Netw Spat Econ (2012) 12:75–107
DOI 10.1007/s11067-010-9148-1

A Large-Scale Spatial Optimization Model of the European Electricity Market

Florian U. Leuthold · Hannes Weigt · Christian von Hirschhausen

Published online: 5 October 2010

Abstract In this paper, we present a large-scale spatial model of the European electricity market including both generation and the physical transmission network (DC Load Flow approach). The model was developed to analyze various questions on market design, congestion management, and investment decisions, with a focus on Germany and Continental Europe. It is a bottom-up model combining electrical engineering and economics: its objective function is welfare maximization, subject to line flow, energy balance, and generation constraints. The model provides simulations on an hourly basis, taking into account variable demand, wind input, unit commitment, start-up costs, pump storage, and other details. Various forms of spatial price discrimination can be implemented, such as locational marginal pricing ("nodal pricing"), or zonal pricing. With over 2,000 nodes and over 3,000 lines, this is one of the largest models developed to date, and allows a highly differentiated spatial analysis. We report modeling results regarding efficient congestion management for Germany and Europe, optimal network expansion under the aspect of increased wind energy production, and the impact of network constraints on location decisions of generation investments.

Keywords Network modeling · Electricity markets · Spatial price discrimination

F. U. Leuthold · H. Weigt · C. von Hirschhausen (✉)
Faculty of Business and Economics, Chair of Energy Economics and Public Sector Management, Dresden University of Technology, 01069 Dresden, Germany
e-mail: cvh@mailbox.tu-dresden.de

1 Introduction

Electricity markets around the world are still in a state of flux, even two decades (the UK market), one decade (for some U.S. markets) or a couple of years (continental Europe) into the reform process. In Europe, the reform momentum has accelerated in the second half of this decade. In fact, the "Acceleration Directive" (2003/54/EC) has been followed by a more coherent attempt of moving toward a single European market. Yet central reform steps such as vertical unbundling, incentives for cross-border transmission investment, and the integration of large-scale renewable electricity into the network are still in the making. Evidence of this process is provided by the discussions of the "3rd Energy Package" of the European Union, providing energy policy guidelines for the next decade.

In order to understand the impact of different reform proposals and to simulate diverse development scenarios, the Chair of Energy Economics and Public Sector Management (EE^2) has developed a model of the European electricity market(s) based on a DC Load Flow model, called ELMOD (Fig. 1). The model was initiated by Leuthold et al. (2005) for the German electricity market. Weigt et al. (2006) continued this work and extended the model by including France, Benelux, Western Denmark, Austria and Switzerland. Weigt (2006) broadened the scope to a timeframe of 24 h to simulate variable demand and wind input as well as unit commitment, start-up and pumped storage issues. The model was subsequently extended to cover the entire European UCTE (Union for the Co-ordination of Transmission of Electricity) electricity markets (essentially Central and Western Europe). Today, this is one of the larger engineering-economic models, with a very high granularity, allowing differentiated spatial price and flow analysis.

While unit commitment decisions can be included in ELMOD, it is not a typical day-ahead unit commitment model. ELMOD is intended to represent supply and demand behavior on a typical day of a season, assuming that consumers have had time to adjust to a certain market mechanism. ELMOD then calculates the market outcomes under different market assumptions and can be used as analysis tool for researchers and policy makers.

The model is unique in that it combines a large number of specifics from the electricity sector with a large-scale spatial representation of the pan-European high-voltage electricity network. Thus, in addition to the modeling formulation, the results are directly applicable to a large variety of lines (more than 2,000) and nodes (more than 4,000). The flexibility of the model can thus be used to obtain an optimal degree of detail vs. computational speed.

This paper summarizes the model and provides an in-depth description of model assumptions and specifics. We start out with an overview of the literature on network modeling of electricity (Section 2), and then proceed with the technical and economic details of ELMOD (Section 3). Section 4 presents the data used, the underlying assumptions, sources, et cetera. In

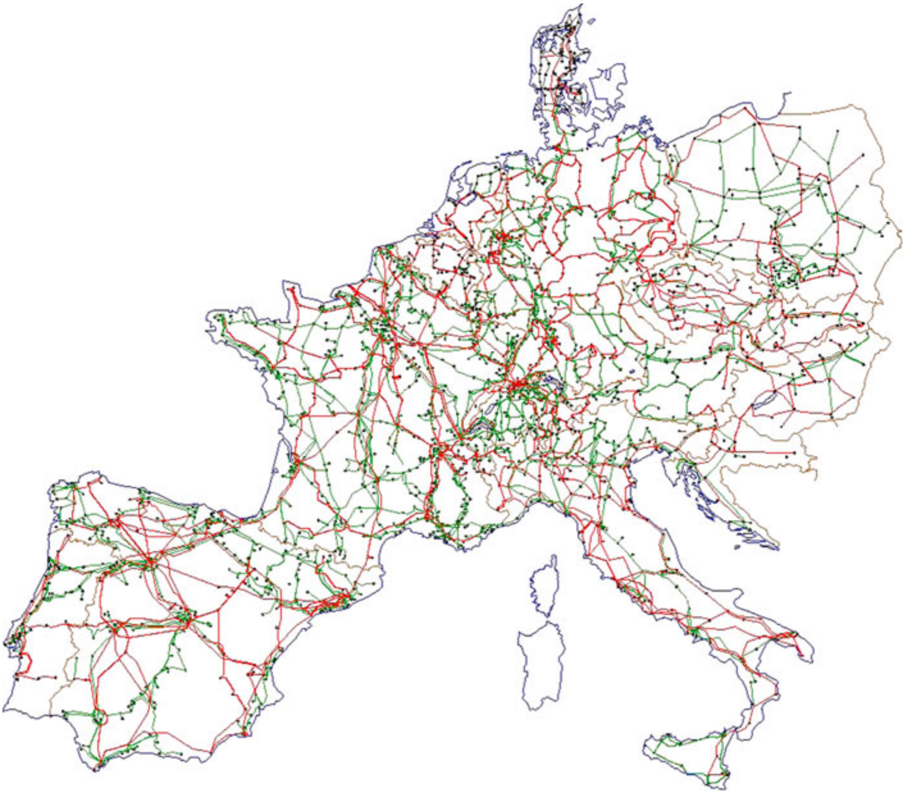

Fig. 1 ELMOD representation of the European high voltage grid. Source: own presentation

Section 5 we discuss various applications of the model, including congestion management issues, wind integration, and generation capacity expansion. Section 6 concludes and sketches out topics for further research.

2 Background of the model

2.1 Survey on modeling electricity markets

The objective of electricity market reforms is generally to replace monopolistic structures with competition and—where natural monopolies prevail—with more efficient regulation. In Europe, several Directives were issued since 1996 to advance on this reform path. In addition, the discussion of climate change has added further elements to energy policy, such as the European

Emissions Trading System (ETS), and the ambitious targets for electricity from renewable energy sources, mainly wind. Thus, Germany and Spain have introduced generous feed-in tariffs for onshore and offshore wind energy that the network operators have to integrate in their network management. All in all, there is a strong interest of firms, regulators and scientists in electricity market models taking into account these new challenges of liberalization and changing generation and demand structures.

Ventosa et al. (2005) provide a detailed overview of market modeling tendencies. They point out three trends: optimization models, equilibrium models and simulation models. Optimization models can either apply a profit maximization of a single firm or a welfare maximization approach under perfect competition. Ventosa et al. (2005) distinguish two types of models for a single-firm optimization problem: either the price is an exogenous parameter or determined via a function of the demand supplied by the firm. In contrast, equilibrium models take into account that a firm is able to influence the price by its output decision. The market behavior of all players can then be modeled. Market equilibria problems differ in their assumptions regarding strategy space, ranging from classical Bertrand and Cournot to more complex supply function equilibria (SFE) (compare Klemperer and Meyer 1989; Green and Newbery 1992; Day et al. 2002). For the time being, equilibrium problems taking into account strategic behavior of many players while considering network constraints are very hard to solve. Ventosa et al. (2005) state that in this case, simulation models can be applied.

Another overview is provided by Smeers (1997) distinguishing between perfect competition models and imperfect competition paradigms. The most simple approach to an ex post analysis of markets is to use perfect competition models.[1] Smeers (1997) regards them as very useful since they can handle large data. Imperfect market characteristics can be introduced into these models as well by taking into consideration quantitative restrictions or mark-ups. Furthermore there exists another category of single-staged equilibrium models containing standard imperfect competition paradigms such as the Cournot or Bertrand paradigm and models for system operation (for example Metzler et al. 2003). The former models being used for ex ante analysis of new institutions like the introduction of a pool or power exchange for electricity. The basis for the latter models was introduced by Schweppe et al. (1988), making reference to the concept of economic dispatch: short run operations are assumed to be perfectly regulated, hence its aim is operational. Since electricity cannot be stored, generation and demand have to be equilibrated at any time, making some kind of central control necessary. Smeers (1997) notices

[1]Note that even these 'simple' perfect competition models can become computationally challenging if certain nonconvex characteristics of electricity markets such as start-up (e.g., Gabriel et al. 2004; García-Bertrand et al. 2005) or minimum profit constraints (e.g., García-Bertrand et al. 2006) are included.

that the usual approach to determine generation operations is an economic dispatch model. A third type of models can be found in the multistage equilibrium models being the most complicated and less developed ones (for example Hobbs et al. 2000). Recently, Shanbhag et al. (2009) have focused on two-stage stochastic equilibrium problems in a power market setting. This appears to be amongst the few instances of a scalable algorithm for computing equilibrium in multi-stage settings.

In other model reviews such as in Kahn (1998) numerical techniques to analyze market power are examined. Day et al. (2002) provide a a detailed comparison of equilibrium models. Classifications are grouped regarding the clearing process used in the power market model (centralized/decentralised) and the nature of interaction among rival generators (from strong competition to collusion). Eight types of equilibrium models are defined including the conjectured supply function. Applications of each model type are indicated. Day et al. (2002) observe that DC load flow approximations are common among these models.

Due to the existence of a great variety of market designs both Hogan (2003) and Ma et al. (2003) describe the development towards a standard market design proposed and used in various regions (e.g., already implemented in PJM). In the last decades, market designs and thus electricity market models drifted into two directions: on the one hand reliability-driven and on the other hand pricing-driven. After this partial co-existence an optimal Standard Market Design (SMD) was proposed claiming a coordinated spot market for energy and ancillary services. The SMD framework shall include bid-based, security-constrained, economic dispatch implementing locational marginal prices and in particular the introduction of financial transmission rights (Hogan 2002). Joskow (2005) argues in a similar manner that pure economic models have to be expanded to take the complexity of electrical constraints accurately into account.

