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The Complexity in the Study of Spatial Networks: an Epistemological Approach

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Abstract Provided that the study of complex networks has reached a crucial measure to be considered by many researchers as a separate discipline, the so-called Network Science (NS), the assessment of this field under the epistemological perspective serves substantial self-definition purposes. This paper takes its inspiration from both the review articles of Barthelemy in Phys Rep, 499, 1–101, 2011, and of Ducruet and Beauguitte in Netw Spatial Econ, 14, 297–316, 2014, and attempts to provide an integrated epistemological consideration for the complex study in the scientific subfield of spatial networks. Within the concept of epistemology, which is the theory examining the ways that knowledge is being achievable, the major research question that is tried to be answered in this paper is "whether there is convergence or divergence in the evolution of the spatial networks' study, in terms of cognition, methods, and applications". The further purpose of this paper is to shape an integrated methodological framework for the study of spatial networks, incorporating the existing structural, functional and socioeconomic approaches. The overall consideration aspires to contribute to the comprehension and the standardization of the procedures in the analysis of spatial networks, to promote the interdisciplinary research of complex networks, and to introduce the dialogue about how Complex Network Analysis (CNA) and NS can move methodically from the research field into the academic didactics.

Keywords Spatial embedding · Complex network analysis · Network science · Network modeling · Didactics

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1 Introduction

Modeling communication systems is an important and complex process. Its importance is related to the immanent human need of communication, whereas its complexity regards the variety of factors contributing to their configuration and evolution. Provided that geography sets inevitable constraints in the conduct of communication, many of such systems can be considered as spatial. In terms of the Network Science (NS) (Barthelemy 2011; Barabasi 2013; Brandes et al. 2013; Ducruet and Beauguitte 2014), which is an emerging discipline using the network paradigm to model complex communication systems as pair-sets of interconnected nodes and their linkages (edges), the communication systems embedded in the physical (geographical 2d or 3d) space are defined as spatial networks.

For a network, the spatial property is accompanied with a set of characteristics affecting either its structural configuration, or its functionality, or its attributes, or even its evolution (Barthelemy 2011; Ducruet and Beauguitte 2014). For example, according to Watts and Strogatz (1998), space constraints the process of preferential attachment that often occurs over shortest distances and is responsible for the development of hierarchies in networks' structure. Barthelemy (2011) notes in his work that spatial constraints affect the organization of a network, its centrality, the typology of its degree distribution, and the correlations captured between network topology and traffic. Additionally, Ducruet and Beauguitte (2014) observe some disciplinary driving forces on scholars studying spatial networks, which diversify the way that such networks are interpreted and thus they complicate the integration in this research field.

Moreover, node functionality in spatial networks usually appears quite different compared to this of networks not embedded in space (Barthelemy 2011; Ducruet 2013; Ducruet and Beauguitte 2014). For example, in a social network the nodes (representing either individuals or social configurations) operate directly as actors, which are ruled by behavioral and cognitive driving forces. On the contrary, in a spatial network, the functionality of nodes (such as are transport terminals, for instance) is the resultant of the actions performed by many different actors and not by "one actor" per se, and thus it is indirectly related to the social behavior and human cognition.

Within this framework, space obviously matters in networks and towards this direction numerous researches have already provided rich theoretical and empirical documentation (Barrat et al. 2005; Boccaletti et al. 2006; Bagler 2008; Barthelemy 2011; Wang et al. 2011; Jia and Jiang 2012; Ducruet 2013; Ducruet and Beauguitte 2014). Among them, the works of Barthelemy (2011) and Ducruet and Beauguitte (2014) suggest two fundamental reviews addressing this issue. The first focuses on the theoretical and empirical framework of spatial network analysis, described mainly under the physicists' perspective, whereas the second on how this complex research is integrated into geography and regional science. These two articles appear to have introduced the epistemological dialogue in the study of spatial networks, which was so far absent or scattered in the relevant research (Ducruet and Beauguitte 2014).

This paper takes its inspiration from both these review articles (Barthelemy 2011; Ducruet and Beauguitte 2014) and attempts to provide an integrated epistemological consideration in the complex study of spatial networks. The basic research question is whether the evolution in the study of spatial networks converges or diverges, in conceptual, methodological, and disciplinary (through the way that scholars approach them) terms. The further purpose of this paper is to shape a complete methodological framework for the study of spatial networks, incorporating the existing structural, functional and socioeconomic approaches. This framework can be used as a procedures' manual describing the successive steps necessitating to study networks embedded in space. It generally aspires to contribute to the comprehension and the standardization of the procedures in the analysis of spatial networks, providing utility to real world applications, to promote the interdisciplinary research of complex networks, and to introduce the dialogue about how Complex Network Analysis (CNA) and NS can move methodically from the research field into the academic didactics.

The remainder of this paper is structured as follows; section two distinguishes the aspects composing so far the epistemological framework in the study of spatial networks, whereas section three proposes a methodological approach integrating these considerations. Finally, the conclusion section discusses potential utility and addresses of further research of the proposed consideration.

2 The Epistemology of Spatial Networks

The epistemological consideration of networks becomes an emerging necessity today, mainly due to their voluminous research material and their interdisciplinary nature (Easley and Kleinberg 2010; Barthelemy 2011; Ducruet and Beauguitte 2014). Generally, Epistemology studies the origin and the production mechanisms of scientific knowledge (Goldman 1986). Since network research is already considered by many researchers as a separate discipline (Brandes et al. 2013), such an approach is expected to serve self-definition purposes to this emerging scientific field. Further, the multi-collective material of the NS necessitates methodical epistemological evaluation to define and upgrade the common communication code among its diverse contributors.

Within this framework, this section discusses existing epistemological approaches that cover different aspects in the study of spatial networks. The overall consideration attempts to answer at the main research question and to distinguish what is really spatial in them, jointly conceptually, methodologically, and through the way that scholars approach such concepts. The examined approaches are classified according to their typology in the categories shown in Table 1, where each of them is discussed separately at the following subsections.

2.1 Disciplinary Approaches

The review article that appears to introduce the per se disciplinary dialogue in the study of spatial networks is the work of Ducruet and Beauguitte (2014), which deals with how complex network research has been integrated into Geography and Regional Science. This work provides an epistemological consideration to the field of spatial networks, discussing the diachronic evolution, the involving disciplines, the contributors, and the mismatches emerged in the configuration of this research field. Because of the broad approach provided by these authors, this subsection will not be such extensive, but will attempt to abstract the highlights of this issue and to complement the commentary where it is necessary, according to the diagram shown in Fig. 1.

Table 1 Classification of existing epistemological approaches in the study of complex networks

Typology	Sub-category	Criterion	Class	
2.1 Disciplinary approaches		Discipline	(i)	Geography and Earth Sciences
			(ii)	Physics and Mathematics
			(iii)	Sociology and Humanity Sciences
			(iv)	Computer Sciences
2.2 Network structure	(a)	Links	(i)	Immaterial
			(ii)	Physical
	(b)	Nodes	(i)	Social
			(ii)	Biological
			(iii)	Technological
			(iv)	Infrastructure
			(v)	Trade
			(vi)	Conceptual
2.3 Spatial embedding	(a)	Space	(i)	Topological
			(ii)	Metric
	(b)	Planarity	(i)	Planar
			(ii)	Non-plannar
2.4 Methodological approaches	(a)	Type of research	(i)	Theoretical
			(ii)	Empirical
			(ii.a)	Empirical vs theoretical
			(ii.b)	Empirical vs empirical
			(iii)	Review
	(b)	Number of layers	(i)	Monolayer
			(ii)	Multilayer
2.5 Conceptual elements	(a)	Conceptual component	(i)	Structural
			(ii)	Functional
			(iii)	Socioeconomic/Ontological

source: own elaboration based on examined literature

Based on the work of Ducruet and Beauguitte (2014) and on other reviews relevant to spatial and network analysis (Freeman 2004; Boccaletti et al. 2006; Bascompte 2007; Fortunato 2010; Barthelemy 2011; Boccaletti et al. 2014; Kivela et al. 2014), *four classes of disciplines* can be considered as major contributors in the research field of spatial networks; *Physics and Mathematics*, *Geography and Earth Sciences*, *Sociology and Humanity Sciences*, and *Computer Sciences*. However, these disciplines appeared to incorporate asynchronously and separately the network paradigm in their research matter.

Sociology started from the 30s to develop tools for social network analysis (SNA), focusing exclusively on non-planar graphs (Wasserman and Faust 1994). In that period, Jakob Moreno (1934) introduced the term "sociometry" to describe the broad research effort that included all four of the defining features of the today's social network

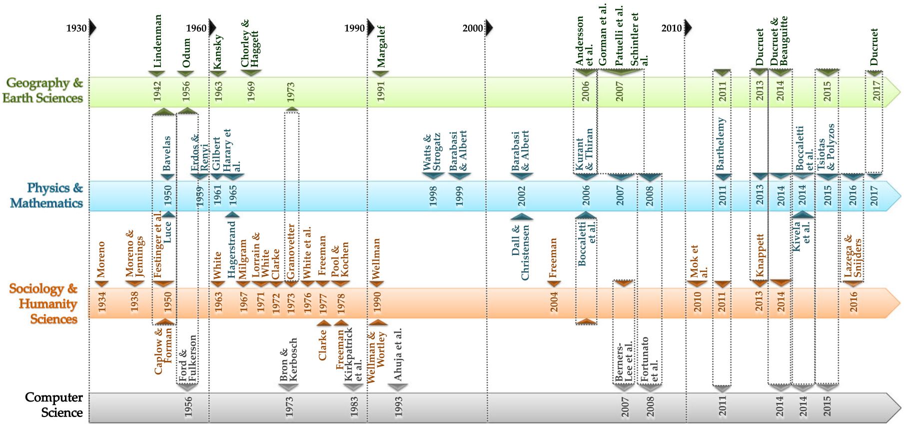

Fig. 1 Time diagram of representative scientific papers that contributed since the 30s to the configuration of the framework of spatial network analysis utilizing CNA (multiple marks indicate multidisciplinary approaches)

analysis, namely the *structural intuitions*, the *systematic empirical data*, the *graphic imagery* and an *explicit mathematical modeling* (Moreno and Jennings 1938; Freeman 2004). In 1936 this inspired effort turned into a journal, the "*Sociometric Review*", which at the next year (1937) was renamed to "*Sociometry*" and kept this name for almost 20 years (Freeman 2004). However, the sociometry's research collapsed quickly in the 40s (see Freeman 2004) and the SNA approach remained embryonic for the sociologists until the 1960s, when the Harvard School, leaded by Harrison White, started reactivating towards this direction (White 1963; Lorrain and White 1971; White et al. 1976). At the period from 40s–60s, which is called as "dark ages" for the SNA (Freeman, 20,004), the sociology seemed to be aware of the fact that space affects social interaction, as it can be traced in some works, such as of Festinger et al. (1950) and of Caplow and Forman (1950) (Wong et al. 2006).

