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A Credit-Based Congestion Management Scheme in General Two-Mode Networks with Multiclass Users

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Abstract This paper examines the design of the credit-based congestion management schemes that achieve Pareto-improving outcome in general two-mode networks. It is assumed that transit is a slower but cheaper alternative to driving alone. The distributional welfare effects of congestion pricing on users with the different value of time (VOT) in Liu and Nie (Trans Res Board 2283:34–43, 2012) are used in developing Pareto-improving credit schemes. We show that, similar to the single-mode model, the sufficient and necessary condition for the existence of a discriminatory Pareto-improving credit scheme is the reduction in the total system cost. A sufficient condition for the existence of an anonymous Pareto-improving credit scheme is also derived. A cross-OD subsidization scheme is proposed when the sufficient condition is not satisfied for each origin-destination (O-D) pair. Numerical experiments on the expanded Sioux Falls networks with a log-normal VOT distribution demonstrate that the proposed Pareto-improving scheme can generate positive net revenue in the presence of good transit coverage.

Keywords Pareto-improving · Congestion pricing · Travel credit scheme · Multiclass · Transit

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1 Introduction

Congestion pricing has been promoted as an effective instrument to manage travel demand and road congestion. It has been extensively studied in the literature (see, e.g. Pigou 1920; Beckmann et al. 1956; Vickrey 1969; Verhoef 1996; Yang and Huang 2005), and today, it has been implemented in the various forms around the world. As a successful example, Singapore targets to implement the second generation of Electronic Road Pricing system which will be based on the Global Navigation Satellite System and admit great flexibility in terms of different forms of congestion pricing (e.g., the distance-based pricing) and parking pricing.

A large body of literature studies the distributional welfare effects of congestion pricing when users value their time differently. Road pricing appears to bring a direct loss to those who do not value their travel time saving high enough to justify the paid toll, or those who are "tolled off" the road and have to use undesirable alternatives. Therefore, road pricing may be viewed as a regressive policy (Evans 1992; Arnott et al. 1994; Hau 1998), i.e., its benefit rises with user's income. Accordingly, improving system efficiency as a whole is rarely a sufficient argument, unless the majority of the demand do not suffer direct loss from the policy.

The distributional impacts of congestion pricing have been studied using both static network models and the bottleneck model. Using static models, Verhoef and Small (2004) found user's loss (or gain) from the first-best pricing, in a two-link network, is monotonically decreasing (or increasing) in the value of time. However, in the second-best pricing, the users with intermediate value of time (VOT) suffer most or gain least. Liu et al. (2009) and Nie and Liu (2010) examined a two-link (expressway and transit) model with a continuous VOT distribution and showed that the user who is indifferent to the two modes suffers the greatest loss. Liu and Nie (2012) extended the model of Liu et al. (2009) to a general two-mode network with multiclass users. The traffic assignment solution structure is analyzed and the welfare effects of congestion pricing and transit service are examined. They conclude that the critical class (the class suffer the greatest loss from pricing) could be the rich, the poor, or the middle class depending on their origin-destination (O-D) pairs. Using the bottleneck model, Arnott et al. (1994) concluded a toll without revenue recycle benefits drivers with high values of travel time and schedule delay, and hurts those with low values. The analyses based on two parallel bottlenecks in Liu and Nie (2011) indicate that the social inequity issue from System Optimum (SO) toll is even worse when route choice is taken into account. Van den Berg and Verhoef (2011) showed in the case with a continuous VOT distribution, in the network with a tolled bottleneck and an untolled alternative road in parallel, users with an intermediate value of schedule delay and the lowest value of time for that value of schedule delay suffer the greatest welfare loss.

It has been suggested that the public may be more willing to support road pricing if it is *Pareto-improving*, i.e., reducing the total congestion at nobody's cost. Daganzo and Garcia (2000) showed, using the bottleneck model, a fraction of commuters can be chosen to use the road for free, so that the morning peak is smoothed out and everyone benefits from such a strategy. While a carefully designed pricing scheme may achieve this goal on its own (see e.g. Lawphongpanich et al. 2004;

Wu et al. 2011; Lawphongpanich and Yin 2010), Pareto-improving may also be accomplished by some form of the refund, i.e., returning toll revenues to road users. Toll revenue refunding can be implemented in various ways. A popular strategy is to provide a lump sum travel credit, which the traveler can either cash out by staying off the toll road, or use for covering, often partially, the toll expense (Small 1992; DeCorla-Souza 1995; Adler and Cetin 2001; Kockelman and Kalmanje 2005). In these schemes, net revenues are typically assumed to be positive in order to maintain a financially sustainable system. The questions are: under what conditions such a credit scheme can ensure Pareto-improving outcome with positive net revenue, and how it can be implemented.

Eliasson (2001) proved that, for the network with a single O-D pair, a pricing-refunding scheme will make everyone better off, if (1) the total travel time is reduced, and (2) the toll revenue is refunded *anonymously* to all users (that is, everyone receives an equal credit). Guo and Yang (2010) extended Eliasson's result to general networks. They pointed out that a cross-OD anonymous refunding scheme (toll revenue collected between one O-D transferred to subsidize users between other O-D pairs) is needed when the travel time is not reduced for every O-D pair, and showed how such a scheme might be implemented. Alternatively, they proved that a Pareto-improving discriminitory pricing-refunding scheme exists if and only if the congestion pricing reduces the total system cost. Unfortunately, implementing such a refunding scheme turns out to be difficult because it is class-specific, or *discriminatory*, in that it depends on individual characteristics not typically observable such as VOT (Yang and Huang 2005). Xiao and Zhang (2013) proposed a Pareto-improving time-varying toll in morning commute problem. They further showed that, for a step-toll scheme, transit subsidy is necessary to achieve Pareto-improving outcome. Liu et al. (2009) examined the existence of Pareto-improving refunding schemes in a two-mode network. By explicitly modeling the difference in operating cost of the two mode, the study reveals that merely reducing total travel time is no longer sufficient to guarantee Pareto-improving by an equal lump sum refunding. Instead, the existence of a Pareto-improving refunding scheme depends on the VOT distribution of the commuters. Nie and Liu (2010) used the same bi-modal network. They showed that the existence of a Pareto-improving refunding scheme is guaranteed for several special functional forms, but not for the more realistic VOT distributions such as the log-normal distribution. Zheng et al. (2016) show in the numerical experiments that by integrating incentive program for transit usage both high-income and low-income groups can benefit from congestion pricing.

Another stream of studies in congestion management policy is the tradable credit system. Tradable credit system allows users to trade the credits in a market that are initially distributed by the government to all registered users (see, e.g., Yang and Wang 2011; Nie and Yin 2013; Zhu et al. 2014). Given an initial credit endowment, it is expected that the unequitable welfare effects of congestion pricing can be alleviated through the tradable credit program. Different from our proposed credit-based scheme, the tradable credit system constrains limits the total number of credits and lets the market determine the credit price. Xiao et al. (2013) explored the various credit allocation schemes corresponding to different measures of equity in the

bottleneck model. Wu et al. (2012) examined the trade-off between the equity and the social welfare of tradable credit schemes in multimodal networks.

This paper further examines the two-mode general network model in Liu and Nie (2012) and proposes a credit-based scheme which integrates the demand management of road and transit system. Transit is explicitly modeled as a slower but cheaper alternative to driving alone. The operating cost is introduced as the out-of-pocket cost, i.e., transit fare for transit users, and fuel, insurance, and car deprecation costs for driving alone. Given a toll scheme, we analyze the possibility of allocating initial credits to travelers in order to offset the welfare effect of toll and to achieve a win-win solution. Along the line of Guo and Yang (2010) which studies the single-mode network, we show that a sufficient and necessary condition for the existence of a discriminatory Pareto-improving credit scheme is the reduction of the total system cost (the sum of the total operating cost and the monetary value of travel time) by the pricing intervention. We also propose a sufficient condition for the existence of an anonymous Pareto-improving credit scheme. It depends on the change of O-D specific total travel time, as well as the VOTs of two special user classes: the class which is indifferent to transit and highway at no-toll equilibrium, and the class which suffers the most from pricing at tolled equilibrium. Furthermore, unlike in the simple cases such as discussed in Nie and Liu (2010), however, this condition is no longer necessary. Finally, a cross-OD subsidization scheme is also proposed which allows toll revenue from one O-D pair to be transferred to subsidize the other O-D pairs when the above sufficient condition is not fulfilled for each O-D pair.

The proposed credit-based schemes enable the effective road congestion management without making anyone worse off. It reduces the complexity of implementation and does not require the unobservable information such as the VOT of every user. The proposed credit scheme can be simply computed according to the change of travel time and the VOTs of two special classes. The satellite-based navigation technology can be a useful instrument to access the necessary travel time information. Furthermore, the VOTs of the two special classes can be also found out according to the analytical results in Liu and Nie (2012) which will be briefly reviewed later. Perhaps more importantly, the cross-OD and cross-mode subsidization enabled by a credit-based scheme opens the door to subsidizing the use of alternative, more sustainable transportation modes such as public transit, and thereby promoting their use.

In what follows, Section 2 introduces the two-mode network model and makes necessary definitions; Section 3 reviews the analytical properties of its equilibrium solution and welfare effects from toll in Liu and Nie (2012); Section 4 proposes discriminatory and anonymous Pareto-improving credit schemes, with or without cross-OD subsidization; Section 5 provides numerical results on a single O-D example and an expanded two-mode Sioux Falls networks to demonstrate the analyses; Implementation procedure in the computational study is provided in Section 5.3; Section 6 concludes our findings and remarks on future research.

2 Model Settings

Consider a general network $G(\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ represents a set of nodes and $\mathcal{A}$ represents a set of links. Let x_a and $t_a(x_a)$ be the total link flow and travel time on link $a \in \mathcal{A}$, where $t_a(\cdot)$ is an increasing and non-negative function. A discrete class setting is considered for heterogeneous travelers. Users in class m are associated with value of time (VOT) β_m. Without loss of generality, it is assumed that $0 < \beta_1 < \beta_2 < ...\beta_M$. Let $\mathcal{R}$ and $\mathcal{S}$ denote the sets of origin and destination nodes, respectively. The trip rate (demand) between O-D pair r-s for group m is d_{rs}^m. The set of paths that connect an O-D pair rs is denoted by $\mathcal{K}_{rs}$, and the flow of group m on path k $\forall k \in \mathcal{K}_{rs}$ is represented by f_{rs}^{km}.

