

Design of Sustainable Cordon Toll Pricing Schemes in a Monocentric City

Zhi-Chun Li · Ya-Dong Wang · William H. K. Lam ·
Agachai Sumalee · Keechoo Choi

Published online: 6 October 2013
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Abstract This paper addresses the design issue of sustainable cordon toll pricing schemes in a monocentric city. An analytical model that maximizes the total social welfare of urban system is first proposed for simultaneous optimization of the cordon toll location and charge level. The solution properties of the model with/without considering traffic congestion and/or environmental effects are explored and compared analytically. The proposed model is then extended to explicitly incorporate the effects of subsidizing the retrofit of old vehicles on reduction in average vehicle emissions. The optimal subsidy scheme for maximizing the social welfare of the system is also determined. Finally, a numerical example is given to illustrate the model applications. Insightful findings are reported on the interrelationships among cordon toll scheme, traffic congestion and environmental effects, urban population distribution, and subsidy scheme as well as their implications in practice.

Z.-C. Li (✉) · Y.-D. Wang
School of Management, Huazhong University of Science & Technology, Wuhan 430074, China
e-mail: smzcli@hust.edu.cn

W. H. K. Lam · A. Sumalee
Department of Civil & Environmental Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China

W. H. K. Lam
e-mail: william.lam@polyu.edu.hk

A. Sumalee
e-mail: ceasumal@polyu.edu.hk

W. H. K. Lam
School of Traffic & Transportation, Beijing Jiaotong University, Beijing 100044, China

K. Choi
TOD-based Sustainable Urban Transportation Center, Department of Transportation Engineering,
Ajou University, San 5 Woncheon-Dong, Yeongtong-Ku, Suwon 442-749, South Korea
e-mail: keechoo@ajou.ac.kr

Keywords Monocentric city · Sustainable cordon toll pricing · Traffic emissions · Air pollution cost · Social welfare · Subsidy

1 Introduction

The past decade has witnessed the rapid urban expansion due to urbanization and population growth in many densely populated Chinese cities such as Beijing, Shanghai, and Wuhan. This rapid urban expansion has caused a huge pressure to urban transportation systems, resulting in increasingly serious traffic congestion and environmental issues. It was shown that vehicular use contributes to a major proportion of air pollution, including 30 %–50 % of hydrocarbon, 40 %–60 % of nitrogen oxide, and 80 %–90 % of carbon monoxide emissions (USEPA 1991, 1992). The growth rate of carbon emissions has increased from less than 1 % in the 1990s to 3.5 % since 2000 (IPCC 2007), exacerbating the global warming and climate change. Therefore, it is necessary to take measures to develop a sustainable and low-carbon urban transportation system.

As an efficient traffic demand management measure, cordon toll pricing schemes, in which each vehicle passing through a specified cordon is charged a fixed toll, are often adopted in practice because of their ease of implementation and potential to internalize the congestion and environmental externalities (Li et al. 2012a). Typical examples include those schemes adopted in Singapore, London, Hong Kong, Oslo, Trondheim, and Bergen (Zhang and Yang 2004; Rouwendal and Verhoef 2006). More recently, project certification works on the cordon toll pricing schemes are being conducted in some major Chinese cities, such as Shanghai, Shenzhen, and Hangzhou. In these project certifications, an important issue that the authority faces is how to specify the cordon toll location and toll level so as to maximize the efficiencies of the urban transportation systems.

Over the past decades, significant research efforts have been made towards the cordon toll pricing problems. The modeling methods adopted in the previous related studies can be classified into two major categories: discrete network modeling approach and continuum modeling approach. Example studies on the discrete network toll pricing models include May et al. (2002a,b), Verhoef (2002), Zhang and Yang (2004), Shepherd and Sumalee (2004), Sumalee (2004, 2007), Yang and Huang (2005), Akiyama and Okushima (2006), Szeto and Lo (2006), Ekstrom et al. (2012), and Hartman (2012). The discrete network models are useful for practical applications, but they are inadequate for revealing general properties of a problem because their results depend very much on the network topology structures. Moreover, they cannot be used to assess analytically the effects of the urban spatial configuration. However, an empirical study for eight English towns conducted by Santos et al. (2001) showed that the efficiency of cordon tolls is closely related to the urban form. These issues can be addressed analytically by the continuum modeling approach so as to make a general conclusion regarding a policy directly from the properties of the continuum models. Sample studies on the continuum toll pricing models include Mun et al. (2003, 2005), Ho et al. (2005, 2007), Verhoef (2005), Chu and Tsai (2008), and Li et al. (2012b). For comprehensive reviews of recent methodological advances in road toll pricing issues, readers can refer to Yang and Verhoef (2004), Tsekeris and

Voß (2009), and de Palma and Lindsey (2011). In this paper, the continuum modeling approach is adopted.

These previous studies on the continuum models have mainly focused on the optimization of the cordon toll location and/or the toll level. Little attention has been paid to the environmental externality problem caused by the vehicular pollutant emissions, which is an especially important issue in the era of climate change (Szeto et al. 2012). In fact, motorized vehicles produce various emissions, such as Carbon dioxide (CO₂), Carbon monoxide (CO), Ozone, Lead, Nitrogen oxides (NO_x), and Sulfur oxides (SO_x). These emissions can have harmful impacts on human health and ecosystems. Recently, the local governments in many large Chinese cities, such as Beijing and Shanghai, have launched the air cleanse action program for controlling the air pollution level and improving the air quality. The measures adopted in this program include subsidizing the use of clean energy (e.g. electric and natural gas vehicles), retrofit of old motorized vehicles, and the purification of vehicular pollutant emissions (e.g. free-of-charge supply of the vehicular exhaust purifier). To keep the financial sustainability of the air cleanse action program, it has been proposed to utilize the toll revenue via a redistribution or subsidy scheme to fund such a program in the future. The redistribution of the toll revenue has been recognized to be an important issue that affects the implementation of congestion pricing schemes (Adler and Cetin 2001). It is therefore of great importance to consider the vehicular environmental externality and the subsidy policy in the cordon toll pricing problems so as to achieve environmentally and financially sustainable urban transportation systems.

In addition, the existing relevant studies did not address the effects of urban configuration in terms of urban population distribution on the optimal cordon toll location and toll level. Obviously, the urban population distribution governs the level of potential travel demand and thus of traffic congestion. A high-density large-scale city implies a high potential travel demand level and a high traffic congestion level, and thus plausibly a high toll level and a close-to-CBD (central business district) cordon location, and vice versa. It is thus important to reveal the interrelationship between the urban configuration and the cordon toll pricing schemes.

In light of the above, this paper proposes an analytical model for designing sustainable cordon toll pricing schemes in a monocentric city, as shown in Fig. 1. The objective of the proposed model is to maximize the total social welfare of the urban system by determining the cordon toll location and toll level. This paper made three major extensions to the previous related studies. First, the proposed model extends traditional cordon toll pricing models to consider the congestion and environmental externalities due to vehicular use. The solution properties of the model with/without considering traffic congestion and/or environmental externalities are analyzed and compared. Second, the effects of subsidizing the retrofit of old vehicles via the redistribution of the toll revenue on reduction in vehicle emissions are explored, and the optimal subsidy scheme is also determined. Third, the effects of the urban form in terms of urban population distribution on the optimal cordon toll pricing schemes are examined. Sensitivity analyses of some key parameters of the model, such as population density gradient and level of subsidy for retrofitting old vehicles, are also carried out.

The remainder of this paper is organized as follows. In the next section, some basic components of the proposed model are described, including the basic assumptions, travel demand and the air pollution cost. Section 3 formulates the optimization model

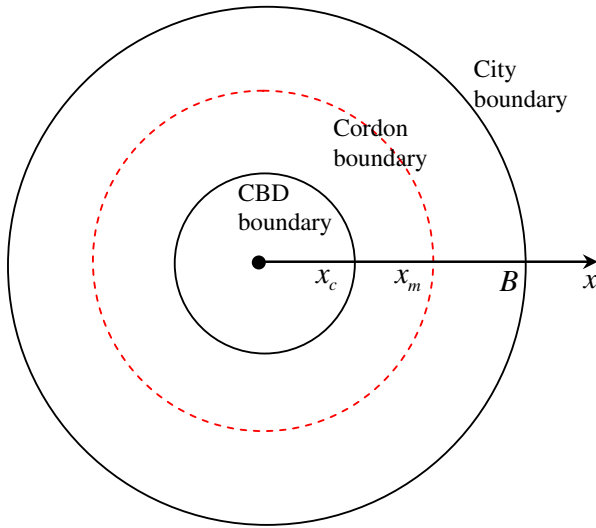


Fig. 1 Cordon toll pricing system in the monocentric city

for the cordon toll location and toll level. The solution properties of the model with/without considering the traffic congestion and/or environmental effects are presented and compared. In Section 4, the proposed model is extended to explicitly consider the effects of the subsidy policy. In Section 5, a numerical example is provided to illustrate the essential contribution of the proposed model. Finally, conclusions are given in Section 6 together with recommendations for further studies.

