

Dynamic Vulnerability Analysis of Public Transport Networks: Mitigation Effects of Real-Time Information

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Abstract In this paper, a dynamic and stochastic notion of public transport network vulnerability is developed. While previous studies have considered only the network topology, the granular nature of services requires a more refined model for supply and demand interactions in order to evaluate the impacts of disruptions. We extend the measures of betweenness centrality (often used to identify potentially important links) and link importance to a dynamic-stochastic setting from the perspectives of both operators and passengers. We also formalize the value of real-time information (RTI) provision for reducing disruption impacts. The developed measures are applied in a case study for the high-frequency public transport network of Stockholm, Sweden. The importance ranking of the links varies depending on the RTI provision scheme. The results suggest that betweenness centrality (passenger/vehicle flows) may not be a good indicator of link importance. The results of the case study reveal that while service disruptions have negative effects and RTI may have significant positive influence, counter examples also exist due to secondary spillover effects.

Keywords Vulnerability · Public transport · Disruption · Transit assignment · Network centrality · Critical links · Real-time information · Mitigation

1 Introduction

1.1 Vulnerability Analysis of Transport Systems

Public transport is a vital component of urban transport systems. In tackling the challenges of increasing congestion and negative environmental impacts, shifting trips from personal cars to public transport options is generally seen as one of the most

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important strategies. For public transport to be an attractive option for travellers, the system needs to be efficient as well as robust. Efficiency means that travel should be fast, convenient, affordable and comfortable under normal operating conditions. Robustness means that the system should be able to withstand or quickly recover from disturbances such as infrastructural and vehicular malfunctions. In order to ensure that the system is robust, it is first necessary to analyse the system-wide impacts of potential disruption scenarios for travellers and operators. This enables the identification of problematic scenarios, for example expressed as a set of important network links where disruptions would be the most severe. When the scenarios have been identified, appropriate actions can be taken to reduce the problems and improve the robustness.

The research field concerned with the risk of severe transport network disruptions for the society is commonly called vulnerability analysis (Berdica 2002). Until now, most work in transport network vulnerability analysis has focused on degradations of the physical infrastructure and major incidents, in particular for the road network and personal vehicle travel (Scott et al. 2006; Jenelius and Mattsson 2012; Taylor and Susilawati 2012). Both theoretical analysis and numerical applications have increased the understanding of how supply and demand together determine vulnerability, through the redundancy of the network and the travel patterns of the users.

Much less is known about the vulnerability of public transport networks (PTN), where services are superimposed on roads and railways. Some studies have looked at how degradations of physical infrastructure links in a particular modal network affect connectivity and distances between stations (Angeloudis and Fisk 2006; Criado et al. 2007). PTN configuration plays a key role in determining the impacts of prospective service disruptions. Graph theory provides alternative measures of link importance that were applied on PTN worldwide (von Ferber et al. 2009, 2012). These studies considered the number of immediate connections (node degree) and the betweenness centrality measure which corresponds to the share of shortest paths between nodes which go via a certain node. They concluded that network vulnerability in terms of the size of the largest connected subset is more sensitive to betweenness centrality than to node degree. The same conclusion was reached when vulnerability was defined in terms of the speed in which the system becomes fragmented (Colak et al. 2010). Notwithstanding, PTN varied with respect to the impact of various attack scenarios (von Ferber et al. 2012). It was suggested that robustness of metro systems corresponds to the number of cyclic paths available in the network (Derrible and Kennedy 2010). While some general conclusions can be drawn, such analyses cannot capture many features of PTN that we believe are essential in order to describe their vulnerability properly.

The analysis of PTN vulnerability considers disruptions that imply a substantial reduction in the capacity of system components and hence their incapability to fulfil the purpose of the system. Disruptions of PTN need not be caused by degradations of the underlying physical infrastructure, but can also arise from degradations of the services, for example crew strike or limited infrastructure capacity (stops or tracks).

PTN are characterized by greater complexity than road networks due to the importance of transfers, multi-modality, transport hubs and the intermediate walking links. These network characteristics suggest that PTN are made up of links that belong to distinguished sets. The connectivity of the PTN is lower than that of road networks in both the spatial and the temporal domains: PTN are less dense than road networks, and

the level of service varies non-continuously according to the time tables within the day and between days of the week. Together, these factors imply that PTN are more dependent on few critical network elements and hence possibly more vulnerable. At the same time, the multimodality of PTN can potentially allow alternative modes to provide redundant capacity.

1.2 The Dynamic Approach Towards Public Transport Vulnerability

The aim of this paper is to develop a dynamic and stochastic notion of PTN vulnerability. The granular nature of public transport services requires a more refined model for emulating system supply. Supply dynamics imply that the travel costs associated with alternative paths are time-dependent, which influences adaptive passengers' path decisions. The spatial distribution of network vulnerability may therefore vary over time due to variations in service capacity and reliability. Hence, the dynamics of both public transport supply and demand are modelled as well as their interactions in order to evaluate the impacts of disruptions. None of the evaluations carried out in previous studies have taken into consideration these underlying system dynamics.

A dynamic notion of public transport service disruption takes into account its accumulated effect on system performance. A service disruption implies that public transport vehicles can neither progress along nor enter a disrupted network element. The disruption has a certain duration after which the system is expected to gradually recover back to normal conditions. These conditions could be defined in terms of the flow of supply (vehicles) or demand (passengers). Compared with the case of road networks, service disruption in the PTN has wider direct implications. While service disruption is associated with the immediate effect on the disrupted network element, the dynamic nature of public transport supply results in escalating impacts on service availability and capacity further downstream. Depending on its duration, it may also impact service availability upstream and even on other lines due to its impact on vehicle scheduling.

The impacts of service disruptions depend on local crowding levels as well as on how the demand reacts to changes in supply. Although previous studies have not stated their behavioural assumptions, they all share the assumption that all passengers have perfect knowledge of system conditions and that they always choose the shortest path available. These assumptions are relaxed here by adopting a more realistic behavioural representation. A probabilistic path choice process is used in order to model passenger decisions. The evaluation of alternative paths depends on passenger's preferences and perceptions. The latter is determined by prior knowledge and traveller's access of real-time information (RTI) on system conditions.

In this study, a dynamic public transport operations and assignment model, *BusMezzo*, is used as the evaluation tool. The model represents the interactions between traffic dynamics, public transport operations and traveller decisions. The different sources of public transport operations uncertainty including traffic conditions, vehicle capacities, dwell times, vehicle schedules and service disruptions are modelled explicitly. A dynamic path choice model considers each traveller as an adaptive decision maker. Travellers' progress in the public transport system consists of successive decisions based on anticipated downstream attributes. Factors such as timetables, transfers and walking distances are used for the predictions of passenger loads under various scenarios.

Advanced public transport systems (APTS) have the potential to improve system robustness. This includes real-time control and management strategies. The public transport system may also become more robust by informing passengers on downstream conditions. Previous studies demonstrated the effects of RTI on passenger decisions under service disruption scenarios. However, the analysis also highlighted that as passengers are more informed, passenger loads are subject to more fluctuation and therefore may counteract system robustness (Cats et al. 2011a).

The remainder of the paper is organized as follows. The proposed methodology for public transport vulnerability analysis is described in Section 2. Section 3 describes a case study for the high frequency PTN of Stockholm, Sweden. Section 4 presents the application results followed by a discussion of the benefits of vulnerability analysis in planning, operations and management and concludes the paper.

2 Methodology

2.1 Public Transport Network Definition

The physical PTN is defined by a directed graph $G(S, E)$, where the node set S represents stops and rail stations (all called stops here for simplicity), and the link set $E \subseteq V \times V$ represents direct connections between stops. The number of stops and links are denoted $|S|$ and $|E|$, respectively.

Each link $e \in E$ may be operated by one or several public transport lines. A line l is defined by a sequence of stops $l = (s_{l,1}, s_{l,2}, \dots, s_{l,|l|})$, where $o_l = s_{l,1}$ is the origin terminal and $d_l = s_{l,|l|}$ is the destination terminal. The set of all origin–destination (OD) terminals is denoted $S_T \subseteq S$; the set of lines between origin terminal $o \in S_T$ and destination terminal $d \in S_T$ is denoted L_{od} and the set of all lines is denoted L . We let $e \in l$ mean that link e is in line l , i.e., that $e = (s_{l,i}, s_{l,i+1})$ for some i .

Each link e is associated with a *riding time*, which is the time from the departure from the upstream stop to the arrival at the subsequent downstream stop. The riding time may vary systematically between different lines and also between different trips and between days depending on the current traffic conditions. The riding time from stop s to the next stop on line l at time-of-day t is denoted $RT_{sl}(t)$. If day-to-day variability in traffic conditions is considered, $RT_{sl}(t)$ may be considered a stochastic variable. Similarly, each stop is associated with a *dwelling time*, which is the time required for a vehicle to stop for boarding and alighting. Like the riding times, the dwelling times may vary between lines, trips and days, depending on the current number of passengers, vehicle type, etc. The dwelling time at stop s for line l at time t is denoted $DT_{sl}(t)$ and may be considered stochastic.