To study the multi-mode networks (transit and automobile), an exclusive transit link is added between each O-D pair in the road network, which has a constant travel time γ_{rs}. Also, we introduce mode-specific operating cost, denoted by o_{rs}^T for transit and highway users, respectively. The mode-specific operation cost here represents a transit or automobile user's out-of-pocket cost, exclusive of toll. It is assumed to be identical for all classes between the same O-D pair. The operation cost for highway users include the costs of owning and using a car, i.e., car maintenance and depreciation, auto insurance premiums, fuel costs and others. The operation cost for transit refers to the transit fare. We assume that transit is always a cheaper alternative, i.e., $o_{rs}^C > o_{rs}^T, \forall rs$. Further, let $\Delta_{rs} = o_{rs}^C - o_{rs}^T$. The flow of transit users of class m between O-D pair rs is denoted as q_{rs}^m, and $q_{rs} = \sum_m q_{rs}^m$ is the aggregated flow on the transit line between r and s. Let w_{rs} denote the aggregated highway flow between rs, where $w_{rs} = \sum_m \sum_k f_{rs}^{km}$.

To overcome the distributional welfare effects of road pricing, we propose a credit-based scheme as a policy instrument, which initially issues a certain amount of credits to users. Users can spend the credits by traveling on charged links, purchase more credits from the government, or redeem the unused credits for cash rebate. The price of unit credit is predetermined as one dollar per credit. At the end of each implementation period (e.g., a month), the toll revenue generated by the credit-based system, inclusive of the credit purchase and rebate, should be nonnegative so as to maintain a sustainable financial system. The initial credit allocation scheme may be either anonymous or discriminatory. A discriminatory scheme is class-specific, i.e., the amount of initial credits π_{rs}^m varies across classes within the same O-D pair according to the VOT of class m. In contrast, an anonymous scheme is O-D specific, i.e., all users receive equal credits π_{rs}, regardless of their VOTs. Clearly, an anonymous scheme can be viewed as a special case of the discriminatory scheme where $\pi_{rs}^m = \pi_{rs}^l, \forall m, l$. Let $\Pi_{rs} = \sum_m \pi_{rs}^m d_{rs}^m$ be the total credits initially issued to the O-D pair rs, and note that in the anonymous case, $\Pi_{rs} = d_{rs}\pi_{rs}$. Also, let $R_{rs} = \sum_a \sum_m \sum_k u_a \delta_{rs}^{ka} f_{rs}^{km}$ denote the total credits collected from the highway paths serving the O-D pair rs. The total collected credits and the total initial issued credits are denoted as $R = \sum_{rs} R_{rs}$ and $\Pi = \sum_{rs} \Pi_{rs}$, respectively. If link a is charged by a toll equal to u_a credits, the tolled user

equilibrium (TUE) problem with multi-class users in the two-mode network model can be formulated as follows:

$$\min \sum_a \int_0^{x_a} t(w)dw \;+\; \sum_{rs}\sum_m q_{rs}^m\left(\gamma_{rs}+\frac{o_{rs}^T}{\beta_m}\right) + \sum_m\sum_{rs}\sum_k\left(f_{rs}^{km}\frac{o_{rs}^C}{\beta_m}+\sum_a\frac{u_a}{\beta_m}\delta_{rs}^{ka}f_{rs}^{km}\right) \quad (1)$$

subject to:

$$\sum_k f_{rs}^{km} + q_{rs}^m = d_{rs}^m, \qquad \forall m, \forall rs \in \mathcal{RS} \tag{2}$$

$$\sum_m\sum_k\sum_{rs} f_{rs}^{km}\delta_{rs}^{ka} = x_a, \qquad \forall a \in \mathcal{A} \tag{3}$$

$$f_{rs}^{km} \geq 0; q_{rs}^m \geq 0, \qquad \forall m, \forall rs \in \mathcal{RS}, \forall k \in \mathcal{K}_{rs} \tag{4}$$

where $\delta_{rs}^{ka} = 1$ when link a is on the path k connecting O-D pair rs; otherwise, $\delta_{rs}^{ka} = 0$. Constraint (2) represents the flow conservation condition; Constraint (3) is known as the path-link incidence relationship. The KKT conditions of the above optimization problem include the following complementarity conditions:

$$c_{rs}^k + \frac{1}{\beta_m}(v_{rs}^k + o_{rs}^C) \geq \mu_{rs}^m, \; f_{rs}^{km}\left(c_{rs}^k + \frac{1}{\beta_m}(v_{rs}^k + o_{rs}^C) - \mu_{rs}^m\right) = 0 \quad (5)$$

$$\text{and} \quad \gamma_{rs} + \frac{o_{rs}^T}{\beta_m} \geq \mu_{rs}^m, \; q_{rs}^m\left(\gamma_{rs} + \frac{o_{rs}^T}{\beta_m} - \mu_{rs}^m\right) = 0, \forall m, \forall rs \in \mathcal{RS}, \forall k \in \mathcal{K}_{rs} \; (6)$$

where $c_{rs}^k = \sum_a t_a(x_a)\delta_{rs}^{ka}$ denotes the travel time on path k connecting O-D pair rs, $v_{rs}^k = \sum_a u_a\delta_{rs}^{ka}$ denotes the total credits charged on highway path k, so that $\frac{1}{\beta_m}(v_{rs}^k + o_{rs}^C)$ is the equivalent time of out-of-pocket cost for class m on path k, and $\gamma_{rs} + \frac{o_{rs}^T}{\beta_m}$ represents the travel cost in time unit for class m if they uses transit. Therefore, the Lagrange multiplier μ_{rs}^m associated with the flow conservation constraint (2) can be interpreted as the minimum travel cost in time unit of class m between O-D pair rs, inclusive of cost of travel time, toll and operation cost. Thus, $\beta_m\mu_{rs}^m$ corresponds to the minimum travel cost in monetary unit for the user between the O-D pair rs. It is worth noting that neither anonymous nor discriminatory credit allocation scheme would affect the flow pattern at the tolled equilibrium, because no one has the incentive to change his/her route choice when the initial credit allocation is irrespective of route. Hence, the travel cost including the effect of the initial credits can be computed by $\beta_m\mu_{rs}^m - \pi_{rs}^m$. We will introduce the credit allocation scheme in Section 4.

The system efficiency can be measured either by system cost or system time. The system cost measures the overall economic efficiency (users' welfare), while the system time indicates the level of system disutility in terms of time, which is useful when the focus is to alleviate traffic congestion or mitigate effects highly related to

congestion (e.g. mobile emissions and fuel consumptions). To differentiate these two measures, we shall use G to denote the system time, and C to denote the system cost:

$$G = \sum_a t_a(x_a)x_a + \sum_{rs}\sum_m q_{rs}^m \gamma_{rs} = \sum_m \sum_{rs} \sum_k f_{rs}^{km} c_{rs}^k + \sum_{rs} \sum_m q_{rs}^m \gamma_{rs} \quad (7)$$

$$C = \sum_m \sum_{rs} \sum_k \beta_m f_{rs}^{km} c_{rs}^k + \sum_{rs} \sum_m \beta_m q_{rs}^m \gamma_{rs} + \sum_{rs} (w_{rs} o_{rs}^C + q_{rs} o_{rs}^T) \quad (8)$$

It is worth emphasizing that "travel cost" of an individual either in time unit (μ_{rs}^m in (5)) or monetary unit ($\beta_m \mu_{rs}^m$), includes both toll and travel time, while "system cost" defined in (8) excludes net revenue, because net revenue is assumed to be transferred within the transport system or the society.[1] Since in our model the transit users also receive initial credits, part of the toll revenue collected from the road is actually recycled to subsidize mass transit system.

Before the credit-based system is introduced ($u_a = 0, \forall a$), i.e., no-toll equilibrium (NTE), Conditions (5)–(6) are reduced to:

$$\bar{c}_{rs}^k + \frac{1}{\beta_m} o_{rs}^C \geq \bar{\mu}_{rs}^m, \; \bar{f}_{rs}^{km}\left(\bar{c}_{rs}^k + \frac{1}{\beta_m} o_{rs}^C - \bar{\mu}_{rs}^m\right) = 0, \forall m, \forall rs \in \mathcal{RS}, \forall k \in \mathcal{K}_{rs} \quad (9)$$

$$\gamma_{rs} + \frac{o_{rs}^T}{\beta_m} \geq \bar{\mu}_{rs}^m, \; \bar{q}_{rs}^m\left(\gamma_{rs} + \frac{o_{rs}^T}{\beta_m} - \bar{\mu}_{rs}^m\right) = 0, \forall m, \forall rs \in \mathcal{RS}, \forall k \in \mathcal{K}_{rs} \quad (10)$$

where $\bar{\mu}_{rs}^m$ is the individual travel cost in time unit at NTE. NTE is considered as the benchmark scenario and $\bar{\mu}_{rs}^m$ is the reference point when user's loss or gain is defined and computed later. Throughout the paper, a variable headed with bar is associated with NTE. For example, $\bar{G}$ denotes the total system travel time at NTE. In the objective (1), the link travel time function $t(\cdot)$ is assumed to be positive, separable and strictly increasing, so that both tolled and no-toll problems have a unique link flow solution. The class-specific link flows, as well as path flows, are not unique in general (Yang and Huang 2005). The notation is summarized in Table 1.

Now Pareto-improving credit-based scheme is defined for the proposed two-mode general network:

Definition 1 (Pareto-improving credit-based scheme) A credit-based scheme is Pareto-improving if it guarantees that (1) nobody is worse off, i.e. $\beta_m \mu_{rs}^m - \pi_{rs}^m \leq \beta_m \bar{\mu}_{rs}^m$, $\forall m, rs \in \mathcal{RS}$ and (2) the net revenue is non-negative, i.e., $\Pi \leq R$.

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[1] Of the 2008 net revenues (about US $222 million) of congestion pricing at London, 82 percent went for bus improvements, 9 percent for roads and bridges, and the remaining 9 percent for road safety (Arnold et al. 2010).