2 Basic Considerations

2.1 Assumptions

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made in this paper.

A1 The city concerned is assumed to be monocentric, in which all job opportunities are located in the urban CBD. A one-cordon tolling scheme is applied to the city. Auto drivers who originate from outside of the cordon and head for the CBD are required to pay a toll when they pass through the cordon.

A2 The population (or residential) density in the city is specified as a linear function, as assumed in Mun et al. (2003) and Chu and Tsai (2008). We represent the population density, $m(x)$, at location x from the CBD as

$$m(x) = b - a(x - x_c), \quad \forall x \in [x_c, B], \quad (1)$$

where B is the city boundary and x_c is the CBD boundary, i.e. the radius of the CBD (see Fig. 2). b is the population density at the CBD boundary and a (≥ 0) is the density gradient describing how rapidly the density falls as the distance from the CBD increases. The larger the value of a , the smaller the population density at the city boundary and the more compact the city. On the contrary, a smaller

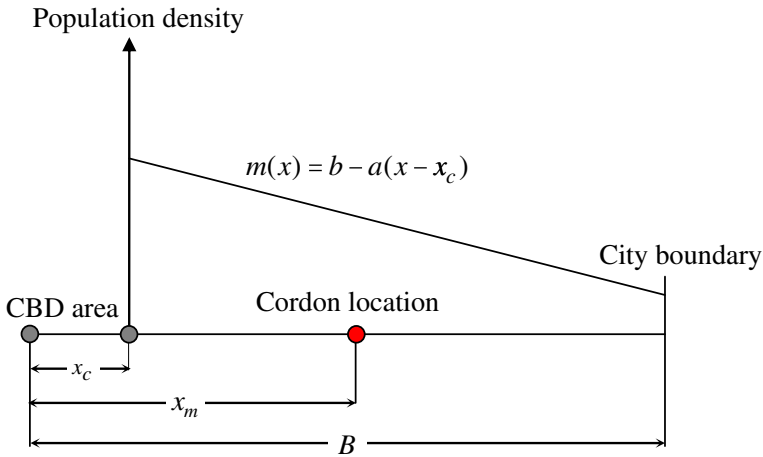


Fig. 2 Linear population density function

value of a means a more decentralized city. In particular, when a equals 0, the linear population density function is reduced to a uniform one. With this assumption, the relationship $N = \int_{x_c}^B 2\pi x m(x) dx$ holds, where N is the total number of population in the planning area.

A3 A linear elastic travel demand density function (Kocur and Hendrickson 1982; Li et al. 2012c) is used to capture the responses of travelers to the quality of the transportation service, which is measured by a generalized travel cost plus the cordon toll (if any). The responses include switching to alternative modes (e.g., bus, subway, or walk) and not making the journey at all (Lam and Zhou 2000; Li et al. 2007).

A4 The vehicular pollutant emission is estimated by the macroscopic model suggested by Penic and Upchurch (1992). It has been applied in some related studies, such as Rilett and Benedek (1994), Benedek and Rilett (1998), Yin and Lawphongpanich (2006), Nagurney et al. (2010), and Li et al. (2012a).

2.2 Travel Demand

Referring to Fig. 1, the CBD area represents the work locations, and the city’s residents distribute between the CBD boundary and city boundary. Every morning, commuters make a journey to their workplaces located in the CBD area. x_m represents the radius of the cordon which divides the region between the CBD and city boundaries into two rings: one is from the CBD boundary to the cordon toll location, and the other is from the cordon toll location to the city boundary. The cordon location x_m satisfies the relationship: $x_c \leq x_m \leq B$. As $x_m = x_c$, all the commuters have to pay the toll. As $x_m = B$, no commuters need to pay the toll.

Travel demand is usually sensitive to the level of traffic congestion, and is thus elastic. Let $q_1(x)$ be the density of travel demand (i.e. the number of travelers) at location $x \in [x_c, x_m]$, and $q_2(x)$ be the density of travel demand at location $x \in [x_m, B]$.

According to **A3**, a linear elastic travel demand density function is used to model the effects of travel demand elasticity as follows

$$q_i(x) = m(x)(1 - \alpha_p p_i(x)), \quad i = 1, 2, \tag{2}$$

where $m(x)$ can be determined by Eq. (1), $p_i(x)$ is the travel disutility from location x to the CBD, and α_p is the sensitivity parameter for the travel disutility. The subscripts “1” and “2” denote the “inside” and “outside” of the cordon, respectively.

The disutility, $p_i(x)$, of travel from location x to the CBD is defined as the sum of the generalized travel cost and the cordon toll (if any), i.e.

$$\begin{cases} p_1(x) = u(x), & \forall x \in [x_c, x_m], \\ p_2(x) = u(x) + \tau, & \forall x \in [x_m, B], \end{cases} \tag{3}$$

where τ is the (nonnegative) cordon toll (i.e. $\tau \geq 0$), and $u(x)$ is the generalized travel cost, which consists of the travel time cost and monetary travel cost, expressed as

$$u(x) = \beta_t T(x) + c_v x + c_f, \quad \forall x \in [x_c, B], \tag{4}$$

where $T(x)$ is the travel time from location x to the CBD, and β_t is travelers’ value of time. The sum of the second and third terms on the right-hand side of Eq. (4) represents the monetary travel cost from location x to the CBD, which is assumed to be a linear function of the distance traveled (Wang et al. 2004; Liu et al. 2009; Li et al. 2012b). c_f and c_v are, respectively, the fixed component (e.g., parking charge in the CBD per work trip) and variable component (e.g., auto fuel cost per unit of distance) of the monetary travel cost.

Travel time $T(x)$ depends on the level of traffic congestion at location x , and is defined as

$$T(x) = \begin{cases} \int_{x_c}^x t(Q_1(w))dw, & \forall x \in [x_c, x_m], \\ \int_{x_c}^{x_m} t(Q_1(w))dw + \int_{x_m}^x t(Q_2(w))dw, & \forall x \in [x_c, B], \end{cases} \tag{5}$$

where $Q_1(x)$ is the total traffic volume (i.e. total number of vehicles) at location $x \in [x_c, x_m]$ and $Q_2(x)$ is the total traffic volume at location $x \in [x_m, B]$. $t(Q_i(x))$ is the travel time per unit of distance around location x . It is assumed to be a strictly increasing function of total traffic volume $Q_i(x)$ at location x . Similar to Mun et al. (2003) and Chu and Tsai (2008), $t(Q_i(x))$ can be estimated by a linear function as follows

$$\begin{aligned} t(Q_i(x)) &= t_0 + \mu Q_i(x) \\ &= \frac{1}{V_0} + \mu Q_i(x), \quad i = 1, 2; x \in [x_c, B], \end{aligned} \tag{6}$$

where t_0 is the free-flow travel time per unit of distance, V_0 is the free-flow travel speed per unit of distance, and μ is the marginal travel time with respect to traffic volume at location x .

In order to determine $Q_i(x)$, we first determine the total traffic volume $\widehat{Q}_i(x)$ along the circular arc with a length of θx at location x , as shown in Fig. 3. It can be expressed as

$$\begin{cases} \widehat{Q}_1(x) = \int_x^{x_m} \theta y q_1(y) dy + \int_{x_m}^B \theta y q_2(y) dy, & \forall x \in [x_c, x_m], \\ \widehat{Q}_2(x) = \int_x^B \theta y q_2(y) dy, & \forall x \in [x_m, B]. \end{cases} \tag{7}$$

The traffic volume $Q_i(x)$ at location x in Eqs. (5) and (6) is then equal to traffic volume $\widehat{Q}_i(x)$ divided by the arc length θx , i.e.

$$Q_i(x) = \frac{\widehat{Q}_i(x)}{\theta x}. \tag{8}$$

Substituting Eq. (7) into (8) yields

$$\begin{cases} Q_1(x) = \frac{1}{x} \left(\int_x^{x_m} y q_1(y) dy + \int_{x_m}^B y q_2(y) dy \right), & \forall x \in [x_c, x_m], \\ Q_2(x) = \frac{1}{x} \int_x^B y q_2(y) dy, & \forall x \in [x_m, B]. \end{cases} \tag{9}$$

Proposition 1 (i) Traffic volume function $Q_i(x)$ and travel time function $t(Q_i(x))$ are continuous at cordon location x_m .