Each line l is operated with a set of vehicle trips according to a schedule. The departure time of trip k of line l from the origin terminal is in general a function of a scheduled departure time and the arrival time of the previous trip, which may be stochastic due to the stochastic riding and dwelling times. The set of vehicle trips covering some part of line l during a certain time interval $(t, t + \tau)$ is denoted $K_l(t, \tau)$. The number of trips during the time interval, $|K_l(t, \tau)|$, may be stochastic since it is an outcome of underlying stochastic variables.

Travel demand is connected to the network through a subset of origin–destination (OD) nodes, $S_{od} \subseteq S$. The set of travellers from origin $o \in S_{OD}$ to downstream destination $d \in S_{OD}$ during time interval $(t, t + \tau)$ is denoted $N_{od}(t, \tau)$; the set of travellers between all OD pairs is denoted $N(t, \tau)$. Passengers departure rates are assumed to be inelastic, that is, not affected by changes in travel times etc. However, the number of travellers during the time interval may be stochastic to represent day-to-day variations.

Similarly to the vehicle lines, the physical path of a traveller is defined by a sequence of stops from the origin to the destination, that is, $j = (s_{j,1}, s_{j,2}, \dots, s_{j,|j|})$, where $o_j = s_{j,1}$ is the origin stop and $d_j = s_{j,|j|}$ is the destination stop. The set of all physical traveller paths between origin o and destination d is denoted J_{od} . In general, which physical path a given traveller chooses a given day and time-of-day will depend on the properties of the different public transport lines and on the conditions that day, according to the preferences of the individual. Considering the dynamics and the stochasticity of the system, the probability that traveller n uses physical path j is denoted $p_n(j)$. In Section 2.3 a dynamic public transport route choice model is presented that is used in this paper to calculate $p_n(j)$.

2.2 Network Centrality

The ability of a transport network to withstand degradations has clear connections to the structure of the network. It has long been recognized that central links, in the sense that many paths between pairs of nodes must cross those links, are often also critical with respect to degradations. This kind of network centrality is commonly referred to as *betweenness* centrality (Freeman et al. 1991; Crucitti et al. 2007). For a link e , the betweenness centrality is the fraction of shortest paths, where path length is measured as the number of intermediate nodes, between all pairs of nodes in the network that contain the link; if there are multiple shortest paths between a pair of nodes, the fraction of those paths that contain e is calculated.

If $f_{s_1, s_2}(e)$ denotes the fraction of shortest paths between stop s_1 and stop s_2 that contain link e , the traditional betweenness centrality of link e in the PTN is

$$BC(e) = \frac{1}{|S|(|S|-1)} \sum_{s_1 \in S} \sum_{s_2 \in S \setminus s_1} f_{s_1, s_2}(e) \tag{1}$$

This simple network measure has a number of limitations which may reduce its relevance for identifying central links in real-world PTN. First, it assumes that all node pairs are equally important for the centrality of a link. Second, the only relevant paths between a pair of links are the shortest paths in terms of the number of intermediate nodes; this implicitly defines the path choice model for the network. In the following the betweenness centrality measure is developed for PTN by taking into account some of the features highlighted in Section 1:

- Dynamic system—the demand and supply between nodes may change with time
- Probabilistic path choice—not all passengers choose the same path between two nodes
- Stochastic system—there is inherent variability in demand and supply between days

Furthermore, different centrality measures may be relevant depending on the perspective from which the system is viewed. In particular, one may focus on the operations of the vehicle fleet, or on the passengers. Hence, we extend the traditional betweenness centrality measure with focus on vehicles and passengers, respectively. In each case, we show that the extended betweenness measure corresponds to a measure of flow across the link. Since the path choices of the passengers may be influenced by real-time information, the betweenness centrality measures will in general depend on the RTI provision scheme. We keep this dependence implicit in the formulas here for simplicity.

2.2.1 Centrality for Vehicles

From an operator’s perspective, a network link should be considered central if a large number of lines and vehicle trips traverse the link. Therefore, it is relevant to weight each pair of stops with the number of lines and, for each line, the number of vehicle trips, between the stops. To avoid double-counting the same trip passing several stops along the path, only the origin terminal o_l and destination terminal d_l of each line l should be considered in the aggregation.

The fact that the line schedules may vary by time-of-day and day-of-week means that the betweenness centrality of a link may vary depending on which time interval $(t, t+\tau)$ is considered. Hence, the betweenness centrality measure becomes dynamic. Furthermore, stochastic riding, dwell and departure times mean that the number of vehicles on line l entering a link e during the time interval, $|K_{le}(t, \tau)|$, and the total number of vehicles along line l , $|K_l(t, \tau)|$, are stochastic. To obtain a deterministic betweenness centrality measure we consider the *expected* number of departures during the time interval, $E[|K_{le}(t, \tau)|]$ and $E[|K_l(t, \tau)|]$, respectively. The betweenness centrality measure from the operator’s perspective is

$$OBC(e|t, \tau) = \frac{\sum_{o \in S_T} \sum_{d \in S_T} \sum_{l \in L_{od}} E[|K_{le}(t, \tau)|]}{\sum_{o \in S_T} \sum_{d \in S_T} \sum_{l \in L_{od}} E[|K_l(t, \tau)|]} \tag{2}$$

The summation across origin and destination terminals can be avoided by simply aggregating the measure across all public transport lines L . An equivalent formulation is thus

$$OBC(e|t, \tau) = \frac{\sum_{l \in L} E[|K_{le}(t, \tau)|]}{\sum_{l \in L} E[|K_l(t, \tau)|]} \tag{3}$$

Note that the numerator is simply the total number of vehicle trips entering link e during the time interval, while the denominator is the total number of vehicle trips in the system during the same period. By dividing both the numerator and the denominator by the interval duration τ , the measure can be expressed in terms of vehicle flows instead of number of vehicles.

2.2.2 Centrality for Passengers

When focus is on the travellers, a network link may be considered central if a large number of passengers traverse the link. It is therefore relevant to weight each pair of

stops with the number of travellers between the stops. To avoid double-counting the same traveller passing several stops along the path, only the origin and destination of each traveller should be considered in the aggregation.

Since travel demand levels and line schedules may vary with time, the passenger betweenness centrality measure, like the operator betweenness centrality measure, is dynamic. Day-to-day variations in demand and supply further imply that the number of passengers from o to d entering a link e during the time interval, $|N_{ode}(t, \tau)|$, and the total number of passengers travelling between o and d , $|N_{od}(t, \tau)|$, are stochastic. The stochasticity arises in part from the fact that variations in traffic conditions (in-vehicle time, waiting times, etc.) influence the path choice probabilities $p_n(j)$ for a given individual. The betweenness centrality measure from the travellers' perspective is

$$PBC(e|t, \tau) = \frac{\sum_{o \in S_{OD}} \sum_{d \in S_{OD}} E[|N_{ode}(t, \tau)|]}{\sum_{o \in S_{OD}} \sum_{d \in S_{OD}} E[|N_{od}(t, \tau)|]} \quad (4)$$

Note that the numerator is simply the expected total number of travellers entering link e during the time interval, while the denominator is the expected total number of travellers in the system during the same period. By dividing both the numerator and the denominator by the interval duration τ , the measure can be expressed in terms of traveller flows instead of number of passengers.

2.3 Disruption Scenarios and Impacts

To evaluate any kind of change to a system, a general approach is to describe both the new state and the baseline, reference state in terms of *scenarios*, and to evaluate the difference in some system performance measures between the two states. The scenario definitions must include all relevant dimensions of the system that are different between the new state and the old state, but does not need to include any dimensions that are unchanged. The characterization of a scenario therefore differs depending on the focus of the analysis. In our case, we are interested in the impacts of network disruptions, which means that the scenarios should contain a representation of such events. Furthermore, some appropriate performance measures with which to evaluate the scenarios need to be defined (Jenelius and Mattsson 2014).

Previous vulnerability studies of PTN have considered the system as static and deterministic (Angeloudis and Fisk 2006; Berche et al. 2009; Derrible and Kennedy 2010). Also, very simple measures of network performance have been used. In particular, the scenarios considered have been complete removals of nodes or links from the network, and the performance of the system has been evaluated as the number of interconnected nodes in the largest network component, and the mean distance (in terms of number of intermediate links along the shortest path) between all node pairs in the network. Hence, the performance of other links is independent of the disruption, and only node pairs with the disrupted link in their shortest paths are affected by the disruption. The implication that the remaining network functions as normal, even the disconnected parts of disrupted lines, is highly unrealistic as a model for unplanned network disruptions.

In Section 2.1 the analysis of link centrality is adapted to account for dynamic and stochastic demand and supply and probabilistic path choice. Here the analysis of disruptions is adapted in the same direction. Like the centrality measures, different impact measures can be defined depending on the perspective of the analysis.

2.3.1 Impacts for Vehicles

From a private operator's perspective, disruption impacts are intuitively linked to the *costs* associated with the disruption. The causes and sizes of the costs may depend on the operator's fleet management, contractual obligations etc. One important component may be the total travel time of the vehicles, since that affects fuel and labour costs and potentially on-time arrival performance penalties. Given that riding times and dwell times are stochastic, the operator's cost in a scenario is also stochastic. We assume here that the total operating cost $OC(\sigma|t, \tau)$ in scenario σ during some time interval $(t, t+\tau)$ can be expressed as the sum of the operating cost of each vehicle trip during the interval. To have a deterministic value for the evaluation we consider the expected operating cost, that is,

$$OC(\sigma|t, \tau) = E[\sum_{l \in L} \sum_{k \in K_l(t, \tau)} OC_k(\sigma)] \quad (5)$$

where $OC_k(\sigma)$ is the operating cost associated with vehicle trip k in scenario σ .