Up to the 60s, in the field of mathematics, the work(s) of Erdos and Renyi (1959) complemented the graph theory with the probabilistic perspective, while Gilbert (1961) examined planarity for random networks, and several other mathematicians showed interest in the social value of graph theory, such as Bavelas (1950), at the MIT, Luce (1950), Harary et al. (1965), and they provided seminal papers towards this direction. In informatics, early studies concerned the construction of algorithms on network flows maximization and minimal cuts estimations in transportation networks (see Ford and Fulkerson 1956). However, network analysis in geography and earth sciences remained simple and was restricted in the use of graph theoretic measures in applications of transport geography (Ducruet and Beauguitte 2014). An early reflection of the network paradigm in ecology can be found at the works of Lindeman (1942) and Odum (1956), who used elements of network modeling to represent and describe food webs that were being within the interest of ecology for several decades (Bascompte 2007).

The first station in geography was the work of Kansky (1963), which explored the relationship between transportation networks and regional economic characteristics. The author pioneered in using a combined graph theoretic and probabilistic consideration in transport geography. He modeled the distribution of transportation routes considering that nodes with more economic activities are more probable to belong to a transportation network (Makkonen et al. 2013). Station works at this period in geography were also of Hagerstrand (1967) and of Haggett and Chorley (1969), which stimulated a growing amount of research. The first author modeled the processes of diffusion of two classes of innovation (the first was stockbreeding and agricultural and the second technological and banking services innovation) and he used a pair of approaches, a descriptive and inductive model to describe the diffusion's spatial stages and a series of Monte Carlo simulation models (Hagerstrand 1967), concluding that private communication is more powerful for the diffusion than public announcement. This work succeeded to surpass the field of innovation infusion and to contribute more to research in geography. The second work of Haggett and Chorley (1969), entitled "Network Analysis in Geography", deals with theoretical concepts and networks models mainly in the application areas of streams, transportation, and geography. This book provided an integrated review of the basic elements composed the field that was being called at that time as "New Geography" (Knappett 2013).

At the same period, in sociology, Milgram (1967) presented the distinguished "six degrees of separation" problem (formally the small-world phenomenon), interpreting that six persons only (as a binary distance) are needed to connect any two persons in the world. This paper, although it did not mobilize a graph theoretical approach, highlighted the spatial dimension in SNA, but it succeeded little attention from geographers (Ducruet and Beauguitte 2014). Some years later, Granovetter (1973) published a remarkable work entitled "*The strength of weak ties*", dealing with the impact of the so-called dyadic ties on the diffusion of influence and information, mobility, and organization, where, among others, it highlighted the importance of weak ties for urban communities (and thus it involved the spatial dimension in SNA). Of great interest is also the work of Freeman (1977), which introduced the family of betweenness centrality measures. At that time, in archaeology some elements of the network approach can be traced in the works of David Clark (1972, 1977), examining the spatial information of archaeological sites, under the influence of that period's New Geography (Haggett and Chorley 1969; Knappett 2013). Further, computer science was focusing more on technical (such as computational and optimization) approaches in NS (i.e. Bron and Kerbosch 1973; Kirkpatrick et al. 1983), while physics until the late 90s remained indifferent to CNA (Ducruet and Beauguitte 2014).

In the early 90s, an interesting case in ecology is the work of Margalef (1991), entitled "Ecological Networks", which is characteristic not only for the description of ecological systems in terms of network modeling, but also for the comparison applied between different network types (an approach that is a common practice in complex network analysis) and for the scale invariant properties detected for ecological networks (this idea is related to network heterogeneity that yet enjoys of current research in complex networks) (Bascompte 2007). From the perspective of Informatics, the work of Ahuja et al. (1993) provided a reference book configuring the (until that time) framework on network flows computation. In the late 90s, the physicists showed intense interest in networks, which initiated a new era for the NS because this discipline studied networks by incorporating the graph theoretic analysis with statistical mechanics. Station works at this period were the papers of Watts and Strogatz (1998) and Barabasi and Albert (1999), which provided sufficient documentation on the small-world (SW) and the scale-free (SF) models and ignited further research. At that period, many papers from the sociologist Barry Wellman also highlighted the spatial dimension of social networks (Wellman 1990; Wellman and Wortley 1990; Wellman and Tindall 1993; Mok et al. 2010).

According to Ducruet and Beauguitte (2014), while the physicists were increasingly integrating the space in their works (see i.e. Dall and Christensen 2002), geographers and regional scientists paid limited attention to complex networks research, with some exemptions the works of Andersson et al. (2006), Patuelli et al. (2007), Gorman et al. (2007), and Schintler et al. (2007), which tried to incorporate the CNA with econometric approaches of spatial interaction. Regardless the massive research produced at the first decade of the twenty-first century in CNA, the multidisciplinary consideration in the study of spatial networks remained deficient. Perhaps this was due to the necessity of CNA to reach its critical self-definition measures, since enough works attempted this period to exceed the mono-disciplinarily borders (Boccaletti et al. 2006; Kurant and Thiran 2006a; Berners-Lee et al. 2007; Fortunato 2010).

The first work that presented a review integrating all the four above-mentioned disciplines was of (Barthelemy 2011) in 2011, entitled "*Spatial Networks*". Given that this paper was written by a physicist, its major disciplinary orientation is the physics, however it definitely succeeded to communicate the physicists' cognition that space is a concept immanent to complex networks and that the study of spatial networks includes multidisciplinary components. Two years later, Knappett (2013) communicates the message of the Society for American Archaeology that network analysis is an effective tool in the diverse fields of archaeologic research and, from the geographer's perspective, Ducruet (2013) provided a comprehensive review for the use of multilevel networks in geography. At the next year, the review article of Ducruet and Beauguitte (2014) addressed the interdisciplinary coupling of Spatial Science and NS, introducing the epistemological dialogue in the field of spatial networks.

The reviews of Boccaletti et al. (2014) and Kivela et al. (2014) shaped the framework with the necessary mathematical rigor in the study of multilayer networks, which suggests an emerging field in NS due to its eruptive development and to the increase in its level of complexity. In a multidisciplinary attempt, Tsiotas and Polyzos (2015c) showed how Web Science and CNA can collaborate in an empirical model providing utility in transportation research (commuting analysis). From the Statistics perspective, Lazega and Snijders (2016) proposed recently an integrated statistical framework for multilevel network analysis, but their spatial aspect seems to be quite low. Finally, from the aspect of geography, Ducruet (2017) has just published a paper dealing with the multilayer dynamics of complex spatial networks, focusing on the case of maritime flows.

From the aspect of network visualization and manipulation, the first network analysis software were the packages "Ucinet" (Borgatti et al. 1992), "Pajek" (Batagelj and Mrvar 1998) and "Graphviz" (Ellson et al. 2001), which were launched in the market at 90s, perhaps in the chronological order as being mentioned, where only Ucinet was for commercial use (the others were freeware). All these packages were primarily used as tools for social network analysis, without having any built in features of spatial embedding (this potential was available for those supporting the open-source architecture, but it necessitated advanced computing knowledge). In 2009, the "Gephi" software (Bastian et al. 2009) launched the market and was open-source, freeware, and ready for plugins supporting spatial layouts and embedding. Recent software on CNA is "MuxViz" (De Domenico et al. 2015) and the "Vistorian.net" web application, developed by J-D. Fekete's team (Bach et al. 2015), which both can account for spatial information and embedding.

2.2 Network Structure

Based on the substance of the communication medium (links), networks can be distinguished amongst *immaterial* and *physical* (Goldman 1986; Sgroi 2008). Immaterial networks have non-physical links that can be either conceptual (Fauconnier and Turner 1998), such as are the semantic and language networks (Gurevych 2005), or relational links (Goldman 1986), such as are the social networks, where connections may represent either friendship, or relationship, or collaboration, etc. On the contrary, in physical networks (Sgroi 2008) connections are material (of every possible state), such as in the case of road networks (Strano et al. 2009; Crucitti et al. 2006; Barthelemy 2011), where links are infrastructural, or in the case of telecommunication networks (Barthelemy 2011; Kivela et al. 2014), where signal is transmitted electromagnetically. According to this classification, spatial constraints are immanent (mostly directly) to physical networks, since every real world network is embedded in the physical space, whereas in immaterial networks such constraints are either absent or indirect.

However, a spatial reference can be indirectly found even in "purely" immaterial networks, whether taking into consideration the interactive role between space and human cognition (Levinson 2003). For example, in social networks, the geographical space suggests a basic source for the so-called "*baseline homophily*", namely the tendency of social actors to be associated with similar others, because of the conditions or restrictions existing in their surrounding social framework (Wong et al. 2006). The works of Festinger et al. (1950) and Caplow and Forman (1950) are from the early studies addressed this issue, which showed (both examining student communities) that the spatial arrangement of rooms was an important factor for the students to develop strong or weak ties. Ever since, many other studies reached to similar conclusions, finding that the majority of personal friendships in social networks are "local" (see Wong et al. 2006; Mok et al. 2010; Ducruet and Beauguitte 2014), even in the cases of recent social networks supported by technology.