2.2 Technical specifics

Network models have to take into account physical laws when determining prices making electricity an unusual commodity. Electricity cannot be stored, thus requiring demand and supply to equal each other. Furthermore the electricity network transporting electricity from the point of injection to the point of withdrawal has to cope with line capacity limitations, thermal line restrictions, line losses, and security constraints. However, generation and load at any node within the considered network influences the flow on each line, thus demanding quite complex calculations. The use of Kirchhoff's and Ohm's laws is necessary. They include both real and reactive power flows, called AC load flow. An approximation of these load flows for economic modeling can be found in Schweppe et al. (1988), the DC load flow model (DCLF). AC models extend a model's calculation time immensely. In contrast, DCLFs consider only real power equations and can thus reduce the problem size (Overbye et al. 2004).[2] Stigler and Todem (2005) give a brief but informative insight how to derive the DCLF equations from physical fundamentals. There are two basic assumptions: the voltage angle differences between nodes of the network must be presumed to be very small and the voltage amplitudes to be constant (compare Section 3.2). The main advantage in using a DCLF is its applicability to large scale problems with many capacity constraints (Day et al. 2002).

3 Model description

In its basic formulation, ELMOD can be classified as a large non-linear optimization model maximizing social welfare under the assumption of perfect competition taking into account technical constraints. It is solved in GAMS (General Algebraic Modeling System). In order to increase readability of the model formulation, exogenously given parameters are denoted by capital letters while endogenously determined variables are denoted by lower case letters.

3.1 Notation

Indices:

$l \in L$	line within the network
$n, i, j, k \in N$	nodes within the network
$s \in S$	power plant unit
$t, \tau \in T$	time periods

Sets:

L	set of all lines
N	set of all nodes
S	set of all power plants
T	set of all time periods

Parameters:

B_{ni}	relative susceptance between two adjacent nodes n and i
$\widetilde{B}_{jk}$	series susceptance between two adjacent nodes j and k
BL_l	series susceptance of line l
$\widehat{C}_{st}$	marginal generation cost of plant s in time period t
$\widetilde{C}_s$	cost block occurring for start-up of plant s
$\overline{G}_{ns}$	available maximum generation level of plant s at node n
$\underline{G}_{ns}$	required minimum generation level of plant s at node n
$\widetilde{G}_{jk}$	series conductance between two adjacent nodes j and k

[2]The name 'DC load flow' is due to historical origins and does not refer to the use of direct current in the electricity network.

$\overline{PS}_n$	maximum working capacity of a pumped storage hydro plant (PSP) at node n
$\overline{P}_l$	maximum available power flow capacity over line l
R_{jk}	series resistance between two adjacent nodes j and k
R_l	series resistance of line l connecting nodes j and k
$\lvert\underline{U}_{nt}\rvert$	absolute value of the complex voltage vector at node n in time period t
WI_{nt}	wind input at node n in time period t
$\overline{\vartheta}_s$	minimum online duration of plant type s
$\underline{\vartheta}_s$	minimum offline duration of plant type s

Variables:

$c_{nst}(g_{nst})$	generation cost of plant s at node n in time period t as a function of g_{nst}
g_{nst}	generation of plant s at node n in time period t
ni_{nt}	net grid input at node n in time period t
on_{nst}	binary variable describing status of plant s at node n in time period t
$p_{nt}(q_{nt})$	linear inverse demand function at node n in time period t
$pslevel_{nt}$	filling level of a PSP at node n in time period t
$\tilde{p}_{jkt}$	power flow between two adjacent nodes j and k in time period t
p_{lt}	power flow over line l in time period t
$\overleftarrow{ps}_{nt}$	energy produced by a PSP at node n in time period t
$\overrightarrow{ps}_{nt}$	energy demanded to fill a PSP at node n in time period t
q_{nt}	demand quantity at node n in time period t
su_{nst}	start-up costs of plant s at node n in time period t
$\widehat{su}_{nst}$	start-up costs of plant s at node n in time period t
tl_{jkt}	transmission losses between two adjacent nodes j and k in time period t
up_{nst}	variable to determine whether a plant s at node n was switched on in time period t or not
θ_{jkt}	phase angle difference between two adjacent nodes j and k in time period t
θ_{nt}	phase angle difference at node n with respect to the swing bus in time period t
λ_{nt}	dual variables for energy balance constraints
$\overline{\mu}_{lt}$	dual variables for line flow constraints in positive direction
$\underline{\mu}_{lt}$	dual variables for line flow constraints in negative direction

3.2 DC load flow model

Conceptually ELMOD is based on the work of Schweppe et al. (1988) and Stigler and Todem (2005). Schweppe et al. (1988) have provided seminal economic analysis of electricity networks. They apply it to their nodal approach for electricity pricing. Stigler and Todem (2005) describe the way from the physical fundamentals to the DCLF equations.[3] Equation 1 of the so-called 'decoupled' AC model builds the foundation of all further assumptions and calculations. Power flow $\tilde{p}_{jkt}$ depends on the conductance $\widetilde{G}_{jk}$, the susceptance $\widetilde{B}_{jk}$, and the voltage angle difference θ_{jkt} between nodes j and k as well as on the voltage magnitudes $|\underline{U}_{jt}|$ and $|\underline{U}_{kt}|$:

$$\tilde{p}_{jkt} = \widetilde{G}_{jk}\, |\underline{U}_{jt}|^2 - \widetilde{G}_{jk}\, |\underline{U}_{jt}|\, |\underline{U}_{kt}|\, \cos\theta_{jkt} + \widetilde{B}_{jk}\, |\underline{U}_{jt}|\, |\underline{U}_{kt}|\, \sin\theta_{jkt} \tag{1}$$

Schweppe et al. (1988) assume that the voltage angle difference θ_{jkt} is very small and that the voltage magnitudes $|\underline{U}|$ can be standardized to per unit calculations.[4] $|\underline{U}_{jt}|$ and $|\underline{U}_{kt}|$ are thus assumed to be equally 1 at each node j and k during all time periods t. Using the first order terms of the Taylor series approximation the following simplification can be made:

$$\cos\theta_{jkt} \approx 1 \tag{2a}$$

$$\sin\theta_{jkt} \approx \theta_{jkt} \tag{2b}$$

Equation 1 can then be simplified to become:

$$\tilde{p}_{jkt} = \widetilde{B}_{jk}\theta_{jkt} \tag{3}$$

Equation 3 is the core of the DCLF as it shows the interdependence of demand and generation—determined via θ_{jkt}—and the resulting physical flows $\tilde{p}_{jkt}$. In addition, line losses have not been considered yet. However, in real networks the sum of total generation does not equal the sum of total demand due to transmission losses. Thus, transmission lines are stressed by demand plus losses. In order to approximate the losses on a line, Eq. 2a must be complemented by the second order term of the Taylor series approximation:

$$\cos\theta_{jkt} = 1 - \frac{(\theta_{jkt})^2}{2} \tag{4}$$

[3]Overbye et al. (2004) come to the conclusion that the DCLF is adequate for modeling nodal prices albeit there are some buses with a certain price deviation. The latter occurs particularly on lines with high reactive power and low real power flows.

[4]If the model includes more than one voltage level as it is the case within ELMOD, the standardization works by choosing a reference voltage level and then convert all other line parameters by a conversion factor. For example, the factor to express 220 kV parameters in 380 kV terms would be approximately 0.58. Hence, resistances and reactances of the 220 kV lines would have been divided by this factor. However, as the maximum power capacity has to be converted, too, one could also define a reference power flow level, relate a line's power capacity to this predefined level, and convert all parameters accordingly. In any case, regarding the conversion factors, one has to be aware that the power capacity is a quadratic function of the voltage magnitude.

Then, after some further assumptions and conversions following Stigler and Todem (2005) transmission losses can be calculated via the power flow $\tilde{p}_{jkt}$ and the line series resistance R_{jk}:

$$tl_{jkt} = R_{jk} \left(\tilde{p}_{jkt}\right)^2 \tag{5}$$

The implementation of the DCLF into ELMOD requires the modeling line specific flows. Hence, the power flow p_{lt} on a line l must be derived from the power flow $\tilde{p}_{jkt}$ between two nodes j and k using a mapping. Within ELMOD this is achieved by using a network incidence matrix (IM_{ln}), stating which lines l connect nodes j and k:

$$IM_{ln} = \begin{cases} 1 \text{ for } n = j \\ -1 \text{ for } n = k \\ 0 \text{ otherwise} \end{cases}, \forall l$$

Common in DCLF modeling is the definition of a slack bus, swing bus, or hub, respectively (e.g., Christie et al. 2000). For this swing bus i', the voltage angle is defined to be 0. Hence, the voltage angle difference $\theta_{ni't}$ can be rewritten as θ_{nt}, thus, being the voltage angle relative to the voltage angle at the swing bus. The physical line parameters are implemented by using a network transfer matrix H_{ln} and a network susceptance matrix B_{ni} which can be derived using the line susceptances BL_l of each line l:[5]

$$H_{li} = BL_l IM_{li} \tag{6a}$$

$$B_{ni} = \sum_l (IM_{ln} H_{li}) \tag{6b}$$

Other models using the DCLF replace the network transfer matrix H and network susceptance matrix B by a power transfer distribution factor matrix (PTDF) (e.g., Christie et al. 2000; Delarue et al. 2007). A PTDF contains factors that quantify the impact of an injection or withdrawal at a certain location on all lines within the network. When using a PTDF, the voltage angle θ_{nt} does not have to be included into the optimization problem. The PTDF can be multiplied with the generation and demand values at each node in order to find the network flows. However, we believe that our approach is more general and leads to a greater flexibility, e.g., when developing models where B and H are not constant.

3.3 Optimization problem

The standard version of ELMOD uses a welfare maximizing approach taking into account line flow, energy balance and generation constraints. Welfare

[5]The line susceptances BL_l are real parameters that can be observed using voltage level, length, number of circuits, and material of a transmission line (compare Section 4).