In a dynamic setting, an important aspect of a disruption is the *recovery time*, that is, the time from the beginning of the disruption at t_D until the system has recovered to operating normally again at some time $t_D + \tau_R$. The recovery time τ_R will be different for different scenarios and is determined by the dynamic interactions of supply and demand. Before the disruption and after the recovery, the expected operating cost is by definition the same in the disruption scenario and in the baseline scenario. To evaluate the impacts of a disruption scenario σ , it is therefore sufficient to compare the operating cost with that in the baseline scenario σ_0 during the recovery time. For simplicity, we write the operating cost during the recovery time as $OC(\sigma)$. The impact of disruption scenario σ from the operator's perspective is then

$$\Delta OC(\sigma) = OC(\sigma) - OC(\sigma_0) \quad (6)$$

2.3.2 Impacts for Passengers

From the perspective of the travellers, evaluating the impacts of network disruptions involves comparing and summing the various aspects of the impacts for different travellers. The impacts must therefore be expressed in units such that interpersonal comparisons and summations are meaningful. For many reasons, not least in cost-benefit analyses of robustness-improving investments, it is desirable to express the disruption impacts in economic terms. This allows prevention, repair and restoration costs to be added and compared to other impacts such as late arrivals (Jenelius and Mattsson 2014).

With these aims, it is reasonable to express the impacts for passengers in terms of changes in *welfare*, essentially the total utility of the passengers expressed in monetary

terms. With $W_n(\sigma)$ denoting the welfare of passenger n in scenario σ , the total welfare during time interval $(t, t+\tau)$ in scenario σ is

$$W(\sigma | t, \tau) = E \left[\sum_{o \in S_{od}} \sum_{d \in S_{od}} \sum_{n \in N_{od}(t, \tau)} W_n(\sigma) \right] \tag{7}$$

Again for simplicity, we write the total welfare for all passengers during the recovery time as $W(\sigma)$. The impact of disruptions scenario σ from the passengers’ perspective is thus the total change in welfare from the baseline scenario during the recovery time, i.e.,

$$\Delta W(\sigma) = W(\sigma) - W(\sigma_0) \tag{8}$$

In the case study of this paper, welfare is evaluated as a generalized cost function which is a linear combination of four factors: in-vehicle time, waiting time, walking time and number of transfers.

2.3.3 Element Importance and the Value of Real-Time Information

In this paper we are primarily interested in two dimensions of the disruption scenarios: the network element that is disrupted, i.e., the set of disrupted links and nodes, and the type of RTI provided to the passengers (different information provision schemes are described in more detail in Section 3). Other factors such as the start time and duration of the disruption are held fixed in all scenarios. A disruption scenario σ involving network element δ and RTI scheme r can then be summarized as the pair $\sigma = (\delta, r)$. Let $\delta = 0$ denote a scenario with no disruption and $r = 0$ a scenario with no RTI.

Following Nicholson and Du (1994) and Jenelius et al. (2006), the *importance* of a network element is defined as the impact of a disruption of the element. Many other terms have been used in different fields for the same concept, including “criticality” (Taylor and Susilawati 2012), “vitality” (Ratliff et al. 1975; Ball et al. 1989), “vulnerability” (Murray-Tuite and Mahmassani 2004), “significance” (Sohn 2006), “delta centrality” and “information centrality” (Latora and Marchiori 2007). The main purpose behind the importance measure is to compare and rank different elements. This allows, for example, the identification of locations in the transport system where disruptions would be particularly severe. Disruptions of such elements represent worst-case scenarios and the elements can also be considered potential targets for antagonistic attacks on the system.

Identifying important elements means that targeted measures can be taken to reduce the risk (i.e., the probability and/or consequences) of disruptions in those locations. More generally, the importance of each element combined with the probability of the element being disrupted is useful when allocating resources to reduce the overall vulnerability of the society.

In the present framework, the importance of an element is evaluated conditional on a certain RTI provision scheme. Given scheme r , the importance of network element δ is then, from the operator’s perspective,

$$OI(\delta | r) = OC(\delta, r) - OC(0, r) \forall r, \forall \delta \neq 0 \tag{9}$$

and from the passengers' perspective,

$$PI(\delta|r) = W(\delta, r) - W(0, r) \forall r, \forall \delta \neq 0 \tag{10}$$

One of the possible ways to reduce the impacts of network disruptions is to provide RTI to the passengers. This provides information about the expected arrival times of different lines, and may be provided at various levels of completeness. RTI can allow passengers to choose alternative paths to their destinations, avoiding the lines that are negatively affected by the disruption. For a given disruption scenario δ , the value of RTI scheme r is

$$VRTI(r|\delta) = W(\delta, r) - W(\delta, 0) \forall r \neq 0, \forall \delta \tag{11}$$

2.4 Public Transport System Model

A dynamic public transport operations and an agent-based assignment model called BusMezzo (Cats 2013) was used in this study for representing public transport supply and demand and their interactions. The dynamic public transport model is suitable for evaluating the centrality measures and disruption impacts defined above. An implementation of the model is used in the case study described in Section 3.

2.4.1 Supply

The supply of the public transport system consists of network configuration and service availability with their respective attributes. The supply is represented in terms of individual vehicle runs, where each run has a corresponding timetable that is used for control (e.g. dispatching, holding at time points) and the calculation of measures of performance (e.g. on-time performance). In addition, public transport vehicles follow a schedule that consists of a sequence of trips. The chain of trips that a vehicle undertakes is modelled explicitly in order to capture the dependency between successive trips through the propagation of delays from trip to trip.

The departure time of trip k of line l from the origin terminal is calculated as the scheduled exit time, or the time the vehicle is available to depart after it completed its previous trip and some stochastic recovery time, if that occurs later. Vehicle trip travel times then consist of two parts: riding times between departure time from stop s and arrival time at stop $s+1$, denoted RT_{sl}^k , and dwell times at stops, denoted DT_{sl}^k . The exit time from stop s is thus

$$ET_{sl}^k = \sum_{s'=s_1}^{s-1} RT_{s'l}^k + \sum_{s'=s}^s DT_{s'l}^k \tag{12}$$

Dwell times depend on the number of passengers boarding and alighting and are also stochastic. Riding times are composed of running times on links and delays at intersections. In this paper the effects of background traffic are modelled implicitly by representing link travel times as random variables with distributions that are derived from empirical travel times of public transport vehicles. Delays at intersections are determined by individual stochastic queue servers that generate service times following pre-defined distributions. The framework and details of the supply representation in BusMezzo are described in Toledo et al. (2010).

2.4.2 Demand

In this study, travel demand is represented as a time-dependent OD matrix. Travellers are assigned with pre-defined origins and destinations in the network which may correspond to any location that is accessible to or from a public transport stop. Hence, travellers are typically able to choose different access stops and reach their destination through using different egress stops. Each passenger is assigned with a departure time based on a random Poisson process.

Each traveller undertakes successive path choice decisions that are triggered by the evolving public transport system conditions. The evaluation of alternative actions depends on travellers' preferences and expectations. The latter are determined by prior knowledge, experience (e.g. elapsed waiting time) and the availability of RTI. It is assumed that in the context of high-frequency urban public transport systems, travellers have a prior knowledge of network topology, timetable travel times and planned headways. The information that is available to a traveller when making a certain decision is determined by the dissemination means and their locations, and by individual characteristics.

A path alternative $a \in A^{od}$ is a member of the path set for origin o to a destination d and is defined by an ordered set of stops ($S_a \subseteq S$), lines ($L_a \subseteq L$) and connection links ($C_a \subseteq C$). Connection links are access, egress and transfer links that can be traversed by various non-public transport travel modes (e.g. walking, cycling, park-and-ride). S , L and C are the sets of all the stops, lines and connection links in the network, respectively.

A graph representation of the path consists of nodes that correspond to stops and origin and destination locations, while links are either segments of public transport lines or connection links. Note that each element in the path alternative is a set. This definition makes it possible to group several public transport lines that provide an equivalent connection between a pair of public transport stops or several public transport stops which are connected by the same public transport lines. Hence, alternatives that imply that passengers are indifferent towards them are defined as a single path.

The dynamic path choice model includes three decision models: *connection*, *boarding* and *alighting*. A connection decision takes place when the traveller chooses at which public transport stop to initiate the trip, and also each time the traveller alights from a public transport vehicle. A boarding decision is made for each arriving public transport vehicle when the traveller waits at a stop. Once on-board a vehicle, a traveller makes an alighting decision immediately upon boarding and may reconsider this decision in light of new information.

Making a decision corresponds to restricting the initial path set to a subset of paths that are still feasible for the remaining trip. Let us consider the general decision case where individual decision maker n is at certain location o with path set A^{od} . The individual has to choose an action g from the set of alternative actions G . The path set associated with action g is denoted $A^g \subset A^{od}$. The utility of passenger n associated with path $a \in A^g$ is denoted $v_{a,n}$. The utility attached to action g is then given by the logsum over the path set A^g ,

$$v_{g,n} = \ln \sum_{a \in A^g} e^{v_{a,n}} \tag{13}$$

In the context of the proposed path choice model, the logsum term expresses the utility of an action as a function of the utilities of the associated path alternatives.