Table 1 List of notation

β_m	value of time of class m
β_{rs}^t	value of time of the indifferent class of rs at tolled equilibrium
β_{rs}^e	value of time of the indifferent class of rs at the no-toll equilibrium
β_{rs}^p	value of time of the critical class
z	objective function at no-toll equilibrium
$\bar{G}$	system travel time at no-toll equilibrium(NTE)
G	system travel time at tolled equilibrium
$\bar{C}$	system travel cost at NTE
C	system travel cost at tolled equilibrium
o_{rs}^j	operating cost of mode j in monetary unit, $j = T, C$
Δ_{rs}	difference of operating between two modes, $o_{rs}^C - o_{rs}^T$
w_{rs}	highway flows of O-D pair rs, $w_{rs} = \sum_m \sum_k f_{rs}^{km}$
q_{rs}^m	flows of class m traverse on the transit line of O-D pair rs
q_{rs}	flows on the transit line of O-D pair rs, $q_{rs} = \sum_m q_{rs}^m$
γ_{rs}	constant travel time on transit line between O-D rs
f_{rs}^{km}	k-th path flow of class m and O-D pair rs
δ_{rs}^{ka}	$\delta_{rs}^{ka} = 1$ when link a is on the path k connecting O-D pair rs; otherwise, $\delta_{rs}^{ka} = 0$
x_a	flow on link a, $x_a = \sum_m \sum_k \sum_{rs} f_{rs}^{km} \delta_{rs}^{ka}$
d_{rs}^m	demand of class m between O-D pair rs, $d_{rs}^m = \sum_k f_{rs}^{km} + q_{rs}^m$
$t_a(x_a)$	link travel time as a function of x_a
u_a	toll on link a
v_{rs}^k	total toll(credit) collected on path k, $v_{rs}^k = \sum_a u_a \delta_{rs}^{ka}$
c_{rs}^k	travel time on path k of O-D pair r-s, $c_{rs}^k = \sum_a t_a(x_a) \delta_{rs}^{ka}$
$\bar{c}_{rs}$	minimum travel time between O-D pair rs at NTE
μ_{rs}^m	minimum travel cost in time unit of class m users between O-D pair rs
	at tolled user equilibrium (TUE)
$\bar{\mu}_{rs}^m$	minimum travel cost in time unit of class m users between rs at NTE
R_{rs}	total credits collected from users between O-D pair rs
Π_{rs}	total credits issued to users between rs

Condition (2) in Definition 1 simply requires that the net revenue considering credit purchase and rebate should be nonnegative. Intuitively, achieving Pareto-improving is easier by a discriminatory scheme, because it is more flexible and can compensate each class differently according to welfare loss for each class user. However, differentiating allocation of credits based on such user characteristics as VOT is generally regarded as unrealistic and infeasible. VOT is not easy to observe and may not be a constant value even for the same person (Arnott and Kraus 1998).

The condition (2) in Definition 1 enables cross-OD subsidization, i.e., the excess toll revenue collected between one O-D can be used to compensate users between other O-D pairs. It would be a useful and relatively realistic instrument to achieve

Pareto-improving goal, particularly when pricing imposes unevenly distributed spatial impacts, which favor certain O-D pairs over the others. Without cross-OD subsidization, the total issued credits for each O-D pair has to be less than or equal to the credits collected within the O-D pair, and Condition (2) becomes $\Pi_{rs} \leq R_{rs} \forall rs$. This requirement is evidently harder to satisfy than $\Pi \leq R$ (note that $\Pi_{rs} \leq R_{rs} \forall rs \Rightarrow \Pi \leq R$ but not vice versa).

3 Review in Welfare Effects of Pricing

The properties of the solution structure of multi-class assignment and welfare effects of congestion pricing are analyzed using the Pareto frontier in Liu and Nie (2012). It is revealed that at no-toll equilibrium, the travel time is identical across classes. At tolled equilibrium, the travel time of each class is monotonic in the value of time, i.e., users with higher VOT always choose a path with equal or lower travel time. Therefore, the critical class, i.e., the class suffer the greatest loss due to congestion pricing, could be rich, poor, or middle class depending on the change of travel time on the nondominated paths. In this section, we will review the relevant results in Liu and Nie (2012) which will be used for the credit scheme design in the following section.

We first define two special classes as follows.

Definition 2 (Indifferent class) At NTE or TUE, the class of users between O-D pair rs who has exactly the same travel cost on both modes is the indifferent class, and their VOT is denoted by β_{rs}^e at NTE and β_{rs}^t at TUE.

Definition 3 (Critical class) At TUE, the class of users between O-D pair rs subject to the highest increase in travel cost is called the critical class and their VOT is called critical VOT β_{rs}^p.

Note that an indifferent class does not always exist for two reasons. First, corner solutions may arise, where all users between an O-D pair prefer one mode to another. Second, in our discrete setting, it is likely that none of the existing discrete VOT exactly satisfies the indifferent condition exactly. The critical class is of interest partly because all classes should not be worse off as long as the critical class is properly compensated by an anonymous credit allocation.

The following Lemmas 1 and 2 summarize the discussions of the equilibrium solution structure in Liu and Nie (2012). In general, an individual traveler will choose paths among the nondominated paths by evaluating the trade-offf between time and money according to his/her own value of time. When multiple nondominated paths exist for certain origin and destination, users will be distributed according to their VOTs, the higher VOT corresponding to the lower travel time. The NTE case is simple: users either use transit or the highway paths with identical travel time c_{rs}^-.

Lemma 1 *At the no-toll equilibrium, all highway users between a O-D pair rs, regardless of their VOT, have the same equilibrium travel time $\bar{c}_{rs}$.*

Lemma 2 *At tolled or no-toll equilibria, the travel time of each class is monotonically decreasing in value of time. If class m uses path/mode k, any class with higher VOT $\beta_l > \beta_m$ would use path/mode with equal or lower travel time, and any class with lower VOT $\beta_l < \beta_m$ would also use path/mode with equal or higher travel time.*

The following Lemma 3 and Proposition 1 summarize the results in the welfare effects of congestion pricing. Specifically, Proposition 1 shows how the critical class can be determined from the changes of travel time from NTE to TUE. As mentioned in Section 1, with the satellite-based navigation technology, which enables access to travel time information, the critical class can be determined according to Proposition 1.

Lemma 3 *Consider two users who use highway at both NTE and TUE. The user with higher VOT always has equal or less (more) travel cost increase due to pricing compared to the other if the travel time of both users at TUE is lower (higher) than at NTE.*

Proof see Liu and Nie (2012).   □

Proposition 1 *Given a toll scheme on a two-mode network, the alternatives include a set of highway paths $\{1, 2, ..., p\}$ and transit if it is available. The **critical VOT** between the O-D pair is*

1   *the lowest VOT if transit is not available or not used and travel time decreases on all the non-dominated paths: $\bar{c}_{rs} > c_{rs}^1 > c_{rs}^2 > \cdots > c_{rs}^p$*
2   *the highest VOT if travel time increases on all the non-dominated paths: $c_{rs}^1 > c_{rs}^2 > \cdots > c_{rs}^p > \bar{c}_{rs}$*
3   *either the highest VOT among all users of path k or the lowest VOT among all users of path $k + 1$, if $\gamma_{rs} > c_{rs}^1 > \cdots > c_{rs}^k > \bar{c}_{rs} > c_{rs}^{k+1} > \cdots > c_{rs}^p$*

Proof see Liu and Nie (2012).   □

Proposition 1 is used to design the anonymous credit scheme in Fig. 3 later. In this paper, we further analyze the welfare effects of a pricing scheme in a multi-class two-mode network. The curve of welfare loss from toll (increase of travel cost) is found concave with respect to VOT for users who use automobile at NTE.

Proposition 2 *Between each O-D pair, except users who use transit at NTE, the curve of individual welfare loss is a concave function of value of time.*

Proof the discrete setting of VOT makes the curve of welfare effects not derivable in the interval $[\beta_1, \beta_M]$. The curve can be plotted by connecting class m and $m + 1$ with a line ($m = 1, ..., M - 1$). As shown in Fig. 1, each point represents a user class with value of time (X-axis) and welfare loss (Y-axis). Without loss of generality, it is assumed that class $n1$ and $n2$ use path l and class $m1$ and $m2$ use path k. It can
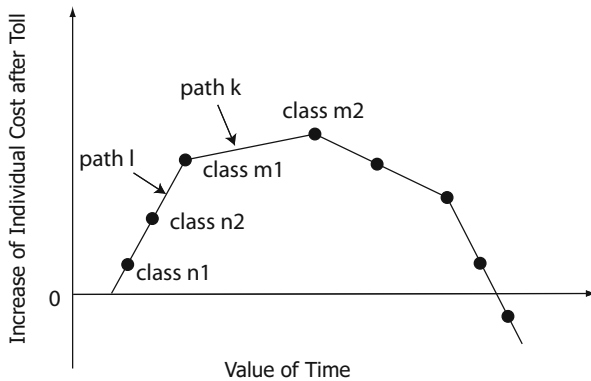
Fig. 1 Welfare Effects of Toll as Concave Function of VOT

be easily verified the slope of path l in Fig. 1 is $(t_l - \bar{c})$, where t_l denotes the travel time on path l and $\bar{c}$ denotes the travel time at NTE. Note that only highway time $\bar{c}$ is used here because the welfare loss may not be concave when including the VOT interval of transit users at NTE. Also those who use transit at NTE do not concern us here, since they either break even or benefit after toll. Similarly, the slope of path k is $(t_k - \bar{c})$, where t_k denotes the travel time on path k. According to Lemma 2, $t_k \leq t_l$, so that the slope of path k is lower. Therefore, it can be proved that the curve constituted by two paths next to each other is concave, which can be generalized for all the other the other paths. □

In the single-OD model with two parallel routes and continuous VOT distribution, Verhoef and Small (2004) found users with intermediate VOT suffer the greatest welfare loss (the critical class defined above) from the second-best pricing. Liu et al. (2009) showed the users indifferent to the two alternative routes are mostly worst off. We prove here, in general network setting, the welfare effects patterns can be described as a concave function of VOT. However, different from the one-OD case, the users with lowest VOT or highest VOT, instead of the intermediate VOT, could be the critical class too.

The above results suggest that the highway users whose experienced path travel time at TUE is closer to that at NTE generally suffer higher welfare loss, and that the critical class is always among those whose pricing-induced change in travel time is smallest.

4 Pareto-improving Credit Schemes

4.1 Discriminatory Credit Scheme

As aforementioned, a discriminatory scheme allows the flexibility to allocate initial credits according to individual's VOT. Compared with anonymous scheme in

Section 4.2, the condition to achieving Pareto-improving by discriminatory scheme is weaker. Therefore, it serve as a optimal and benchmark case. That is, if a Pareto-improving discriminatory scheme does not exist, a Pareto-improving anonymous scheme surely does not exist. We extend the result of Guo and Yang (2010) for the single-mode model as follows.

Proposition 3 *A discriminatory Pareto-improving credit scheme without O-D cross-subsidization exists if and only if the system cost for each O-D pair is reduced by toll, i.e.,*

$$\bar{C}_{rs} \geq C_{rs} \tag{11}$$

Proof The result can be proved similarly as the single-mode model in Guo and Yang (2010). For convenience, the proof is given in Appendix. □

Note that the proof of sufficiency of the above result offers a revenue-neutral design scheme in which the amount of initial allocated credits of each user equals his/her welfare loss plus a positive term $a_{rs}^m(\bar{C}_{rs} - C_{rs})$ where $\sum_m a_{rs}^m = 1, a_{rs}^m > 0$. Therefore, nobody is worse off. When the reduction of system cost is not realized for each O-D pair, cross-OD subsidization can be implemented in a similar fashion as suggested in Guo and Yang (2010), i.e., uniform Pareto-improving scheme that gives users perfect equity. All the users have the same percent of welfare gain independent of their O-D pairs and VOTs. In this case, Pareto-improving scheme exists as long as the total system cost is reduced.