With criterion the substance of nodes, networks may attain a variety of classifications, such as *social*, *biological*, *technological*, *infrastructure*, *trade* and *conceptual* networks (Fauconnier and Turner 1998; Gurevych 2005; Boccaletti et al. 2006; Sgroi 2008; Easley and Kleinberg 2010; Barthelemy 2011) or a mixture of them (i.e. a transportation network may be considered jointly as social, technological, infrastructure and trade). Substantially, the kind of nodes (or actors) composing a network identifies its type and considerably contributes to its structural attributes (Boccaletti et al. 2006; Easley and Kleinberg 2010; Barthelemy 2011; Kivela et al. 2014). According to this classification, space also matters in the network configuration, either directly or indirectly. For example, relevant research in social networks (i.e. cell phone contacts network, bloggers network, social media network) has shown that connectivity decays with distance (see Barthelemy 2011; Ducruet and Beauguitte 2014), complying with the intuition that most of friends and relatives are usually not physically distant to each other. In all the categories that is either directly or circularly applicable, the spatial constraint is related to the cost of overcoming the so-called "spatial impedance" (Polyzos et al. 2014) or "transportation cost" (Ducruet and Beauguitte 2014), implying the energy demanded for transmitting the communication signals or flows through space. This practically interprets that the links between distant nodes cost more than those between neighbor nodes (Barthelemy 2011) and consequently their construction is beneficially when serving greater demand (Polyzos 2011).

From a disciplinary perspective, the role that space has to the configuration of network structure has been studied and highlighted by geographers at the so-called spatial interaction models (SIMs), which are generally used in the social sciences to describe phenomena (i.e. flows) of social interaction, such as trade, commuting, migration, etc. These models are expressed as product $Y_n = b{\cdot}\prod_n X_n^{a_n}$ of power-law factors of the form $b_n{\cdot}X_n^{a_n}$, which are calibrated empirically on socioeconomic and spatial data (Polyzos 2011). The major representative of the SIMs is the so-called "gravity model" (Polyzos et al. 2014; Ducruet and Beauguitte 2014) that emulates spatial interaction using a gravitational rule, namely proportionally to the mass (usually population) of the interacting units and under an inverse (approximately square) analogy of their distance. Despite the fact that SIMs do not utilize a graph theoretic or CNA approach in their spatial interaction modeling, their logic is compatible with the CNA's, since it is based on the analysis of flow matrices (see Ducruet 2013, for an example of commodity flow analysis) similarly to the way that CNA is based on calculations of adjacency matrices (see Diestel 2005; Barthelemy 2011).

Within the concept of spatial interaction, it can be said that the analysis of spatial networks is not an effort that can be exclusively attributed to physics in 90s, but some of its elements can be found to geographers, even back to Tobler (1970), who developed their own (free of complex networks) econometric-based approaches (see Miller 1999; Thomas 2002; Reggiani et al. 2011; Ducruet and Lugo 2013) to model spatial interaction phenomena (Ducruet and Beauguitte 2014). In geography, the concept of network emerged in many discussions concerning the spatial analysis of global production, commodity, value, supply chain, and corporation networks, but all such cases were focusing on the social (actors), strategic, and territorial aspects, rather than on the analysis of network topologies per se (Jacobs et al. 2010; Ducruet and Beauguitte 2014). Towards the direction of integrating approaches of spatial analysis in geography with CNA we can find the works of Andersson et al. (2006), who highlighted the benefits of using CNA complementarily to SIMs and to multiplicative growth models for urban growth research, of Patuelli et al. (2007) and Gorman et al. (2007), who used CNA and SIMs in the study of commuting networks in Germany, of Schintler et al. (2007), who attempted to link raster-based Geographic Information System (GIS) approach and CNA for road and railway network analysis in Florida, of Ducruet et al. (2011), who applied a Multiple Regression Quadratic Assignment Procedure (QAP) to capture correlations between network topologies on different levels of node aggregation, and of Tsiotas and Polyzos (2015a), who used CNA measures as variables participated in a spatial econometric model describing the interaction between a national maritime network with its socioeconomic framework.

According to the previous approaches, space obviously affects the network structure, which is a broad concept including material, typological, and geometric (spatial) terms. Provided that the network topology is an aspect immanent to network structure (Easley and Kleinberg 2010; Newman 2010), spatial constraints are consequently expected to interact with network topology. This interpretation suggests another research perspective that this paper aims to highlight at the following paragraphs.

2.3 Spatial Embedding

Spatial embedding is a critical concept in the study of spatial networks. It is related to the spatial receptor where a network is embedded, which causes some consequent structural and functional properties detected in a network. Depending on the space of embedding, networks are divided into two major categories, those that are embedded in a *topological* and a *metric* space. Loosely speaking, the *topological space* illustrates how a set of elements is arranged in a dimensionless (non-metric) space when subjected to a rule, such as neighborhood, hierarchy, etc. (Hausdorff 1957; Newman 2010). The topology is the prime structure describing a network before it is embedded to a metric space. By equipping the topological space with a metric function, this dimensionless space is transformed into a metric space (Hausdorff 1957) and thus distance measurement is henceforth possible.

Metric and particularly Euclidean spaces are where by default spatial networks are embedded. Depending on the dimensions of the metric space, spatial networks are divided into two sub-categories, the *planar* and *non-planar spatial networks* (Boccaletti et al. 2006; Barthelemy 2011; Ducruet and Beauguitte 2014). Planar networks are restricted in the two-dimensional (2d) Euclidean space and they have no crossing edges (Ducruet and Beauguitte 2014), whereas non-planar networks are embedded in the three-dimensional (3d) (or theoretically higher, *n*d) Euclidean space and they may also have intersecting edges. Land transportation networks belong mostly to the first category, whereas maritime and air transport networks are non-planar by default (Boccaletti et al. 2006; Barthelemy 2011).

Intuitively, the spatial embedding is the procedure that defines the concepts of "topology" and "geometry" in a network, where the first illustrates the relational configuration of the network elements, whereas the second delineates how these relations are transformed when they are embedded in a metric space. Fig. 2 illustrates two paradigms towards this discrimination. On the one hand, when studying a network embedded in a topological space someone is interested in whether the nodes are connected or not and by how many successive steps one node can reach another. This is the concept defining the network topology, illustrating the "map of relations" (here connectivity) that the network nodes may have in an inductive plane, where distance is topological and counts steps (degrees) of separation (see Milgram 1967). Some early works in social network analysis also use the term "social space" to describe the network topology (see i.e. Pool and Kochen 1978), which was a concept introduced by Emile Durkheim in the 1890's (Rood 1982). On the other hand, when a network is embedded in a metric space the shape and the arrangement of such linkages may alter due to the spatial constraints, where distance this time is measured in geometric units (such as km, nm, etc.). In Fig. 2, the example (a) illustrates how such differences may be for a star-like graph and (b) for a national railway network.

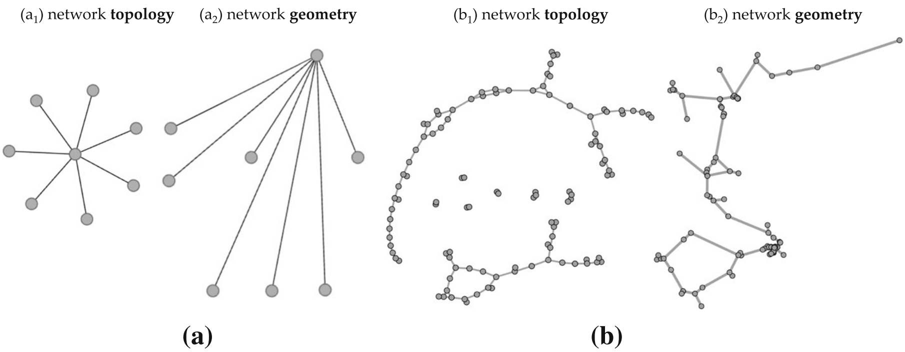

Fig. 2 A pair of paradigms used for the conceptual discrimination between network "topology" and "geometry". In example (**a**) a star graph is illustrated in (a_1) a topological and (a_2) in a metric space. In case (**b**) a railway network (in Greece) is illustrated in (b_1) the topological and (b_2) its physical (geometric) space (source: own elaboration)

Within this concept, the spatial constraints cause some typical effects to network topology and traffic, such as:

- *Effects of space in the degree* $p(k)$ and *edge distribution* $p(e)$: Space undermines the development of links (and thus connectivity) in networks, mainly due to transportation cost. This is usually reflected on the typology of the degree distribution that is peaked around the average (Barthelemy 2011). Such constraints are more intense in planar networks (Barthelemy 2011; Ducruet and Beauguitte 2014), whereas in non-planar networks are smoother resulting many times to different typologies than of peaked distribution (Guida and Maria 2007; Bagler 2008; Barthelemy 2011; Ducruet and Beauguitte 2014; Tsiotas and Polyzos 2015a). Spatial constraints also affect the edge length and thus the typology of the edge distribution $p(e)$. For planar networks, the $p(e)$ has a peaked pattern, whereas patterns of non-planar networks are smoother (Barthelemy 2011).
- *Effects of space in network assortativity and clustering:* Spatial constraints undermine the development of hubs and the extent of preferential attachment that occurs usually over shortest distances. Long distances are developed mainly between hubs serving high traffic needs leading to the planar behavior of the assortativity distribution and to the emergence of cliques increasing the network clustering (Watts and Strogatz 1998; Barthelemy 2011).
- *Effects of space in average path length*: Average path length in planar networks scales according to a regular lattice ($\sim n^{1/2}$). When shortcuts are connecting non-successive nodes, such as in cases of small-world networks (Watts and Strogatz 1998), this square root scale becomes a logarithmic one ($\langle l \rangle \sim \log n$). However, in some 3d–space cases (such as in biological networks) is difficult to distinguish between the logarithmic ($\langle l \rangle \sim \log n$) and exponential ($\langle l \rangle \sim n^{1/3}$) behavior (Barthelemy 2011).
- *Effects of space in the relation between topology and traffic:* The spatial constrains reinforce the local activation of preferential attachment favoring the development of regional hubs. This induces large fluctuations to betweenness centrality (C^b) that is geographically-controlled and concentrated near the network barycenter. Additionally, it yields higher values of spatial strength (s) than those expected in topological

spaces, leading to non-linear correlations ($b > 1$) between network *connectivity* (*degree* - k) and *distance* (*strength* - s) (Barthelemy 2011).