Hence, it reflects the joint utility for a bundle of alternatives. Note that since A^{od} is divided into mutually exclusive and collectively exhaustive subsets for the respective actions, each alternative path in A^{od} is taken into consideration exactly once in the choice probability.

All traveller decisions are represented with multinomial logit models. Hence, the probability to choose action g is

$$p_n(g) = \frac{e^{v_{g,n}}}{\sum_{g \in G} e^{v_{g,n}}} \quad (14)$$

The independence of irrelevant alternatives (IIA) property of the multinomial logit model implies that the model does account for correlation between alternatives in the choice-set. This modelling drawback is counteracted by (a) the nested choice-tree structure bundles path alternatives (A^S) based on the respective action alternatives; (b) the path-set generation model consolidates paths that have common lines or stops, as described above. These features reduce the potential bias induced by network overlapping in the choice model.

The probability that individual n follows a certain physical network path alternative j , $p_n(j)$, can be formulated as the joint conditional probability of intermediate decisions g_1, g_2, \dots, g_m that lead to the composition of this specific path,

$$p_n(j) = p_n(g_1) \cdot p_n(g_2|g_1) \cdot p_n(g_3|g_1, g_2) \cdot \dots \cdot p_n(g_m|g_1, \dots, g_{m-1}) \quad (15)$$

2.4.3 Supply and Demand Interaction

Supply and demand of public transport systems exercise a dynamic and mutual interaction. The effects of demand on supply are primarily manifested through the dwell time, which increases with the number of boarding, alighting and on-board passengers. The flow-dependent dwell time function trigger a positive feedback loop in supply variations causing service reliability degradation known as bunching (Cats et al. 2011b).

Modelling individual passengers that undertake successive decisions enables the analysis of the interaction of passenger decisions and public transport performance. Passengers' path choices are triggered and influenced by how the public transport system evolves. Traveller's ability to carry out a decision is also subject to vehicle capacity constraints. Passengers experience denied boarding in the case of overcrowded conditions. RTI is generated based on the instantaneous supply conditions.

3 Application

3.1 Network Description

The model was applied to the Stockholm inner-city rapid public transport system. The system consists of the seven Metro lines (10–11, 13–14, 17–19), four high-demand trunk bus lines (1–4) and one light rail train (LRT) line (22). The complete network of these lines was coded in BusMezzo with the real-world timetables, vehicle schedules

and walking distances between platforms and stops. The network is shown in Fig. 1 where nodes correspond to stops and links to line segments.

During the morning peak period (6:00–9:00) there are approximately 700 service trips carried out by over 200 vehicles. The scheduled headways are 5 min for each metro line, between 4 and 6 min for each trunk bus line and 6–8 min for the LRT. This implies an hourly capacity of 9,600, 1,200 and 1,600 passengers per line-segment, respectively. Given the peak-period headway of the rapid transport network, all travellers are assumed to depart randomly and have prior knowledge of planned headways and in-vehicle times according to the time table. The three different public transport modes have distinguished vehicle types, operating speeds, travel time variability, dwell time functions and are operated with different holding control strategies. These operational attributes yield different reliability and capacity levels depending on service design and right-of-way.

To allow a warm-up period for the public transport supply, passenger demand was simulated only for the peak hour (7:00–8:00). Approximately 125,000 passenger trips are initiated during this hour. The passenger demand was extracted from data collected at entrance barriers at Metro stations, passenger counts at transfer locations and LRT stations (SL 2009) and automatic passenger counts on trunk buses. The passenger stop-to-stop OD matrix was obtained by applying an iterative proportional fitting method for the trip distribution procedure.

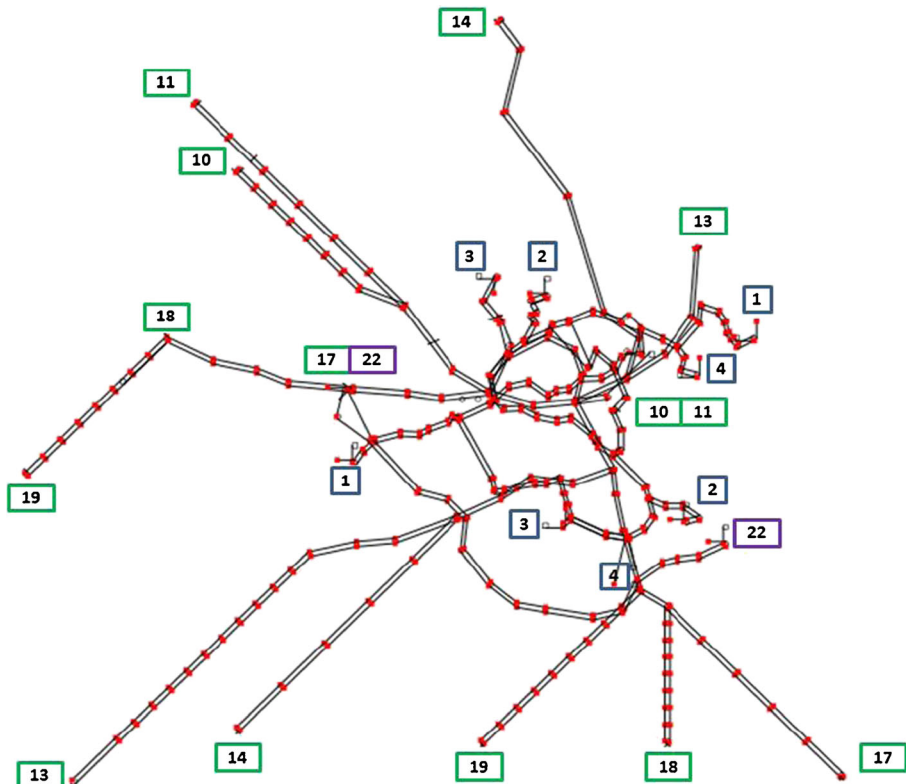


Fig. 1 Stockholm's inner city rapid public transport network as displayed by BusMezzo

A rule-based choice set generation algorithm was used as a pre-process step to the simulation runs. BusMezzo generates path alternatives by executing a recursive search method and applying a series of logical, behavioral and dominancy rules (Cats et al. 2011a). It resulted in 99,270 alternative paths for the entire network. This master set was used in the construction of action-dependent choice sets throughout the simulation. The parameters of the choice set generation model and the dynamic path choice model were estimated based on a stated-preferences survey on public transport route choice decisions (Cats 2011). The utility associated with path $a \in A^g$ is defined as

$$v_{a,n} = \beta_a^{wait} t_{a,n}^{wait}(t) + \beta_a^{ivt} t_{a,n}^{ivt}(t) + \beta_a^{walk} t_{a,n}^{walk} + \beta_a^{trans} trans_a \tag{16}$$

where $t_{a,n}^{wait}(t)$ and $t_{a,n}^{ivt}(t)$ are the time-dependent anticipated waiting time and in-vehicle time, respectively. $t_{a,n}^{walk}$ is the expected walking time and $trans_a$ is the number of transfers involved with the path alternative. $\beta_a^{ivt} = -0.04$, $\beta_a^{wait} = -0.07$, $\beta_a^{walk} = -0.07$ and $\beta_a^{trans} = -0.334$ are the corresponding coefficients. This reflects a ratio of 1.75 between in-vehicle and waiting or walking times and a transfer penalty equivalent to approximately eight in-vehicle minutes. The anticipated values depend on travellers' prior knowledge and the level of information that is available to them when making the decision.

The average values assigned to the coefficients were derived from the stated-preferences survey. Each individual is assigned with coefficients sampled from a normal distribution to account for the heterogeneity of preferences in the population. The trip fare is fixed for the entire network and hence does not affect passenger path decisions.

3.2 Applying the Betweenness Centrality Measures to the Stockholm Network

The case study considered the case of normal operations and disruptions on selected links. The passenger betweenness centrality (PBC) measure (Eq. 4) was used for identifying the candidate important links. The base case scenario of normal operations and the existing information conditions (RTI provision is available at all rapid public transport stops) for the Stockholm network was simulated to allow the calculation of the PBC measure across the network. Table 1 presents the five network segments with the highest ranking which were selected for further analysis. Segments were defined as a sequence of consecutive links with similar PBC values while assuring that there is no other link that has a higher PBC value that is not included in the five selected segments. The average passenger load during the rush hour is equivalent to the numerator of the PBC measure for $t = 7:00$ AM and τ equal to one hour.

All five segments are in the core of the network where there are rapid public transport alternatives. Furthermore, all of them are metro segments. Figure 2 illustrates the network core and allows identifying the five segments as well as the availability of the number and complexity of alternative paths. Main transfer locations are highlighted. The two busiest segments are the two metro lines that enter the inner city from the south. This reflects the distribution patterns of population and employment in the Stockholm area.