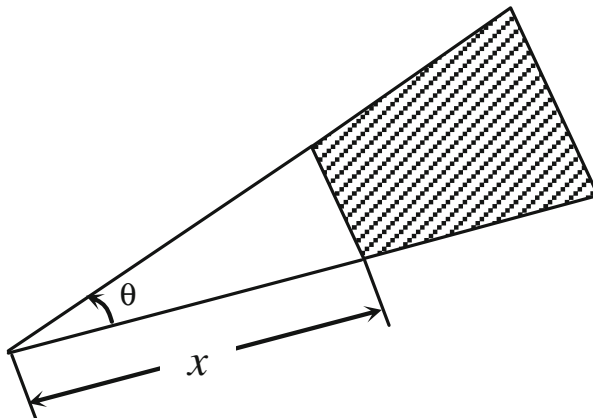


Fig. 3 A typical fan-shaped element

(ii) As $\tau > 0$, travel demand density function $q_i(x)$ is discontinuous at x_m .

Proof From Eq. (9), we have

$$Q_1(x_m) = Q_2(x_m) = \frac{1}{x_m} \int_{x_m}^B yq_2(y)dy. \tag{10}$$

This means that $Q_i(x)$ and thus $t(Q_i(x))$ in terms of Eq. (6) is continuous at cordon location x_m .

From Eq. (3), one obtains

$$p_2(x_m) = p_1(x_m) + \tau. \tag{11}$$

This implies that as $\tau > 0$, the travel disutility function $p_i(x)$ is discontinuous at x_m . From Eqs. (2) and (11), we have

$$q_2(x_m) = q_1(x_m) - \tau \alpha_p m(x_m). \tag{12}$$

Apparently, as $\tau > 0$, the travel demand density function $q_i(x)$ is discontinuous at x_m . This completes the proof of this proposition.

2.3 Air Pollution Cost

Vehicle air pollution cost refers to motorized vehicle air pollutant damages, including human health, and ecological and esthetic degradation (see VTPI 2011). It can also be termed as damage cost. We denote $\widehat{C}(x)$ as the average air pollution cost of vehicles traveling from location x to the CBD. It can be given by

$$\widehat{C}(x) = \gamma E(x), \quad \forall x \in [x_c, B], \tag{13}$$

where γ is the air pollution cost per unit of vehicular pollutant emissions, measured in dollars per gram, and $E(x)$ is the average amount of traffic emissions of vehicles traveling from x to the CBD, measured in grams per vehicle, which can be calculated by

$$E(x) = \int_{x_c}^x e(x)dx, \quad \forall x \in [x_c, B], \tag{14}$$

where $e(x)$ is the average amount of traffic emissions of vehicles per unit of distance at location x , measured in grams per vehicle per kilometer. According to Penic and Upchurch (1992), $e(x)$ can be estimated by

$$\begin{aligned} e(x) &= \frac{\rho_1}{V(x)} \exp(\rho_2 V(x)) \\ &= \rho_1 t(\cdot) \exp\left(\frac{\rho_2}{t(\cdot)}\right), \quad \forall x \in [x_c, B], \end{aligned} \tag{15}$$

where $V(x)$ is the average travel speed of vehicles per unit of distance around location x , and travel time $t(\cdot)$ can be given by Eq. (6). ρ_1 and ρ_2 are constants, and their values are

$\rho_1=11.063927$ and $\rho_2=0.008493$, respectively. When converted to feet and feet per second, their values are the same as those given by Penic and Upchurch (1992). As $t(\cdot)$ is a continuous function over the region $[x_c, B]$, as shown in Proposition 1, $e(x)$ is also continuous over $[x_c, B]$.

Proposition 2 *If the free-flow travel speed of vehicles V_0 satisfies $V_0 \leq 1/\rho_2$, then the average amount of traffic emissions per unit distance, $e(x)$, at location x increases with the increase in the traffic volume $Q_i(x)$ but decreases with the increase in the average vehicle travel speed $V(x)$.*

Proof From Eq. (15), the first-order partial derivative of $e(x)$ with respect to $V(x)$ is

$$\frac{\partial e(x)}{\partial V(x)} = \frac{\partial e(x)}{\partial t(x)} \frac{\partial t(x)}{\partial V(x)} = \frac{\partial e(x)}{\partial t(x)} \left(-\frac{1}{V^2(x)} \right) = \rho_1 \left(-\frac{1}{V^2(x)} \right) \left(1 - \frac{\rho_2}{t(x)} \right) \exp\left(\frac{\rho_2}{t(x)}\right). \tag{16}$$

The first-order partial derivative of $e(x)$ with respect to $Q_i(x)$ in terms of Eqs. (6) and (15) is

$$\frac{\partial e(x)}{\partial Q_i(x)} = \frac{\partial e(x)}{\partial t(x)} \frac{\partial t(x)}{\partial Q_i(x)} = \frac{\partial e(x)}{\partial t(x)} \mu = \mu \rho_1 \left(1 - \frac{\rho_2}{t(x)} \right) \exp\left(\frac{\rho_2}{t(x)}\right). \tag{17}$$

According to Eq. (6), $t(x) \geq 1/V_0$ always holds. Thus, as $V_0 \leq 1/\rho_2$, we have

$$1 - \frac{\rho_2}{t(x)} \geq 1 - \rho_2 V_0 \geq 0. \tag{18}$$

Therefore, $\frac{\partial e(x)}{\partial V(x)} \leq 0$ and $\frac{\partial e(x)}{\partial Q_i(x)} \geq 0$ hold in terms of Eqs. (16) and (17), respectively. This implies that $e(x)$ increases with the increase of $Q_i(x)$ but decreases with the increase of $V(x)$. This completes the proof of the proposition.

This proposition reveals that the amount of traffic emissions varies with the traffic congestion level and the average vehicle travel speed. Specifically, a heavy traffic congestion level and a low vehicle travel speed may lead to the large amount of traffic emissions, and vice versa.

3 Model Formulation

3.1 Social Welfare

The social welfare is defined as the sum of consumer surplus and producer surplus. The consumer surplus is the difference between what consumers would be willing to pay for travel and what they actually pay. The producer surplus is the difference of total cordon toll revenue minus total air pollution cost. The social welfare maximization problem can be expressed as

$$\begin{aligned} & \max_{x_m, \tau} \bar{\Phi}(x_m, \tau) \\ & = \int_{x_c}^{x_m} 2\pi x \left(\int_0^{q_1(x)} p_1(q) dq - p_1(x) q_1(x) \right) dx + \int_{x_m}^B 2\pi x \left(\int_0^{q_2(x)} p_2(q) dq - p_2(x) q_2(x) \right) dx \\ & \quad + \int_{x_m}^B 2\pi x q_2(x) \tau dx - \int_{x_c}^{x_m} 2\pi x q_1(x) \widehat{C}(x) dx - \int_{x_m}^B 2\pi x q_2(x) \widehat{C}(x) dx, \end{aligned} \tag{19}$$

where the first two terms on the right-hand side of Eq. (19) are the surpluses of consumers (or trip-makers) inside and outside the cordon, respectively. The third term is the total cordon toll revenue from the trip-makers who are bound for the CBD from the locations outside the cordon, and the final two terms are the air pollution costs inside and outside the cordon, respectively.

Substituting Eq. (3) into (19), we have

$$\begin{aligned} & \max_{x_m, \tau} \Phi(x_m, \tau) \\ &= \int_{x_c}^{x_m} 2\pi x \left(\int_0^{q_1(x)} p_1(q) dq - q_1(x)u(x) \right) dx + \int_{x_m}^B 2\pi x \left(\int_0^{q_2(x)} p_2(q) dq - q_2(x)u(x) \right) dx \\ & \quad - \int_{x_c}^{x_m} 2\pi x q_1(x) \widehat{C}(x) dx - \int_{x_m}^B 2\pi x q_2(x) \widehat{C}(x) dx. \end{aligned} \tag{20}$$

Equation (20) indicates that the cordon tolls do not appear in the social welfare explicitly. This is because the payment of the cordon tolls is only a transfer of money from the travelers to the authority inside the system instead of a deadweight loss.