According to the previous consideration, space appears to affect the network topology, but also to interact with it in the extent of favoring or restricting the appearance of certain topological patterns or motifs. For instance, it is more likely for a spatial network to develop a lattice-like than a scale-free or a small-world topology and this is more possible to occur for a road or railway network than for a maritime or an air transport network. Further, it is more likely for land transport hubs to appear in central geographical places than in peripheral locations (Polyzos 2011; Barthelemy 2011; Ducruet and Beauguitte 2014). However, regardless that space is an aspect being either directly or indirectly immanent in the network structure and topology, the study of the spatial dimension is often ignored when using CNA. As being evident from the review of Ducruet and Beauguitte (2014), many scholars from many disciplines when studying spatial networks usually focus on the examination of network topology and not (either at all or equivalently to the topology) on its spatial embedding (e.g. the distance parameters are not included in the analysis), so as finally to ignore space in the study of networks with the spatial property. This is perhaps due to the considerable SNA deposits in the CNA measures' toolbox, which disproportionally highlight the significance of network topology against its spatial embedding (i.e. the network geometry), or due to interdisciplinary difficulties in integrating the research and cognition in networks. Towards this integration several scholars worked for exploring the influence of the spatial structure to the network topology (see Ducruet and Beauguitte 2014) and this paper devotes the purpose of its epistemological consideration.

2.4 Methodological Approaches

2.4.1 Type of Research

According to the type of research, the methodological approaches in the study of spatial networks can be distinguished to *theoretical*, *empirical* and *reviews* (Barthelemy 2011; Ducruet and Beauguitte 2014). Theoretical approaches are the earliest and responsible for configuring the basis for the development of CNA in spatial networks. They are based mainly on graph theory, statistical and computational mechanics, physics, and simulation. However, their extensive presentation falls out of the purpose of this paper and concerns more the broader field of complex networks (Berners-Lee et al. 2007; Easley and Kleinberg 2010; Newman 2010; Brandes et al. 2013). Overall, the theoretical approaches have so far contributed to the in depth knowledge of the structure and functionality of networks and have provided reference models that are useful in pattern recognition and in further empirical research (Fortunato 2010; Barthelemy 2011; Tsiotas and Polyzos 2015b).

Based on the empirical research in complex networks (Albert and Barabasi 2002; Boccaletti et al. 2006; Barthelemy 2011; Ducruet and Beauguitte 2014), two major methodological approaches are evident; the first regards the *comparison of empirical networks with reference models*, whereas the second *compares empirical networks with other empirical models* already studied in the literature. The common feature in both these approaches is the configuration of a spatial interaction system into a graph model, which is a complex procedure depending on the available data, the computational effort, and the researcher's experience. These two approaches are briefly described as follows:

(a) *Comparison of empirical network models with theoretical reference networks*: This approach regards the calculation of network measures and attributes of empirical network models and the comparison of the results with those corresponding to equivalent theoretic (reference) networks. Comparisons may provide indications whether the empirical network that is under examination is related to a theoretical pattern. Some indicative examples drafted mainly from (Barthelemy 2011) indicate that an empirical network:

- is *planar* when $m \leq 3n\text{-}6$, where n is the number of nodes and m the number of edges in the network,
- is a *giant lattice* when the inverse eccentricity (i.e. *maximum* path length from a node) is zero,
- has *random* characteristics when the binary average edge length is $\langle d_e \rangle \approx \ln n / \ln \langle k \rangle$, where $\langle k \rangle$ is the average node degree,
- has *a star topology* when the transitivity (clustering coefficient) is $C \approx 0$ (Tsiotas and Polyzos 2015b),
- is described by *SF attribute* when the degree distribution $p(k)$ follows a power-law rule $p(k) \sim k^{-a}$, with typical exponent lying within the interval $2 \leq a \leq 4$ or when its diameter scales as $d(G) \sim \ln(\ln n)$,
- is submitted to *intense spatial constraints* when it has a peaked degree distribution,
- is described by the small-world attribute when its average path length scales as $\langle l \rangle_{bin} \sim \log n$.

Indicative papers using (partially or exclusively) this method have studied road (Lammer et al. 2006), railway (Sen et al. 2003), commuter (Chowell et al. 2003; de Montis et al. 2007), cargo ship (Hu and Zhu 2009), and air transport (Guida and Maria 2007; Bagler 2008) networks. A major concern in this procedure regards the generation of reference networks, the so-called null models (Fortunato 2010), which are computationally generated graphs matching the empirical graphs either in the number of nodes and edges or more restrictively in the degree distribution. Some basic algorithms for generating such networks is the algorithm of Albert and Barabasi (2002), for obtaining a SF network with the same number of nodes with the empirical network and with probability of attaching new edges proportional to the each time nodes' degree, the random algorithms of Maslov and Sneppen (2002) and Newman and Girvan (2004), for obtaining random networks preserving the degree distribution of the empirical network, and the "latticization" algorithm of (Rubinov and Sporns 2010), which produces an equivalent to the empirical network lattice with the same degree distribution.

Some major advantages of this methodological approach concern light data modeling requirements (it necessitates the construction of a single graph), availability of theoretical research, standardization, and easy documentation based on theoretical reference. On the other hand, considerable disadvantages are that is being submitted to approximations due to the asymptotical ($n \rightarrow \infty$) definitions of some concepts, such as the SF and SW attributes, and that it lacks of direct physical interpretation due to theoretical reference (Boccaletti et al. 2006; Fortunato 2010; Barthelemy 2011; Ducruet and Beauguitte 2014). This approach is considered incomplete to stand solely, but it is rather effective whether it is used in conjunction with others.

(b) *Comparison of empirical network models with other empirical models:* In this approach network measures and attributes are being compared between empirical networks. Comparisons are made either by using data from an empirical network that was previously studied in the literature (and it is considered as a reference network) or by constructing two or more empirical models. The empirical results may either verify findings of other studies or provide new insights that are further being submitted under consideration for the formulation of empirical rules and theory.

Some indicative studies of first category (empirical models vs empirical networks already studied in literature) examine urban road (Crucitti et al. 2006) and commuting networks (de Montis et al. 2007; Strano et al. 2009; Polyzos et al. 2014), global cargo ship network (GCSN) (Kaluza et al. 2010), and air transport networks, such as the worldwide aviation network (WAN) (Guimera and Amaral 2004; Guimera et al. 2005), the USA Aviation Network (USAN) (Barrat et al. 2005; Guimera et al. 2005), the Air Transport Network of China (ATNC) (Wang et al. 2011), the Italian Aviation Network (IAN) (Guida and Maria 2007), and the Aviation Network of India (ANI) (Bagler 2008). Additionally, some indicative examples of the second category (empirical models vs empirical models) are the studies of Buhl et al. (2006) comparing topological characteristics among road networks of cities worldwide (Algeria, Belgium, France, Germany, India, Spain, Iran, Italy, Syria), of Cardillo et al. (2006) comparing also topological characteristics among city road networks worldwide (India, Spain, Brazil, Egypt, USA, France, Korea, Italy), and of (Hu and Zhu 2009) comparing two versions of the GCSN, the first embedded in the L- and the second in the P- space representation.

Overall, some major *benefits* of this methodological approach concern implementation to real-world networks producing practical results of physical interpretation, existence of links between network topology and socioeconomic framework, and availability of empirical research. On the other hand, some disadvantages are the condition of models compatibility (i.e. accepted comparisons should occur between two undirected or two directed networks and not between a directed and an undirected network), data availability, and sensitivity to modeling conditions (Boccaletti et al. 2006; Fortunato 2010; Barthelemy 2011; Ducruet and Beauguitte 2014).

2.4.2 Number of Layers

Multilayer modeling is a way to express different aspects of a network as layers included in a grouped representation called multilayer network (Boccaletti et al. 2014; Kivela et al. 2014). Ducruet (2013) notes that early elements of such an approach can be found by sociologists in the 1960s (Mitchell 1969; Burt 1992), while examining both personal and professional relations between individuals belonging in career networks. This possibility to model more than one relation in a single edge was expressed in social network analysis with the term "two dimensional edges" and networks having such edges were studied using conventional network analysis at each type of edges (Wasserman and Faust 1994; Newman 2010; Ducruet 2013).

In the engineering science and especially in the transportation sector, the early multilayer network modeling appeared as a coupling of different networks in terms of interdependency, mutual specializations, and vulnerabilities (Van Geenhuizen 2000; Zhang et al. 2005; Ducruet 2013). The coupled network was considered as a "network of networks" suggesting a promising approach in many applications, such as for the modeling of the multimodal aspects of global urban centrality (Ducruet et al. 2011) or of the confidence relationships among individuals in firms (Ducruet 2013). Towards this direction many empirical research targeted to examine the effect of network coupling on the structure of the aggregate network (Ducruet 2013, 2017), such as are indicatively the works of:

- Rosato et al. (2008), who studied the electrical grid of a telecommunication network in Italy,
- Bogart (2009), who detected inter-modal influences of road, canal, and port infrastructure in England at the period 1760–1830,
- Parshani et al. (2010), who studied inter-similarities of coupled worldwide air and sea transport networks,
- Jin et al. (2010), who examined local effects of network coupling compared to local socioeconomic characteristics,
- Ducruet et al. (2011), who investigated the role of air and sea transport networks in the formulation of the global hierarchy of urban accessibility,
- Tavasszy et al. (2011), who developed a strategic network choice model incorporating maritime and inland links for predicting global container flows, and of
- Ducruet (2013), who examined the coupled global network of the respective weighted graphs of solid bulk, liquid bulk, container, general cargo, and passenger/vehicles vessel inter-port movements.