Corollary 1 *A discriminatory Pareto-improving credit-based scheme with cross-OD subsidization exists if and only if*

$$\sum_{rs} (\bar{C}_{rs} - C_{rs}) \geq 0 \, or \, \bar{C} \geq C \tag{12}$$

Corollary 1 can be viewed as an extension to Theorem 1 in Guo and Yang (2010). The major difference is that the reduction of operating cost $((q_{rs} - \bar{q}_{rs})\Delta_{rs} \geq 0)$ is now accounted in the system cost. Moreover, because pricing may produces positive term $(q_{rs} - \bar{q}_{rs})\Delta_{rs}$, Pareto-improving scheme may exist even if the cost associated with system time increases. Those conditions can be adopted to guide the design of toll scheme. A toll scheme reducing system cost could be easily found, such as system optimum (SO) toll which minimizes system cost.

4.2 Anonymous Credit Scheme

Because a discriminatory scheme is usually difficult to implement in practice, this section proposes an alternative anonymous scheme that obviates class-specific credit allocation. A sufficient condition is first derived given O-D toll revenue is equally allocated as credits to users between that O-D pairs. The sufficient condition depends on reduction of system time, transit flow at NTE and TUE, and two special VOTs defined in Section 3.

Proposition 4 *An anonymous Pareto-improving credit scheme without cross-OD subsidization exists if the following condition holds for each O-D pair*

$$\bar{G}_{rs} - G_{rs} + \Delta_{rs}\left(\frac{q_{rs}}{\beta^p_{rs}} - \frac{\bar{q}_{rs}}{\beta^e_{rs}}\right) \geq 0 \tag{13}$$

where β^p_{rs} is the VOT of the critical class and β^e_{rs} is the VOT of the indifferent class at the no-toll equilibrium.

Proof we shall show that the critical class p is better off given Condition (13). Because all users within the same O-D pair receive identical travel credit under the anonymous scheme, everyone is better off if the critical class is better off.

The critical class may either use a highway path k or transit at the tolled equilibrium. By Wardrop's equilibrium principle, we have

$$\beta^p_{rs}\mu^p_{rs} - \pi_{rs} \leq \beta^p_{rs}c^j_{rs} + v^j_{rs} + o^C_{rs} - \pi_{rs}, \text{ for } f^{kp}_{rs} > 0 \,\forall j \in K_{rs}$$

Multiply both sides by the aggregated tolled equilibrium path flow $f^j_{rs} = \sum_m f^{jm}_{rs}$ and sum over all $j \in K_{rs}$:

$$\beta^p_{rs}\mu^p_{rs}w_{rs} - \pi_{rs}w_{rs} \leq \beta^p_{rs}\sum_j c^j_{rs}f^j_{rs} + R_{rs} + o^C_{rs}w_{rs} - \pi_{rs}w_{rs} \tag{14}$$

where the aggregate highway flow between rs $w_{rs} = \sum_j f^j_{rs}$ and the toll revenue (the total credits collected) between rs $R_{rs} = \sum_j v^j_{rs}f^j_{rs}$. Similarly, regardless of the mode choice of the critical class, we have the following by the equilibrium conditions:

$$\beta^p_{rs}\mu^p_{rs} - \pi_{rs} \leq \beta^p_{rs}\gamma_{rs} - \pi_{rs} + o^T_{rs}$$

Multiply both sides by the aggregated transit flow $q_{rs} = \sum_m q^m_{rs}$:

$$\beta^p_{rs}\mu^p_{rs}q_{rs} - \pi_{rs}q_{rs} \leq \beta^p_{rs}\gamma_{rs}q_{rs} - \pi_{rs}q_{rs} + o^T_{rs}q_{rs} \tag{15}$$

Adding the conditions (14) and (15), with the total allocated credits $\Pi_{rs} = \pi_{rs}(w_{rs} + q_{rs})$, yields:

$$(\beta^p_{rs}\mu^p_{rs} - \pi_{rs})d_{rs} \leq \beta^p_{rs}G_{rs} + R_{rs} - \Pi_{rs} + o^C_{rs}w_{rs} + o^T_{rs}q_{rs}$$

The financial feasibility requires that the toll revenue should be sufficient to compensate the total allocated credits, i.e., $R_{rs} \geq \Pi_{rs}$, as defined in Definition 1. Considering the revenue-neutral case ($R_{rs} = \Pi_{rs}$), we obtain:

$$(\beta^p_{rs}\mu^t_{rs} - \pi_{rs})d_{rs} \leq \beta^p_{rs}G_{rs} + o^C_{rs}w_{rs} + o^T_{rs}q_{rs} \tag{16}$$

Plugging Condition (13) into the above inequality, we have the following

$$(\beta_{rs}^p \mu_{rs}^p - \pi_{rs})d_{rs} \le \beta_{rs}^p \left(\bar{G}_{rs} + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right) + o_{rs}^C w_{rs} + o_{rs}^T q_{rs}$$

$$= \beta_{rs}^p \left[(\bar{c}_{rs} \bar{w}_{rs} + \gamma_{rs} \bar{q}_{rs}) + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right] + o_{rs}^C w_{rs} + o_{rs}^T q_{rs}$$

$$= \beta_{rs}^p \left[\left(\bar{c}_{rs} \bar{w}_{rs} + \left(\bar{c}_{rs} + \frac{\Delta_{rs}}{\beta_{rs}^e} \right) \bar{q}_{rs} \right) + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right] + o_{rs}^C w_{rs} + o_{rs}^T q_{rs}$$

$$= \beta_{rs}^p \bar{c}_{rs} d_{rs} + o_{rs}^C d_{rs}$$

$$\Rightarrow \beta_{rs}^p \mu_{rs}^p - \pi_{rs} \le \beta_{rs}^p \bar{c}_{rs} + o_{rs}^C = \beta_{rs}^p \bar{\mu}_{rs}^p \qquad (17)$$

where the first equality holds by the definition of system cost $\bar{G}_{rs} = \bar{c}_{rs} \bar{w}_{rs} + \gamma_{rs} \bar{q}_{rs}$ and the second equality holds by $\gamma_{rs} = \bar{c}_{rs} + \Delta_{rs}/\beta_{rs}^e$, which is derived from the equilibrium conditions (9)-(10) for indifferent class β_{rs}^e at no-toll equilibrium; the last inequality (17) suggests that the tolled critical class t is better off with initial credits, when Condition (13) is satisfied. By Definition 3, all users are better off when Condition (13) is satisfied. $\qquad \square$

Remark 1 Inequality (17) indicates the critical class is better off only when the critical class uses highway at NTE. In the case that the critical class prefers transit at NTE, they will break even (without initial credit allocation) in the worst case. If they can stay on transit after toll, their travel cost remains the same before redeeming the unused credits for cash rebate. That is, the condition for Pareto-improving is trivial in this case.

Remark 2 Condition (13) is similar to those given in Liu et al. (2009) and Nie and Liu (2010). Unlike in the simple two-link model employed in the aforementioned studies, however, this condition is sufficient but no longer necessary. In other words, even if the sufficient condition above is not satisfied, Pareto-improving scheme may still exist.

The sufficient condition is derived based on the revenue-neutral assumption, i.e., the net revenue between each O-D pair is zero. In fact, when the left-hand side of Condition (13) is strictly larger than zero, it is possible to withhold part of the toll revenue while still achieving the Pareto-improving goal. The positive net revenue can be devoted to infrastructure improvement or compensating users of other O-D pairs between which the toll revenue is not sufficient to make up welfare losses from pricing. We shall discuss the cross-OD subsidization in the following. However, the long-term impacts of investment of toll revenues on infrastructure improvement are not considered here.

When Condition (13) is not satisfied for all the O-D pairs, and cross-OD subsidization is allowed, each O-D pair may receive an external subsidy ϕ_{rs} in addition to the total allocated credits which are equivalent to its own toll revenue $\Pi_{rs} = R_{rs}$. ϕ_{rs} could be positive (receiving subsidy from other ODs) or negative (subsidizing other ODs).

$$\phi_{rs} = -\beta_{rs}^p \left(\bar{G}_{rs} - G_{rs} + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right), \qquad (18)$$

which would ensure that the critical class is better off, as shown below. Subtracting the subsidy ϕ_{rs} on both sides of the the inequality (16), we have

$$\left(\beta_{rs}^p \mu_{rs}^p - \pi_{rs} - \frac{\phi_{rs}}{d_{rs}} \right) d_{rs} \leq \beta_{rs}^p G_{rs} + o_{rs}^C w_{rs} + o_{rs}^T q_{rs} - \phi_{rs} \tag{19}$$

$$\Rightarrow \left(\beta_{rs}^p \mu_{rs}^p - \pi_{rs} - \frac{\phi_{rs}}{d_{rs}} \right) d_{rs} \leq \beta_{rs}^p \left(\bar{G}_{rs} + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right) + o_{rs}^C w_{rs} + o_{rs}^T q_{rs} \tag{20}$$

$$\Rightarrow \beta_{rs}^p \mu_{rs}^p - \pi_{rs} - \frac{\phi_{rs}}{d_{rs}} \leq \beta_{rs}^p \bar{c}_{rs} + o_{rs}^C = \beta_{rs}^p \bar{\mu}_{rs}^p \tag{21}$$

where the inequality (20) is derived similarly as the proof of Proposition 4. When an O-D pair meets Condition (13), ϕ_{rs} is negative, and it may contribute funding for other O-D pairs. When an O-D pair does not meet Condition (13), ϕ_{rs} is positive and extra funding is needed. Whether or not the proposed anonymous Pareto-improving scheme above is valid depends on, of course, if the total additional funding designated in this way is less than or equal to zero, i.e., $\sum_{rs} \phi_{rs} \leq 0$, so that the net revenue of the whole network at the end is non-negative (Condition (2) in Definition of Pareto-improving), $\sum_{rs}(\Pi_{rs} + \phi_{rs}) \leq \sum_{rs} R_{rs}$. It educes the following corollary which applies when O-D cross-subsidization is considered.

Corollary 2 *An anonymous Pareto-improving credit scheme with cross-OD subsidization exists if*

$$\sum_{rs} \beta_{rs}^p \left(\bar{G}_{rs} - G_{rs} + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right) \geq 0 \tag{22}$$

where β_{rs}^p is the VOT of the critical class.

The sufficient condition above is directly derived from the financial requirement $\sum_{rs} \phi_{rs} \leq 0$ for the scheme proposed in (18). The conditions above can be used to examine the existence of Pareto-improving toll scheme if the reduction of congestion, transit riderships, critical and indifferent VOT can be estimated. The proposed scheme reduces the complexity of implementation because it does not require to track the travel cost of people with all possible VOTs. We consider that the two special VOTs are relatively easy to compute or estimate from the results in Proposition 1 and travel time data. And the satellite-based navigation technology is available to access the necessary travel time information. Our numerical experiments show that the sufficient condition is satisfied and Pareto-improving credit schemes do exist when toll scheme and transit network are properly designed. The total net revenue (NR) generated from this scheme is

$$NR = -\sum_{rs} \phi_{rs} = \sum_{rs} \beta_{rs}^p \left(\bar{G}_{rs} - G_{rs} + \Delta_{rs} \left(\frac{q_{rs}}{\beta_{rs}^p} - \frac{\bar{q}_{rs}}{\beta_{rs}^e} \right) \right) \tag{23}$$

Note that an external subsidy lower than the level determined by (18) may still neutralize the critical class because (22) is sufficient but not a necessary condition as discussed before. It is illustrated with an example in Section 5.2.