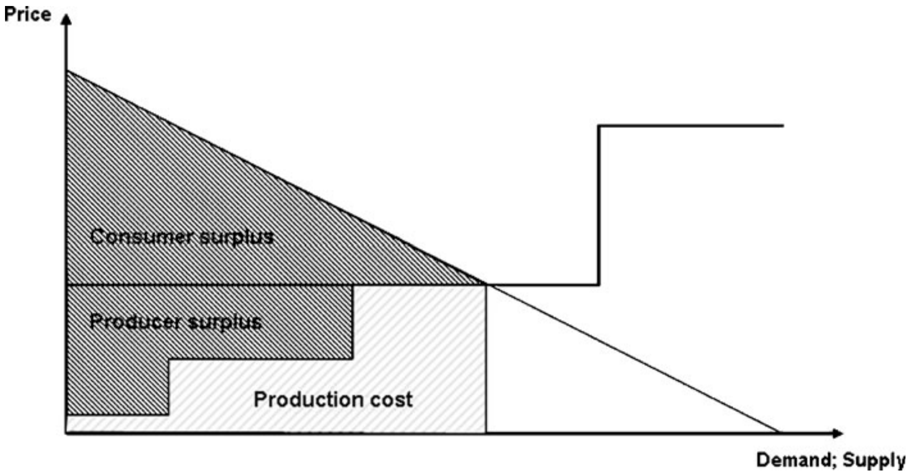

Fig. 2 Welfare in an electricity market. Source: own presentation based on Todem (2004)

is obtained using a linear demand and a supply function and is calculated subtracting the cost of generation from the area below the demand function (Fig. 2 and Eq. 7). At each node reference demand, reference price and elasticity (see Section 4.3) are estimated in order to identify demand via a linear demand function.[6] Generation cost are determined by an individual cost function for each node. In its simplest form, this cost function is composed of a stepwise function (so-called merit-order-function) representing the different constant marginal cost $\widehat{C}_{st}$ per plant s in time period t. However, depending on the application of the model this function can be complemented towards a decreasing marginal cost function (due to partial load conditions) and cost-blocks for the start-up of power plants. Hence, if start-up costs are included, the unit commitment process is integrated into the cost function $c_{nst}(g_{nst})$.

In order to include technical network limits, a line flow constraint, an energy balance, and a generation constraint are integrated into the model. Through the line flow constraints (Eqs. 9a and 9b), a maximum absolute amount of power $p_{lt} = \sum_i H_{li}\theta_{it}$ transported on line l during period t is determined, constrained by the thermal limit of each line $\overline{P}_l$. The multipliers on these constraints measure the scarcity of transmission capacity. If shadow price $\overline{\mu}_{lt} > 0$, line l is congested in time period t by flows in positive direction. If shadow price $\underline{\mu}_{lt} > 0$, the line l is congested in time period t by flows in negative direction.[7]

[6]It can be easily seen that ELMOD could also be run as cost minimization model by fixing the reference demand values at each node.

[7]The definition of positive and negative direction is arbitrary.

The energy balance (Eq. 8) states that at a node n the difference between total generation and demand has to be balanced by injections into or withdrawals from the grid, respectively, adjusted by the amount of occurring losses. The shadow price λ_{nt} on this equation equals the electricity price at node n in time period t. The shadow price $\lambda_{n't}$ for the swing bus n' is called system marginal price for time period t. In time periods with congestion, the prices differ node by node. Hence, total welfare is derived summing over all hours:

$$\max_{g_{nst},q_{nt}} \left\{ w = \sum_{n,t} \left(\int_0^{q_{nt}} p_{nt}(q_{nt})\,\mathrm{d}q_{nt} - \sum_s (c_{nst}(g_{nst})) \right) \right\} \tag{7}$$

$$\sum_s g_{nst} + WI_{nt} + \overleftarrow{ps}_{nt} - \overrightarrow{ps}_{nt} - q_{nt} - ni_{nt} = 0 \quad \forall n,t \quad (\lambda_{nt}) \tag{8}$$

$$p_{lt} \leq \overline{P}_l \quad \forall l,t \quad (\overline{\mu}_{lt}) \tag{9a}$$

$$-p_{lt} \leq \overline{P}_l \quad \forall l,t \quad \left(\underline{\mu}_{lt}\right) \tag{9b}$$

$$pslevel_{nt} = 0.75\,\overrightarrow{ps}_{nt} - \overleftarrow{ps}_{nt} + pslevel_{n(t-1)} \quad \forall n,t \tag{10a}$$

$$\overrightarrow{ps}_{nt} + \overleftarrow{ps}_{nt} \leq \overline{PS}_n \quad \forall n,t \tag{10b}$$

$$\overleftarrow{ps}_{nt} \leq pslevel_{n(t-1)} \quad \forall n,t \tag{10c}$$

$$on_{nst}\underline{G}_{ns} \leq g_{nst} \leq on_{nst}\overline{G}_{ns} \quad \forall n,s,t \tag{11a}$$

$$on_{nst} - on_{ns(t-1)} \leq on_{ns\tau}, \tag{11b}$$

$$\tau = t+1, \ldots, min\{t+\overline{\vartheta}_s, T\}$$

$$on_{ns(t-1)} - on_{nst} \leq 1 - on_{ns\tau}, \tag{11c}$$

$$\tau = t+1, \ldots, min\{t+\underline{\vartheta}_s, T\}$$

$$su_{nst} = \tilde{C}_s\underline{G}_{ns}up_{nst} \quad \forall n,s,t \tag{11d}$$

$$up_{nst} \geq on_{nst} - on_{ns(t-1)} \quad \forall n,s,t \tag{11e}$$

$$\widehat{su}_{nst} = \tilde{C}_s g_{nst}(on_{nst} - on_{ns(t-1)}) \quad \forall n,s,t \tag{11f}$$

$$c_{nst} \geq 0 \quad \forall n, s, t \tag{12a}$$

$$g_{nst} \geq 0 \quad \forall n, s, t \tag{12b}$$

$$\overleftarrow{ps}_{nt} \geq 0 \quad \forall n, t \tag{12c}$$

$$\overrightarrow{ps}_{nt} \geq 0 \quad \forall n, t \tag{12d}$$

$$q_{nt} \geq 0 \quad \forall n, t \tag{12e}$$

$$su_{nst} \geq 0 \quad \forall n, s, t \tag{12f}$$

$$\widehat{su}_{nst} \geq 0 \quad \forall n, s, t \tag{12g}$$

$$up_{nst} \geq 0 \quad \forall n, s, t \tag{12h}$$

$$on_{nst} \in [0, 1] \quad \forall n, s, t \tag{12i}$$

Generation consist of the sum of fossil generation $\sum_s (g_{nst})$ and wind input WI_{nt}. Pumped storage plant generation is added if the pumped storage plant generates electricity $\overleftarrow{ps}_{nt}$. If the pumped storage needs to be filled with water this required electricity $\overrightarrow{ps}_{nt}$ is subtracted (see also Section 3.5). Generation must equal all withdrawals resulting from demand q_{nt} and the so-called net input ni_{nt}. The net input incorporates the DCLF and is determined by the voltage angle θ_{it}: $ni_{nt} = \sum_i B_{ni}\theta_{it}$.[8] By choosing the voltage angle at a node, the model defines a node's grid withdrawal or injection, respectively, and associated line flows hereof as well as demand and generation. Hence, the definition of the net input is an important specificity of ELMOD. Incorporating the DCLF (flows and losses) and possible imbalances between node demand and generation into the energy balance constraint (Eq. 8) provides great modeling flexibility. In addition, there is no need of a system wide energy balance constraint.[9] The generation constraint in Eq. 11a assures on the one hand that a power plant s will be turned off if generation is below a minimum generation $\underline{G}_{ns}$ necessary to obtain workable technical conditions, and on the other hand that it does not exceed its maximum capacity $\overline{G}_{ns}$. Each of the constraints must hold for each hour t.

3.4 Time constraints, unit commitment, and optimal dispatch

Electricity cannot be stored on a large-scale basis. Therefore demand and generation always have to equal each other. Demand is not constant over

[8] In the case of including grid losses, the line losses for l are assigned equally to the nodes connected by this line via the net input: $ni_{nt} = \sum_i B_{ni}\theta_{it} - 0.5 \sum_l (IM_{ln} p_{lt})^2 R_l$.

[9] However, another constraint of the form $\theta_{i't} = 0$ needs to be implemented in the model which has been left out above for simplicity reasons. This constraint is used to define an arbitrary node i' within the system to be the so-called slack bus or hub. Consequently, all θ_{it} values in the system are relative in respect to this slack bus which ensures that all single net inputs ni_{nt} sum up to zero for the entire system and, thus, a system-wide energy balance is established. An exhaustive discussion of this modeling approach can be found in Schweppe et al. (1988).

time, but varies in the course of the day, the week and the season. In Europe, demand is higher in winter than in summer mainly influenced by the weather. On workdays more electricity is consumed than on weekends because of a decrease of industrial demand and changed household behavior. To incorporate these characteristics ELMOD can model a 24 h timeframe.

Unit commitment describes the decision process on whether and when a power plant is running in order to contribute to the satisfaction of demand. Unit commitment identifies those plants available for the following dispatch process in which the output of each plant is determined ex-ante according to the actual electricity demand, technical needs and the plant cost function. As plants need time to be launched ranging from some minutes for small gas turbines up to several days for large nuclear plants ("start-up"), timing is essential for obtaining a cost minimal dispatch as well as maintaining system stability. ELMOD solves unit commitment within the social welfare optimization process. The optimal output for each plant is determined taking into account the minimal output level to be reached to put a plant online and a certain time for starting up the plant. This introduces a binary variable on_{nst} to the calculation process to determine whether a plant is online or offline. Following Takriti et al. (1998), a minimum online and offline constraint can then be defined as displayed in Eqs. 11b and 11c. These equations link the hours of the day in order to include online and offline constraints for power plants, respectively. Since the time increment in ELMOD is one hour, it is reasonable to only include the offline constraint (Eq. 11c). Hence, it is assumed that each plant can be shut down after the end of each hour. Once a plant was shut down, it cannot be turned on again immediately depending on the plant type. Therefore, conditions are introduced to keep plants switched off for a certain time interval $\underline{\vartheta}_s$. Further, in order to reduce the calculation effort, each plant is assigned to one group out of three possible groups following Voorspools and D'haeseleer (2003):

- the must-run units: base load plants that supply the grid with a constant output covering thus the base load which is always demanded;
- the test group: medium load plants that provide the increasing electricity demand during the day and are switched on in the morning hours and shut down during the night;
- the peak units: peak load plants that are crucial to satisfy various demand peaks during the day. Peak load plants can be turned on within a short timeframe.