A characteristic application in this field is the work of Ducruet et al. (2011), examining the participation of cities in worldwide air and sea networks. The purpose of the study was to reveal complementarities between these transport networks in shaping an urban hierarchy and to evaluate the influence of their aggregation in the topology of these networks. To do so, the authors considered a six-level network divided into two groups (air and sea transport) and within each group into three classes representing different levels of scale: *cities*, *regional areas* and *megalopolises*. The analysis indicated that the chosen multi-level structure grasped some important topological and geographical properties of these networks, which were interdependent to urban development. It also showed that global players in the transport industry tend to follow the main paths of urban development and that the majority of the urban regions and megalopolises were benefited from a combined air and sea network, whereas such co-existence in the city level was rather competitive than cooperative. Finally, the analysis highlighted that the largest nodal regions were specializing in air traffic and that independent cities tend to be connected with those of similar specializations as an effect of the distinctive geographic coverage between air and sea networks.

All relevant to the previously mentioned studies can be considered as early contributors to the multilayer network modeling, because, although they manipulate coupled networks connected with diverse types of interconnections and thus they are inspired by a multilayer rationale in their modeling, they did not yet follow a universal formalism at their multilayer consideration. According to Ducruet (2013), such studies were absent of a specific mathematical definition of the properties of multigraphs, motivating this author to review how such coupled networks have been analyzed across these diverse fields. One of the first studies that introduced the term "multi-layer" in their analysis and attempted to provide basic terminology was the work of Kurant and Thiran (2006b), entitled "Layered Complex Networks". However, the works that filled the literature gap that existed in the formalism of multilayer modeling were these of Boccaletti et al. (2014) and Kivela et al. (2014), who presented simultaneously review articles on multilayer networks equipped with the necessary mathematical rigor originating mainly from the set theory (see Hausdorff 1957). Within this framework, it can be said that the multilayer modeling suggests a modern perspective that emerged due to the evolution in the study of complex networks and to the consequent increase of their level of complexity is multilayer modeling (Boccaletti et al. 2014; Kivela et al. 2014; Tsiotas and Polyzos 2015b).

Intuitively, multilayer networks are models representing jointly a group or family of networks (called layers) that suggest either different or complementary aspects of the same communication system. More rigorously, according to Boccaletti et al. (2014) and Kivela et al. (2014), a multilayer network $\mathscr{M}$ with p layers is defined as the pair set $\mathscr{M} = (\mathcal{G}, \mathcal{V})$, where $\mathcal{G} = \{G_a, \text{ with } a = 1,2,\ldots,p\}$ is a family of graphs $G_a = (V_a,E_a)$ and $\mathcal{C} = \{E_{a\beta} \subseteq V_a \times V_\beta, \text{ with } a,\beta \in \{1,2,\ldots,p\} \text{ and } \alpha \neq \beta\}$ is the number of the interconnections attached between nodes of different layers G_a, G_β, with $\alpha \neq \beta$. Cases where $V_a \equiv V_\beta$, for $a,\beta \in \{1,2,\ldots,p\}$, are defined as multiplex networks, which are of particular interest in transportation research (Ducruet 2013; Boccaletti et al. 2014; Kivela et al. 2014; Tsiotas and Polyzos 2015b).

A representative couple of studies was presented by Cardillo et al. (2013a,b), who examined the structural properties of the European Air Transport Network (EATN), consisting of 37 layers, each representing either a major or an LCC aviation enterprise. In their first work (Cardillo et al. 2013a), the researchers studied the changes in topology of the EATN, which were induced by the successive random merging of layers. This approach indicated that EATN has some structural attributes similar to the WAN, such as are the "rich-club" property, because of the contribution of major airlines, the path redundancy, as a result of the cooperative aggregation of major and LCC firms, and the small-world property, where LCC seem to contribute significantly. The authors also noted that the topologic properties of the EATN depend more on the multilayer structure of this network, rather on the layer topologies separately. In their second study (Cardillo et al. 2013b), the authors examined the structure of multilayer EATN under the perspective of its resilience to failures, examining the network's resilience against to the deletion of an edge (cancelation of a flight). Here, the layers express the potential of alternative flights for reaching one's destination, in case this is impossible through in-layer rescheduling. In this approach the authors observed that also the structures of the multilayer and overlaid differ, concluding that multilayer structure is more resilient to failures, since it allows movements through different layers.

From another perspective, Tsiotas and Polyzos (2015b) studied the topology of the multilayer *Greek Aviation Network* (GAN), expressed as a multiplex graph with layers representing networks of the companies operating in the domestic Greek aviation market. In this approach, homologue vector variables participated to a multivariate regression analysis applied per network attribute, considering cases of the overlaid network as response and of the layers as predictor variables. The results showed which layer contributes more to the configuration of every network attribute of the overlaid GAN and they provided insights about the strategic plans that the aviation companies follow comparatively with their contribution to regional development and aviation policy.

In methodological terms, it can be said the multilayer consideration advances the study and management of more complex sets of network data, since it suggests a hybrid approach allowing joint manipulation of multilayer and monolayer models. This contributes to the increase of resolution and to the universality of the multilayer modeling allowing the formation of a single network model, regardless the type of the examined communication system. On the other hand, it necessitates greater volume of network data and advanced computing knowledge, which restricts its applicability (Ducruet 2013; Boccaletti et al. 2014; Kivela et al. 2014; Tsiotas and Polyzos 2015b; Ducruet 2017). However, it suggests a promising modeling field and the conceptual basis where the proposed methodological framework was structured.

2.5 Conceptual Elements

According to Easley and Kleinberg (2010), the concept of "network" includes a *structural* and a *functional* (or *behavioral*) component. The structural component represents the set of the network infrastructures and suggests the constructed background supporting the conduct of communication among the network nodes. All the other factors that are related with the communication flows represent the functional component of the network. Especially for the case of the WWW, Berners-Lee et al. (2007) presented a similar conceptual model describing this superior technological and socioeconomic network as a composition of a structural (the *Internet*, i.e. the hardware of the WWW), an operational (the *Web*, i.e. the software being responsible for the information exchange within the internet), and a social layer (i.e. the human *society* utilizing this global network as a consumption good).

Under this perspective, Tsiotas and Polyzos (2015c) proposed an expansion of Easley's and Kleinberg's (Easley and Kleinberg 2010) dichotomous conceptual model, including a third "ontological" component to the concept of "network" (Fig. 3), which refers to the amount of *socioeconomic*, *cultural*, *ethical*, *cognitive* and relevant attributes that describe the network nodes (actors). This approach highlights the importance of the socioeconomic framework in the network configuration, which, according to the authors of Ducruet and Beauguitte (2014), is an aspect that has not yet been sufficiently studied by the network scientists, attracting more the interest of geographers.

Some representative studies examining in common topological and socioeconomic aspects of networks are the works of Crucitti et al. (2006) studying centrality of urban road networks comparatively with commerce vitality, land-use separation and urban crime, of Ducruet and Notteboom (2012) examining the regional dynamics of the GCSN, of Wang et al. (2011) examining the economic pattern of the network topology

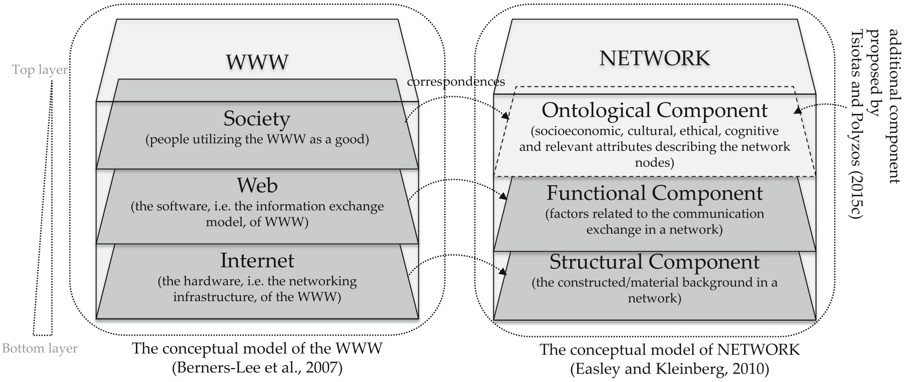

Fig. 3 Analogies between the conceptual models of "WWW" (Berners-Lee et al. 2007) and "NETWORK" (Easley and Kleinberg 2010) (source: own elaboration)

of Air Transport Network of China (ATNC), of Ducruet et al. (2011) examining the participation of cities in worldwide air and sea networks, of Tsiotas and Polyzos (2015a) examining the socioeconomic factors immanent in a national maritime network, and of Tsiotas and Polyzos (2015b) detecting strategic plans of aviation firms from their air transport network topology.

Within this framework, the conceptual model of Tsiotas and Polyzos (2015c) seems to serve the interdisciplinary demand noted by the authors of Ducruet and Beauguitte (2014) and thus to be more equipped for studying spatial networks. According to this consideration, a complete analysis of a spatial network should be conducted under a joint *structural*, *functional* (or *behavioral*) and *ontological* aspect, which is the basis of the proposed methodological framework described in the following section.

3 Integrating the Topology, Space, and Socioeconomic Framework in the Study of Spatial Networks

As previously mentioned, the multidisciplinary nature of NS, the volume of relevant research, and the diversity of disciplinary approaches illustrate the complexity in the study of spatial networks, which necessitates a joint consideration. Being inspired by the review articles of Barthelemy (2011) and Ducruet and Beauguitte (2014), this subsection attempts to integrate existing approaches in the study of spatial networks into a single methodological framework. The proposed framework is both of practical and instructive purpose, aspiring to standardize the procedures in the analysis of spatial networks, to contribute to their comprehension, and to promote the interdisciplinary research of complex networks. Additionally, this approach pushes further the epistemological dialogue about how NS can move from research to the academic didactics.