5 Numerical Experiments

5.1 One O-D Example 1

Consider a network with one O-D pair connected by two highway paths and one transit line, as shown in Fig. 2. Transit line has constant travel time $\gamma = 16$ and the link performance functions are $t_1 = 7 + x_1$, $t_2 = 2 + 5x_2$, $t_3 = 3x_3$, where x_i and t_i are total flow and travel time on link i. There are two classes of users. The demand of class 1 $d_1 = 3$ with VOT $\beta_1 = 1$; the demand of class 2 $d_2 = 1$ with VOT $\beta_2 = 2$. The two highway paths, 1 and 2, use links 1 and 2 respectively. The operating cost is $o^C = 5$ for highway and $o^T = 2$ for transit. Let f^{km} be the flow of class $m = 1, 2$ on highway path $k = 1, 2$, and q^m be the flow class $m = 1, 2$ on the transit line.

At NTE, the link flows are obtained as

$$x_1 = 0.65, x_2 = 1.13, x_3 = 1.78, x_T = q^1 = 2.22$$

Note that the class-specific link flows and path flows on the highway network are not unique. The total travel time $\bar{G}$ of the entire system is 58.65. Several toll schemes are considered in the following, and all of them charge a positive toll on link 2, which has a significantly higher externality, and there is no toll on the other links. Let u_2 be the toll on link 2.

When $u_2 = 5.39$, the tolled equilibrium flow pattern can be found as

$$x_1 = 1.01, x_2 = 0.66, x_3 = 1.67, x_T = q^1 = 2.33$$

$$f^{11} = 0.67, f^{21} = 0, f^{12} = 0.34, f^{22} = 0.66$$

The users' travel costs at no-toll, tolled and under the credit scheme equilibria are presented in Table 2, where c_m represents the travel cost of class m. Clearly, both classes break even after pricing (the row "Tolled"). The total travel time of the system is decreased from 58.65 ($\bar{G}$) to 57.22 (G). Thus, it is Pareto-improving even without issuing initial credits. The total toll revenue is $R = 3.56$. If it is distributed equally to all users as initial credits, everyone benefits, as shown in the last row in Table 2.

As u_2 increases, the system time first decreases and then begins to rise again. As shown in Table 2, when $u_2 = 11.5$, the total system travel time returns to the level at NTE. As u_2 futher increases, the flow on path 2 keeps decreasing until it is reduced to zero when $u_2 = 13.5$. Interestingly, the two classes' travel costs remain the same for all levels of u_2. The reason is that the tolled indifferent class (which is also the critical class) uses transit at NTE, which satisfies the description of Remark 1 for Proposition 4. As the cost of using transit never increases by definition, so is the

Fig. 2 One O-D network
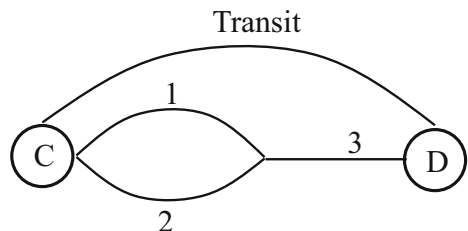
Table 2 Travel cost at no-toll, tolled and credit-based equilibria in Example 1

	$u_2 = 5.36, R = 3.56$			$u_2 = 11.5, R = 1.5$			$u_2 = 13.5, R = 0$		
	c_1	c_2	G	c_1	c_2	G	c_1	c_2	G
No-toll	18	31	58.65	18	31	58.65	18	31	58.65
Tolled	18	31	57.22	18	31	58.65	18	31	60.5
Credit scheme	17.11	30.11	57.22	17.63	30.6	58.65	18	31	60.5

cost of using the slow highway path (note that these users are indifferent to the two options). Since there is no toll on the slow highway path, its travel time must remain unchanged too. That is, charging toll on the fast path shifts certain amount of users from the fast path to the slow path and the same amount of users from the slow path to the transit. Apparently, this phenomenon has to do with the fact that there are only two classes in this example. In the next section, we will show that very different results emerge when one more class is added.

5.2 One O-D Example 2

This example demonstrates that travel time on one of the highway paths may be increased after toll even though the total system time is reduced. As a result, according to Proposition 1, the indifferent class (whose VOT denoted as β^t) may not be the critical class (whose VOT denoted as β^p). The same network in Fig. 2 is used in this experiment, with different parameters. The travel time on transit is 9 and link performance functions become: $t_1 = 3 + 2x_1$, $t_2 = 3 + 3x_2$, and $t_3 = 1$. There are three classes of users: the demand of class 1 $d_1 = 2$ with VOT $\beta_1 = 1.5$ (the poor class); the demand of class 2 $d_2 = 2$ with VOT $\beta_2 = 2$ (the middle class); the demand of class 3 $d_3 = 1$ with VOT $\beta_3 = 3$ (the rich class). Moreover, $o^C = 4$, $o^T = 2$. The toll on link 1 and 2 is 0.25 and 3.5, respectively.

The class-specific path flows at NTE and tolled equilibrium are reported in Table 3, and the total toll revenue is 4.35. At NTE, the poor class is indifferent to transit and path 1 ($\beta^e = 1.5$). The other two classes are split between path 1 and path 2. At the tolled equilibrium, the middle class becomes the indifferent class ($\beta^t = 2$). The poor class prefers transit, while the rich class prefers highway. The path travel time at NTE is ($\gamma = 9, c^1 = 7.67, c^2 = 7.67$); the path travel time after toll is ($\gamma = 9, c^1 = 7.875, c^2 = 6.792$), where γ is the transit time and c^k denotes the time on path k. Thus, the travel time on path 1 increases by 0.205 after toll, yet the

Table 3 Class-specific path flow at no-toll and tolled equilibria in Example 2

	Poor class			Middle class			Rich class		
	transit	path 1	path 2	transit	path 1	path 2	transit	path 1	path 2
No-toll flow	1.94	0.06	0	0	1.19	0.81	0	0.59	0.41
Tolled flow	2	0	0	0.13	1.87	0	0	0.07	0.93

Table 4 Travel cost before and after toll in Example 2

	Poor	Middle	Rich
No-toll equilibrium	15.5	19.33	27
Tolled equilibrium	15.5	20	27.75
Cost increase	0	0.67	0.75

total system time is still reduced from 40.93 to 40.77. As shown in Table 4, both the average and rich classes are worse off, while the poor class breaks even. Table 4 also reveals that the tolled indifferent class (the middle class, $\beta^t = 2$) is not the critical class. The critical class is the rich class (i.e. $\beta^p = 3$), essentially because the travel time on path 1 increases after toll (i.e., the case 3 in Proposition 1).

If all toll revenues (4.35) are equally distributed to users, each user would receive a credit $\pi = 4.35/5 = 0.87$, which will ensure everyone is better off. However, the sufficient condition (13) does not hold

$$\bar{G} - G + \Delta_{rs} \left(\frac{q}{\beta^p} - \frac{\bar{q}}{\beta^e} \right) = 40.93 - 40.77 + (4-2) \left(\frac{2.13}{3} - \frac{1.94}{1.5} \right) = -1.00 < 0$$

This result is consistent with the analysis in Remark 2 of Section 4.2 that Condition (13) is not necessary. That is, it is possible to find a Pareto-improving credit scheme even if the sufficient condition (13) is not met.

5.3 Implementation Issues

We provide implementation details in this section that enable a comprehensive computational study of the proposed credit schemes in general networks. We will explain the choice of toll schemes, the solution algorithm, and flow chart of the credit scheme design. In Section 5.4, the proposed scheme is demonstrated using expanded Sioux Fall network.

5.3.1 Choice of Pricing Schemes

Up to now we always assume that link tolls u_a are given as exogenous variables. The simplest choice may be the marginal-cost toll scheme, since such a scheme provides the optimal result for the efficiency gain from pricing. Yet the definition of the marginal-cost toll scheme in a multi-class model depends on whether the system efficiency is measured by system cost (cost-based) or system time (time-based). Yang and Huang (2004) and Yang and Huang (2005) showed that the cost-based system optimum (SO) flow pattern can be decentralized by the marginal-cost link toll that is uniform to everyone who uses the same link regardless of their VOTs, whereas the marginal cost toll that decentralizes the time-based SO flow has to be class-specific and hence is more difficult to implement in practice.[2] In addition, the cost-based SO is a more natural choice because it minimizes the total perceived cost of time, i.e.,

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[2]For the case where toll can be charged on all links in the network, Yang and Huang (2004) provides a method to obtain anonymous tolls for time-based SO. However, this method is not applicable in our study because toll is not allowed on transit links.

the total system travel disutility measured in monetary unit (Yang and Huang 2004). Therefore, unless otherwise specified, the toll schemes considered in what follows are always based on the cost-based SO. The cost-based SO (CSO) toll can be found by solving the following program.

$$\min \sum_{a} \sum_{m} \left(\beta_m t_a(x_a) x_a^m \right) + \sum_{rs} \sum_{m} \left[q_{rs}^m \left(\beta_m \gamma_{rs} + o_{rs}^T \right) + \sum_{k} \left(f_{rs}^{km} o_{rs}^C \right) \right] \quad (24)$$

subject to: Constraints (2–4)

5.3.2 Solution Algorithms

All multi-class traffic assignment models discussed above, tolled and no-toll user equilibrium as well as cost-based SO, are solved using Dial's algorithm B (Dial 2006), which belongs to a class of highly efficient bush-based assignment algorithms. Algorithm B was implemented in Nie (2010) and compared favorably with other bush-based algorithms on various standard single-class traffic assignment problems. In order to apply the algorithm in the multi-class case with transit links, the original network is expanded to accommodate the transit links (see details in Liu and Nie 2012). And the original Algorithm B operates on a set of bushes, each corresponding to an origin node. In the multi-class implementation, each class associated with the same origin has its own bush,[3] which is equilibrated according to its own VOT.

It is well known that both the class-specific link flows and the class-specific path flows are not unique under the user equilibrium conditions. To ensure the uniqueness and the stability of route flow solutions (see, e.g., Shu and Nie 2010; Bar-Gera et al. 2010), we use the following proportionality assumption.

Assumption 1 (Proportionality) The distribution of vehicle flows on any pair of alternative route segments, or PAS, should be identical among behaviorally similar travelers, regardless of their origin and destination.

Bar-Gera (2010) shows that unique route flows may be solved by iteratively checking and enforcing the proportionality condition on pairs of alterative route segments. The idea is adopted and implemented in a research tool called Talex, which can be downloaded for free at http://translab.civil.northwestern.edu/nutrend_install/TalexSetup.msi.

