

$$\frac{dp_k}{p_k} = \frac{dw_k}{w_k}$$

$$dq_k = dq_{kk} + dq_{kl}$$

$$\frac{dq_{11}}{q_{11}} = \frac{dE_{1,0}}{E_{1,0}} - \sigma \frac{dp_1}{p_1} - \frac{n_1 (p_1)^{1-\sigma}}{n_1 (p_1)^{1-\sigma} + n_2 (\tau p_2)^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right) - \frac{(1-\sigma) n_2 (\tau p_2)^{1-\sigma}}{n_1 (p_1)^{1-\sigma} + n_2 (\tau p_2)^{1-\sigma}} \frac{dp_2}{p_2}$$

$$= \frac{dE_{1,0}}{E_{1,0}} - \frac{n_1 (p_1)^{1-\sigma} + \sigma n_2 (\tau p_2)^{1-\sigma}}{n_1 (p_1)^{1-\sigma} + n_2 (\tau p_2)^{1-\sigma}} \frac{dp_1}{p_1} - \frac{(1-\sigma) n_2 (\tau p_2)^{1-\sigma}}{n_1 (p_1)^{1-\sigma} + n_2 (\tau p_2)^{1-\sigma}} \frac{dp_2}{p_2} - \frac{n_1 (p_1)^{1-\sigma}}{n_1 (p_1)^{1-\sigma} + n_2 (\tau p_2)^{1-\sigma}} \frac{dn_1}{n_1}$$

$$\frac{dq_{22}}{q_{22}} = \frac{dE_{2,0}}{E_{2,0}} - \sigma \frac{dp_2}{p_2} - \frac{(1-\sigma) n_2 (p_2)^{1-\sigma}}{n_2 (p_2)^{1-\sigma} + n_1 (\tau p_1)^{1-\sigma}} \frac{dp_2}{p_2} - \frac{n_1 (\tau p_1)^{1-\sigma}}{n_2 (p_2)^{1-\sigma} + n_1 (\tau p_1)^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right)$$

$$= \frac{dE_{2,0}}{E_{2,0}} - \frac{n_2 (p_2)^{1-\sigma} + \sigma n_1 (\tau p_1)^{1-\sigma}}{n_2 (p_2)^{1-\sigma} + n_1 (\tau p_1)^{1-\sigma}} \frac{dp_2}{p_2} - \frac{n_1 (\tau p_1)^{1-\sigma}}{n_2 (p_2)^{1-\sigma} + n_1 (\tau p_1)^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right)$$

$$\frac{dq_{12}}{q_{12}} = \frac{dE_{2,0}}{E_{2,0}} - \sigma \frac{dp_1}{p_1} - \frac{n_1 (\tau p_1)^{1-\sigma}}{n_1 (\tau p_1)^{1-\sigma} + n_2 (p_2)^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right) - \frac{(1-\sigma) n_2 (p_2)^{1-\sigma}}{n_1 (\tau p_1)^{1-\sigma} + n_2 (p_2)^{1-\sigma}} \frac{dp_2}{p_2}$$

$$= \frac{dE_{2,0}}{E_{2,0}} - \frac{n_1 (\tau p_1)^{1-\sigma} + \sigma n_2 (p_2)^{1-\sigma}}{n_1 (\tau p_1)^{1-\sigma} + n_2 (p_2)^{1-\sigma}} \frac{dp_1}{p_1} - \frac{(1-\sigma) n_2 (p_2)^{1-\sigma}}{n_1 (\tau p_1)^{1-\sigma} + n_2 (p_2)^{1-\sigma}} \frac{dp_2}{p_2} - \frac{n_1 (\tau p_1)^{1-\sigma}}{n_1 (\tau p_1)^{1-\sigma} + n_2 (p_2)^{1-\sigma}} \frac{dn_1}{n_1}$$

$$\frac{dq_{21}}{q_{21}} = \frac{dE_{1,0}}{E_{1,0}} - \sigma \frac{dp_2}{p_2} - \frac{(1-\sigma) n_2 (\tau p_2)^{1-\sigma}}{n_2 (\tau p_2)^{1-\sigma} + n_1 (p_1)^{1-\sigma}} \frac{dp_2}{p_2} - \frac{n_1 (p_1)^{1-\sigma}}{n_2 (\tau p_2)^{1-\sigma} + n_1 (p_1)^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right)$$

$$= \frac{dE_{1,0}}{E_{1,0}} - \frac{n_2 (\tau p_2)^{1-\sigma} + \sigma n_1 (p_1)^{1-\sigma}}{n_2 (\tau p_2)^{1-\sigma} + n_1 (p_1)^{1-\sigma}} \frac{dp_2}{p_2} - \frac{n_1 (p_1)^{1-\sigma}}{n_2 (\tau p_2)^{1-\sigma} + n_1 (p_1)^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right)$$