Within the 24 h model, base load plants such as nuclear and lignite plants are constantly producing at least their minimum required output which basically means that they are by definition online over all time periods. Hydro plants and gas turbines are supposed to be able to go online within one hour. Hence Eq. 11c is not binding for them. For the remaining plant types (hard coal plants, oil and gas steam plants, and combined cycle gas turbine plants) the start-up decisions are made endogenously during the optimization.

Regarding start-up, one can distinguish cold, warm and hot start-up, according to the time since the last shut down. If a plant has recently gone offline, it can be started much faster than a 'cold' plant. This is due to the remaining heat level in the plant, while a 'cold' plant has to entirely build up the necessary starting heat.

For the time being, the maximum considered time period within the model is one day (24 h). Therefore the necessary information to decide on the right kind of start-up may not be available. Also, the calculation effort increases as logic operations have to be considered. Thus for those plants where Eq. 11c applies, the start-up is supposed to be a warm start-up. For gas plants, all start-ups are supposed to be cold start-ups.[10] The start-up times $\underline{\vartheta}_s$ are based on Schröter (2004). Taking these constraints into account, the model calculates the status and the output for each plant in each hour.

Within ELMOD, start-up costs can be determined in two different ways depending on the type of mathematical program to be solved (compare Section 3.6). In the case that the unit commitment is endogenous (Eqs. 11d–11e), the start-up costs su_{nst} are included by assuming that there is a per MW cost $\widetilde{C}_s$ associated with the start-up of $\underline{G}_{ns}$ MW of plant s in the time period t in which this plant has been switched on determined by up_{st}. In the case that the unit commitment has been determined in a previous model run, the start-up cost $\widehat{su}_{nst}$ is then calculated by Eq. 11f. In this case there is a per MW cost $\widetilde{C}_s$ associated with the start-up of g_{nst} MW of plant s in the time period t in which this plant has been switched on determined by the fixed status variables on_{nst}. The respective start-up costs su_{nst} or $\widehat{su}_{nst}$ are added to the cost function $c_{nst}(g_{nst})$ in the objective function of the model.

3.5 Modeling pumped storage and wind energy plants

Pumped storage hydro plants (PSP) as well as wind energy plants cannot be modeled as normal thermal plants. PSPs can either inject to or withdraw energy from the grid. The peculiarity of wind energy is its priority in feed-in. Subsequently, the implementation of these production types in ELMOD is explained in further detail.

PSPs constitute the only way to store larger amounts of electrical energy. These plants can run either in pumping mode, filling a storage basin by using electricity, or in generation mode, using the stored water like a classical hydro plant. The electrical energy is thus actually stored in form of potential energy of the water. These PSP facilities are crucial for system stability, as they can start-up rapidly and therefore cancel out fluctuations. In general they pump water during night time and weekends and start producing electricity generation during the peak periods. Within the model, PSPs can either demand electricity $\overrightarrow{ps}_{nt}$ and fill their storage or use the stored energy and generate electricity $\overleftarrow{ps}_{nt}$. In the model, PSPs start with an empty storage at 8pm. An overall degree

[10] This is irrelevant for the time constraint but important for the cost estimation.

of efficiency is implemented in Eq. 10a by only adding 75% of that energy $\overrightarrow{ps}_{nt}$ to the storage $pslevel_{nt}$ that is actually withdrawn from the network. In return, the model treats pumped storage production as lossless. Consequently, pump storage plants are assumed to have an overall degree of efficiency of 75% for pumping and generating, together.[11] Hence, there is no own cost function needed for PSPs. The model can use the PSPs to intertemporally shift electricity from low priced (off-peak) hours to higher priced (peak) hours. The cost for this shift is endogenously included by taking into account the generation costs for filling the storage and the loss of electricity due to the 75% efficiency level of PSPs.

Equations 10b and 10c define the capacity constraints of the storage facilities. The pumped or generated amount is limited by the plant's working capacity $\overline{ps}_n$. Moreover, the storage level $pslevel_{n(t-1)}$ of a PSP facility at the end of a previous period $t-1$ defines the upper bound for the available generation from that facility $\overleftarrow{ps}_{nt}$ for the current period t.[12]

Regarding renewable energy production, wind has become a major part in the German generation mix with 20.6 GW installed capacity by the end of 2006.[13] Also on the European level, wind energy is the fastest growing renewable energy source with 48 GW installed in 2006.[14] Due to the dependence of wind turbines upon wind speed, there is no active control of energy output like in a fossil plant. Only by setting a turbine offline, a minimal active control can be achieved. Because of the feed-in guarantees provided by most European countries, wind energy has to be accepted as priority energy source by the TSO and is thus a fixed exogenous parameter for the model. Wind speeds change over time according to meteorological conditions and so does the energy input from wind turbines. In times of high generation by wind turbines, conventional plants must reduce output, while in times of low wind input fossil plants have to compensate the shortfall. A consequence could be additional line flows within the transmission grid, particularly in times of high wind input and low demand.

Wind forecasts play a major role in determining the wind input and therefore the plant schedule for the next hours or day. The differences between forecasted wind input and realized input have to be compensated in order to maintain system stability. The operating reserve that must be provided is not considered in the model. While fossil plants are running in constant mode at an optimal load level whenever possible, wind turbines often run in partial load mode and can change output within hours up to 100%. These changes cause

[11] According to Müller (2001), modern PSPs have an average efficiency between 70% and 80%.

[12] The maximum timeframe modeled with ELMOD for the time time being is 24 h. Hence, modeling the storage behavior might be simplified as the storage process also takes place at weekend nights. In addition, the hourly increment used in ELMOD may result in a biased representation of PSPs as one of their main tasks is to react in case of rapidly changing conditions. However, these simplifications could be included within the model framework by extending the timeframe beyond 24 h.

[13] Compare DEWI (2007).

[14] Compare EWEA (2007).

an increased need of backup plants to be able to start-up or reduce output according to the wind input. Within the model, the wind input is calculated for each hour and node and given as an external parameter included in the energy balance (see Eq. 8).[15] However, our model does not deal with intermittency and stochasticity of wind integration, which puts a caveat on the results.

3.6 Problem types

ELMOD can either be used as a classical dispatch or market clearing model for short term analyses. In addition, long term aspects can be included via a scenario method by simulating differently weighted representative days to make up an entire year. The full representation of ELMOD as displayed in the set of Eqs. 7 to 12i would result in a non-linear mixed-integer program (MINLP) which is computationally challenging, particularly for a large scale network. Thus when it comes to applying ELMOD, the entire set of restrictions is normally not included (see Section 5). In principle, ELMOD can be clustered around the following problem types. The basic version of ELMOD includes Eqs. 7 to 9b, and 11a only regarding a single period t. In this case, the pumped storage representation is neglected ($\overleftarrow{ps}_{nt} = \overrightarrow{ps}_{nt} = 0$) and also unit commitment is not included (on_{nst} fixed to 1 and $\underline{G}_{ns}$ to 0). Thus, the model becomes a non-linear program (NLP). The single period formulation can be extended by incorporating more periods and also include the pumped storage restrictions (constraints (10a)–(10c)). This formulation remains a NLP. If the unit commitment (constraints (11a)–(11c)) is to be included, the problem type changes into a mixed-integer linear program (MIP) by fixing the demand levels q_{nt} and thus making the problem a cost minimization approach. Furthermore, losses are then not included as they contribute a quadratic element. If both, an elastic demand and the full unit commitment are to be modeled, ELMOD is solved in a two-stage process: First the unit commitment is conducted by fixing the demand variables to the reference demand values and ignoring line losses which results in a MIP. The result of the MIP run is the plant statuses (on_{nst} variables) which are held fixed in the second welfare maximizing market clearing run. By fixing the binary variables, allowing for a elastic demand, and accounting for line losses, the second run is a NLP.

The different problem versions of ELMOD are coded and solved in GAMS. The NLPs are solved using CONOPT and the MIPs are solved using CPLEX. The MIP model versions are typically solved within minutes whereas the NLP need several hours on a standard desktop computer systems.[16] A discussion

[15] This constraint can become critical if the grid is not capable of transporting all wind energy. Then the only way to fulfill the energy balance constraint is the increase of local demand even if prices become negative. For the time being, in reality other measures are taken in order to avoid such situations. Possibilities in order to manage such extreme cases are the shut-down of certain wind parks and other technical measures. Such short-term measures are not included in ELMOD.

[16] For clarity, we do not describe the problem sizes of all possible applications. However, the following problem sizes might help to get a rough idea: ELMOD for one hour without unit

about the reliability, advantages, and disadvantages of different standard solvers and algorithms is not in the scope of this paper. We typically test our model results by using different starting points for the optimizations. However, the model outcomes do normally not differ significantly.

4 Data

4.1 Network

The underlying network is based on the European high voltage grid (UCTE 2004; VGE 2005). Substations, line voltage level and line length were uploaded into a digital map, making it possible to add and remove additional lines and nodes. An underestimation of line length can occur, since altitude differences have not been considered. Since no data about the system state is publicly available, all lines connected to a node are assumed to be connected with one another. Also, no information about the transformation capacities of the substations is available. Security constraints are considered by a 20% transmission reliability margin. Thus, no line within the modeled grid will be stressed with more than 80% of their thermal capacity limit.