5.3.3 Design the Anonymous Credit Scheme

After solving the multi-class assignment (both NTE and TUE) with Algorithm B and adjusting the solution by the proportionality condition, the following quantities are first calculated.

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[3]From the computational performance point of view, this may not be the most efficient strategy. It is very likely that classes with similar VOT would use the exactly same bush, and consequently, the number of bushes actually needed per origin is far less than the number of classes. We leave this refinement to a future algorithmic study.

- The aggregated class-specific travel time and travel cost for each O-D pair. Note that class-specific costs can be easily calculated from equilibrium link costs using a shortest path algorithm. While calculating class-specific travel times requires more disaggregated analysis, it can be done efficiently by taking advantage of the bush structure, without enumerating all paths.
- Each O-D's total and class-specific flow on the transit line, which can be easily retrieved from the bush-based solution.
- The VOT of critical class, β_{rs}^p. Although β_{rs}^p may be determined according to Proposition 1, a simpler method for our analysis would be directly comparing the class-specific costs at NTE and TUE.[4] If the cost of the critical does not increase, initial credit is necessary for Pareto-improving.
- The VOT of the indifferent class at NTE β_{rs}^e. Depending on the structure of the equilibrium solution, β_{rs}^e may be determined as follows. From KKT condition (9)–(10) at NTE:

$$\bar{c}_{rs} + \frac{o_{rs}^C}{\beta_{rs}^e} = \gamma_{rs} + \frac{o_{rs}^T}{\beta_{rs}^e} \Leftrightarrow \beta_{rs}^e = \frac{o_{rs}^C - o_{rs}^T}{\gamma_{rs} - \bar{c}_{rs}}$$

where $\bar{c}_{rs}$ and γ_{rs} represents the travel time on highway and transit respectively.

Figure 3 gives a flow chart for calculating O-D specific credit with cross-OD subsidization using the proposed anonymous scheme. As shown in the chart, no credit allocation is needed when the number of highway users increase or all users are on transit at both NTE and TUE. In these cases, highway flow increases, no user is worse off, so that no credit is provided. Cross-OD subsidization defined in (18) is implemented for all the other cases. Subtracting credit from travel cost of each class at tolled scenario gives the final travel cost at credit-based scenario. Subtracting total credit allocated from toll revenue gives the net revenue (NR at the end of Fig. 3). It implies Pareto-improving outcome if the net revenue is non-negative.

5.4 Expanded Sioux Falls Network

We further test the Sioux Falls in Fig. 4. There are 528 O-D pairs and 76 links. The BPR function $t = t_0(1 + 0.15(x/C))^4$ is used to model link travel time, where t_0 is the free flow travel time and C is the link capacity. Let Γ_r denote the set of all O-D pairs that originate from r and have a transit connection, c_{rs}^0 be the free flow travel time between O-D pair rs, l_{rs}^0 be the shortest distance between O-D pair rs, and σ_t and σ_c are unit operating cost for transit and highway respectively. We set the constant travel time on the transit line as

$$\gamma_{rs} = \xi_{rs} c_{rs}^0, \tag{25}$$

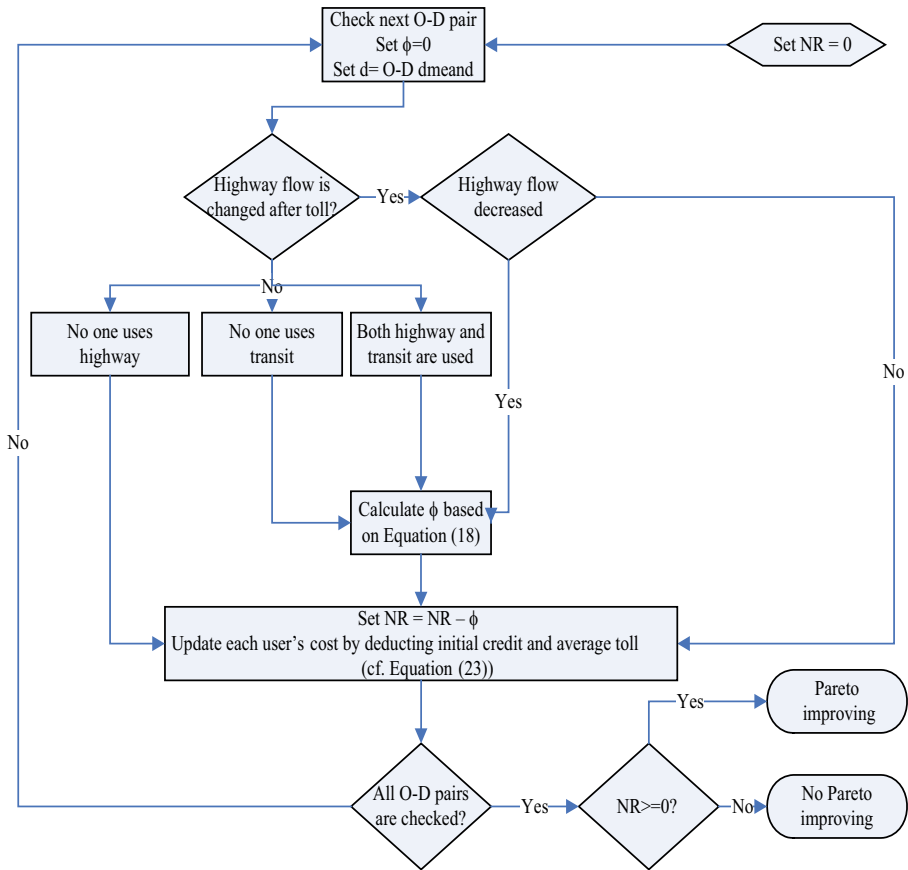[4]We note that the direct comparison may be infeasible if VOT is continuously distributed.

Fig. 3 Implementation flow chart of the anonymous credit scheme

$\xi_{rs} > 1$ is a positive scalar, so that transit is slower than highway. The operating costs for transit and highway are calculated based on the average shortest distance from origin r:

$$o_{rs}^C = \sigma_c \sum_{rs \in \Gamma_r} l_{rs}^0 / |\Gamma_{rs}|, \quad o_{rs}^T = o_{rs}^C \frac{\sigma_t}{\sigma_c}. \tag{26}$$

where $|\Gamma_{rs}|$ denotes the number of O-D pairs in the set Γ_{rs}. In the following scenarios, σ_t is \$0.3 per mile per passenger and σ_c is between the range [0.31, 0.6] per mile per passenger, and δ_{rs} is set to be 2. Equation (26) suggests that all O-D pairs having a transit line and originating from the same origin would share the same operating costs for both highway and transit, which is assumed in the process of the network expansion. For the O-D pairs that do not have transit, their highway operating cost is set to zero, because it has no impact on equilibrium solutions anyway.[5]

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[5]Note that the operating cost in our model is not affected by route choice, which is certainly not the case in reality. However, our focus here is to examine the the impact of operating cost on the mode choice.
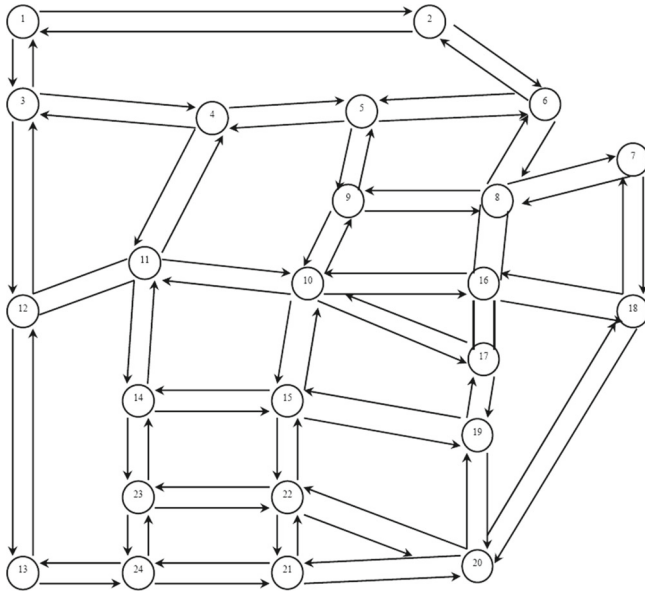
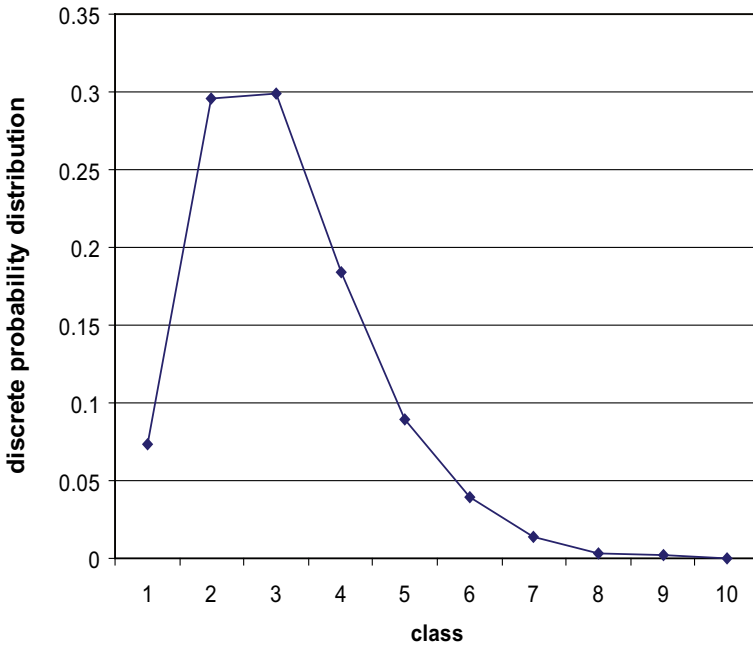**Fig. 4** Original Sioux Falls network

Fig. 5 Class-specific demand in a discrete log-normal distribution

VOT is assumed to follow a log-normal VOT distribution in the population, with an average of \$21/hour and a variance \$110/hour. These parameters are adopted from a study of the commuters on State Route 91 (Lam and Small 2001). Users are discretized into 10 classes according to the log-normal curve, as shown in Fig. 5.

5.4.1 Impacts of Transit Network

Sioux Falls networks with different configuration of transit network are tested to see how the number (density) of transit lines would affect pricing's distributional welfare impacts. For the original Sioux Falls network (no transit), we descendingly sort the 528 O-D pairs according to the loss of individual traveler after SO toll and then select the first N O-D pairs to include transit lines. The impacts of different transit configuration, decided by the input parameter N, are presented in Table 5. In Table 5, the system performance in terms of system cost and time is given for each transit configuration. It is always improved by the SO toll which minimizes the system cost. As shown in last column of Table 5, the net revenue of the proposed credit system is generally increasing with the growth of transit network. Therefore, dense and competitive transit system helps to achieve the Pareto-improving goal. When the city is covered with a good transit system ($N = 300, 400, 528$), the credit system can reduce system time/cost and generate positive net revenue while making all the travelers benefit. With a poor transit system ($N = 0, 100, 200$), the toll revenue is not sufficient to cover the distributional effects (net revenue is negative). And the Pareto-improving Condition (22) is not satisfied when $N = 0, 100, 200$. Thus, the proposed scheme does not exist in those cases.