$$(\alpha + \beta q_1) dn_1 + n_1 \beta dq_1 - \frac{g_{\gamma_1}}{w_1} \frac{dw_1}{w_1} = L_1 \left(1 - \frac{\gamma_1 \lambda_1 L_1}{2}\right) d\lambda_1$$

$$n_2 d\beta q_2 - \frac{g_{\gamma_2}}{w_2} \frac{dw_2}{w_2} = L_2 \left(1 - \frac{\gamma_2 \lambda_2 L_2}{2}\right) d\lambda_2$$

$$\frac{dg}{g} = -\frac{dF_1 + dF_2}{L_1 + F_1 + L_2 + F_2}$$

$$\frac{dg}{g} = -\frac{dF_1 + dF_2}{L_1 + F_1 + L_2 + F_2}$$

Taking into account the symmetry and free-entry conditions

$$\frac{dE_{k,0}}{E_{k,0}} = \frac{dF_k}{L_k + F_k} - \frac{dt}{1-t}, k = 1, 2$$
$$\frac{dw_k}{w_k} = \frac{\gamma_k \lambda_k L_k}{2-\gamma_k \lambda_k L_k} \frac{d\lambda_k}{\lambda_k}, k = 1, 2$$
$$\frac{dF_k}{F_k} = \left(\frac{w_k + 1}{w_k - 1}\right) \frac{dw_k}{w_k} - \frac{dt}{1-t}, k = 1, 2$$
$$\frac{dp_k}{p_k} = \frac{dw_k}{w_k}$$
$$dq_k = dq_{kk} + dq_{kl}$$
$$\frac{dq_{11}}{q_{11}} = \frac{dE_{1,0}}{E_{1,0}} - \frac{1 + \sigma\tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \frac{dp_1}{p_1} - \frac{(1-\sigma)\tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \frac{dp_2}{p_2} - \frac{1}{1 + \tau^{1-\sigma}} \frac{dn_1}{n_1}$$
$$\frac{dq_{22}}{q_{22}} = \frac{dE_{2,0}}{E_{2,0}} - \frac{1 + \sigma\tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \frac{dp_2}{p_2} - \frac{\tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right)$$
$$\frac{dq_{12}}{q_{12}} = \frac{dE_{2,0}}{E_{2,0}} - \frac{\sigma + \tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \frac{dp_1}{p_1} - \frac{1-\sigma}{1 + \tau^{1-\sigma}} \frac{dp_2}{p_2} - \frac{\tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \frac{dn_1}{n_1}$$
$$\frac{dq_{21}}{q_{21}} = \frac{dE_{1,0}}{E_{1,0}} - \frac{\sigma + \tau^{1-\sigma}}{1 + \tau^{1-\sigma}} \frac{dp_2}{p_2} - \frac{1}{1 + \tau^{1-\sigma}} \left(\frac{dn_1}{n_1} + (1-\sigma) \frac{dp_1}{p_1}\right)$$
$$(\alpha + \beta q_1) dn_1 + n_1 \beta dq_1 - \frac{g_{\gamma_1}}{w_1} \frac{dw_1}{w_1} = L_1 \left(1 - \frac{\gamma_1 \lambda_1 L_1}{2}\right) d\lambda_1$$
$$n_2 d\beta q_2 - \frac{g_{\gamma_2}}{w_2} \frac{dw_2}{w_2} = L_2 \left(1 - \frac{\gamma_2 \lambda_2 L_2}{2}\right) d\lambda_2$$
$$\frac{dg}{g} = -\frac{dF_1 + dF_2}{L_1 + F_1 + L_2 + F_2}$$
$$d\pi_k = \frac{\beta w_k q_k}{\sigma-1} \frac{dq_k}{q_k}$$