4.1.1 Germany

The most detailed region mapped in the model is Germany with 365 nodes: 336 regular nodes representing substations and 29 auxiliary nodes. Three different reference line characteristics, one for each voltage level, are considered based on Fischer and Kießling (1989). Three main technical factors are included: maximum thermal limit, line resistance and line reactance. The values differ significantly for the three voltage levels. To obtain the values for lines with more circuits, the impedances have been calculated according to a parallel combination. Thus, the interaction of multiple circuits has been neglected. The data source for the line characteristics is based on the UCTE-network map (UCTE 2004). As cross-border flows and transactions play an important role in electricity markets, nine country nodes are added, representing the neighboring countries and 81 cross-border nodes to simulate the import and export, as well as cross-border flows. The model contains 271 lines of the 220 kV and 309 lines of the 380 kV level as well as six lines with 110 kV. In addition, 50 country tie-lines with unlimited capacity are included, connecting the cross-border nodes with the neighboring country node and representing the grid

commitment and PSP for the German network has about 2,000 variables; ELMOD for one hour without unit commitment and PSP for the European network has about 10,000 variables; ELMOD for 24 h including unit commitment and PSP for the European network has about 400,000 continuous and 40,000 discrete variables.

of the respective country. Cross-border lines between countries are modeled according to their length and voltage level.[17]

4.1.2 The European grid

The European UCTE-grid is modeled in a similar way, though with a slightly lower level of detail concerning demand estimations, installed generation capacity, and wind facilities. The entire high voltage grid in Europe is contained in ELMOD based upon the UCTE-network map (UCTE 2004) as well. The model then covers Portugal, Spain, France, the Netherlands, Belgium, Luxembourg, Western Denmark, Germany, Switzerland, Austria, Italy, Poland, Czech Republic, Slovakia, Hungary and Slovenia. This accounts for about 2,120 substations (nodes) and about 3,150 lines of the three highest voltage levels (Fig. 1). Regarding line characteristics, the same assumptions as for Germany are made.

4.2 Generation

4.2.1 Capacities

Generation is divided into different plant types: nuclear, lignite, coal, oil and gas steam plants, combined cycle gas turbines (CCGT), open cycle gas turbines (OCGT), hydro, pumped storage, and combined heat power plants. Wind capacity is addressed separately in a paragraph subsequently (Section 4.2.3). Power plant capacities are based on VGE (2005). The current database includes all active plants for 2006 with a generation capacity greater than 100 MW. Each plant is assigned to one node. In the case of unclear grid integration, plants are allocated to the geographically closest node. There can be more than one plant assigned to a node.

Since thermal plants need a certain heat level to produce electricity, a minimal capacity is defined for each plant class according to DEWI et al. (2005). These values are identical for every thermal power plant. If output drops below this level, the plant has to be turned off. These values are used for defining the binary plant condition variable indicating whether the plant is on- or offline.

Combined heat and power plants (CHPs) often deliver long-distance heat or are integrated in a thermal production process in industries, thus producing electricity as a byproduct. These cogeneration plants were grouped

[17] It must be noticed that the implementation of neighboring countries has an impact on the welfare calculation. As they are part of the overall optimization problem, their demand and generation adds to the total system welfare. Due to energy exports and imports, it is not possible to calculate the welfare for Germany only when including neighboring countries. This must be taken into account while regarding welfare effects. However, as long as only Germany is modeled in detail and the other countries are aggregated to a few nodes, the values should largely reflect changes in Germany.

corresponding to their primary output in heat- and power-operated plants. Due to legal guidelines an additional must-run condition was implemented in ELMOD to take into account that energy produced by this type of plant has to be fed-in prior to other energy types. The generation behavior of the 'heat-operated' power plants follows the same criteria as other power plants of the same type but they are assumed to be like base load plants in terms of unit commitment. Thus they constantly produce at least at their minimum output levels which is assumed to correspond to the specific required heat levels.[18] This may lead to an overestimation of output during night times and an underestimation during day times.

4.2.2 Costs

For each plant type a reference efficiency value and marginal cost are estimated based on different fuel types. Depending on the output level a mark-up can be added if the output is lower than the reference efficiency value in order to allow for efficiency losses. The mark-ups have been transformed into cubic polynomials. An additional cost block is added if a thermal plant has to start-up. Hence, cost functions vary between the different plant classes. Also, costs of plants from the same type differ since efficiency levels are not identical. In general, modern plants have a higher efficiency than older ones. However, the construction of the power plant cycle, the actual level of output and external conditions like cooling water availability influence the efficiency as well.

The generation costs are calculated on a marginal cost basis via a step-wise fuel cost function plus a start-up cost block. The step-wise function can additionally be extended by including markups to account for partial load conditions.[19] ELMOD provides the possibility to additionally account for efficiency losses if the output is lower than the optimal output level (assumed to be the maximum output). Three mark-ups are defined: one for steam plants, one for CCGT plants and one for OCGT plants. The mark-ups depend on the output level in relation to the maximal output. The increase of specific heat consumption due to operating below the optimal output is referred to as partial load conditions. The increase in specific heat consumption can be transformed into a decrease of the plants efficiency as more thermal energy (fuel) is required to produce the same amount of electricity (Fig. 3). The partial load conditions can be modeled by cubic equations and, thus, introduce non-linear elements to the generation cost function $c_{nst}(g_{nst})$ and thus is not suitable for MIP versions of ELMOD.

The impact is rather low for classical steam plants, but becomes important for peak load units like gas turbines and therefore is crucial in times of rapidly changing wind input conditions. The mark-up for CCGT-plants is based on

[18]Heat demand curves are not included; the actual output is approximated via seasonal factors.

[19]The actual generation costs are derived from the plants efficiency η_s and the input fuel price fp_{st}: $\widehat{C}_{st} = \frac{fp_{st}}{\eta_s(g_{nst})} g_{nst} + su_{nst}$.

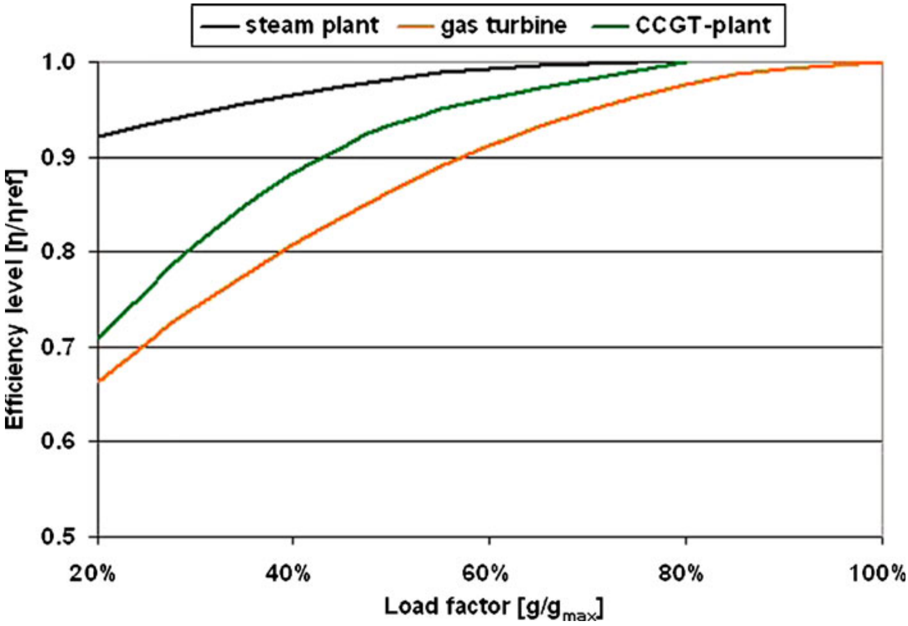

Fig. 3 Partial load efficiency. Source: own calculations based on Kehlhofer et al. (1984), Baehr (1985), and VDI (2000)

VDI (2000) assuming reference efficiency at maximum output of 52.5 % (Müller 2001). The efficiency of gas and oil fired gas turbines depend on the compressor inlet temperature. Based on a reference efficiency of 34.5% (Müller 2001) and a temperature level of 15°C, the partial load efficiency is taken from Kehlhofer et al. (1984). For steam plants, a functional interrelationship of specific heat consumption and partial load can be obtained from Baehr (1985). Nuclear plants may have additional drawbacks due to the necessary security constraints that are not considered within the model formulation.

Based on the above described assumptions it is possible to estimate the impact of varying wind energy on the total system costs. Although wind energy has no marginal generation costs inherently, it causes fossil plants to reduce generation and therefore operate under partial load conditions thus increasing their costs.[20] ELMOD uses the simplified partial load curves in order to

[20] A simple example reveals the impact: Assume a 1,000 MW fossil plant with generation costs of 10 €/MWh that has to reduce its output because 200 MW wind energy are available and need to be fed into the grid. Running at 80% of optimal output causes the efficiency to drop and thereby the costs to rise to 10.07 €/MWh. The cost reduction therefore is not 2,000 €/h, but only 1944 €/h. The difference could be considered as the indirect marginal cost of wind energy. In reality, a clear cost allocation of wind energy is not possible, because changes in demand modify the operation of the fossil plants. Furthermore, the indirect cost of wind generation is not constant but changes with the load situation of the fossil power plants.

calculate the cost of wind energy and neglects further wind specific additional costs. Nonetheless the overall impact on welfare is considered. Moreover, prices for CO_2 allowances are included into the generation costs. Therefore the plant specific CO_2 emissions are calculated based on efficiency and plant type.[21] Prices for CO_2 allowances are exogenous to the model and are adjusted according to most recent data for different applications of ELMOD (compare Section 5).

Additional costs occur if a thermal plant has to start-up or go offline. Fossil plants generate electrical energy through transforming heat energy. This heat has to reach a certain level before generation can start and has to be cooled down in a controlled process after generation is stopped. The cool-down phase is assumed to be mainly affected by fixed cost parameters. Since ELMOD uses a marginal cost approach, it does not take into account cooling down specifically in its optimization. The start-up costs are mainly driven by fuel prices, as a certain amount of fuel has to be consumed before the heat level is high enough to start electricity generation. The cost estimations for start-up are taken from DEWI et al. (2005). As base load plants are assumed to be must-run plants they do not have start-up costs.[22]

4.2.3 Wind

Since wind turbines have relatively small installed capacities, not all of them can be considered individually. To obtain a realistic distribution of wind capacities in Germany a map representing the installed capacity based on 10 km^2 squares is used.[23] Each square—meaning a certain capacity value—is attached to the geographical closest node. This has been done for each federal state separately to obtain a percentage distribution which can then be updated with the actual wind capacities of the federal state. This distribution mechanism makes it also possible to increase the installed capacities without the necessity to reallocate each node individually assuming that installed capacities represent the suitability of a region for the use of wind turbines. As wind input depends on the wind speeds and largely differs between regions, a simplified classification scheme is used. Therefore six different wind zones have been defined using hourly wind speed (v_{ref}) information covering the time from 2002 to 2004 from seven representative stations.[24] Since these reference stations are located approximately 10 meter above ground (h_{ref}),

[21] Compare Gampe (2004).

[22] This may lead to biased results in the long run, but should not influence the price and welfare calculation within the modeled reference timeframe.

[23] Compare ISET/IWES (2002).