The case with 528 transit lines presents a successful example. It produces a positive net revenue \$40.67 and nobody is worse off. Among the \$145.39 toll revenue (without counting the initial credit allocation), 72% is proposed to allocate to users as initial credits. Because we can track the cost of any user before and after pricing by solving the traffic assignment problem, another anonymous credit scheme can be designed in such a way that the critical class in each O-D pair exactly breaks even in the credit system. This break-even scheme generates the maximal possible net revenue among all the anonymous Pareto-improving credit schemes. It may not

Table 5 System time&cost and net revenue for different transit configurations

No of transit lines	System time(min)		System cost($)		Net revenue ($)
	no-toll	SO toll	no-toll	SO toll	
0	4488	4335	1554	1477	−121.16
100	3468	3193	1823	1707	−39.87
200	3105	2864	1658	1530	−2.07
300	2991	2829	1565	1437	15.88
400	2899	2812	1462	1345	32.51
528	2905	2837	1400	1282	40.67
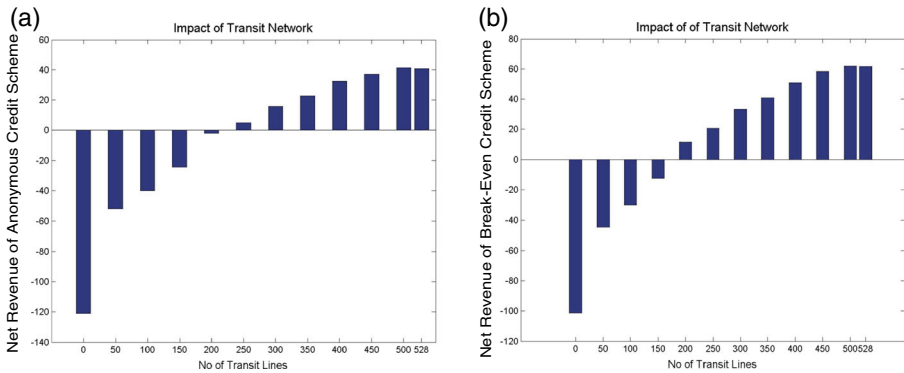
Fig. 6 Net revenue of two credit schemes $Oc = 0.5$

be easily calculated in practice since the data for assignment and VOT distribution is prerequisite. However, since we can compute the traffic assignment problem here, we can examine how close is our proposed credit scheme in (18) (Fig. 6a) to the optimal scheme (the critical class break even, Fig. 6b). Also it reveals that even in the context with full information where the break-even scheme is tractable, for the case with 528 transit lines, still 58% of toll revenue needs to be recycled to maintain Pareto-improving outcome.

5.4.2 Impacts of Scaled SO Toll

Even though designing toll scheme, especially the second-best pricing,[6] is not within the scope of this study, this section demonstrates how the magnitude of toll would affect travelers' welfare and the existence of Pareto-improving credit scheme. Using the network configuration with 528 transit lines and operation cost parameter $o_c = 0.6(\$/mile)$, the system optimum link toll is simply scaled down by a number less than 1, which leaves a lower toll scheme but still improves both system time and system cost comparing with no-toll baseline. It is found that the distribution of welfare is rendered more inequitable when toll goes up in our scaled setting. To maintain Pareto-improving outcome, the SO Toll does not generate the maximal net revenue among those schemes. It is mainly due to that fact that, the higher toll results in bigger welfare gap between the rich and the poor (Liu and Nie 2012).

For each toll scheme, represented by the scale in x-axis, the net revenue (y-axis) after the anonymous credit allocation is plotted in Fig. 7. The toll schemes with positive net revenue present successful examples by using proposed credit scheme with cross-OD subsidization proposed in Section 4.2. The net revenue is negative when the scale is less than 0.5, which indicates that the sufficient condition is not satisfied for these toll schemes and thus it is not Pareto-improving. As shown in Fig. 7, the SO toll scaled by 0.7 generates the maximal net revenue after taking into account of

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[6]The second-best pricing problem has gained ample interests in the literature, that is how much toll to impose on the roads when much of network is liable to remain untolled.
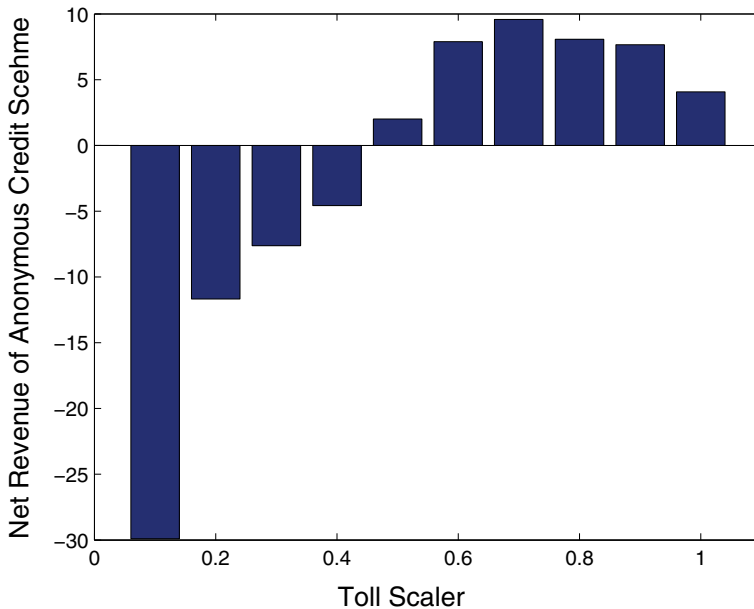
Fig. 7 Net revenue of scaled toll schemes $o_c = 0.6$

the credit allocation. Such a moderate pricing allows improvement in system performance and user welfare simultaneously. A toll scheme designed in the way that only optimizes system performance as a whole might lose the capability to find a Pareto-improving credit scheme. In the sensitivity tests, the parameter of highway operation cost is tested within the range [0.31, 06] and the findings above are generally robust and the conclusions also apply for those cases.

5.5 Disaggregated Welfare Effects

In this section, we will look into the disaggregated welfare effects across different O-D pairs and different user classes. The SO toll scheme is applied to the expanded Sioux Falls network with 528 transit lines. The operation cost parameter $o_c = 0.5(\$/mile)$. Considering the no-toll equilibrium as the baseline, we first compute the change of the individual cost for each class between each O-D pair caused by congestion pricing (x-axis in Fig. 8). By integrating the anonymous credit allocation proposed in Section 4.2, we can compute the change of individual travel cost with initial credits (y-axis in Fig. 8). Figure 8 describes the spatial distribution of welfare with and without credit allocation. Each point in figure represents a user class between a O-D pair. All users within each class between the same O-D pair have identical travel cost at NTE and TUE.

First, Fig. 8a reveals that most of class 1 users (lowest VOT) are not affected by pricing ($x = 0$), mainly due to the fact that they use public transport regardless of toll. Therefore, they actually benefit from the credit scheme ($y < 0$) which could be considered as subsidy to reduce transit fare in practice.
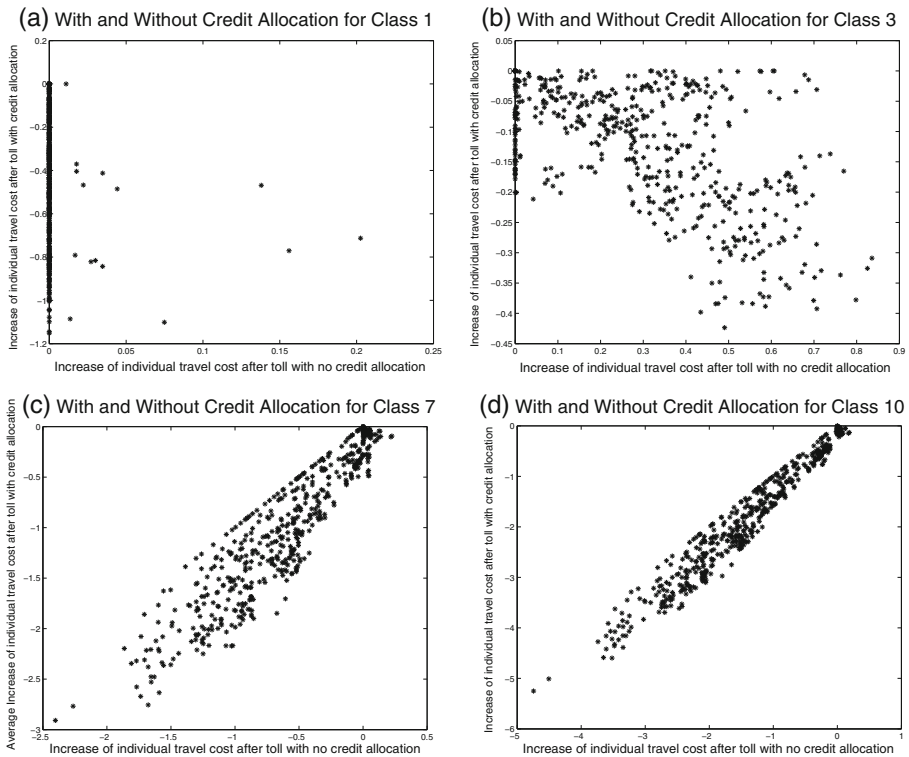
Fig. 8 The spatial distribution of welfare effects of credit scheme $o_c = 0.5$

Second, Fig. 8b shows that class 3 users between some O-D pairs are worse off after pricing ($x > 0$), and between the other O-D pairs break even because of using public transit before toll ($x = 0$). With credit allocation, they all benefit and receive welfare gain ($y < 0$).

Finally, Fig. 8c and d demonstrate the welfare effects for high-income classes, i.e., class 7 and 10. Most of them receive welfare gain even without credit allocation ($x < 0$). Thanks to their high VOTs, the benefit from travel time saving is large enough for them to justify the paid toll. Another important observation here is x and y are highly concentrated. Furthermore, the credit scheme does not overly compensate the high-income users. Most of the benefits they enjoy are actually from time-saving associated with congestion pricing. In contract, class 3 in Fig. 8b indicates many O-D pairs of class 3 have welfare significantly changed by credit allocation, mainly because they are the critical classes among the ten classes.