Table 1 also includes the average number of vehicles entering the segment during the rush hour, which corresponds to the nominator of the operator's betweenness

Table 1 Network segments selected for vulnerability analysis

PBC ranking	Metro line	Direction	Segment	Average passenger load during the rush hour (unnormalized PBC)	Average number of vehicles entering vehicles during rush hour (unnormalized OBC)	Average traditional betweenness centrality $BC(e)$
1	Green (17,18,19)	Northbound	Gullmarsplan →Hötorget	27,186	36.1	0.01575
2	Red (13,14)	Northbound	Liljeholmen →Centralen	18,363	24.0	0.02781
3	Green (17,18,19)	Southbound	Alvik →Centralen	14,785	32.8	0.01012
4	Blue (10,11)	Southbound	Fridhemsplan →Centralen	11,510	16.9	0.01617
5	Red (13,14)	Southbound	Centralen →Hornstull	10,508	24.0	0.02781

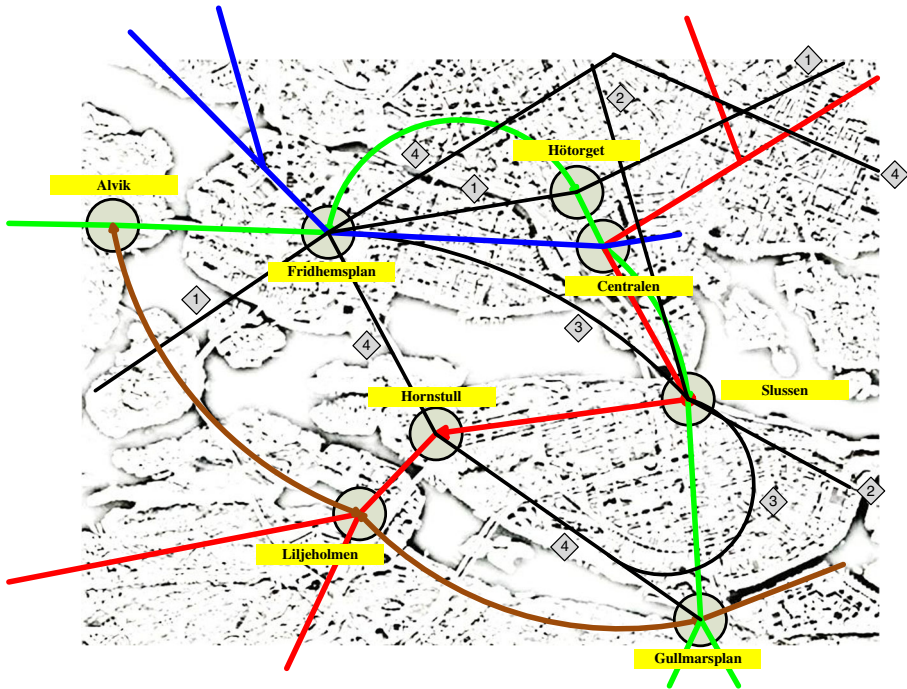


Fig. 2 Schematic representation of the network core and stations of interest (The three metro corridors: Green (17–19), Blue (13–14) and Red lines (10–11), the orbital LRT (22) in brown and the 4 trunk bus lines in black with their line numbers marked)

centrality (OBC) measure (Eq. 3). Note that the OBC ranking differs from the PBC ranking as supply is not perfectly adjusted to passenger flows. Hence, the selected disruption scenarios do not reflect the candidate important links for the public transport rolling stock.

Figure 3 presents the relationship between relative values of PBC and OBC for all network links which exercise a correlation of $r_{PBC,OBC}=0.24$. Displayed values correspond to percentage-wise deviation from the respective average value. A large number of links have relatively low PBC with average or slightly below average OBC values (bottom-left quadrant) which corresponds to bus links. The mismatch between these two variables is partially attributed to varying vehicle capacities. Moreover, links with high OBC values have a large range of PBC as passenger loads vary considerably along the line, in particular towards line edges. Furthermore, the conventional betweenness centrality (BC) measure (Eq. 1) based solely on network topology is also shown in Table 1 for each segment. This measure has low correlations with the dynamic betweenness centrality measures ($r_{BC,PBC}=0.18$, $r_{BC,OBC}=0.005$).

3.3 Scenarios Design

3.3.1 Disruption Scenarios

The case study considers a short-horizon and unplanned disruption. For each of the above five segments, a disruption that takes place between 7:15 and 7:45 was

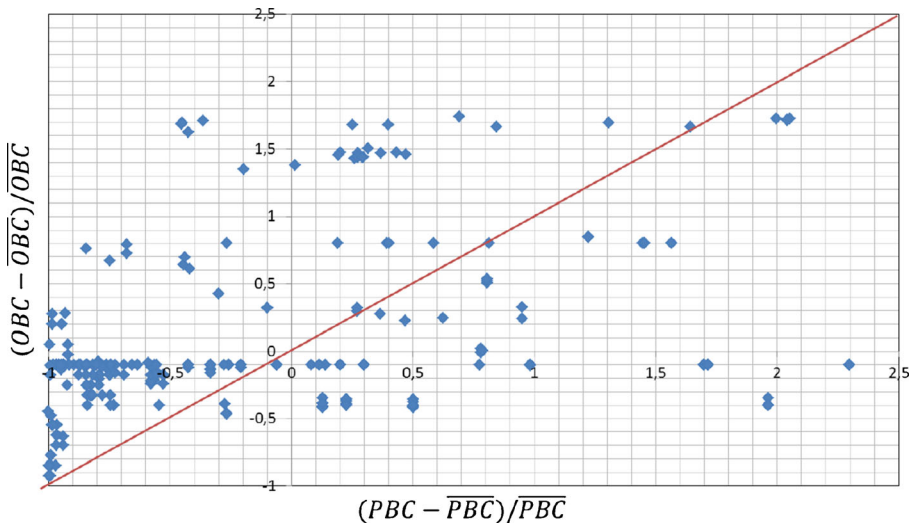


Fig. 3 Relative values of PBC versus OBC (passenger and operator betweenness centrality, respectively) for all network links

simulated. The disruption of a certain segment could be a result of an unpredicted incident—either a random technical problem (e.g., signal failure, vehicle breakdown) or human-caused closure (e.g., accident, suspicious item). Previous studies often assumed that the remaining sections continue to function regularly as two independent routes (e.g. von Ferber et al. 2012). In contrast, since the disruption is short-horizon and unplanned, it is assumed here that the system operators cannot employ any special measures to mitigate the impacts of the disruption such as providing a replacement service or operating the remaining disconnected parts of the line as independent lines. Note that segment closure on a certain line does not imply disruptions on other lines since each metro line has a distinguished infrastructure (tracks and platforms) and fleet.

The disruption scenarios were modelled in BusMezzo by specifying the incident start time, duration and the effected links. The simulation model then prevents vehicles from traversing to the disrupted link as long as the disruption is in effect. This implies that upstream public transport vehicles progress until they queue upstream of the link closure. On-board passengers are unable to alight while passengers waiting at downstream stops (including stops along the disrupted segment) can reconsider and revise their travel decision (e.g., walk to a nearby stop). In addition to the passengers directly affected by the disrupted segment, spillover secondary effects are caused by supply interactions (e.g. upstream and downstream stops, vehicle scheduling) and passenger rerouting (e.g., delays, denied boarding due to capacity constraints).

3.3.2 Real-Time Information Provision

The provision of RTI could mitigate the impacts of disruptions by allowing passengers to make more informed decisions and choose alternative paths. The case study considers passenger information systems with RTI regarding the next vehicle arrival time at various levels of coverage and comprehensiveness. The simulation of traffic dynamics and public transport operations in BusMezzo emulates public transport performance

and the production of automated data collection (ADC) methods, such as automatic vehicle location (AVL) and automatic passenger counts (APC). This data can be processed in order to generate predictions on future public transport conditions that will be disseminated to travellers.

The prediction scheme used by BusMezzo is designed to replicate the method that is commonly used by public transport agencies for generating real-time arrival information. In the case of service disruption, it is assumed that the RTI generator can approximate its anticipated duration. The provisioned RTI concerning stops along the disrupted segment as well as further downstream enables passengers to update their waiting time expectations and adapt their travel choices. In contrast, the simulated RTI does not help passengers upstream of the disruption segment to avoid a service heading towards the disruption segment, since it does not refer to in-vehicle time. The following RTI provision scenarios were simulated:

- No-RTI: Passengers have no access to RTI, all travel decisions rely on prior knowledge
- Stop-RTI: RTI is available at stops and rail stations regarding all public transport services departing from the a specific rail platform or bus stop
- Cluster-RTI : RTI is available at stops and rail stations regarding all public transport services departing from all platforms and bus stops within a single station/hub or a walking distance of up to 500 m
- Network-RTI: RTI is available regarding all public transport services in the network to all individuals through personal mobile devices

The availability of RTI influences the anticipated attributes of alternative paths and ultimately passenger flows. In order to examine the full mitigation potential of RTI dissemination, it is assumed that passengers perceive RTI as credible and therefore incorporate it into their decisions. The availability of RTI is not equivalent to perfect knowledge because of the prevailing service uncertainty. The impact of RTI is manifested through the assignment of the values provided by the RTI (e.g. waiting time or in-vehicle time) in the path utility function (Eq. 16). Note however that the comprehensiveness of the RTI provision determines which path segments are effected. For example, consider a traveller that does not have a personal mobile device with access to RTI. Hence, the walking decision from the origin to the first public transport stop is based on the traveller's prior knowledge. If the traveller arrives at a stop with real-time arrival information on the local stop, then the traveller's boarding decision relies on the RTI waiting times while the remaining travel attributes are based on prior knowledge. In case there is no RTI on-board, the alighting decision relies entirely on prior knowledge. When the traveller alights at a certain stop, a connection decision takes place. If the transfer stop is equipped with RTI display that covers also nearby stops then the immediate waiting time component in this decision is based on RTI.