[24] Compare DWD (2005).

an estimation regarding the speed values in the turbine height is applied. In general, wind speed and height follow a logarithmic function (Hau 2003):

$$v(h) = v_{ref} \frac{ln\left(\frac{h}{z_0}\right)}{ln\left(\frac{h_{ref}}{z_0}\right)} \tag{13}$$

Wind speed $v(h)$ depends on the absolute height of the turbine above ground h and the local conditions like the building density, hillsides or forests that influence the roughness length z_0. To obtain average values a roughness length of 0.2, representing farm land with trees and bushes but without surrounding buildings, is defined for all nodes. The height of all turbines is assumed to be 60 meters, based on average values for mid-sized turbines. Calculating the speed values for all zones shows a clear separation between the coastal area in the North and the Southern areas.

For wind capacities in Europe, the World Energy Outlook (IEA 2007) and the Wind Force 12 study (Greenpeace International and EWEA 2005) are chosen. Although both studies analyze the energy sector developments on a global level and for different time horizons it is possible to extract data for continental Europe.[25] Wind capacities are allocated according to federal states or similar administrative areas taking into account political, geographical and meteorological framework conditions.

4.3 Demand

In order to derive a node-specific demand, ELMOD assumes a positive correlation between economic income and total electricity demand. This relation is modeled in greatest detail for Germany, where demand is differentiated into consumption of industries, services and households: electricity is consumed to around 46% by the industrial sector, 27% by households and 21% by services (Eurostat 2004).[26] Standard load profiles for households (H0) and services (G0)[27] are applied and calculated for typical winter and summer workdays. Since various different load profiles exist in the industry sector, the industry consumption is approximated by taking real electricity consumption of a typical winter and summer workday from UCTE (2006) and deduct power of households and services according to the standard load profiles. Consequently, the difference indicates the industry consumption. Load profiles are calculated on an hourly basis and are normalized to the overall consumption of electricity made by each sector as stated above.

[25] Further data are derived from EMD (2005), EWEA (2005), IGW (2005), and WSH (2005).

[26] The remaining electricity consumption is used by agriculture, transportation, the energy sector and others. Since these sectors amount only for a small part of the overall consumption, they are not taken into account separately.

[27] Compare VDEW (1999).

To weight the sector specific consumption with the amount of this sector on a specific node, the gross value added of industry and services and the gross domestic product of households are used. The gross value added is available at Euro NUTS[28] 3 level for larger countries and Euro NUTS 2 for smaller countries. Each district is assigned to a node. In case there are different nodes in one district, the entire gross value is divided by the number of nodes. In case there is no node in the district, the gross value added is distributed to all neighboring districts with nodes. The share of a node of the whole gross value added is calculated and applied to the overall electricity consumption by industry and services, respectively. Regarding the node-specific consumption of households, they are deduced distributing the inhabitants of an administrative district to the node in the same manner as the gross value added for industry and services are assigned to. In a second step, the annual energy consumption of the households is assigned to the nodes according to the node's share in the whole gross domestic product. This, subsequently, yields a reference demand per node. On the basis of this reference demand, a reference price (e.g., average EEX price for Germany) and the assumption of a demand elasticity at this reference point,[29] a linear demand function can be estimated.

For the remainder of Europe, demand is based on UCTE data. For ELMOD applications with focus on Germany the neighboring countries are condensed in single nodes, thus a separation of demand according to industry, commerce and residential is not necessary. Reference prices are taken from the national electricity exchanges.[30] A linear demand behavior is obtained in the same way as for Germany. For calculations covering more countries a node specific demand is derived by using the gross value added as key for a distribution of load to different districts. Thus, a separation of household, service and industrial demand is not considered for the rest of Europe.

5 Applications of ELMOD

This section shows different applications of the ELMOD model and its value, e.g., for modeling the impact of policy decisions and fundamental economic questions regarding market design and renewable energy integration. It should noted that not for each study the full ELMOD model was applied. It is rather the case that a subset of functionalities was chosen according to the focus of the respective application.

[28]NUTS (Nomenclature des Unites Territoriales Statistiques) is a geographical code standard developed by the EU for statistical reasons: http://ec.europa.eu/eurostat/ramon/nuts/introduction_regions_de.html.

[29]Green (2007) includes different assumptions about demand point elasticities in his nodal pricing analysis of a simplified network of the UK. Based on his study, the default demand elasticity in ELMOD is −0.25. However, this value can be altered easily for different model applications - normally between 0 and −0.25.

[30]In case no national price is available, a European average price is calculated based on the existing national prices.

5.1 Congestion management in a German context

One of the first uses of the model was to study different congestion management schemes for the German electricity market, particularly the problem of integrating large scale offshore wind projects as presented in DEWI et al. (2005). Leuthold et al. (2005) demonstrated that nodal pricing is superior to uniform pricing in welfare terms given the German market environment.[31] They analyze the implication of additional wind supply into the German high-voltage grid concluding that a nodal pricing scheme provides a welfare increases between 0.6% and 1.3% or about 350 million € per year compared to the reference uniform prices German market system. They also illustrated that there is an additional welfare increase of about 1% on average in case of additional offshore wind input into the German power grid due to reduced generation costs. Their results also show that there is a limit of wind energy injection at about 8 GW offshore capacities without additional network extensions.

Weigt (2006) extended the model developed by Leuthold et al. (2005) to include a timeframe of 24 h in order to simulate variable demand and wind input as well as unit commitment, start-up and pump storage issues.[32] The timeframe allowed a more differentiated assessment of the price impact due to network restrictions (Fig. 4). While the average price during off-peak times is on an equal level under uniform and nodal pricing, prices greatly diverge during peak times. Moreover, although specific nodes face higher prices under nodal pricing than under the current uniform system the average price level is much lower during peak hours resulting in a welfare gain of about 100 million € per year. Weigt (2006) also analyzed the impact of varying wind input on the price pattern in Germany. In general a higher wind input leads to a price decrease. During low load phases the additional wind energy can help to reduce grid load due to decentralized generation and a large availability of transmission capacities due to low power flow levels. However, price increases are also possible under certain conditions. In peak load situations the power flow level causes congestion within Germany which can be aggravated by a high wind input leading to a large price divergence between North Germany with a high share of wind capacities but low demand and South Germany with a high demand level but a small share of wind capacities. An extension of wind capacities including offshore wind tightens the grid situation.

[31] For this application, inter-temporal and integer constraints were not included and the grid coverage was restricted to Germany. Thus, the model is solved as a NLP using the CONOPT solver in GAMS.

[32] For this application, the model is solved as a two-stage process. First the unit commitment under fixed demand and ignoring line losses is conducted as MIP using the CPLEX solver in GAMS. After fixing the plant statuses, the welfare maximizing market clearing is conducted as a NLP using the CONOPT solver in GAMS.

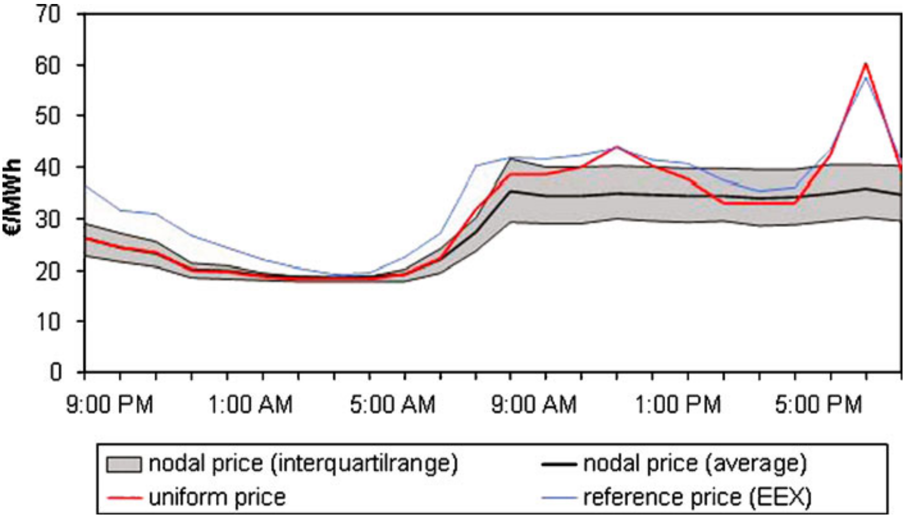

Fig. 4 Price comparison nodal versus uniform pricing in Germany. Source: Weigt (2006)

5.2 Wind integration in Europe

The two above mentioned studies did not take into account cross-border flows affecting neighboring countries. However, power flows follow physical laws and thus a changed injection pattern in Germany can have an impact on the entire European grid. Thus, Weigt et al. (2006) continued the extension of ELMOD by including France, Benelux, Western Denmark, Austria and Switzerland in order to examine cross-border flow issues.[33] They point out that even under status quo conditions, the price situation in Benelux is affected by high wind input in Germany. However, the price increase is rather low on average. In times of high wind input mainly Belgium and Southern parts of the Netherlands face slightly higher prices whereas the Northern and Western parts of the Netherlands profit from increased local wind input. This situation is bound to aggravate if the planned wind capacity extension will be realized without proper grid adjustments. In particular the Northern parts of the Netherlands will face price increases caused by high wind input in Northern Germany. Therefore grid extensions in Germany are urgent to prevent further congestion. The work of Weigt et al. (2006) is the first approach to model the effects of nodal pricing in combination with increased wind energy on the North-Western European grid. Leuthold et al. (2008) build on the aforementioned works in order to recommend nodal pricing for electricity market analysis particularly in a European context.

[33]For this application, inter-temporal and integer constraints were not included. Thus, the model solved as a NLP using the CONOPT solver in GAMS.

Fig. 5 HVDC overlay-grid with three direct connections. Source: UCTE (2004) and own representation

As wind energy will play a major role in Germany's electricity policy for the coming decades the question of adequate grid extensions remains in focus. However, centralized production of highly intermittent wind generation raises two transmission concerns: network integration, and "bridging" between supply and demand. In Germany, the discrepancy between generation located in the North Sea and load concentrated in industrialized regions hundreds of kilometers away is particularly striking. Weigt et al. (2010) took up this problem and analyzed the possibility of integrating large scale offshore capacities using high voltage direct current (HVDC) lines in order to transport the energy to demand centers in the South and West of Germany (Fig. 5).[34] They showed that not only Germany benefits from the HVDC approach due to less congestion and lower prices, particularly in the South, but also the Benelux benefit from the reduction in cross-border flows leading to congestion in their markets.