The methodological framework is illustrated at the flow chart in Fig. 4 and it is structured into discrete parts (A, B, A_1, A_{21}, A_{111}, etc.) for readability purposes. The framework starts with the decision of the network number being under examination. Whether the researcher decides to study a single network ($SN = 1$), the procedure goes to the alternative "A" (single network analysis), otherwise (when $SN > 1$) it goes to

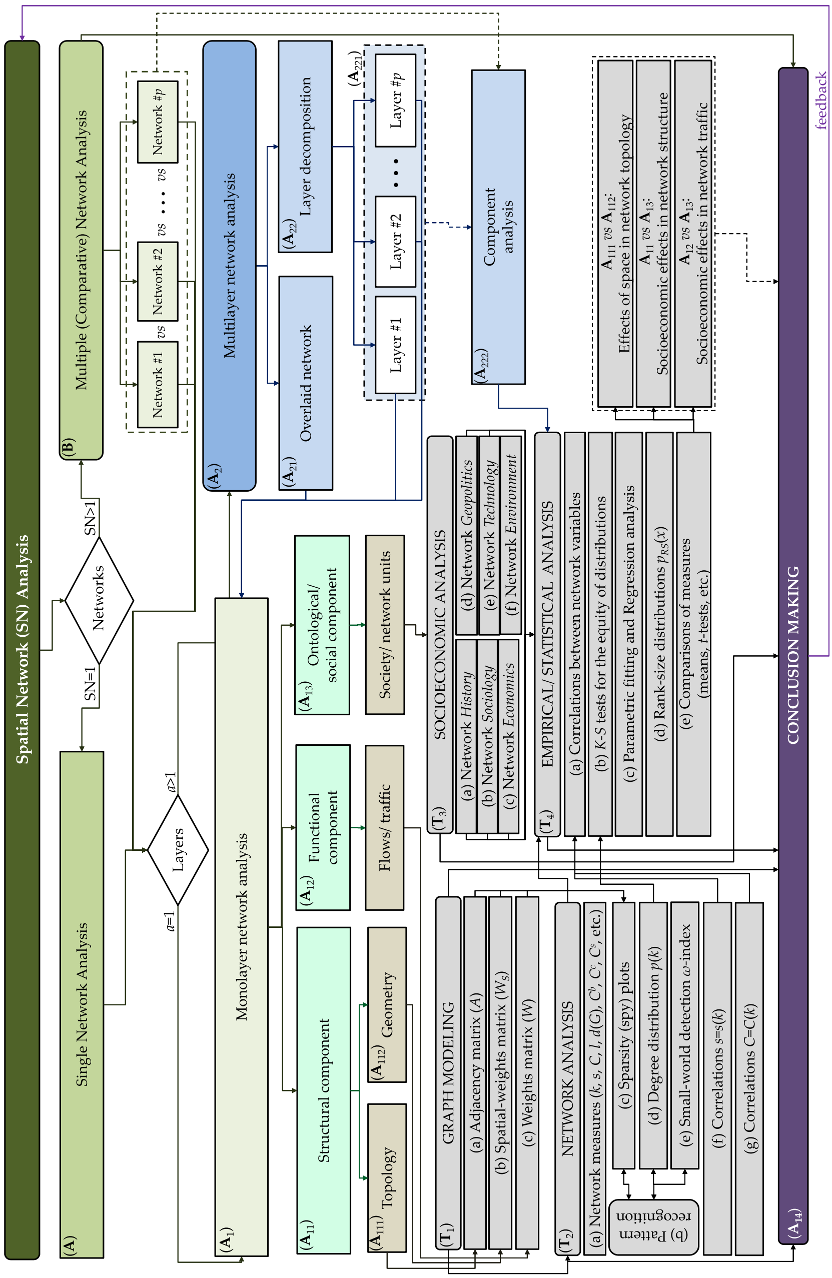

Fig. 4 Flow chart of the proposed methodological framework (source: own elaboration)

alternative “B” (multiple/ comparative network analysis). This decision depends on the specific purpose of the study and mainly on the availability of data.

In the study of a single network (alternative “A”) a second decision appears concerning the chosen number of layers (monolayer/A_1 or multilayer/A_2 analysis). This is a multi-criteria decision, depending jointly on the physical or socioeconomic network structure, on the node attributes, and on the availability of data (Boccaletti et al. 2014; Kivela et al. 2014; Tsiotas and Polyzos 2015b). Indicatively, cases of either disconnected graphs consisting of p in number sub-graphs or of network nodes V that can be divided into p groups may favor the multilayer modeling that strongly depends on data availability. For example, an air transport system with available either non-traffic or aggregate (i.e. just origin-destination that cannot be grouped) traffic data is not suitable for multilayer modeling. In contrast, the multilayer representation is suggested when air traffic data per aviation firm are available for the network (Tsiotas and Polyzos 2015b). In general, modeling a spatial network in multilayer terms is interdependent to the environment of the spatial system, but it also significantly depends on the researcher’s experience and skills (for further information see Ducruet et al. 2011; Boccaletti et al. 2014; Kivela et al. 2014; Tsiotas and Polyzos 2015b).

According to the proposed framework, both of the $SN = A \vee B$ and $A = A_1 \vee A_2$ decisions drive cyclically to the monolayer network analysis (procedure A_1). This implies that monolayer modeling is a fundamental procedure in CNA, since every layer represents also a single network. The monolayer network analysis (A_1) is divided into three conceptual parts (A_{11}, A_{12}, and A_{13}), which correspond to the components of the conceptual model proposed by Tsiotas and Polyzos (2015c), and into four technical (methodological) parts (T_1–T_4), each including a different aspect of the spatial networks analysis (1: Graph Modeling, 2: Network Analysis, 3: Socioeconomic Analysis, and 4: Empirical/Statistical Analysis). When it is applicable, comparisons between homologue features that are computed on different components (sub-procedure T_4.e) may provide insights about the components’ interrelations.

The first part of the monolayer (spatial) network analysis regards the study of the structural component (A_{11}), which is distinguished into the concepts of network “topology” (A_{111}) and “geometry” (A_{112}). This discrimination allows capturing some latent effects caused by the spatial embedding of the network (through comparisons T_4.e), since the topology refers to the dimensionless arrangement of the network elements in a topological space, whereas the geometry refers to their metric configuration (see Fig. 2). A keynote concept in this procedure that affects the functionality of many network measures and attributes is *distance*; the topological analysis considers the so-called “binary distances” that measure steps of separation or number of intermediating links between two nodes, whereas spatial (geometric) analysis is implemented on metric or geographical distances.

In technical terms, the topological modeling of the spatial network is implemented with the construction of the adjacency (or connectivity) matrix A (sub-procedure T_1.a), in which non-zero elements are equal to one (=1) and imply the existence of a connection between the nodes corresponding to the matrix’s coordinates. On the other hand, the spatial (geometric) modeling is implemented on a weights matrix (T_1.b), the so-called “spatial-weights matrix” (W_S), in which the non-zero elements that are homologue to those of the adjacency matrix are this time equal to the geographical distances between the corresponding nodes (for elements of graph theory see Diestel

2005). Network measures (T_2.a) that are computed on the adjacency matrix and count distances are considered as "binary", because path lengths in the adjacency matrix represent number of steps, whereas those that are computed in the spatial-weights matrix are considered as "spatial-weighted", because path lengths in the spatial-weights matrix express (geo)metric distances. Within this context, comparisons between homologue features computed on binary and metric distances (T_4.e) can illustrate the effect of space in network topology (Tsiotas and Polyzos 2015a). For example, while row (or column) summations in the adjacency matrix produce degrees expressing connectivity, in the spatial weights matrix they produce strengths expressing neighborhood dominance.

However, those network measures that do not compute distances in their formula are by default indifferent to this discrimination (topological vs geometric) and they may be considered as just topological measures. For example, the node degree (k) is a measure counting node connectivity (i.e. the number of edges that are adjacent to a node), which gives the same result either it is computed on the adjacency or the spatial-weights matrix. On the other hand, the node strength (s) counts the total of geometric neighborhood distances when it is computed on the spatial-weights matrix, whereas it is transformed to the measure of node degree when it is computed on the adjacency matrix. According to this consideration, the capturing of spatial information from the CNA measures is a demanding procedure necessitating from the researcher to be familiar with the conceptual framework and the functionality of such measures. In order to contribute towards this familiarization, Table 2 shows the functionality classifications of the network measures according to a set of major characteristic network attributes, such as the *network scale*, the *type of network elements*, the *centrality*, the *space of embedding*, the *graph type*, and the *computational structure*. These attributes are drafted from the relevant literature (Diestel 2005; Koschutzki et al. 2005; Barthelemy 2011; Rozenblat and Melancon 2013; Boccaletti et al. 2014; Kivela et al. 2014; Tsiotas and Polyzos 2015b) and they are presented in Table 2 with a short description, in respect to their classification criterion.

In Table 2, there are cases of measures belonging to more than one class, such as the transitivity (clustering coefficient, C) that has both global (i.e. when it counts triplets and triangles in the whole network) and local (i.e. when the counting occurs for a neighborhood) expressions, and other cases where the measure-classes may overlap due to the modeling conditions (e.g. the network may be with both spatial and non-spatial flow weights and thus the same measure, such as the node strength, may attain different expressions) or due to definition (e.g. the mixing space and topology measures may be conditionally considered as topological measures, or the binary measures are also topological measures). Also, the local measures may provide aggregate information when they are being averaged along the total of nodes (e.g. the average path length $\langle l \rangle$, the average clustering coefficient $\langle s \rangle$, etc.).