The experimental design consists of six network operational conditions: normal operations (D0) and disrupted segments corresponding to their PBC ranking (D1–D5), and four levels of RTI provision: No-RTI, Stop-RTI, Cluster-RTI and Network-RTI. The combinations of operational conditions and RTI provision defined 24 scenarios. For each scenario, ten simulation runs were conducted for a three hours period with a uniform passenger demand during the peak hour. This number of replications yielded a

maximum allowable error of less than 5 % for the average passenger travel time based on the method proposed by Dowling et al. (2004). The execution time for a single run was less than 1 min on a standard PC.

4 Results

4.1 Passengers' Travel Times

Table 2 presents the average passenger travel time under each combination of operational and information provision conditions. Under normal operational conditions, the average travel time in the case study network is 24 min which consists of waiting times, in-vehicle times and walking times at transfer locations. As expected, service disruptions result in considerably longer travel times. Longer travel times can be caused by either primary or secondary disruption impacts. An increase of 1 % induces 500 additional passenger hours during the rush hour only. Moreover, upon disruption, passengers tend to deviate to alternative paths which involve higher complexity. This is especially pronounced in the case of scenario D1 (disruption of the Green line at Gullmarsplan-Hötorget) as it forces a large number of passengers that travel from the south to the city center to take either the orbital trunk bus or the orbital LRT from Gullmarsplan and then change to a radial line (see Fig. 2).

The individual-based representation of passengers demand enables the disaggregate analysis of travel times. The impact of disruptions on travel times were further investigated by constructing the distribution of unweighted total travel time for each of the operational scenarios with no-RTI (Fig. 4). It is evident that travel times vary significantly among travellers. The variation depends on the origin–destination pair, service availability and reliability, and path choice decisions. Service disruption results in a larger share of the passengers experiencing long or very long travel times compared with the non-disrupted scenario (D0). The share of trips that take between 40 min and 60 min increases from 12 % in D0 to 16 % in D1, and from 2 to 5 % for trips longer than 60 min, respectively. In the context of this case study, 1 % of all trips correspond to 1,250 travelers.

In addition to the increase in average travel time (Table 2), the standard deviation of travel times increases by 2–19 % under disrupted operations. Furthermore, a small group of travellers may benefit from service disruptions as reflected by the left bending of the travel time distribution. A positive secondary disruption impact may occur on a service element that operates independently of the disrupted segment and the latter is a common upstream trip stage for a considerable share of its demand. In such cases, remaining passengers will benefit from fewer delays, higher service reliability and less crowding.

4.2 Effects on Total Welfare and Crowding

The increase in travel time and number of transfers yields a decrease in the overall welfare value (Eq. 16) under all disruption scenarios. Figure 5 presents the change in total welfare, where the scenario of normal operations with the corresponding RTI provision conditions used as the reference value. The welfare reduction is in the range

Table 2 Passenger travel time and relative changes under the different scenarios

Operational scenario (δ)	RTI provision scenario (r)	Unweighted total travel time $t^{walk} + t^{wait} + t^w$ [sec]	Transfers $trans$	Welfare for scenario combination $(\delta, r)W(\delta, r)$	Relative welfare change due to disruption $P(\delta r)/W(0, r)$ [%]	Relative welfare change due to RTI $VRT((+\delta)W(\delta, 0))$ [%]	Normalized RTI mitigation effect $\frac{VRT((+\delta)W(\delta, 0))}{P(\delta r)/W(\delta, 0)}$ [%]
D0	No	1452	1.44	-73.07	-	-	-
	Stop	1427	1.38	-71.79	-	+1.74	-
	Cluster	1424	1.37	-71.65	-	+1.92	-
	Network	1420	1.37	-71.43	-	+2.22	-
D1	No	1607	1.67	-80.91	-10.69	-	-
	Stop	1619	1.65	-81.52	-13.52	-0.77	-5.59
	Cluster	1584	1.60	0.79.72	-11.23	+1.45	13.03
	Network	1576	1.61	-79.32	-11.00	+1.95	17.79
D2	No	1467	1.45	-73.82	-1.03	-	-
	Stop	1483	1.45	-74.65	-3.98	-1.13	-28.21
	Cluster	1503	1.42	-75.63	-5.56	-2.48	-44.10
	Network	1477	1.43	-74.31	-4.03	-0.68	-16.50
D3	No	1561	1.47	-78.52	-7.49	-	-
	Stop	1507	1.44	-77.73	-8.30	+1.00	12.20
	Cluster	1498	1.44	-75.36	-5.18	+4.03	77.57
	Network	1545	1.43	-75.82	-6.16	+3.44	55.81
D4	No	1558	1.44	-78.40	-7.35	-	-
	Stop	1520	1.41	-76.45	-6.52	+2.50	38.33
	Cluster	1525	1.40	-76.71	-7.10	+2.15	30.50
	Network	1503	1.39	-75.62	-5.90	+3.54	60.34
D5	No	1482	1.45	-74.58	2.08	-	-
	Stop	1480	1.44	-74.47	-3.73	+0.15	4.13

Table 2 (continued)

Operational scenario (δ)	RTI provision scenario (r)	Unweighted total travel time $t^{walk} + t^{wait} + t^w$ [sec]	Transfers $trans$	Welfare for scenario combination $(\delta, r)W(\delta, r)$	Relative welfare change due to disruption $P((\delta)r)/W(0, r)$ [%]	Relative welfare change due to RTI $rRTI((\delta)r)/W(\delta, 0)$ [%]	Normalized RTI mitigation effect $\frac{rRTI((\delta)r)/W(0, r)}{P((\delta)r)/W(\delta, 0)}$ [%]
Cluster		1453	1.43	-73.12	-2.03	+1.97	96.00
Network		1456	1.42	-73.28	-2.57	+1.75	67.70

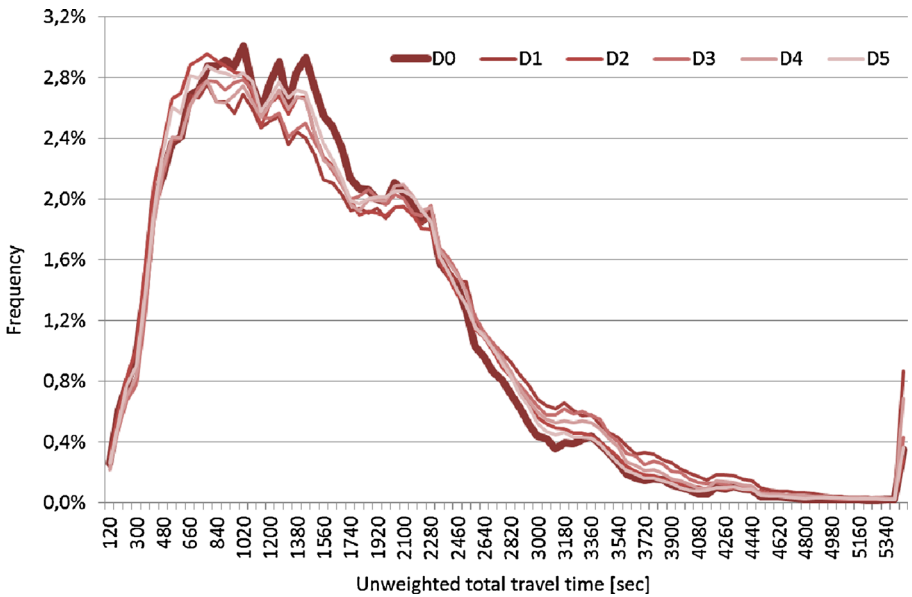


Fig. 4 Unweighted total travel time distribution for the non-disrupted (D0) and all disruption scenarios (D1–D5); all scenarios are with no-RTI

of 1–14 %. D1 results in the most dramatic welfare loss, while D2 yields less severe consequences than the corresponding betweenness centrality would suggest. It should be noted that the impact varies substantially for different origin–destination pairs.