The need for grid investments due to wind extension and cross-border bottlenecks is not only a German problem but the entire UCTE grid will have to be extended if the European Union wants to fulfil its aim of obtaining a single integrated market, and integrating a large share of distributed renewable energy sources.

Therefore, Leuthold et al. (2009) focused on large-scale wind integration in a European context with a particular focus on efficient grid extension measurements.[35] They extended ELMOD to cover all of the European electricity transmission grid and apply a central transmission planning approach to define the economic optimal extension pattern to incorporate expected wind capacities in 2020. They estimate the impact of additional wind energy by analyzing price situations and develop a grid-extension algorithm to extend the grid incrementally until an economically optimal grid status is identified that is capable of carrying the additional wind. Three scenarios were considered by Leuthold et al. (2009): a Benchmark representing 2006, and two extension cases for 2020 based on the World Energy Outlook (WEO) by IEA (2007), and the Wind Force 12 study (WF12) by Greenpeace International and EWEA (2005) which propose 114 GW and 180 GW of wind capacities, respectively. They show that the increase of installed wind capacity in the years ahead leads to electricity price reductions as wind partially replaces conventional generation. Grid expansion on the other hand will not lead to a reduced price level in all European countries (Fig. 6).

The present situation is characterized by congestion at the borders and a market separated into several price zones. If increased network capacity removes some bottlenecks and brings prices closer together, formerly low-price

[34]For this application, inter-temporal and integer constraints were not included and the grid coverage was restricted to Germany and in a simplified way to its neighboring countries. Thus, the model is solved as a NLP using the CONOPT solver in GAMS.

[35]For this application, inter-temporal and integer constraints were not included. The model is solved as a NLP using the CONOPT solver in GAMS. The investment decisions are made in an iterative process solving the NLP repeatedly.

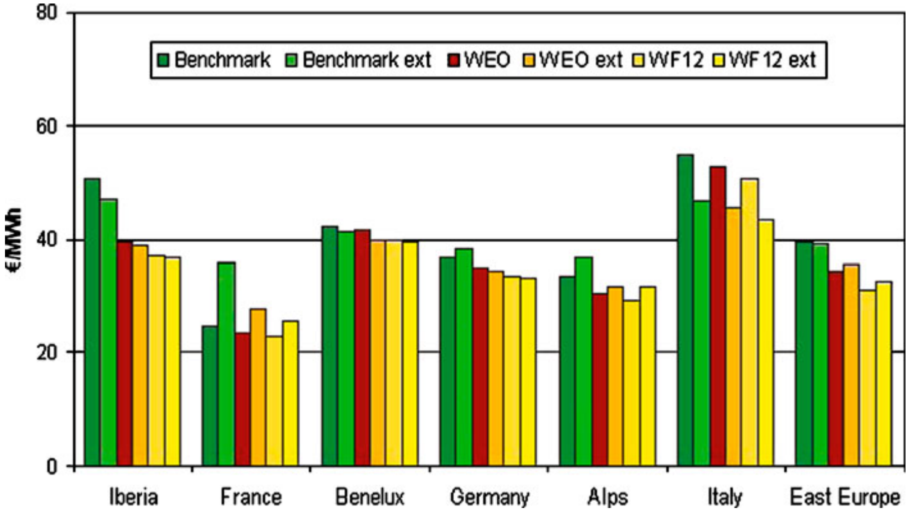

Fig. 6 Average prices before and after the network extension. Source: Leuthold et al. (2009)

regions (e.g., France) will likely encounter higher prices. The welfare properties of the extensions are generally positive in all cases. With a relative low amount of investment costs large welfare gains are achievable A large fraction of this welfare gain is already achieved by the first extensions of highly congested lines. Leuthold et al. (2009) show that developing the network at existing bottlenecks—mainly cross-border connections - should be encouraged by regulatory authorities. With a more moderate wind expansion of 114.5 GW until 2020, the optimal grid investments are even smaller due to resulting counter flows. However, if the additional wind capacity becomes too great (181 GW), the needed grid extensions will increase compared to the actual situation.

5.3 Spatial aspects of generation investments in congested networks

The model can also be applied to identify optimal power plant location decision. Thus, Dietrich et al. (2009) modeled spatial investment location decisions for power plants in the German market up to the year 2012 based on realistic data of planned generation projects.[36] They analyzed where in the current scheme new investment in generation is most likely to take place and compare these results to an optimal investment pattern taking into account network constraints. In this model version, only the plant locations are endogenous

[36]For this application, the model is solved under fixed demand and ignoring losses as MIP using the CPLEX solver in GAMS.

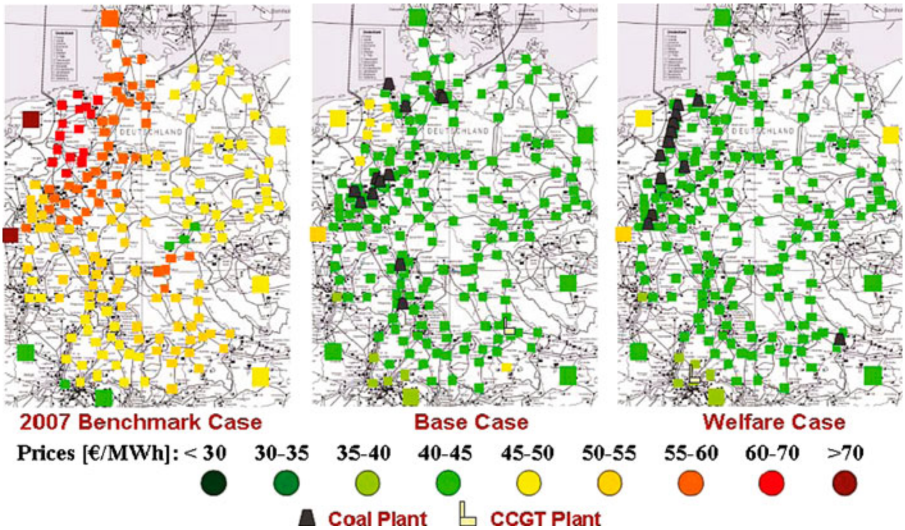

Fig. 7 Average prices and plant locations. Source: Dietrich et al. (2009)

whereas the projected capacity is exogenously defined by planned projects until 2012. The analysis is carried out for different assumptions regarding market representation including a national focus on Germany, a market coupling setting, and a fully integrated European market. They applied nodal pricing to identify local price changes and resulting congestion. Comparing the 2007 market situation with the expected market situation in 2012 (Base Case) shows a large price decrease due to scheduled line extension and the projected power plants (Fig. 7). However, if plant locations are integrated into the welfare algorithm of the model and thus are chosen taking into account network restrictions, the resulting plant locations change in favor of neighboring countries (Welfare Case). The results indicate that new generation capacities are not needed in Germany but in the Benelux area. This becomes evident if the restriction to build plants within Germany is relaxed. In this case 8 out of 13 plants are placed outside of Germany. Although the price impact of the location shift is modest the welfare gain in terms of reduced generation costs is significant and amounts up to 3 billion Euro per year. Their paper demonstrates that great benefits for consumers and producers can be created when physical network restriction are taken into account within a real integrated market.

6 Conclusions

In this paper, we have presented the current version of ELMOD, large-scale a welfare maximizing engineering and economic model of the European electricity market. The model is based on a DC Load Flow approach and captures the essentials of the European electricity markets, even though it lacks some idiosyncrasies of some national markets. ELMOD can be applied to analyze the effect of offshore wind power on the North-West European electricity market, and the effects of congestion between countries and within the German grid. Additionally, the model can also be applied to generation investment issues namely the siting of new power plants under grid constraints. Further development steps are to endogenize investment decisions, in particular the interdependence between investments in generation and in transmission. In the long run, it might be worth the while to integrate strategic behavior of at least one integrated player, and to introduce stochastic elements into the model.

Acknowledgements An earlier version of this paper was presented at the 9th IAEE European Energy Conference (June 10–13, 2007, Florence, Italy). The authors thank Kristin Dietrich, Ina Rumiantseva, Franziska Holz, Christian Todem, Till Jeske, Jürgen Apfelbeck, and Bert Willems for their help and advice on various stages of the model development process.

Appendix: The linear inverse demand function

Assume a linear inverse demand function $p_{nt}(q_{nt})$ of the general known form as given in Eq. 14a with the slope $M \leq 0$ and axis intercept $A \geq 0$. Rearranging yields the demand function Eq. 14b while Eq. 14c displays the standard equation for calculating the price elasticity of the demand.

$$p_{nt} = A + Mq_{nt} \tag{14a}$$

$$q_{nt} = -\frac{A}{M} + \frac{1}{M}p_{nt} \tag{14b}$$

$$\varepsilon = \frac{\partial q_{nt}}{\partial p_{nt}} \frac{p_{nt}}{q_{nt}} = \frac{p_{nt}}{Mq_{nt}} \tag{14c}$$

In order to derive the prohibitive price A and slope M, we assume a known demand elasticity ε at the observed reference point $(P_{nt}^{\text{ref}}, Q_{nt}^{\text{ref}})$. Prohibitive price A and slope M for each node n and time period t can, thus, be calculated on the basis of the given reference values for price and demand according to Eqs. 15a and 15b.

$$M = \frac{P_{nt}^{\text{ref}}}{Q_{nt}^{\text{ref}}} \frac{1}{\varepsilon} \tag{15a}$$

$$A = P_{nt}^{\text{ref}} - MQ_{nt}^{\text{ref}} \tag{15b}$$

Inserting Eqs. 15a and 15b into Eq. 14a yields Eq. 16 which is the equation that is used in ELMOD to determine the linear inverse demand functions per node n and time period t.

$$p_{nt} = P_{nt}^{\text{ref}} - \frac{P_{nt}^{\text{ref}}}{MQ_{nt}^{\text{ref}}} \frac{1}{\varepsilon} Q_{nt}^{\text{ref}} + \frac{P_{nt}^{\text{ref}}}{MQ_{nt}^{\text{ref}}} \frac{1}{\varepsilon} q_{nt} = P_{nt}^{\text{ref}} + \frac{1}{\varepsilon} P_{nt}^{\text{ref}} \left(\frac{q_{nt}}{Q_{nt}^{\text{ref}}} - 1 \right) \tag{16}$$

References

Baehr R (1985) Konzeption und Aufbau von Dampfkraftwerken. In: Bohn T (ed), Handbuchreihe Energie Band 5. Technischer Verlag Reschl/Verlag TÜV Rheinland, Cologne, Germany

Christie RD, Wollenberg BF, Wangensteen I (2000) Transmission management in the deregulated environment. Proc IEEE 88(2):170–195

Day CJ, Hobbs BF, Pang J-S (2002) Oligopolistic competition in power networks: a conjectured supply function approach. IEEE Trans Power Syst 17(3):597–607

Delarue E, Bekaert D, Belmans R, D'haeseleer W (2007) Development of a comprehensive electricity generation simulation model using a mixed integer programming approach. Int J Electr Comput Syst Eng 1(2):92–97

DEWI (2007) Windenergienutzung in Deutschland—stand 31.12.2006. DEWI Magazin 30.