In technical terms, the methods for studying the network topology in CNA are not restricted to the computation of the network measures (T_2.a) that is of course a fundamental procedure in network analysis. Another part in the topological analysis of complex networks is the so-called "pattern recognition" (T_2.b), which concerns the set of procedures, tests, and examinations aiming to detect the pattern (or the typology) of a network's topology (see Albert and Barabasi 2002; Newman 2010; Barthelemy 2011). Some basic aspects that compose the procedure of pattern recognition are the

Table 2 Functionality classification of network measures

Criterion (Sub-criterion)	Measures' Class		Description
Network scale	(i)	Global (aggregate)	Provide information for the whole network.
	(ii)	Local	Describe either a network element, such as a node or edge, or a network component (sub-network).
Type of network elements	(i)	Node	Describe network nodes
	(ii)	Edge (*)	Describe network edges
Centrality	(i)	Degree-based	Illustrate aspects of hierarchy in respect to a network attribute (e.g. degree, closeness, intermediacy)
	(ii)	Closeness-based	
	(iii)	Betweenness-based	
Space of embedding	(i)	Topological	Free of spatial information (they are computed on the adjacency matrix A)
	(ii)	Geometric: mixing space and topology	Include both topological and spatial information (they are computed on the spatial-weights matrix W_S)
Graph type (weights)	(i)	Weighted	Edges have non-spatial weights (they measure flow intensity)
	(ii)	Spatial-weighted	Edges have spatial weights (they measure geographical distance)
	(iii)	Unweighted (binary)	Edges are free of weights (they express connectivity, having binary distances)
Graph type (direction)	(i)	Directed	Edges express one-way (directed) connections
	(ii)	Undirected	Edges express ale-retour (undirected) connections
Computational structure	(i)	Accessibility	Capture aspects of the ease to access (or to move inside) a network.
	(ii)	Neighborhood	Provide information for the first level of connectivity (i.e. the neighbors, which are the direct connections in a node)
	(iii)	Distance	Compute distances (either binary or weighted)
	(iv)	Shortest path	Compute the shortest of the paths existing between pairs of nodes

source: own elaboration based on the references

Diestel (2005); Koschutzki et al. (2005); Barthelemy (2011)

Rozenblat and Melancon (2013); Boccaletti et al. (2014)

Kivela et al. (2014); Tsiotas and Polyzos (2015b)

examination of the sparsity (spy) plots (T_2.c) of the network matrices (T_1), the detection of the typology of the degree distribution $p(k)$ (T_2.d) and the computation of the omega (ω) index for the detection of the small-world property.

The examination of spy plots (T_2.c) (Bishop 2006) regards the visualization of the network's adjacency matrix using a spy plot representing with dots its non-zero elements. The pattern of the empirical network's spy plot is examined comparatively to those of available null-models that are constructed under the $p(k)$-equivalent (i.e. with the same degree distribution) and the n-equivalent (i.e. with the same number of nodes) constraints. A paradigm shown in Fig. 5 illustrates how this approach works, where the empirical network is (a) a national road network (in Greece) and the available

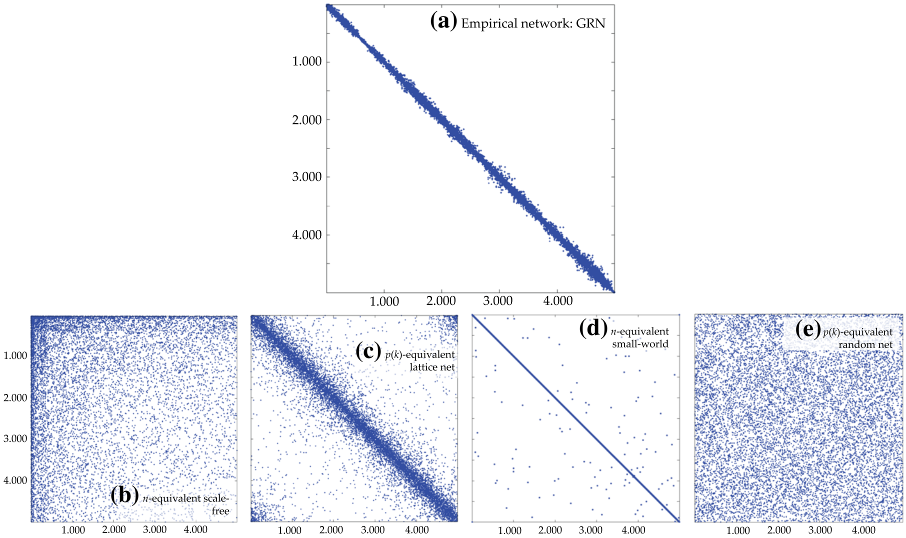

Fig. 5 Sparsity (spy) plots constructed from the adjacency matrices (**a**) of a national road network (in Greece) (with $n = 4.993$ and $m = 6.487$) and of four equivalent null-models, which have respectively the (**b**) *scale-free* (n-equivalent), (**c**) *lattice* ($p(k)$-equivalent), (**d**) *small-world* (n-equivalent), and (**e**) *random* ($p(k)$-equivalent) network property. The examination of the spy plots provides indications about the pattern of the empirical (**a**) network's topology (source: Tsiotas 2016b)

null-models are a $p(k)$-equivalent (c) lattice and (e) random network and an n-equivalent (b) scale-free and (d) small-world network. The null-models are computed using available algorithms (see Maslov and Sneppen 2002; Rubinov and Sporns 2010; Fortunato 2010; Barthelemy 2011). In this paradigm, comparisons indicate that the pattern of the empirical (a) network resembles more to this of the lattice model (c), although the "latticization" algorithm that produced this null-model does not preserve the empirical network's planarity.

Next, a very significant procedure in network analysis (T_2) and especially in pattern recognition (T_2.b) is the examination of the degree distribution $p(k)$. The typology of $p(k)$ provides insights about some structural patterns in networks, such as the SF, the SW, the existence of randomness, or even the intensity of the spatial constraints (Albert and Barabasi 2002; Barthelemy 2011; Stumpf and Porter 2012; Ducruet and Beauguitte 2014). The major concern in this aspect is finding proper methods for detecting the typology of the degree distribution. A pair of effective and most commonly used methods are the *parametric fitting* (Norusis 2005) and the *one-sample Kolmogorov-Smirnov* (*K–S*) *test* (Massey 1951; Conover 1980; Norusis 2005). Technically, parametric fitting (Fig. 4, T_4.c) is the process of estimating a model's coefficients (parameters) that fit to an available set of data. The data are assumed to be of statistical nature, consisting of a deterministic and a random component, according to the semantic diagram shown in Fig. 6. This technique concerns the fitting of a theoretical model (curve) $f(x)$ to a set of empirical data $p(x)$, so as the absolute (alternatively the square) differences $|f(x) - p(x)|$ to be minimized. In this procedure the so-called coefficient of determination (R^2) expresses the degree that the theoretical model describes the variability of the empirical observations (Norusis 2005).

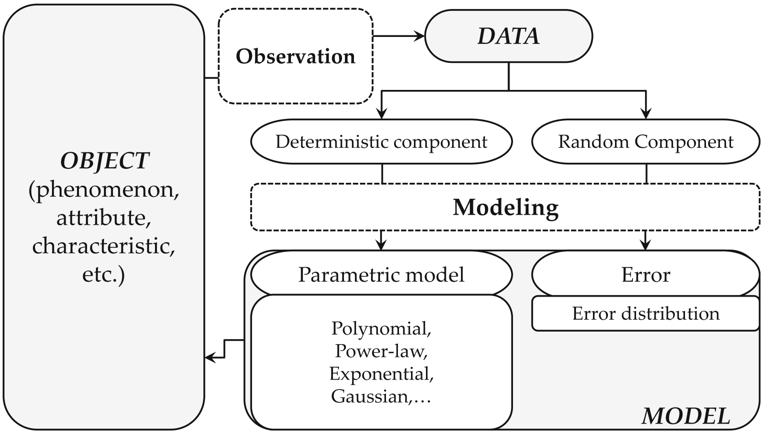

Fig. 6 Semantic diagram illustrating the modeling procedure of a phenomenon (source: own elaboration)

Generally, parametric fitting is effective under the condition of data availability; otherwise a considerable amount of error enters the model (Stumpf and Porter 2012; Tsiotas and Polyzos 2015b). The major concern here is choosing the data size so as to obtain results with satisfactory resolution. A statistically safe choice is to consider more than 30 observations, as it is suggested in statistical inference (Devore and Berk 2012; Walpole et al. 2012). However, having available over 30 discrete degree classes ($k_i = 1,2,\ldots,30$) is impractical for many real-world networks and thus the determination of the fitting data size remains yet an open question, depending on the researcher's modeling decisions. Within this framework, the use of parametric fitting is suggested in conjunction with other approaches.

An alternative method for the degree distribution pattern recognition is the *one sample Kolmogorov – Smirnov* (*K-S*) *test* (Fig. 4, T_4.b) (Massey 1951; Conover 1980; Norusis 2005). This procedure compares the empirical cumulative distribution with a theoretical (Poisson, exponential, uniform, normal, and power-law) and it statistically tests their equivalence. The steps of the *K-S* algorithm are described as follows:

The *K-S* algorithm

Step 1: the empirical observations are sorted into ascending order.
Step 2: the parameters of the theoretical distribution are estimated from the available empirical data.
Step 3: the scores of the theoretic cumulative distributions $\hat{F}(x)$ are calculated based on the parameters resulted from the previous step.
Step 4: the differences $\hat{F}(x_i - 1) - F(x_i - 1)$ are calculated, between successive pairs of theoretical and empirical distributions.
Step 5: the z-score is calculated testing the absolute maximum difference between the compared distributions. The two-tailed significance is estimated using the first 3 terms of the Smirnov's formula.

The *one sample Kolmogorov – Smirnov* (*K-S*) *test* is applied to the observations x_i and not to frequencies of the discrete cases $n(x_{j\le i})$ and thus it utilizes more data for the calculations. Consequently, the *K-S* test is more effective than the parametric fitting in cases of small data availability (Norusis 2005; Stumpf and Porter 2012; Tsiotas and Polyzos 2015b). However, for evaluation purposes, this method is recommended to be used in conjunction with a complementary fitting technique.