Table 2 also provides information about the welfare change due to RTI provision, which was calculated by comparing the welfare to the value obtained under the same disruption scenario when no RTI is available. The availability of RTI is used by passengers for making travel decisions that yield higher utility. For the non-disrupted network (D0) the provision of RTI leads to a reduction of 2 % in the total un-weighted passenger travel times and an equivalent increase in passenger welfare. Interestingly,

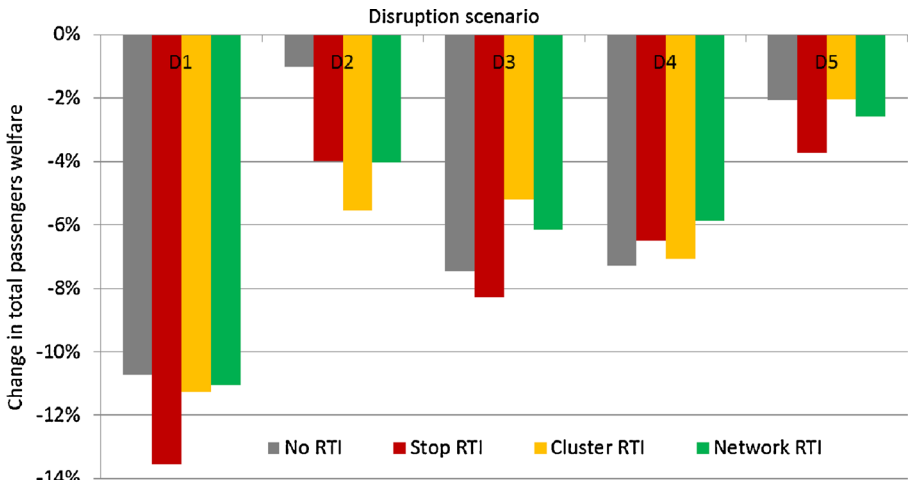


Fig. 5 Comparison of the relative impact of disruption scenarios on total passengers welfare

the number of boardings per trip tends to decrease with greater availability of RTI provision. In the undisturbed case (D0), this decrease is obtained already when RTI is provided at the stop level, suggesting that uninformed travellers make more unnecessary transfers instead of waiting longer for a direct service.

As illustrated in Fig. 6, the impact on RTI provision on passenger welfare varies considerably for different disruption scenarios from a worsening of 2.5 % to an improvement of 4 %. The general trend is that more comprehensive RTI provision results in higher passenger welfare. However, the exact impact depends on network dynamics and the topology of Stockholm’s PTN as it determines the opportunities of gaining from more informed downstream connection decisions. The marginal gain from extending RTI provision thus varies from extremely positive (D3, D4) to harmful (D2) while sometimes being negligible beyond a certain level (D0).

The negative impact of RTI provision in the case of D2 was further analyzed by investigating the path choice changes across the network and the corresponding trip loads. The negative disruption impact in the case of D2 is amplified by the provision of RTI as a consequence of network structure and capacity constraints. The provision of RTI leads more passengers to travel with the orbital LRT (Line 22) and trunk line 4 in order to avoid the disrupted segment. While these passengers gain on average, their rerouting generates negative effects for passengers travelling to or from the stops along both directions of the over-saturated lines. The latter passengers experience increased travel times due to delays and denied boarding as the LRT and bus line cannot accommodate the entire demand shift from the metro line, which has a significantly higher capacity. This negative secondary impact is most pronounced in the case where RTI is available at the cluster-level because all passengers can use RTI concerning alternative stops within a walking distance. Network-RTI enables passengers that are subject to negative secondary effects to choose paths, which reduces the extent of the spillover effect.

The last column in Table 2 presents the share of the disruption impact that is mitigated by the RTI provision, where a ratio of +1 indicates that RTI provision relieves

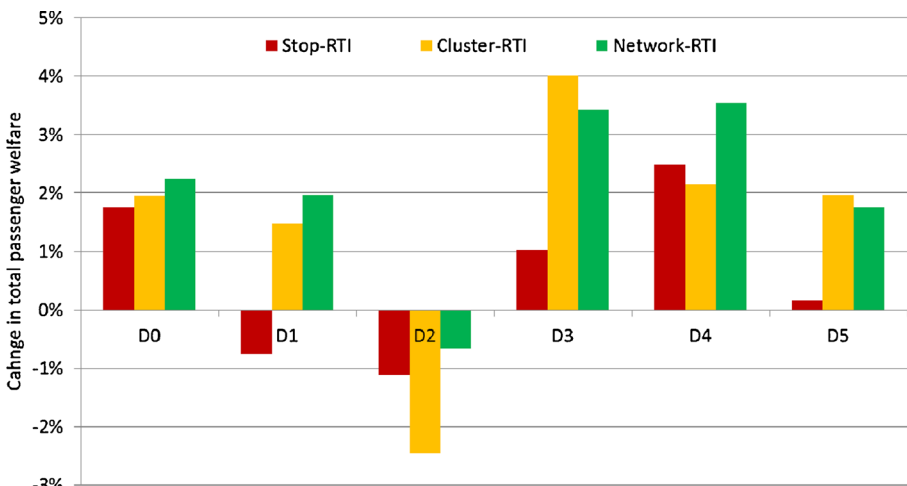


Fig. 6 Comparison of the relative impact of real-time information provision scenarios on total passengers welfare

the entire welfare reduction caused by the service disruption. The mitigation effect of RTI provision varies considerably across disruption and RTI provision scenario combinations. RTI relieves more than 50 % of the disruption impact for D3, D4 and D5 when high levels of RTI comprehensiveness are available, while having a lower magnitude of mitigation for D1. In contrast, RTI provision has a degrading effect in the case of D2. The property that RTI may have negative effects on system performance is not unique to public transport systems; similar phenomena have been observed in simulations of private vehicular traffic (e.g., Mahmassani and Jayakrishnan 1991).

The vulnerability analysis consists of aggregate system measures which arise from the detailed dynamic representation of system supply and demand. In order to illustrate the importance of system dynamics, let us consider the temporal variations in passenger load for trunk line 1 (see Fig. 2). Each line in Fig. 7 corresponds to a rush-hour vehicle trip for the case of no disruption (D0) and disruption on the southbound direction of the Blue line (D4) with the highest level of information provision (RTI-Network). It is evident that even for the case of normal operations there are substantial temporal variations as a result of system dynamics and the interaction between inherent sources of supply and demand variability. This temporal variation is distorted significantly in the case of a disruption on the southbound direction of the blue line (D4). The Green line is the primary alternative for passengers coming with the Blue Line and heading towards the city center with trunk line 1 being a secondary alternative (see Fig. 2). The vehicle capacity constraint is reached for a couple of the trips that arrive at the affected stop, Fridhemsplan, during the disruption (7:15–7:45) and the load is relieved at Hötorget which lies within the city center. As the system recovers from the disruption, passenger loads gradually resemble previous patterns and levels (lighter lines correspond to later trips). This suggests that it is important to capture how the system evolves over time when analyzing the impacts of disruptions on redundant capacity and mitigation strategies.

4.3 Importance Hierarchy

The betweenness centrality measures defined in Section 2 were used in order to identify candidate important network segments. In particular, the five disruption scenarios were defined based on scanning the network for the segments with the highest PBC measure. The results of the simulation model enable us to investigate how well segment betweenness centrality measures correspond to the impact of the corresponding segment closure on the total passenger welfare. This analysis allows us to shed some light on the relation between centrality and importance and to what extent can the former be used as a heuristic for identifying the latter.

Both PBC and OBC indicate that the Green line segment from Gullmarsplan to Hötorget is the most central link. Moreover, the corresponding disruption scenario (D1) resulted with the most severe degradation of passenger welfare, $PI(\delta|r)$, under all RTI conditions. This segment/scenario is therefore used as the benchmark value in Fig. 8 which presents the centrality and importance indices when normalized against the respective D1 values. In line with the PBC index, D1 is clearly the most harmful disruption. In contrast, D2 was ranked second in terms of PBC but results in less significant impacts than segments that were ranked lower. In fact, it ranked lowest among the five disruption scenarios in the case where no RTI is available. Hence, link