According to Eq. (2), the integral $\int_0^{q_i(x)} p_i(q) dq$ can be given by

$$\int_0^{q_i(x)} p_i(q) dq = \int_0^{q_i(x)} \frac{1}{\alpha_p} \left(1 - \frac{q}{m(x)} \right) dq = \frac{1}{\alpha_p} \left(q_i(x) - \frac{q_i^2(x)}{2m(x)} \right) = \frac{m(x)}{2\alpha_p} \left(1 - \alpha_p^2 p_i^2(x) \right). \tag{21}$$

Substituting Eq. (21) into (20) yields

$$\begin{aligned} \max_{x_m, \tau} \Phi(x_m, \tau) &= \int_{x_c}^{x_m} 2\pi x \left(\frac{m(x)}{2\alpha_p} \left(1 - \alpha_p^2 p_1^2(x) \right) - q_1(x)u(x) \right) dx \\ & \quad + \int_{x_m}^B 2\pi x \left(\frac{m(x)}{2\alpha_p} \left(1 - \alpha_p^2 p_2^2(x) \right) - q_2(x)u(x) \right) dx \\ & \quad - \int_{x_c}^{x_m} 2\pi x q_1(x) \widehat{C}(x) dx - \int_{x_m}^B 2\pi x q_2(x) \widehat{C}(x) dx. \end{aligned} \tag{22}$$

In order to obtain the optimal solutions of the cordon toll location and toll level, we set the first-order partial derivatives of Eq. (22) with regard to the toll location and toll level equal to zero, which yields

$$\begin{aligned} \frac{\partial \Phi(\cdot)}{\partial x_m} &= \pi \alpha_p \tau x_m m(x_m) \left(\tau - 2\widehat{C}(x_m) \right) \\ & \quad + \int_{x_c}^{x_m} 2\pi x \frac{q_1(x)}{\alpha_p m(x)} \frac{\partial q_1(x)}{\partial x_m} dx - \int_{x_c}^{x_m} 2\pi x \left(\widehat{C}(x) + q_1(x) \frac{\partial \widehat{C}(x)}{\partial q_1(x)} \right) \frac{\partial q_1(x)}{\partial x_m} dx \\ & \quad + \int_{x_m}^B 2\pi x \left(\frac{q_2(x)}{\alpha_p m(x)} + \tau \right) \frac{\partial q_2(x)}{\partial x_m} dx - \int_{x_m}^B 2\pi x \left(\widehat{C}(x) + q_2(x) \frac{\partial \widehat{C}(x)}{\partial q_2(x)} \right) \frac{\partial q_2(x)}{\partial x_m} dx = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{\partial \Phi(\cdot)}{\partial \tau} = & \int_{x_m}^B 2\pi x q_2(x) dx \\ & + \int_{x_c}^{x_m} 2\pi x \frac{q_1(x)}{\alpha_p m(x)} \frac{\partial q_1(x)}{\partial \tau} dx - \int_{x_c}^{x_m} 2\pi x \left(\widehat{C}(x) + q_1(x) \frac{\partial \widehat{C}(x)}{\partial q_1(x)} \right) \frac{\partial q_1(x)}{\partial \tau} dx \\ & + \int_{x_m}^B 2\pi x \left(\frac{q_2(x)}{\alpha_p m(x)} + \tau \right) \frac{\partial q_2(x)}{\partial \tau} dx - \int_{x_m}^B 2\pi x \left(\widehat{C}(x) + q_2(x) \frac{\partial \widehat{C}(x)}{\partial q_2(x)} \right) \frac{\partial q_2(x)}{\partial \tau} dx = 0. \end{aligned} \tag{24}$$

The economic implications of Eqs. (23) and (24) can be explained as follows. The first term on the right-hand side of Eq. (23) represents the direct marginal effect on the social welfare due to the outward move of the cordon toll location. The second and third terms represent the indirect marginal effects on the consumer surplus and air pollution cost inside the cordon through the change in the demand inside the cordon, respectively. The fourth and fifth terms represent the indirect marginal effects on the consumer surplus and air pollution cost outside the cordon through the change in the demand outside the cordon, respectively. The first term on the right-hand side of Eq. (24) represents the direct marginal effect on the consumer surplus outside the cordon of a marginal increase in the cordon toll level. The other four terms represent the indirect marginal effects of increasing the cordon toll level by one unit on the consumer surplus and air pollution cost through the changes in the demands inside and outside the cordon, respectively. Equations (23) and (24) show that when the optimal solutions of the cordon toll location and toll level are achieved, the sum of the direct and indirect effects on the social welfare is zero.

3.2 Properties of Model Under Three Special Cases

In the following, three special cases are analyzed based on whether the effects of the traffic congestion and/or traffic emissions are considered explicitly.

3.2.1 Case 1: Traffic Congestion Effects can be Ignored

For a low-density city, the effects of the traffic congestion may be ignored due to a low travel demand density. The travel time around a location is thus a constant independent of the traffic flows on that location, that is, the parameter, μ , in Eq. (6) equals zero. Therefore, according to Eq. (5), the travel time from location x to the CBD can be given by

$$T(x) = (x - x_c)t_0, \quad \forall x \in [x_c, B]. \tag{25}$$

Substituting Eq. (25) into Eq. (4), and applying to the system of Eqs. (23) and (24), one obtains

$$\begin{cases} \tau = 2\widehat{C}(x_m), \\ \omega_3 x_m^3 + \omega_2 x_m^2 + \omega_1 x_m + \omega_0 = 0, \quad \forall x_m \in [x_c, B], \end{cases} \tag{26}$$

where $\omega_3, \omega_2, \omega_1$, and ω_0 are, respectively, given by

$$\begin{cases} \omega_3 = -5a, \\ \omega_2 = 8b + 4ax_c - 5aB, \\ \omega_1 = -6bx_c + 8bB + 4aBx_c - 5aB^2, \\ \omega_0 = 3aB^3 + 4ax_cB^2 - 4bB^2 - 6bx_cB. \end{cases} \tag{27}$$

The detailed derivation of Eq. (26) is provided in [Appendix A](#).

In view of the above, we have the following proposition:

Proposition 3 *When the traffic congestion effects can be ignored (i.e. a low population density), the optimal cordon toll location solution x_m^* depends on the population density parameters a and b , CBD boundary x_c , and the city boundary B , and the optimal toll level is twice the associated air pollution cost.*

In particular, when the urban population is uniformly distributed, parameter a in the population density Eq. (1) equals zero. The second equation in (26) can be further written as

$$4x_m^2 + (4B - 3x_c)x_m - (2B^2 + 3x_cB) = 0. \tag{28}$$

The optimal cordon toll location and toll level solutions can then be given by

$$\begin{cases} x_m = \frac{1.5x_c - 2B + \sqrt{12B^2 + 6x_cB + 2.25x_c^2}}{4}, \\ \tau = 2\hat{C}(x_m). \end{cases} \tag{29}$$

Corollary 1 *When the traffic congestion effects can be ignored (i.e. a low population density) and the urban population density is uniformly distributed, the optimal cordon toll location depends only on the CBD boundary x_c and the city boundary B .*

3.2.2 Case 2: Traffic Emission Effects can be Ignored

When the traffic emission effects can be ignored, γ in Eq. (13) equals zero. As a result, Eqs. (23) and (24) are, respectively, reduced to

$$\pi\alpha_p\tau^2x_m m(x_m) + \int_{x_c}^{x_m} 2\pi x \frac{q_1(x)}{\alpha_p m(x)} \frac{\partial q_1(x)}{\partial x_m} dx + \int_{x_m}^B 2\pi x \left(\frac{q_2(x)}{\alpha_p m(x)} + \tau \right) \frac{\partial q_2(x)}{\partial x_m} dx = 0, \tag{30}$$

$$\int_{x_m}^B 2\pi x q_2(x) dx + \int_{x_c}^{x_m} 2\pi x \frac{q_1(x)}{\alpha_p m(x)} \frac{\partial q_1(x)}{\partial \tau} dx + \int_{x_m}^B 2\pi x \left(\frac{q_2(x)}{\alpha_p m(x)} + \tau \right) \frac{\partial q_2(x)}{\partial \tau} dx = 0. \tag{31}$$

The economic implications of Eqs. (30) and (31) are as follows. The first term on the left-hand side of Eq. (30) represents the direct marginal effect on the social welfare when the toll location moves each unit of distance outward. The second

and third terms represent the indirect marginal effects on the surpluses of consumers inside and outside the cordon via the changes of the demands inside and outside the cordon, respectively. The first term on the left-hand side of Eq. (31) represents the direct marginal effect on the consumer surplus outside the cordon due to a marginal increase in the cordon toll level. The second and third terms represent the indirect marginal effects on the consumer surplus of one-unit increase in the cordon toll level via, respectively, the changes of the demands inside and outside the cordon.