6 Conclusion

The welfare effect of congestion pricing has been well studied: pricing tends to favor richer travelers who usually have higher value of time (VOT) while bringing

direct losses to others. Recycling toll revenues in the form of travel credit provides a solution to converting the collective efficiency gain from pricing into tangible individual benefits. Within this context, two questions are of both analytical and practical interests: first, whether or not a congestion management credit scheme exists which generates Pareto-improving outcome; and second, when it does exist, how should such a scheme be implemented? This paper addresses these questions using a network model with multi-class users. The proposed model considers both general network and the impacts of transit. Transit is assumed to be a slower but cheaper alternative which is not subject to congestion pricing.

We showed that a sufficient and necessary condition for the existence of a discriminatory Pareto-improving credit scheme is the reduction of the total system cost (the sum of the total operating cost and the monetary value of trave time) by pricing. This extends the result given in Guo and Yang (2010) by adding transit mode. A discriminatory scheme differentiates credit allocation according to travelers' VOTs, which may not be practical. Therefore, we explore the anonymous credit schemes and derive a sufficient condition that ensures the existence of Pareto-improving outcome. The proposed condition depends on the change of total system travel time, as well as the VOTs of two special user classes: the indifferent class at the no-toll equilibrium, and the critical class at the tolled equilibrium. While it extends the condition proposed in Liu et al. (2009) to general networks, there are two main differences. First, this condition is no longer necessary. Second, it is based on the VOT of the critical class, instead of the indifferent class, at the toll equilibrium, because in the general network setting, the critical class could be the rich, the poor, or the middle class. A cross-OD subsidization scheme is further proposed which allows toll revenue collected from one O-D pair to be transferred to other O-D pairs. The proposed anonymous credit scheme can reduce the complexity of implementation. The credit distribution scheme can be computed from the necessary travel time information which can be accessed via the satellite-based navigation technology.

The numerical experiments are conducted in single O-D pair examples and an expanded Sioux Falls network with discrete users classes representing the log-normal distribution from the study (Lam and Small 2001). The impacts of toll scheme and transit network on users' welfare and Pareto-improving goal are examined. With the intervention of system optimum toll, we found for network with dense transit coverage, the proposed credit scheme generates positive net revenue, which is not true for networks with inadequate transit coverage. By scaling SO toll proportionally, we found that higher toll scheme may not generate the larger net revenue when the proposed credit scheme is implemented to ensure individual user's welfare gain as well. This highlight the trade off between the system efficiency and the equity. The spatial distribution of welfare effects show that the proposed scheme does not overly compensate those high-income users. Most of low-income users are not affected by pricing since they use transit regardless of toll. And they benefit from credit allocation which can be implemented in the form of transit fare subsidy.

Several issues warrant further research. First, it is clearly of practical interest to find a toll scheme which, after credit distribution to achieve Pareto-improving, would optimize certain metrics, such as total net revenue or total travel time. Thus, integrating the design of pricing and credit allocation constitutes an interesting yet

challenging extension. Second, to simplify the analysis, this paper adopts a rather simplistic mode choice model, where transit has a constant travel time. A more realistic option is to set the link performance function on the transit link according to a logit-based choice probability, as shown in Sheffi (1985). The demand is assumed fixed in our model, which could be considered as an elastic function of travel cost in future research. Finally, note that the credit schemes discussed in this paper mostly make use of OD-specific travel times or costs, which are not uniquely determined in the standard multi-class traffic assignment model. It is challenging but also important to understand how this non-uniqueness affects the existence and design of credit schemes.

Appendix

Proof Proof of Proposition 3

Necessity According to Definition 1, at the tolled equilibrium, for those who stay on highway, Pareto-improving scheme requires that:

$$\beta_m \mu_{rs}^m - \pi_{rs}^m = \beta_m c_{rs}^k + v_{rs}^k + o_{rs}^C - \pi_{rs}^m \leq \beta_m \bar{c}_{rs} + o_{rs}^C = \beta_m \bar{\mu}_{rs}^m \Rightarrow v_{rs}^k \leq \beta_m \bar{c}_{rs} - \beta_m c_{rs}^k + \pi_{rs}^m, \forall f_{rs}^{km} > 0$$

Multiplying $f_{rs}^{km} > 0$ on both sides and summing all the inequalities over m and k yields

$$R_{rs} = \sum_m \sum_k f_{rs}^{km} v_{rs}^k \leq \bar{c}_{rs} \sum_m \beta_m \sum_k f_{rs}^{km} - \sum_m \beta_m \sum_k f_{rs}^{km} c_{rs}^k + \sum_m \pi_{rs}^m \sum_k f_{rs}^{km} \tag{27}$$

where $R_{rs} = \sum_m \sum_k v_{rs}^k f_{rs}^{km}$, defined as the total credits collected between rs; the demand of class m $d_{rs}^m = \sum_k f_{rs}^{km}$ here because those users choose highway ($q_{rs}^m = 0$).

We now turn to those "priced-out" users, the transit users at the tolled equilibrium who use highway at NTE. $q_{rs}^m - \bar{q}_{rs}^m$ represents the number of class m users tolled off from highway, and the aggregated users can be denoted by $q_{rs} - \bar{q}_{rs}$, where q_{rs}^m and $\bar{q}_{rs}^m$ specifies the number of transit users of class m at tolled and no-toll equilibrium. To ensure Pareto-improving result requires:

$$\beta_m \gamma_{rs} + o_{rs}^T - \pi_{rs}^m \leq \beta_m \bar{c}_{rs} + o_{rs}^C, \forall m \text{ such that } e \leq m \leq p \Rightarrow 0 \leq \beta_m \bar{c}_{rs} - \beta_m \gamma_{rs} + \pi_{rs}^m + \Delta_{rs}$$

where $\Delta_{rs} = o_{rs}^C - o_{rs}^T$. Multiplying $q_{rs}^m - \bar{q}_{rs}^m$ on both sides and adding all inequalities over m together yields

$$0 \leq \sum_m (q_{rs}^m - \bar{q}_{rs}^m) \beta_m \bar{c}_{rs} - \sum_m (q_{rs}^m - \bar{q}_{rs}^m) \beta_m \gamma_{rs} + \sum_m \pi_{rs}^m (q_{rs}^m - \bar{q}_{rs}^m) + (q_{rs} - \bar{q}_{rs}) \Delta_{rs} \tag{28}$$

Finally, for those who always use transit regardless of toll, the following is always satisfied,

$$\beta_m \gamma_{rs} - \pi_{rs}^m \le \beta_m \gamma_{rs}, \forall m \le e$$

Multiplying $\bar{q}_{rs}^m$ on both sides and adding up all inequalities over all the classes sticking to transit:

$$0 \le \sum_m \bar{q}_{rs}^m \beta_m \gamma_{rs} - \sum_m \bar{q}_{rs}^m \beta_m \gamma_{rs} + \pi_{rs}^m \bar{q}_{rs} \tag{29}$$

Summing up inequalities (27 – 29):

$$\Rightarrow R_{rs} \le \left[\bar{c}_{rs} \sum_m \beta_m \left(\sum_k f_{rs}^{km} + q_{rs}^m - \bar{q}_{rs}^m \right) + \gamma_{rs} \sum_m \beta_m \bar{q}_{rs}^m \right] - \left[\sum_m \beta_m \sum_k f_{rs}^{km} c_{rs}^k \right] \tag{30}$$

$$+ \gamma_{rs} \sum_m \beta_m q_{rs}^m \right] + (q_{rs} - \bar{q}_{rs}) \Delta_{rs} + \Pi_{rs} \tag{31}$$

where $\Pi_{rs} \equiv \sum_m \pi_{rs}^m (q_{rs}^m + \sum_k f_{rs}^{km}) = \sum_m \pi_{rs}^m d_{rs}^m$ is defined as the total credits issued between rs. Let $\bar{f}_{rs}^{km}$ specify the path flow of class m. Recalling the demand constraint (2) for both no-toll and tolled equilibrium: $d_{rs}^m = \sum_k f_{rs}^{km} + q_{rs}^m = \sum_k \bar{f}_{rs}^{km} + \bar{q}_{rs}^m, \Rightarrow f_{rs}^{km} + q_{rs}^m - \bar{q}_{rs}^m = \sum_k \bar{f}_{rs}^{km}$, and the term $(q_{rs} - \bar{q}_{rs}) \Delta_{rs}$ is equivalent to the difference of total operating cost between no-toll and tolled equilibrium: $\bar{w}_{rs} o_{rs}^C + \bar{q}_{rs} o_{rs}^T - w_{rs} o_{rs}^C - q_{rs} o_{rs}^T$, the inequality (31) can be simplified as:

$$R_{rs} \le \left[\bar{c}_{rs} \sum_m \beta_m \sum_k \bar{f}_{rs}^{km} + \gamma_{rs} \sum_m \beta_m \bar{q}_{rs}^m + \bar{w}_{rs} o_{rs}^C + \bar{q}_{rs} o_{rs}^T \right]$$

$$- \left[\sum_m \beta_m \sum_k f_{rs}^{km} c_{rs}^k + \gamma_{rs} \sum_m \beta_m q_{rs}^m + w_{rs} o_{rs}^C + q_{rs} o_{rs}^T \right] + \Pi_{rs}$$

$$\Rightarrow R_{rs} \le \bar{C}_{rs} - C_{rs} + \Pi_{rs} \tag{32}$$

where C_{rs} and $\bar{C}_{rs}$ are defined in (8) as the system cost at tolled and no-toll equilibrium respectively. In the most favorable case where all revenues should be issued as credits i.e., $R_{rs} = \Pi_{rs}$, Pareto-improving requires toll scheme reduces system cost:

$$\bar{C}_{rs} \ge C_{rs}$$

Sufficiency The sufficiency can be proven by designing a Pareto-improving credit scheme, in which each class m user receives the equal lump-sum credits $\pi_{rs} = R_{rs}/d_{rs}$ and the extra class-specific credits.

We now construct the following discriminatory credit scheme. Namely, in addition to π_{rs}, class m receives an extra subsidy ϕ_{rs}^m (positive or negative) and each user within the class receives ϕ_{rs}^m/d_{rs}^m:

$$\phi_{rs}^m = \beta_m \mu_{rs}^m d_{rs}^m - \pi_{rs} d_{rs}^m - \beta_m \bar{\mu}_{rs}^m d_{rs}^m + \alpha_{rs}^m (\bar{C}_{rs} - C_{rs}); \sum_m \alpha_{rs}^m = 1, \alpha_{rs}^m \ge 0, \forall m$$

$$\tag{33}$$

The reader can verify that $\sum_m \phi_{rs}^m = 0$, i.e., the class-specific scheme proposed is revenue-neutral (zero net revenue). Now for any class m in O-D pair rs, we have

$$\beta_m \mu_{rs}^m - \pi_{rs} - \frac{\phi_{rs}^m}{d_{rs}^m} = \beta_m \bar{\mu}_{rs}^m - \frac{\alpha_{rs}^m(\bar{C}_{rs} - C_{rs})}{d_{rs}^m} \leq \beta_m \bar{\mu}_{rs}^m$$

This completes the proof. □

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