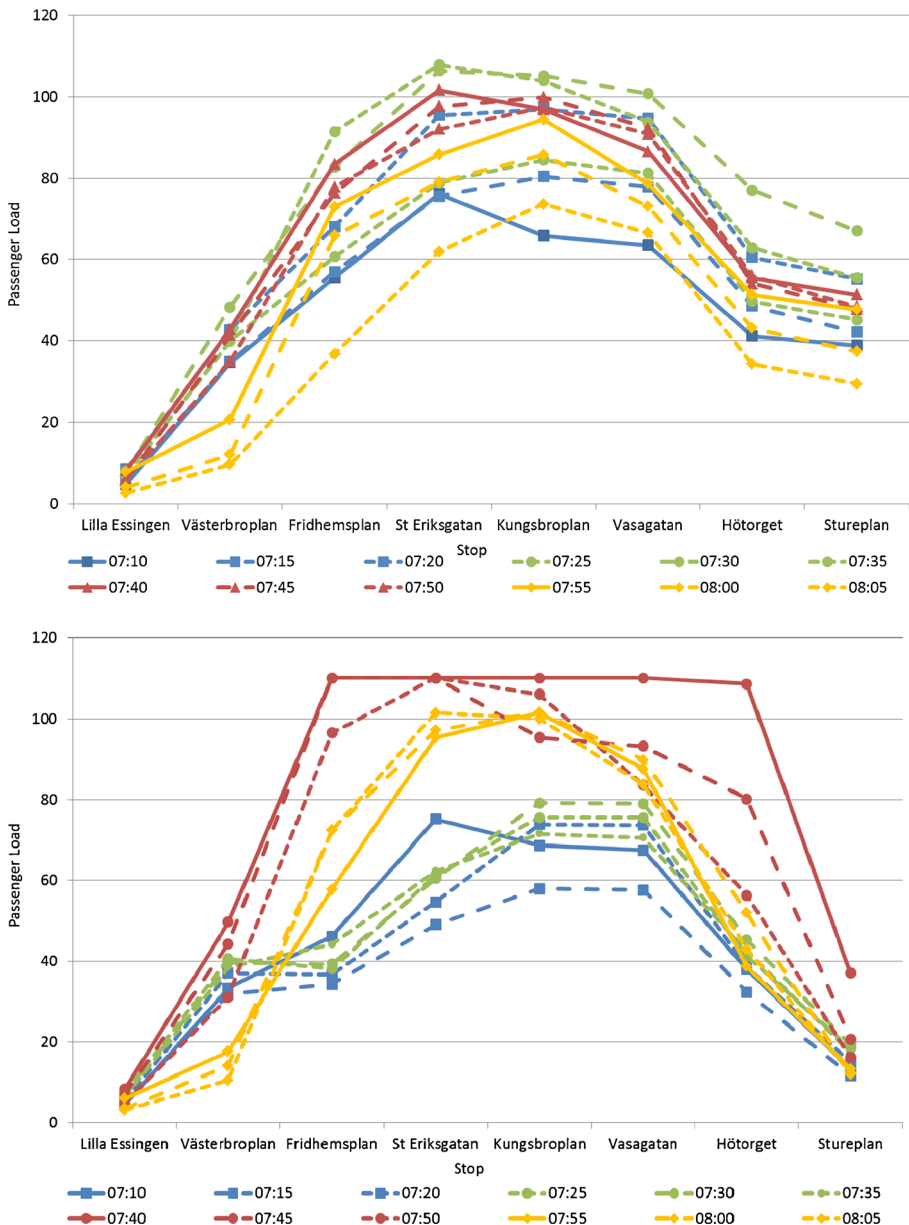


Fig. 7 Passenger loads at key stops along trunk line 1 eastbound during the rush hour for D0, Network-RTI (above) and D4, Network-RTI (below) scenarios. Each line corresponds to a rush-hour vehicle-trip

centrality does not necessarily imply link importance as measured by its impact on total travel welfare. Furthermore, betweenness centrality measure from the operator’s perspective does not provide a more consistent indicator of segment importance. While OBC better explain the more severe implications of D3 over D2, it also associates the same high centrality with D5 which exercises significantly lower importance.

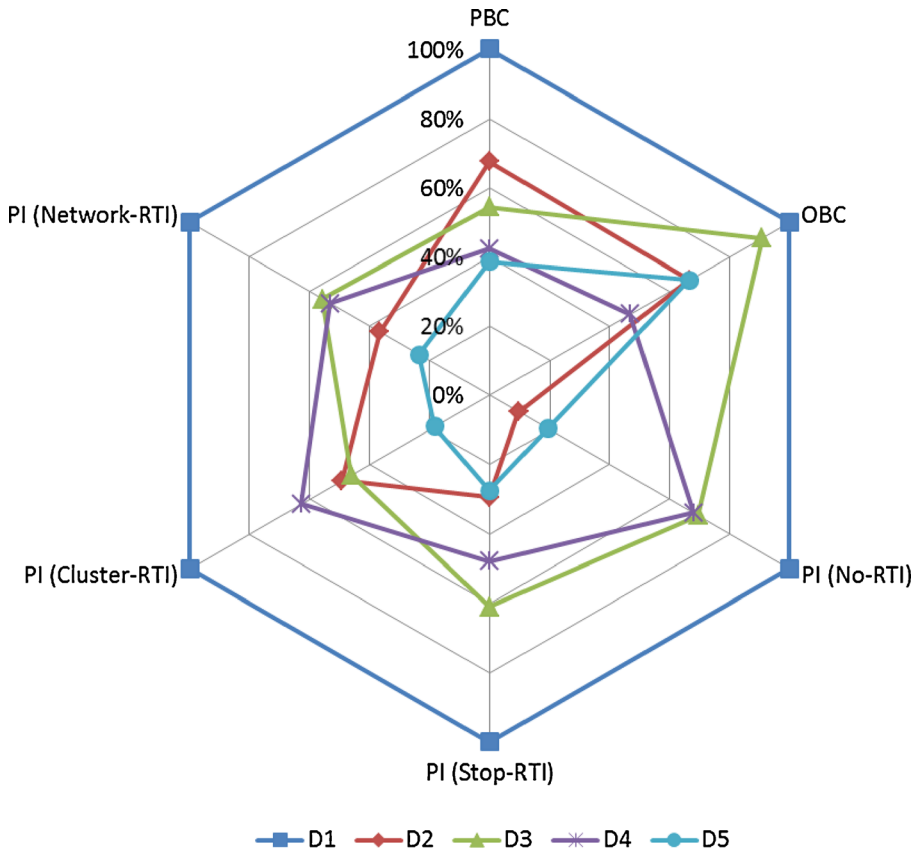


Fig. 8 Relationship between centrality measures and the impact of disruption scenarios on passenger welfare (all values are normalized against D1 values)

The identification of important links depends on passengers’ information. This is clearly visible in Fig. 8. For example, the system is more vulnerable to D3 than to D4 if RTI is provided at the local stop level, while this is reversed if passengers have access also to RTI regarding nearby stops. This suggests that our assumptions about passenger knowledge may have strong implications on which links are to be considered critical for system performance.

5 Conclusions

The performance of PTN is determined by the interaction between supply and demand dynamics. This interaction is manifested in the propagation of supply deterioration and passenger path choice strategies in the case of service disruptions. Hence, the vulnerability analysis of PTN has to consider system dynamics. In this paper, a dynamic and stochastic notion of network vulnerability was presented. This involves time-dependent service availability and passenger demand, the inherent stochastic processes in the public transport system and the accumulated effect of service disruptions. Moreover, the impacts of service disruptions depend on how the demand reacts to changes in

supply. Hence, the underlying assumptions on passengers' knowledge and path choice strategies have great implications on the vulnerability analysis. A probabilistic and adaptive path choice model was integrated into the proposed methodology. The measure of betweenness centrality used in network theory to identify central links was generalized to PTN and defined in terms of rolling stock and passenger flows. Measures of disruption impacts and the importance of network elements were also defined, taking into account the stochastic and dynamic nature of PTN. Moreover, the proposed methodology facilitates the assessment of the value of RTI for mitigating the impacts of service disruptions.

The dynamic approach for network vulnerability analysis was applied to the rapid public transport system in Stockholm, Sweden. Since a complete simulation-based analysis of disruptions on every link in the network would be highly time-consuming, candidate important links were identified based on the different centrality measures obtained from the performance under normal operational conditions. The results of the vulnerability analysis suggest that link centrality does not necessarily imply link importance as measured by its impact on total welfare. In addition, link importance depends on the level of information that is available to passengers as the value of RTI provision varies for different disruption scenarios.

In one of the studied scenarios, RTI is found to worsen the impact of the disruption. This suggests that it may be beneficial to customize the nature and extent of information provision to the characteristics of the location (e.g. capacity on alternative lines) and the disruptive event. How to design effective information provision schemes under significant disturbances is an interesting topic for further research.

The limited correlation between centrality and importance among the top candidate links suggests that using centrality as an indicator for importance may not be a robust way of identifying the most important links. While avoiding a full-range vulnerability analysis, it may be possible to use simulation optimization techniques to identify important links with as few simulation runs as possible (e.g., Carson and Maria 1997). This is an important area for future work. In addition to the impacts of disruptions, more research is also needed in order to understand and model the probability of disruptions occurring with different spatial and temporal extents in the PTN.

A vulnerability analysis provides the background and starting point for an evaluation of various measures to reduce vulnerability, if needed. The next step is thus to analyse how to best manage the vulnerability with emergency preparedness, infrastructural reinforcements and expansions, operations and maintenance procedures etc. That is, given the society's current state of vulnerability to disruptions in the public transport system, what actions should be taken? By evaluating the impacts of disruptions in economic terms as the costs of operators and welfare losses of passengers, the effects of actions to increase robustness can be compared to their costs. This provides a way to integrate vulnerability management in the larger planning process, for example by specifying a certain minimum acceptable level of robustness, or in a cost-benefit analysis framework.

In the planning stage, the identification of critical links can provide guidelines for infrastructure investment decisions, including the design of transfer facilities. Critical links may be prioritized in the allocation of resources for maintenance and upgrades of technical equipment. A vulnerability analysis can also guide the alignment and standard

of a new public transport line, and support the building/operation of new public transport infrastructure/lines that among other benefits provide some redundancy to the existing lines. Robust PTN design, i.e., how to design a PTN from scratch or in the long run with capacity to handle degradation, is an interesting topic for further research.

During the operations and real-time stages, different actions can be taken to reduce the vulnerability depending on the type of identified hazard or threat. As illustrated in the paper, RTI can help mitigating the impacts of disruptions, not only by its dissemination to passengers but also by enabling more proactive fleet management strategies. The public transport simulation model facilitates the evaluation of alternative RTI generation and dissemination schemes. Timetable and vehicle scheduling design which considers risk distribution may reduce the probability of disruption, while better management and restoration strategies, for example by increasing the resources for stand-by maintenance preparedness, may reduce its impact.

The dynamic approach for network vulnerability allows investigating how the system restores under a longer time period. This will require the definition of measures to identify recovery patterns to pre-disruption performance. Future studies may consider different kinds of disruptions such as node closure and random network failures. It is also interesting to further analyse the spatial and demographic distribution of the impacts of service disruption for example by constructing accessibility measures.

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