3.2.3 Case 3: Both Traffic Emission and Congestion Effects can be Ignored

When both traffic emission and congestion effects can be ignored simultaneously, μ in Eq. (6) and γ in Eq. (13) are zero. From Eq. (23), we have

$$\frac{\partial \Phi(\cdot)}{\partial x_m} = \pi \alpha_p \tau^2 x_m m(x_m) \geq 0. \tag{32}$$

Since Eq. (32) is always nonnegative, $\Phi(\cdot)$ always increases with x_m in the region $[x_c, B]$, and thus the maximum value of $\Phi(\cdot)$ is achieved at the bound $x_m=B$ (i.e. the city boundary). However, when $x_m=B$, no toll is required for all the trip-makers in the city. We thus have the following proposition.

Proposition 4 *When both traffic emission and traffic congestion effects can be ignored, no toll is required.*

4 Model Extension

As previously stated, the redistribution of the toll revenue can affect the implementation of the cordon toll pricing schemes. The redistribution schemes include capacity improvement, highway maintenance, direct cash payments to travelers, investment in transit systems, and reduction in general taxes (see, e.g., Goodwin 1989; Small 1992; Hau 1992; King et al. 2007). However, the cordon toll pricing model that is proposed in the previous section does not explicitly consider the impacts of the toll revenue redistribution schemes on reduction in vehicle emissions. In this section, we assume that the government redistributes a proportion of the total cordon toll revenue to the public to subsidize the retrofit of old vehicles (e.g. use of fuel-efficient vehicles) such that the average emission factor of vehicles is reduced.

Let D be the total revenue from the cordon toll pricing, and D_r be the total funds / subsidies used to retrofit the old vehicles, which are, respectively, expressed as

$$D = \int_{x_m}^B 2\pi x q_2(x) \tau dx, \text{ and} \tag{33}$$

$$D_r = \kappa D, \tag{34}$$

where the subscript “ r ” is associated with the retrofit of old vehicles, and κ is the proportion of the subsidy to the total toll revenue used to retrofit the old vehicles.

The use of fuel-efficient vehicles can reduce the average emission factor of vehicles. In this paper, a negative exponential form of function is used to model the

effects of subsidizing the retrofit of old vehicles on the average emissions of vehicles. Let $\widehat{C}_r(x)$ denote the air pollution cost of vehicles traveling from location x to the CBD after retrofitting the old vehicles, which is expressed as

$$\widehat{C}_r(x) = \exp(-\lambda D_r)\widehat{C}(x), \tag{35}$$

where λ is a positive parameter that reflects the sensitivity of the vehicle air pollution cost to the total subsidies devoted to the retrofit of old vehicles. $\widehat{C}(x)$ can be given by Eq. (13). The effects of the subsidy policy on the air pollution cost will be illustrated in the next section.

Let $\Phi_r(\kappa, x_m, \tau)$ be the total social welfare of the system after subsidizing the retrofit of old vehicles. The social welfare maximization problem with considering the effects of the subsidy policy can then be formulated as

$$\begin{aligned} & \max_{\kappa, x_m, \tau} \Phi_r(\kappa, x_m, \tau) \\ & = \int_{x_c}^{x_m} 2\pi x \left(\int_0^{q_1(x)} p_1(q) dq - q_1(x) p_1(x) \right) dx + \int_{x_m}^B 2\pi x \left(\int_0^{q_2(x)} p_2(q) dq - q_2(x) p_2(x) \right) dx \\ & \quad + (1-\kappa)D - \left(\int_{x_c}^{x_m} 2\pi x q_1(x) \widehat{C}_r(x) dx + \int_{x_m}^B 2\pi x q_2(x) \widehat{C}_r(x) dx \right), \end{aligned} \tag{36}$$

where the first two terms on the right-hand side of Eq. (36) are the consumer surpluses inside and outside the cordon, respectively. The third term is the net toll revenue after deduction of the money used to retrofit the old vehicles, and the final two terms represent the air pollution costs inside and outside the cordon after retrofitting the old vehicles, respectively. $\Phi_r(\kappa, x_m, \tau)$ is a function with regard to κ , x_m , and τ . As $\kappa=0$, expression (36) is reduced to expression (19), implying that the maximization problem (19) is a special case of the maximization problem (36).

Let M represent the total air pollution costs before retrofitting the old vehicles, i.e.

$$M = \int_{x_c}^{x_m} 2\pi x q_1(x) \widehat{C}(x) dx + \int_{x_m}^B 2\pi x q_2(x) \widehat{C}(x) dx. \tag{37}$$

The social welfare function $\Phi_r(\kappa, x_m, \tau)$ has then the following properties.

Proposition 5 *Given cordon toll location and charge level, the subsidy scheme and the total social welfare have the following relationship:*

- (i) As $\lambda \leq \frac{1}{M}$, any subsidy scheme for retrofitting the old vehicles would lead to a decrease in the total social welfare.
- (ii) As $\lambda > \frac{1}{M}$, the total social welfare reaches the maximum at the subsidy level of $\kappa = \frac{1}{\lambda D} \ln(\lambda M)$.

Proof According to Eq. (36), the first-order derivative of $\Phi_r(\cdot)$ with respect to κ is

$$\frac{\partial \Phi_r(\cdot)}{\partial \kappa} = -D + D\lambda M \exp(-\kappa \lambda D). \tag{38}$$

- (i) As $\lambda \leq \frac{1}{M}$, then $\frac{\partial \Phi_r(\cdot)}{\partial \kappa} < 0$ always holds for any $\kappa > 0$. This means that any subsidy scheme for retrofitting the old vehicles would cause a decrease in the total social welfare.
- (ii) As $\lambda > \frac{1}{M}$, if $\kappa \leq \frac{1}{\lambda D} \ln(\lambda M)$, then $\frac{\partial \Phi_r(\cdot)}{\partial \kappa} < 0$ always holds. That is, when $0 \leq \kappa \leq \frac{1}{\lambda D} \ln(\lambda M)$, $\Phi_r(\cdot)$ is an increasing function of κ . If $\kappa \geq \frac{1}{\lambda D} \ln(\lambda M)$,

then $\frac{\partial \Phi_r(\cdot)}{\partial \kappa} \leq 0$, and $\Phi_r(\cdot)$ is thus a decreasing function of κ for $\frac{1}{\lambda D} \ln(\lambda M) \leq \kappa \leq 1$. This implies that as κ increases, $\Phi_r(\cdot)$ first increases and then decreases, and reaches the maximum at $\kappa = \frac{1}{\lambda D} \ln(\lambda M)$. This completes the proof of the proposition.

Proposition 5 shows that once the cordon toll location x_m and the charge level τ are given, one can then determine the optimal proportion, κ , of the subsidy to the total toll revenue used to retrofit the old vehicles. Conversely, given the value of κ , one can then determine the optimal values of x_m and τ . The interdependence between the subsidy and cordon pricing schemes leads to a stationary solution with regard to κ , x_m , and τ , which can be solved by the following iterative process:

$$(x_m^{(0)}, \tau^{(0)}) \rightarrow \kappa^{(0)} \rightarrow (x_m^{(1)}, \tau^{(1)}) \rightarrow \kappa^{(1)} \dots \rightarrow (\kappa^*, x_m^*, \tau^*). \tag{39}$$

In the next section, a numerical example will be used to illustrate the application of the iterative method in obtaining the simultaneous optimal solutions κ^* , x_m^* , and τ^* .

5 Numerical Study

To facilitate the presentation of the essential ideas and contributions of this paper, in this section, we employ an example to illustrate the applications of the proposed model. We first compare the optimal solutions with/without considering the traffic congestion and/or environmental effects. We then investigate the effects of the cordon toll pricing schemes on the performance of the urban system, and of the urban population distribution on the optimal cordon toll location and toll level and the system performance. Finally, the effects of subsidizing the retrofit of old vehicles on reduction in average vehicle emissions are explored, and the optimal subsidy scheme is also determined. The baseline values of the model parameters are shown in Table 1.