Simplifications

$$\begin{aligned}
&\text{Let}\,\varphi = \frac{F_k}{L_k + F_k} = \frac{F_l}{L_l + F_l}\\
&\frac{dq_1}{q_1} = \frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-g\varphi}{1-t-t\varphi} + \frac{t(1+\varphi-t)}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{dw_1}{w_1}\\
&+\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-g\varphi}{1-t-t\varphi}\tau^{-\sigma} + \frac{t(1+\varphi-t)}{1-t-t\varphi}\right)\frac{dw_2}{w_2}\\
&-\frac{1+(1+\tau)\sigma\tau^{-\sigma}+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{dw_1}{w_1} + \frac{(\sigma-1)(1+\tau)\tau^{-\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{dw_2}{w_2} - \frac{1+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{dn_1}{n_1}\\
&\frac{dq_2}{q_2} = \frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{g(1+\varphi-t)}{1-t-t\varphi} + \frac{(2+t)(1-t)-t\varphi}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{dw_1}{w_1}\\
&+\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-t\varphi}{1-t-t\varphi} + \frac{t(1+\varphi-t)}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{dw_2}{w_2}\\
&-\frac{(1+\tau)\tau^{-\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\left(\frac{dn_1}{n_1} - (\sigma-1)\frac{dw_1}{w_1}\right) - \frac{1+(1+\tau)\sigma\tau^{-\sigma}+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{dw_2}{w_2}\\
&n(\alpha+\beta q)\frac{dn_1}{n_1} + nq\beta\frac{dq_1}{q_1} - \frac{g_\gamma}{w}\frac{dw_1}{w_1} = \lambda L\left(1-\frac{\gamma\lambda L}{2}\right)\left(\frac{2}{\gamma\lambda L}-1\right)\frac{dw_1}{w_1}\\
&nq\beta\frac{dq_2}{q_2} - \frac{g_\gamma}{w}\frac{dw_2}{w_2} = \lambda L\left(1-\frac{\gamma\lambda L}{2}\right)\left(\frac{2}{\gamma\lambda L}-1\right)\frac{dw_2}{w_2}\\
&d\pi_k = \pi_k\frac{dw_k}{w_k} + \frac{\beta w_k q_k}{\sigma-1}\frac{dq_k}{q_k}
\end{aligned}$$

Resolution

As in symmetric equilibrium, $w = \frac{2}{2-\gamma\lambda L}$

$$\begin{aligned}
\left(\frac{2}{\gamma w^2} + \frac{g_\gamma}{w}\right)\frac{dw_1}{w_1} &= n(\alpha+\beta q)\frac{dn_1}{n_1} + nq\beta\frac{dq_1}{q_1}\\
\left(\frac{2}{\gamma w^2} + \frac{g_\gamma}{w}\right)\frac{dw_2}{w_2} &= nq\beta\frac{dq_2}{q_2}
\end{aligned}$$

Let $\frac{dw_1}{w_1} = \frac{n(\alpha+\beta q)\gamma w^2}{2+\gamma g_\gamma w}\frac{dn_1}{n_1} + \frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}\frac{dq_1}{q_1}$, and $\frac{dw_2}{w_2} = \frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}\frac{dq_2}{q_2}$. So,

$$\begin{aligned}
A_{11}\frac{dq_1}{q_1} - A_{12}\frac{dq_2}{q_2} &= B_1\frac{dn_1}{dn_1}\\
-A_{21}\frac{dq_1}{q_1} + A_{22}\frac{dq_2}{q_2} &= B_2\frac{dn_1}{dn_1}
\end{aligned}$$

with

$$A_{11}=1-\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-g\varphi}{1-t-t\varphi}+\frac{t(1+\varphi-t)}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$+\frac{1+(1+\tau)\sigma\tau^{-\sigma}+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$A_{12}=\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-t\varphi}{1-t-t\varphi}\tau^{-\sigma}+\frac{g(1+\varphi-t)}{1-t-t\varphi}\right)\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$+\frac{(\sigma-1)(1+\tau)\tau^{-\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$B_1=\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-g\varphi}{1-t-t\varphi}+\frac{t(1+\varphi-t)}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{n(\alpha+\beta q)\gamma w^2}{2+\gamma g_\gamma w}$$
$$-\frac{1+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}-\frac{1+(1+\tau)\sigma\tau^{-\sigma}+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{n(\alpha+\beta q)\gamma w^2}{2+\gamma g_\gamma w}$$
$$A_{21}=\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{t(1+\varphi-t)}{1-t-t\varphi}+\frac{(2+t)(1-t)-t\varphi}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$+\frac{(\sigma-1)(1+\tau)\tau^{-\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$A_{22}=1+\frac{1+(1+\tau)\sigma\tau^{-\sigma}+\tau^{1-2\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$-\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{(2+t)(1-t)-t\varphi}{1-t-t\varphi}+\frac{t(1+\varphi-t)}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{n\beta q\gamma w^2}{2+\gamma g_\gamma w}$$
$$B_2=\frac{\varphi}{2(1-t)(1+\tau^{-\sigma})}\left(\frac{w+1}{w-1}\right)\left(\frac{g(1+\varphi-t)}{1-t-t\varphi}+\frac{(2+t)(1-t)-t\varphi}{1-t-t\varphi}\tau^{-\sigma}\right)\frac{n(\alpha+\beta q)\gamma w^2}{2+\gamma g_\gamma w}\frac{dn_1}{n_1}$$
$$-\frac{(1+\tau)\tau^{-\sigma}}{(1+\tau^{-\sigma})(1+\tau^{1-\sigma})}\left(1-(\sigma-1)\frac{n(\alpha+\beta q)\gamma w^2}{2+\gamma g_\gamma w}\right)\frac{dn_1}{n_1}$$
$$d\pi_k=\frac{\beta w_k q_k}{\sigma-1}\frac{dq_k}{q_k}$$

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