DEWI, E.ON Netz, EWI, RWE Transportnetz Strom and VE Transmission (2005) Energiewirtschaftliche Planung für die Netzintegration von Windenergie in Deutschland an Land und Offshore bis zum Jahr 2020. Technical report, German Energy Agency (dena), Berlin, Germany. Internet: http://www.dena.de/de/themen/thema-kraftwerke/projekte/projekt/netzstudie-i/. Accessed: 31 Jan 2009

Dietrich K, Leuthold F, Weigt H (2009) Will the market get it right? The placing of new power plants in Germany. In: Proceedings 6th internationale energiewirtschaftstagung an der TU Wien, 11–13 February 2009, Vienna, Austria

DWD (2005) Datenabgabe 439/05, Wind speed information about 8 stations. German weather service (DWD)

EMD (2005) DK turbine positions. EMD international, Aalborg, Denmark. Internet: http://www.emd.dk/EMD. Accessed: 6 Feb 2009

Eurostat (2004) Versorgung, Umwandlung, Verbrauch—Elektrizität—Jährliche Daten. Statistical office of the European communities. Internet: http://epp.eurostat.ec.europa.eu. Accessed 20 Jan 2007

EWEA (2005) Large scale integration of wind energy in the European power supply: analysis, issues and recommendations. The European Wind Energy Association, Brussels, Belgium. Internet: http://www.ewea.org/fileadmin/ewea/documents/documents/publications/grid/051215_Grid_report.pdf. Accessed 9 Jan 2006

EWEA (2007) Wind power installed in Europe by the end of 2006. The European Wind Energy Association, Brussels, Belgium

Fischer R, Kießling F (1989) Freileitungen—planung, Berechnung, Ausführung. Springer, Berlin, Heidelberg

Gabriel SA, Supat K, Balakrishnan S (2004) A mixed integer stochastic optimization model for settlement risk in retail electric power markets. Netw Spat Econ 4(4):323–345

Gampe U (2004) Umweltaspekte von Energieanlagen. Dresden University of Technology, Chair of Thermal Power Machinery and Plants

García-Bertrand R, Conejo AJ, Gabriel S (2005) Multi-period near-equilibrium in a pool-based electricity market including on/off decisions. Netw Spat Econ 5(4):371–393

García-Bertrand R, Conejo AJ, Gabriel S (2006) Electricity market near-equilibrium under locational marginal pricing and minimum profit conditions. Eur J Oper Res 174(1):457–479

Green R (2007) Nodal pricing of electricity: how much does it cost to get it wrong? J Regul Econ 31(2):125–149

Green RJ, Newbery DM (1992) Competition in the British electricty spot market. J Polit Econ 100(5):929–953

Greenpeace International, EWEA (2005) Wind force 12—a blueprint to achieve 12% of the world's electricity from wind power by 2020. Technical report, Global Wind Energy Council, Brussels, Belgium. Internet: http://www.gwec.net/fileadmin/documents/Publications/wf12-2005.pdf. Accessed 6 Feb 2009

Hau E (2003) Windkraftanlagen, grundlagen, Technik, Einsatz, Wirtschaftlichkeit. Springer, Berlin

Hobbs BF, Metzler CB, Pang J-S (2000) Strategic gaming analysis for electric power systems: an mpec approach. IEEE Trans Power Syst 15(2):638–645

Hogan WW (2002) Electricty market restructuring: reforms of reforms. J Regul Econ 21(1):103–132

Hogan WW (2003) Transmission market design. KSG Working Paper No. RWP03-040, Retrieved on April 1, 2007, from: http://ssrn.com/abstract=453483m
IEA (2007) World energy outlook 2006. OECD/IEA, Paris
IGW (2005) Landkarte. Austrian wind energy association (IG Windkraft). St. Pölten, Austria. Internet: http://igwindkraft.at/index.php?xmlval_ID_KEY[0]=1055. Accessed 6 Feb 2009
ISET/IWES (2002) Räumliche Verteilung der Installierten Nennleistung (Planflächen). Institue of Solar Energy Supply Technology, Kassel, Germany. Internet: http://reisi.iset.uni-kassel.de/pls/w3reisidad/www_reisi_page.show_menu?p_name=121017&p_lang=ger. Accessed 9 Jan 2006
Joskow PL (2005) Patterns of transmission investment. MIT, Joint DAE-CMI Working Paper 0527. Internet: http://www.econ.cam.ac.uk/electricity/publications/wp/ep78.pdf. Accessed 8 Aug 2006
Kahn EP (1998) Numerical techniques for analyzing market power in electricity. Electr J 11(6): 34–43
Kehlhofer R, Kunze N, Lehmann J, Schüller K-H (1984) Gasturbinenkraftwerke, kombikraftwerke, heizkraftwerke und industriekraftwerke. In: Bohn T (ed), Handbuchreihe energie band 7. Technischer Verlag Reschl/Verlag TÜV Rheinland, Cologne, Germany
Klemperer PD, Meyer MA (1989) Supply function equilibria in oligopoly under uncertainty. Econometrica 57(6):1243–1277
Leuthold F, Jeske T, Weigt H, von Hirschhausen C (2009) When the wind blows over Europe: a simulation analysis and the impact of grid extensions. Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-31. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_31_Leuthold_etal_EU_wind.pdf. Accessed 17 Feb 2009
Leuthold F, Rumiantseva I, Weigt H, Jeske T, von Hirschhausen C (2005) Nodal pricing in the german electricity sector—a welfare economics analysis, with particular reference to implementing offshore wind capacities. Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-08a. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_08a_leuthold_rumiantseva_weigt_et
Leuthold F, Weigt H, von Hirschhausen C (2008) Efficient pricing for European electricity networks—the theory of nodal pricing applied to feeding-in wind in Germany. Util Policy 16(4):284–291
Ma, X, Sun DI, Cheung KW (2003) Evolution toward standardized market design. IEEE Trans Power Syst 18(2):460–469
Metzler C, Hobbs BF, Pang J-S (2003) Nash–Cournot equilibria in power markets on a linearized DC network with arbitrage: formulations and properties. Netw Spat Econ 3(2):123–150
Müller L (2001) Handbuch der Elektrizitätswirtschaft, Technische, Wirtschaftliche und Rechtliche Grundlagen, 2nd edn. Springer, Berlin
Overbye TJ, Cheng X, Sun Y (2004) A comparison of the AC and DC power flow models for LMP alculations. In: Proceedings of the 37th Hawaii international conference on system sciences.
Schröter J (2004) Auswirkungen des Europäischen Emissionshandelssystems auf den Kraftwerkseinsatz in Deutschland. Master's thesis, Berlin University of Technology, Institute of Power Engineering. Retrieved 1 Sep 2005, from: http://basis.gruene.de/bag.energie/papiere/
Schweppe, FC, Caramanis MC, Tabors RD, Bohn RE (1988) Spot pricing of electricity. Kluwer, Boston
Shanbhag UV, Infanger G, Glynn PW (2009) A complementarity framework for forward contracting under uncertainty. Internet: https://netfiles.uiuc.edu/udaybag/www/Papers/or_paper_rev2.pdf. Accessed 26 Jun 2010
Smeers Y (1997) Computable equilibrium models and the restructuring of the European electricity and gas markets. Energy J 18(4):1–31
Stigler H, Todem C (2005) Optimization of the Austrian electricity sector (control zone of VERBUND APG) under the constraints of network capacities by nodal pricing. Cent Eur J Oper Res 13(2):105–125
Takriti S, Krasenbrink B, Wu LS-Y (1998) Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem. Oper Res 48(3):268–280
Todem C (2004) Methoden und instrumente zur gesamtsystemischen analyse und optimierung konkreter problemstellungen im liberalisierten elektrizitätsmarkt. PhD thesis, Department of Electricity Economics and Energy Innovation, Graz University of Technology, Graz, Austria

UCTE (2004) Interconnected network of UCTE. Dortmund, Abel Druck
UCTE (2006) Consumption data. Retrieved 19 Jan 2007, from: http://www.ucte.org/statistics/onlinedata/consumption/e_default.asp
VDEW (1999) Lastprofile. VDEW, Frankfurt (Main)
VDI (2000) Energietechnische Arbeitsmappe. Springer, Berlin, Heidelberg, New York
Ventosa M, Baíllo Á, Ramos A, Rivier M (2005) Electricity market modeling trends. Energy Policy 33(7):897–913
VGE (2005) Jahrbuch der Europäischen energie- und Rohstoffwirtschaft 2006. Verlag Glückauf, Essen
Voorspools KR, D'haeseleer WD (2003) Long-term unit commitment optimisation for large power systems: unit decommitment versus advanced priority listing. Appl Energy 76:157–167
Weigt H (2006) A time-variant welfare economic analysis of a nodal pricing mechanism in Germany. Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-11. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_11_weigt_nodal_20%pricing_germany_time.pdf. Accessed 17 Feb 2009
Weigt H, Freund K, Jeske T (2006) Nodal pricing of the European electricity grid - a welfare economic analysis for feeding-in offshore wind electricity. Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-10. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_10_freund_weigt_jeske_nodal_20%pricing_nw_europe.pdf. Accessed 26 Nov 2007
Weigt H, Jeske T, Leuthold F, von Hirschhausen C (2010) Take the long way down: integration of large-scale north sea wind using HVDC transmission. Energy Policy 38(7):3164–3173
WSH (2005) All about wind energy. Wind Service Holland, Woudsend, The Netherlands. Internet: http://home.kpn.nl/windsh/english.html. Accessed 6 Feb 2009