5.1 Effects of Traffic Congestion and Emissions on Cordon Toll Pricing Solution

Table 2 shows the optimal solutions for different scenarios with/without considering traffic congestion and/or traffic emission effects. It can be observed in this table that among all scenarios, the optimal toll level with simultaneous consideration of the traffic congestion and emission effects is the highest (\$20.96). When the traffic emission effects can be ignored, the optimal toll level decreases by \$1.53 (from \$20.96 to \$19.43). In the meantime, the optimal cordon toll location slightly moves away from the CBD by about 0.1 km (i.e. from 3.81 km to 3.90 km). However, when the traffic congestion effects can be ignored, the optimal toll level dramatically decreases to \$0.41 from \$20.96, and the optimal cordon toll location moves outward by 4.25 km (from 3.81 km to 8.06 km). These observations show that traffic congestion externality could have a larger effect on the optimal cordon toll location and charge level compared to traffic environmental externality. This result is consistent with that found in Shepherd (2008).

Table 1 Notation

Symbol	Definition	Baseline value
a	density gradient in the population density function	100
b	population density at the CBD boundary (persons/km ²)	–
B	radius of the city (km)	20
c_f	fixed component of monetary travel cost (\$)	10
c_v	variable component of monetary travel cost (\$/km)	1.0
$\widehat{C}(x)$	air pollution cost (\$/veh)	–
$\widehat{C}_r(x)$	air pollution cost after subsidizing retrofit of old vehicles (\$/veh)	–
D	total revenue from the cordon toll pricing (\$/h)	–
D_r	total funds used to retrofit old vehicles (\$/h)	–
$e(x)$	average amount of vehicle emissions per unit of distance (g/veh-km)	–
$E(x)$	total amount of traffic emissions from x to CBD (g/veh)	–
κ	proportion of subsidy to total toll revenue used to retrofit old vehicles	–
$m(x)$	population density function at x (persons/km ²)	–
N	total number of population in the planning area (persons)	2.0×10^6
$p_f(x)$	travel disutility from x to the CBD (\$)	–
$q_f(x)$	travel demand density at location x (vehicles/km ²)	–
$Q_i(x)$	traffic volume at location x (vehicles)	–
τ_0	free-flow travel time per unit of distance (h/km)	0.02
$t(Q_i(x))$	travel time per unit of distance (h/km)	–
$T(x)$	total travel time from x to the CBD (h)	–
$u(x)$	generalized travel cost (\$)	–
v_0	free-flow travel speed per unit of distance (km/h)	50
$v(x)$	average travel speed of vehicles per unit of distance around location x (km/h)	–
x_c	radius of the CBD (km)	2.0
x_m	location of cordon (km)	–
α_p	sensitivity parameter for travel disutility	0.0125
β_t	value of time (\$/h)	20
γ	air pollution cost per unit of vehicular pollutant emissions (\$/g)	0.1
μ	marginal travel time with respect to traffic volume (h/veh)	0.00001
λ	a positive parameter that reflects the sensitivity of vehicle air pollution cost to the subsidy in retrofitting old vehicles	–
τ	toll level (\$)	–
$\Phi(\cdot)$	social welfare (\$/h)	–
$\Phi_r(\cdot)$	social welfare after subsidizing retrofit of old vehicles (\$/h)	–

5.2 Effects of Cordon Toll Pricing Scheme on Urban System

Table 3 shows the performances of the urban system with and without cordon toll pricing scheme. It can be seen in the table that the total air pollution cost and total resultant travel demand, respectively, decrease by $\$0.7422 \times 10^6$ per hour (from

Table 2 Optimal cordon toll location and charge level solutions under different scenarios

		Traffic emission effects	
		Yes	No
Traffic congestion effects	Yes	$\{x_m^* = 3.81 \text{ (km)} \tau^* = 20.96 \text{ (\$)}$	$\{x_m^* = 3.90 \text{ (km)} \tau^* = 19.43 \text{ (\$)}$
	No	$\{x_m^* = 8.06 \text{ (km)} \tau^* = 0.41 \text{ (\$)}$	Zero toll solution (see Proposition 3)

\$1.2746×10⁶ to \$0.5324×10⁶ per hour) and 2.4346×10⁵ vehicles per hour (from 7.2126×10⁵ to 4.7780×10⁵ vehicles per hour) after implementing the cordon toll pricing scheme. This means that the cordon toll pricing scheme can reduce the traffic emissions and suppress the travel demand, and thus alleviates the traffic congestion in the urban area. In addition, the implementation of the cordon toll pricing scheme can increase the social welfare of the urban system by \$0.4159×10⁷ per hour (from \$1.0042×10⁷ to \$1.4201×10⁷ per hour), implying that the urban efficiency is improved in terms of the social welfare.

Figure 4 plots the travel demand densities before and after introducing the cordon toll pricing scheme. It can be seen in Fig. 4 that the implementation of the cordon toll pricing scheme leads to a discontinuous traffic flow with the cordon location as the discontinuous or interruption point, which illustrates the result presented in Proposition 1. It can also be seen that compared to the no-toll case, the implementation of the cordon toll pricing scheme causes a higher travel demand density inside the cordon but a lower one outside the cordon. This is because the cordon tolls increase the travel cost of commuters originating from the outside of the cordon. Accordingly, the travel demand outside the cordon decreases, leading to a decrease in the level of the traffic congestion inside the cordon. As a result, the travel demand inside the cordon increases.

Figure 5 displays the traffic emission densities before and after the introduction of the cordon toll pricing scheme. The traffic emission density of a location is defined as the product of the average amount of vehicle emissions and the traffic volume at that location, i.e. $Q_i(x)e(x)$. It can be seen in Fig. 5 that the traffic emission densities under both scenarios are continuous. This is because traffic volume function $Q_i(x)$ and travel time function $t(\cdot)$ are continuous over the region $[x_c, B]$ (see Proposition 1). Hence, traffic emission function $e(x)$ in terms of Eq. (15) and thus the traffic emission density $Q_i(x)e(x)$ is also continuous. In addition, for a given location the traffic emission density with the toll scheme is lower than that without the toll scheme. This is because the levy of the cordon tolls can restrain the travel demand and thus reduce

Table 3 Performances of urban system with and without toll scheme

	With toll scheme	Without toll scheme
Total air pollution cost (\$/h)	0.5324×10 ⁶	1.2746×10 ⁶
Total resultant travel demand (vehicles/h)	4.7780×10 ⁵	7.2126×10 ⁵
Social welfare (\$/h)	1.4201×10 ⁷	1.0042×10 ⁷

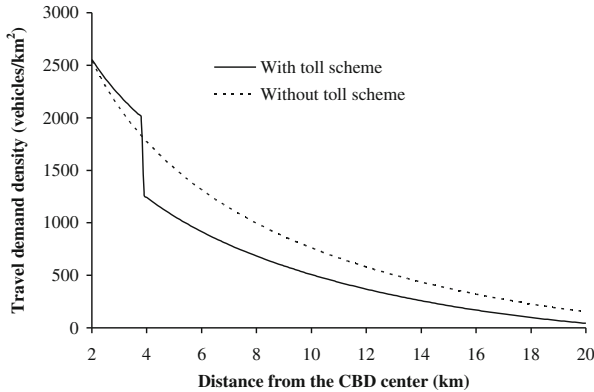


Fig. 4 Profiles of travel demand densities with and without cordon toll scheme

the traffic congestion level and the travel time, leading to a decrease in the vehicle emissions.

5.3 Effects of Urban Form on Optimal Cordon Toll Location and Toll Level

To explore the effects of the urban structure on the optimal cordon toll location and toll level, Fig. 6 shows the profiles of the urban population distributions for five different population density gradients (i.e. $a = 0, 50, 100, 150,$ and 200) for given total number of population of 2,000,000 and city length of 20 km. It can be seen in Fig. 6 that a larger value of density gradient a means a more compact city, whereas a smaller value of density gradient a means a higher level of dispersion in the population distribution. Particularly, when density gradient a equals zero, the urban population is uniformly distributed across the city.