Another approach for pattern recognition in complex networks is the calculation of the *omega* (ω) *index* proposed by Telesford et al. (2011) for the approximate *small-world* detection. According to the network theory, the small-world property is defined on a family of graphs, whether the average path length scales logarithmically $\langle l \rangle = \mathcal{O}(\log n)$ as $n \rightarrow \infty$ (Barthelemy 2011; Porter 2012). Due to the unavailability of studying a family of graphs in empirical cases, the small-world attribute is detected *approximately* using the ω index, which compares the clustering of the empirical network with that of a $p(k)$-equivalent lattice network ($\langle c \rangle_{latt}$) and the empirical network's path length with that of an $p(k)$-equivalent random network ($\langle l \rangle_{rand}$), according to the relation $\omega = (\langle l \rangle_{rand} / \langle l \rangle) - (\langle c \rangle / \langle c \rangle_{latt})$. Values of ω are restricted to the interval $[-1,1]$, where those close to zero illustrate the SW attribute, positive values indicate *random characteristics* and negative indicate more *regular* or *lattice-like* characteristics (Telesford et al. 2011; Tsiotas and Polyzos 2015b).

The second aspect in the study of the structural component (A_{11}) is the spatial/geometric network analysis (alternative A_{112}) that computes spatial (e.g. kilometric) and not binary (i.e. topological) distances, based on the distance weights matrix (W_S) instead of the adjacency matrix (A). Despite that the W_S preserves the properties of symmetry, reversibility, and spectral structure of the adjacency matrix, the measures calculated on this matrix are completely different (either quantitatively or qualitative) from their topological homologues, highlighting some effects of space to network topology. An indicative example of a spatial measure that differs both quantitatively and qualitatively from its respective topological is the network diameter $d(G)$. Qualitatively, the topological diameter ($d(G)$) represents sequence of edges, whereas the spatial diameter ($d_w(G)$) represents geographical distance. Quantitatively, the $d(G)$ is a natural number, whereas $d_w(G)$ is a positive real. Furthermore, these two types of diameter may not refer to the same network path (i.e. the route with the maximum edges may not be the longest kilometric in the network).

An indicative example of a geometric measure that differs only qualitatively from its homologue topological is *betweenness centrality* (C^b). Given that this measure counts shortest paths, both of its topological (C^b) and spatial (in ${C^b}_w$) expressions are quantitatively the same. However, shortest paths counting steps of separation in C^b are qualitatively different than those measuring geographical (spatial) distance in ${C^b}_w$. On the contrary, there are some geometric measures that do not differ from their topological homologues, neither quantitatively or qualitatively. This interprets that space does not affect their configuration. Such measures are defined for the purpose of this paper as *spatially inalterable* or *spatially indifferent network measures*. An indicative example belonging to this family of measures is the node degree (k), which counts both in its topological and spatial expression the number of edges that are adjacent to a given node.

In the procedure A_{112} the edge $p(e)$ and strength $p(s)$ distributions are examined instead of $p(k)$. The edge distribution provides insights about the network scale, whereas the strength distribution is usually correlated with the degree distribution $s = f(k)$ (Fig. 4, T_2.f) and provides insights about the nodes undertaking the distant communication in the network (Barthelemy 2011)]. Provided that network edges and strengths have non-discrete measures, the typology of their distributions is delineated by histograms.

The next alternative (A_{12}) regards the examination of the functional component. A keynote concept in this procedure is the *network flows*, which express the information being exchanged between nodes in the network. In contrast to the structural component, network flows provide a more dynamic consideration illustrating the signal processing mechanism in the network. The modeling of this component starts with the construction of the *weights matrix* W (Fig. 4, T_1.c) that is a weighted expression of the adjacency matrix (including weights $w(e_{ij})$ instead of nonzero elements), which follows the rationale of the procedures A_{111} and A_{112}. In general, the weights matrix corresponds each time to a single flow feature among the variety of characteristics, attributes or aspects describing signal transmission in a network. Results produced by this consideration are completely different from their topological or spatial homologues, providing this time operational information for the network. In this approach, comparisons between A_{11} and A_{12} may illustrate the effects of traffic in the network topology (T_4.e).

The last part of the monolayer network analysis (A_1) concerns the study of the ontological component (procedure A_{13}) and is structured in accordance to the previous procedures (A_{11} and A_{12}). A keynote concept in this part is the *society* describing the units participating as nodes in the network configuration. Under this perspective, the nodes are not being studied impersonally, exclusively as structural entities, but as a social configuration described by a set of specific or latent attributes or characteristics imprinted on the network's structural and functional attributes (Tsiotas and Polyzos 2015c). The study of the ontological component is important at the extent of providing information illustrating the socio-economic aspects or principles that controlled and are still controlling the network construction. Obviously, this approach is more theoretical, since it involves the examination of historic, socioeconomic, technological, and political parameters (Fig. 4, T_3) composing the network's environment, but it contributes to the integration of spatial network analysis. Besides, many socioeconomic attributes composing the ontological component may attain quantitative representation expressed either as categorical, discrete, or continuous variables, and can participate to an empirical analysis (T_4) (see Ducruet and Beauguitte 2014; Tsiotas and Polyzos 2015a, 2015b, 2015c). Comparisons between the ontological (A_{13}) and the other two (A_{11} and A_{12}) components (T_4.e) may illustrate the effects of the (node) society on the network structure and functionality, respectively.

As previously-mentioned, the procedure A_1 facilitates the alternatives B and A_2 that include a "higher order" network consideration. First, the multilayer network analysis (alternative A_2) is implemented in two steps: in the first step the network layers are aggregated into a single layer, called *overlaid network* (A_{21}). In the second step the multilayer network is studied as a family of networks (A_{22}), where every single layer is examined separately. The procedures A_{21} and A_{22} are studied in accordance to the monolayer analysis (A_1), since both the overlaid network and each layer suggest monolayer networks. This consideration produces layer datasets allowing the application of component analysis (procedure A_{222}), where measures and attributes between the overlaid network and the available layers are compared (T_4.e). This approach allows shaping an initial picture about the competitive or collaborative relations between the examined cases (Tsiotas and Polyzos 2015a, 2015b). Some indicative quantitative tools that can be used here are *correlation* (T_4.a) and *regression* (T_4.c) analysis, *rank-size* *distribution* (T_4.d) modeling (Tsiotas 2016a), and *statistical testing* (T_4.b,e) (Norusis 2005; Devore and Berk 2012; Walpole et al. 2012).

After the definition of the steps composing the study of multilayer networks (procedure A_2), the proposed methodological approach is now fully equipped to define comparisons between different networks (alternative B). This part suggests the most voluminous task, because each individual network is studied separately, either as a monolayer or a multilayer network, according to the procedure A (single network analysis). The overall structure of the proposed methodological framework is circularly structured, connecting the starting (*SN* analysis) with the end point (conclusion making), and provides the potential of feedback, evaluation, reconsideration, improvement, and upgrade. Obviously, its application can be customized in accordance to the purpose of the study.

4 Conclusions

This paper examined the complex study of spatial networks under the epistemological perspective, trying to answer the question "whether there is convergence or divergence in the evolution of the spatial networks' study" in conceptual, methodological, and disciplinary terms. The applied joint consideration illustrated that space matters in networks, both in the deep structure of graphs and in the way scholars approach their essence. In epistemological terms, the disciplines of geography, physics, sociology, and computer science seemed to have experienced differences in the way of understanding networks. Provided that Geography is from the early 60s aware of the graph theoretic tools in the study of geographical networks, its interaction with NS until recently appeared very static (Ducruet and Beauguitte 2014). This discipline seems rather indifferent to topological analysis of spatial networks and more interested to their geometric characteristics. However, Geography is very equipped in the research about the socioeconomic framework of (spatial) networks, which suggests an emerging field of NS research. The works of Ducruet and Beauguitte (2014) and Ducruet (2017) appear to enlighten the bridges towards this interdisciplinary coupling and are suggested as references. Sociology experienced since the 1930s the topological research in networks and gradually acknowledges their immanent spatial dimension. It contributed to the study of spatial networks with the SNA tools and shows an evolving consciousness about the spatial effects in social interaction, suggesting a promising methodological tank for the evolution and integration of the analysis of spatial networks.

Computer Science moved in parallel with Sociology, from the study of network topology to the cognition of space in the networks. An interesting case here is the WWW (Berners-Lee et al. 2007), due to its hybrid spatial and dimensionless nature. Similarly to Sociology and Computer Science, Physics experienced first the topological perspective in the study on networks and it afterwards focused on their spatial dimension. However, it showed the most dynamic attitude because, although it was the latest being interested in the CNA field, it was the first that attempted an interdisciplinary integration (Barthelemy 2011), providing a framework in the study of spatial networks. Overall, the disciplinary consideration reveals that complex network analysis in the study of spatial networks moves towards convergence, but obviously this happens slower in comparison with the rates of CNA's evolution.

Towards the epistemological requirement of integration, this paper attempted to configure a methodological framework composing the complex procedures in the study of spatial networks, based on existing relevant approaches. The proposed methodological framework was ruled by a synthetic rationale illustrating the fundamental perspectives that should be taken into consideration when studying spatial networks, the *structural* (both topological and spatial/geometric), *functional* and *socioeconomic* aspect. This framework aspires to contribute to the organization and standardization of the procedures and the inquiries that a researcher is facing during the complex study of spatial networks, and to be of instructive utility in an interdisciplinary level. On the one hand, it advises physicists who are involved with CNA about the importance of the socioeconomic and geographical framework of complex networks, and particularly that the network topology may be a mirror of its socioeconomic framework, suggesting as topics of further research how the aspects composing this socioeconomic environment become such topological reflections. On the other hand, the overall approach advises geographers about the importance of network topology in the study of spatial systems. Network topology is a concept immanent to the geographical configuration of a spatial network but different from it, operating in accordance to the way that the measures of stress and pressure differ in the classical mechanics. The topology illustrates the relational trends existing between network nodes before a network is embedded in a metric space, which may be an insightful perspective to geographers and spatial scientists who study conceptual aspects of spatial systems.

Overall, this paper advances the epistemological dialogue introduced by Ducruet and Beauguitte in Ducruet and Beauguitte 2014 about the coexistence of network science and spatial science and introduces the dialogue about how Complex Network Analysis and Network Science can move methodically from the research field into the academic didactics.

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