Table 4 indicates that as density gradient a increases from 0 to 200, the distance of the optimal cordon toll location from the CBD decreases from 4.20 km to 3.40 km, whereas the optimal toll level increases from \$20.33 to \$21.48. This is because as a increases, the urban form becomes more compact, as shown in Fig. 6. As a result, more residents live nearby the CBD, leading to more potential travel demand. In

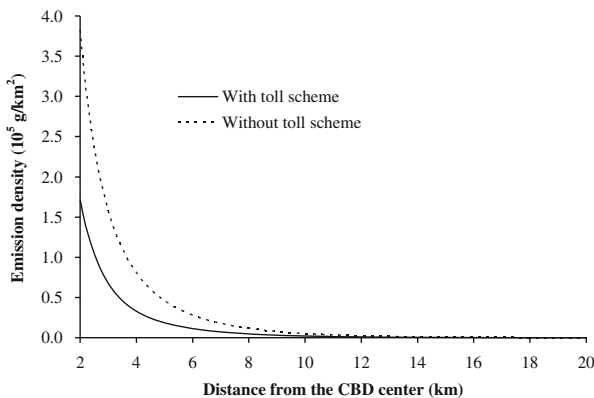


Fig. 5 Profiles of traffic emission densities with and without cordon toll scheme

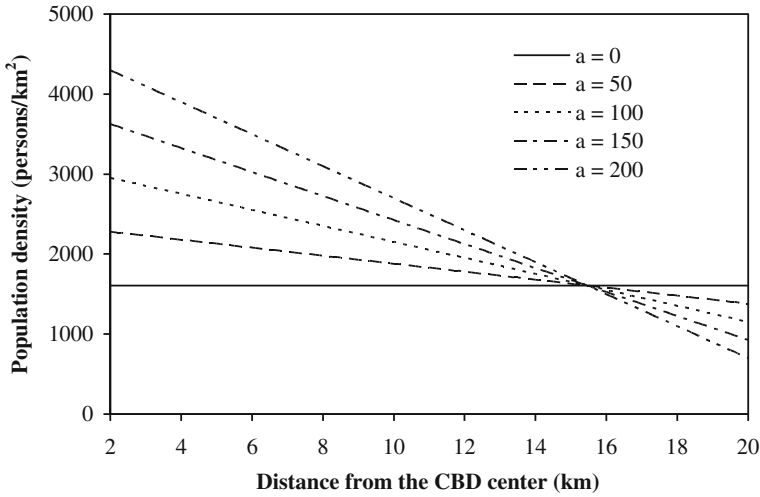


Fig. 6 Urban population distributions with different density gradients

order to mitigate the traffic congestion, the cordon toll location moves toward the CBD and the associated toll level increases. In addition, as density gradient a increases, the total travel demand, total consumer surplus and the social welfare increase. However, the total travel disutility and total air pollution cost also increase. This implies that a more compact city (i.e. a larger value of a) is more efficient in terms of the social welfare but less efficient in terms of the total travel disutility and total traffic emissions compared to a decentralized city.

5.4 Effects of Subsidizing the Retrofit of Old Vehicles

We now examine the effects of subsidizing the retrofit of old vehicles on the urban system. Figure 7 shows the change of the social welfare with the percentage, κ , of the subsidy in the total toll revenue for different values of sensitivity parameter λ when the cordon toll location x_m and the charge level τ are, respectively, fixed as 3.81 km and \$20.96 (i.e. the optimal solutions without the subsidy policy, see Table 2). It can

Table 4 Optimal solutions with different population density gradients

Optimal solution	Population density gradient a				
	0	50	100	150	200
Cordon toll location (km)	4.20	4.10	3.81	3.79	3.40
Toll level (\$)	20.33	20.67	20.96	21.28	21.48
Total travel demand (10^5 vehicles/h)	4.09	4.45	4.78	5.09	5.36
Total air pollution cost (10^5 \$/h)	4.72	5.02	5.32	5.63	5.93
Total travel disutility (10^7 \$)	2.27	2.42	2.56	2.70	2.86
Total consumer surplus (10^6 \$/h)	5.00	5.73	6.30	6.84	7.13
Social welfare (10^7 \$/h)	1.18	1.30	1.42	1.53	1.63

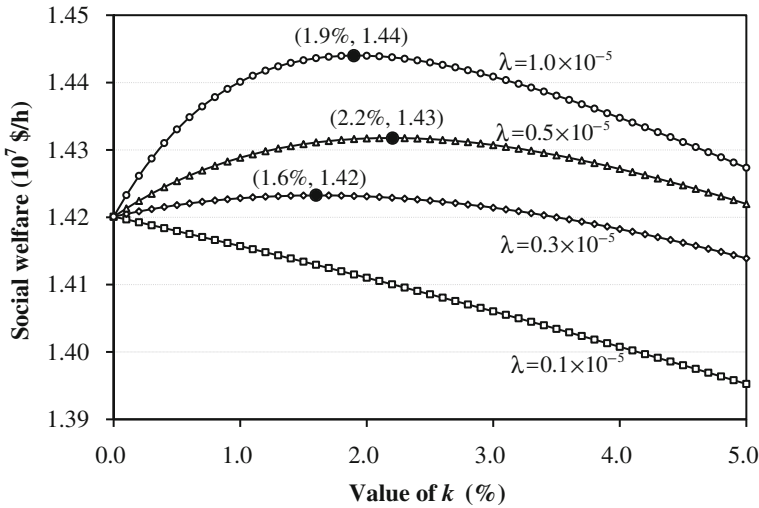


Fig. 7 Change of social welfare with subsidy proportion k for different values of λ ($x_m=3.81$ km and $\tau=\$20.96$)

be observed in Fig. 7 that when λ equals 0.1×10^{-5} , the social welfare is always decreasing with the value of κ , implying that the subsidy policy will not increase the social welfare. This is because the decrease in the total vehicle air pollution cost can not completely cover the total investment cost in retrofitting the old vehicles. However, when λ is increased to 0.3×10^{-5} , as the value of κ increases, the social welfare first increases and then decreases, and reaches the maximum (i.e. $\$1.42 \times 10^7$ per hour) at the level of $\kappa=1.6\%$. This implies that in order to achieve the maximum social welfare, the government should devote 1.6% of the total toll revenue to upgrade the old vehicles to reduce the average vehicle emissions. When λ equals 0.5×10^{-5} and 1.0×10^{-5} , the social welfare maximization solutions, respectively, occur at the levels of $\kappa=2.2\%$ and 1.9% , with the social welfare levels of $\$1.43 \times 10^7$ and $\$1.44 \times 10^7$ per hour. These observations further illustrate the model properties presented in Proposition 5.

Figure 8 shows the change of the total air pollution cost of the system with the proportion of the subsidy to the total toll revenue κ for different values of λ when $x_m=3.81$ km and $\tau=\$20.96$. It shows that for a given value of λ , the total air pollution cost decreases with the increase in the value of κ . On the other hand, for a given value of κ , as the value of λ increases, the total air pollution cost decreases. When κ equals 0 (i.e. no subsidy is utilized to retrofit the old vehicles), the total air pollution cost is highest ($\$5.0 \times 10^5$ per hour). These results illustrate the effects of the subsidy policy on the traffic emissions.

5.5 Simultaneous Optimization of Cordon Toll Location, Toll Level and Subsidy

In the previous subsections, both the cordon toll pricing and subsidy schemes are individually optimized (see, e.g. Tables 2 and 3, and Fig. 7). We now jointly optimize the cordon toll location, toll level and the proportion of subsidy to total toll revenue, as shown in Table 5. From Table 5 and Fig. 7, one can see that the simultaneous optimization can lead to better solution than individual optimization in terms of the social welfare. For example, when λ takes 0.3×10^{-5} , the social welfare under the simultaneous optimization (i.e.

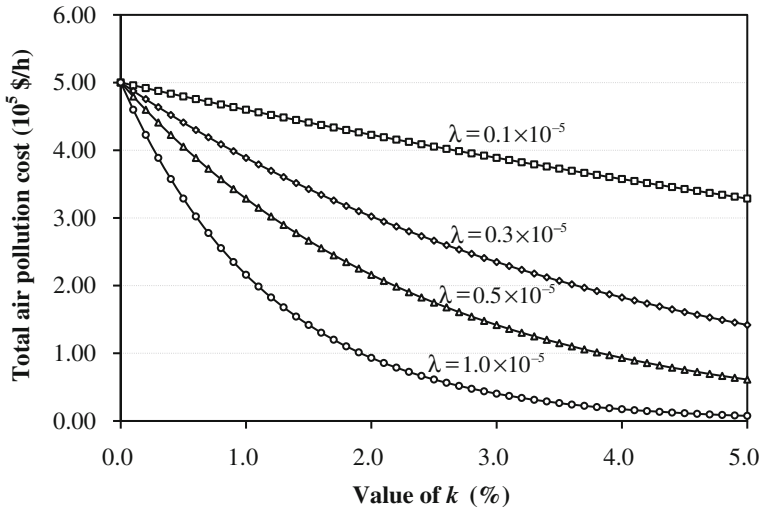


Fig. 8 Change of total air pollution cost with subsidy proportion k for different values of λ ($x_m=3.81$ km and $\tau=\$20.96$)

$\$1.4247 \times 10^7$ per hour) is $\$4.7 \times 10^4$ per hour higher than that with the proportion of subsidy κ as the only optimization variable (i.e. $\$1.42 \times 10^7$ per hour, see Fig. 7). In addition, Table 5 shows that as the value of λ increases from 0.3×10^{-5} to 1.0×10^{-5} , the optimal cordon location and toll level decrease from 3.94 km to 3.88 km and from \$20.37 to \$19.71, respectively. The optimal proportion of the subsidy κ first increases by 0.5 % and then decreases by 0.4 %. This implies that the sensitivity parameter λ has an important effect on the jointly optimal solutions.

6 Conclusion and Further Studies

In this paper, an analytical model was presented to optimize the cordon toll location and toll level in a circular monocentric city. The properties of the model solutions under various cases with/without considering the traffic congestion and/or environmental effects were explored and compared. The proposed model was also extended to reveal the effects of subsidizing the retrofit of old vehicles on reduction in average

Table 5 Optimal solutions of cordon toll location, toll level and subsidy

Optimal solution	Sensitivity parameter λ		
	0.3×10^{-5}	0.5×10^{-5}	1.0×10^{-5}
Cordon toll location (km)	3.94	3.91	3.88
Toll level (\$)	20.37	19.99	19.71
Proportion of subsidy to total toll revenue	2.0 %	2.5 %	2.1 %
Social welfare (10^7 \$/h)	1.4247	1.4346	1.4448

vehicle emissions, and to determine the optimal subsidy scheme that maximizes the social welfare of the system.

It has been shown that the cordon toll pricing destroys the continuity of the travel demand density function (see Proposition 1 and Fig. 4), and the optimal cordon toll pricing scheme is closely related to the levels of traffic congestion and air pollution of the urban system. In practice, whether the authority should explicitly consider the effects of traffic congestion or traffic emissions or both in the cordon toll pricing schemes depends on the actual situation of the city concerned (i.e. the actual levels of traffic congestion and emissions). It has also been shown that the subsidy scheme for retrofitting the old vehicles can reduce the traffic emissions and thus improve urban air quality. Once some condition is satisfied (see Proposition 5), then there is an optimal subsidy scheme in terms of the social welfare. In addition, urban form has a significant effect on the design of the cordon toll pricing scheme and the social efficiency of the urban system. A compact city is more efficient than a decentralized city in terms of the social welfare but less efficient in terms of the total travel disutility and total traffic emissions. The proposed model can serve as a useful tool for design of the cordon toll pricing schemes and for evaluation of the subsidy policy.

Although the proposed model provided some insightful findings for the cordon toll pricing studies, some important extensions should be made in future studies, stated as follows.

- (1) This paper used a linear population density function due to its convenience for analytical tractability. However, other alternative population density functions, such as exponential density function, should be considered in further research.
- (2) This paper assumed that the population distribution was exogenously given and fixed. However, in fact, implementation of cordon toll pricing schemes could induce changes in urban land use and residential distribution (Eliasson and Mattsson 2001). It would therefore be worthwhile to incorporate the effects of the toll pricing schemes on residential relocation into the proposed model.
- (3) This paper only considered the environmental sustainability. In fact, transportation can also create social and economic problems. It is thus necessary to consider social and economic sustainabilities (Szeto et al. 2013) in the cordon toll pricing problems.
- (4) All commuters were assumed to be homogenous in terms of their values of time in this paper. However, previous studies have shown that there is a big difference in the choice behavior of heterogeneous commuters in response to the road toll pricing (de Palma and Lindsey 2004; Xiao et al. 2011). Further studies should thus be carried out to incorporate the effects of the toll pricing schemes on the choice behavior of heterogeneous commuters.
- (5) This paper adopted a negative exponential form of air pollution cost function to model the effects of subsidizing the retrofit of old vehicles on the vehicle emissions. In order to make use of the proposed model for practical applications in reality, there is a need to calibrate empirically the parameters of the air pollution cost function.
- (6) Tradable credit scheme can play the same role as cordon toll pricing scheme in the regulation of congestion and environmental externalities, while stopping the excessively fast growth in the number of vehicles (Yang and Wang 2011; Nie

- 2013). Therefore, the tradable credit schemes should be considered in the cordon toll pricing problems, which is left for further study.
- (7) This paper mainly focused on automobiles, and other types of vehicles (e.g., buses) were not considered. In an urban system, public transport modes can also lead to significant congestion and environmental externalities. It is therefore meaningful to extend the proposed model to consider pollutant emissions from different modes of transportation and to investigate the effects of cordon toll pricing schemes on the multi-modal transport systems.

Acknowledgments The authors would like to thank the guest editor, Dr. W.Y. Szeto, and two anonymous referees for their helpful comments and suggestions on an earlier draft of the paper. The work described in this paper was jointly supported by grants from the National Natural Science Foundation of China (71171013, 71222107), the Research Foundation for the Author of National Excellent Doctoral Dissertation (China) (200963), the Doctoral Fund of Ministry of Education of China (20120142110044), the Fok Ying Tung Education Foundation (132015), the Research Grants Council of the Hong Kong Special Administrative Region, China (PolyU 5196/10E), the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (NRF-2011-0000446), and the Center for Modern Information Management Research at the Huazhong University of Science and Technology.

Appendix A: Derivation of Equation (26)

Substituting Eq. (25) into Eq. (4), one obtains

$$u(x) = (c_v + \beta t_0)(x - x_c) + c_f. \tag{A.1}$$

With the assumption that the traffic congestion effects can be ignored (i.e. $t(\cdot) = t_0$), from Eqs. (13)–(15), the air pollution cost can be expressed as

$$\widehat{C}(x) = \xi(x - x_c), \tag{A.2}$$

where ξ is a constant, which is given by

$$\xi = \rho_1 \gamma t_0 \exp\left(\frac{\rho_2}{t_0}\right). \tag{A.3}$$

As $u(x)$ does not contain x_m , $q_i(x)$ ($i=1,2$) does not also contain x_m in terms of Eqs. (2) and (3). Therefore, we have

$$\frac{\partial q_i(x)}{\partial x_m} = 0, i = 1, 2. \tag{A.4}$$

Substituting (A.4) into (23), we have

$$\frac{\partial \Phi}{\partial x_m} = \pi \alpha_p \tau x_m m(x_m) \left(\tau - 2\widehat{C}(x_m) \right) = 0. \tag{A.5}$$

As $\pi \alpha_p \tau x_m m(x_m) > 0$ always holds, we obtain

$$\tau - 2\widehat{C}(x_m) = 0. \tag{A.6}$$

This means that $\tau = 2\widehat{C}(x_m)$ holds.

On the other hand, as $u(x)$ does not contain τ , from Eq. (2) we have

$$\begin{cases} \frac{\partial q_1(x)}{\partial \tau} = 0, \\ \frac{\partial q_2(x)}{\partial \tau} = -m(x)\alpha_p \frac{\partial p_2(x)}{\partial \tau} = -m(x)\alpha_p. \end{cases} \quad (\text{A.7})$$

When the traffic congestion effects can be ignored, we have $\frac{\partial \widehat{C}(x)}{\partial q_i(x)} = 0$ in terms of Eq. (A.2). Therefore, from Eq. (24), we have

$$\frac{\partial \Phi}{\partial \tau} = \int_{x_m}^B 2\pi x \alpha_p m(x) (\widehat{C}(x) - \tau) dx = 0. \quad (\text{A.8})$$

Substituting Eqs. (1), (A.2) and (A.6) into Eq. (A.8), we have

$$\begin{aligned} & -5ax_m^4 + (8b + 4ax_c)x_m^3 - 6bx_c x_m^2 + (8aB^3 - 12bB^2)x_m \\ & + (4bB^3 + 6x_c bB^2 - 3aB^4 - 4ax_c B^3) = 0. \end{aligned} \quad (\text{A.9})$$

Equation (A.9) can further be written as

$$(x_m - B)(\omega_3 x_m^3 + \omega_2 x_m^2 + \omega_1 x_m + \omega_0) = 0, \quad (\text{A.10})$$

where ω_3 , ω_2 , ω_1 , and ω_0 can be given by Eq. (27). This completes the derivation of Eq. (26).

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