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# R&D for Quality Improvement and Network Externalities

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**Abstract** We investigate the bearings of network externalities on product quality improvements requiring costly R&D investments. The model considers the dynamic behaviour of a monopolist alternatively maximising profits or social welfare. On the one hand, we confirm much of the acquired wisdom from the static literature on the same topic, about the arising of quality undersupply at the private optimum. On the other, we show that the monopoly optimum requires specific viability conditions, while the social optimum is always viable. We also show that the presence of network externalities affects the optimal investment behaviour of the profit-seeking firm but not that of a benevolent planner, who serves all consumers from the outset.

**Keywords** Monopoly · Network externality · Product quality

## 1 Introduction

The analysis of dynamic monopoly is a long standing issue, dating back to Evans (1924) and Tintner (1937), who investigated the pricing behaviour of a firm with convex costs. The analysis of intertemporal capital accumulation appeared later on (Eisner and Strotz 1963). However, several other aspects of

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monopoly behaviour have never been looked upon with the tools of optimal control theory. One such aspect is the provision of product quality, which has been debated in static models to highlight the monopolist's incentive to undersupply quality as compared to the social optimum (Spence 1975; Mussa and Rosen 1978; Itoh 1983; Gabszewicz et al. 1986; Besanko et al. 1987; Champsaur and Rochet 1989).

We develop a monopoly model where the firm may invest to increase quality over time, and consumers enjoy both the utility attached to intrinsic quality and a network effect, whereby the satisfaction of a generic consumer is increasing in the number of individuals purchasing the same good or service (see Cabral et al. 1999; Shy 2000).[1] In a static model with the same ingredients, it is shown that the monopolist trades off quality for quantity as the network effect becomes more relevant (Lambertini and Orsini 2001, 2003). Here, the dynamic formulation of the problem permits to single out some additional features of such a market. While on the one hand there exist precise viability conditions for the monopoly optimum, involving the relative size of product quality and network externalities, on the other hand the social optimum is always viable, without restrictions. As far as the extent of market coverage is concerned, we show that (i) the optimal (private) monopoly output is always increasing in the amount of externalities; yet (ii) the profit-seeking firm never covers the entire market, whatever the network effect is, while the planner serves all consumers from the outset to the steady state.

The remainder of the paper is structured as follows. The basic model is in Section 2. Section 3 contains the analysis of the profit-seeking monopoly equilibrium, while the comparison with the social planner's behaviour is investigated in Section 4. Concluding remarks are in Section 5.

## 2 The setup

Consider a monopoly market over an infinite (continuous) time horizon, $t \in [0, \infty)$. The firm supplies a single good but can modify its features so as to define its quality level. Here, quality is a hedonic index that summarises the presence of a set of desirable product characteristics.[2] Consumers are indexed by their marginal willingness to pay for quality, measured by parameter $\theta$,

[1]The analysis of quality supply is not new in optimal control models (see, e.g., Lambertini 2006, chs. 10 and 11 and the references therein). Our model is close in spirit to a stream of literature where product quality interacts with the formation of goodwill through advertising (see Feichtinger et al. 1994).

[2]This definition of the quality index associated to a given product dates back to Bresnahan (1981), who used it to estimate the performance of the automobile industry in the US.

uniformly distributed with density 1 over $[0, \overline{\theta}]$.[3] Accordingly, the size of the market is $\overline{\theta}$. The generic consumer at $\theta \in \left[0, \overline{\theta}\right]$ buys one unit of the good iff:

$$U(t) = \theta q(t) + \alpha y(t) - p(t) \geq 0 \tag{1}$$

where $p(t)$ and $q(t)$ are the price and the quality of the good supplied by the monopolist at time $t$; $\alpha y(t)$ is the network externality which is assumed to be linear in market demand $y(t)$, with $\alpha \geq 0$. When inequality Eq. (1) is reversed, the consumer located at $\theta$ does not buy and his utility is $U = 0$. Under the uniform consumer distribution and for a given triple $(p(t), q(t), y(t))$, the definition of demand is:

$$d(p(t), q(t), y(t)) = \overline{\theta} - \min\left\{\max\left\{0, \frac{p(t) - \alpha y(t)}{q(t)}\right\}, \overline{\theta}\right\} \tag{2}$$

implying

$$D(p(t), q(t)) = \begin{cases} \overline{\theta} \text{ for all } p(t) \leq \alpha\overline{\theta} \\ \dfrac{\overline{\theta}q(t) - p(t)}{q(t) - \alpha} \text{ for all } p(t) \in \left(\overline{\theta}\alpha, \overline{\theta}q(t)\right) \\ 0 \text{ for all } p(t) \geq \overline{\theta}q(t) \end{cases} \tag{3}$$

In the remainder, we will neglect the trivial cases of full coverage and zero demand, and focus on the case of partial market coverage, i.e., the price range $p(t) \in \left(\overline{\theta}\alpha, \overline{\theta}q(t)\right)$. Note that, in order to have a positive demand, the condition $q(t) > \alpha$ is required. Equivalently, partial coverage obtains if there is a marginal consumer at $\widehat{\theta}(t)$, who is indifferent between buying or not and identifies the lower bound of demand: $y(t) \equiv \overline{\theta} - \widehat{\theta}(t)$. By definition, the indifference condition writes:

$$\widehat{\theta}(t) q(t) + \alpha\left(\overline{\theta} - \widehat{\theta}(t)\right) - p(t) = 0 \Leftrightarrow \widehat{\theta}(t) = \frac{\alpha\overline{\theta} - p(t)}{\alpha - q(t)}. \tag{4}$$

Now, using $D(p(t), q(t)) = y(t) = \left[\overline{\theta}q(t) - p(t)\right] / \left[q(t) - \alpha\right]$ and solving w.r.t. the price, we obtain the inverse demand function:

$$p(t) = \overline{\theta}q(t) + (\alpha - q(t))\, y(t). \tag{5}$$

Quality improvement involves an R&D investment process summarised by the following differential equation, where, for the sake of simplicity, we assume R&D uncertainty away:

$$\dot{q} = bk(t) - \delta q(t), \; b > 0 \tag{6}$$

where $k(t)$ is the instantaneous investment and $\delta \in [0, 1]$ is a constant depreciation rate. This allows us to obtain a closed form solution. On economic grounds, the presence of depreciation can be justified by observing that the lack of innovative activities is seen from the consumers' standpoint as

[3] Parameter $\theta$ can be thought of as the reciprocal of the marginal utility of income, so that high-income consumers are indexed by high levels of $\theta$, and conversely for low-income consumers (see Tirole 1988, ch. 2).

the equivalent of product obsolescence. The instantaneous cost involved by investing $k(t)$ is $C(k(t)) = c[k(t)]^2$. For simplicity, we normalise the marginal production cost of output to zero. This involves no loss of generality concerning the qualitative properties of the ensuing analysis, as long as the production of the final good takes place at a constant marginal cost. Hence, instantaneous monopoly profits are:

$$\pi(t) \equiv p(t)\, y(t) - c[k(t)]^2 \tag{7}$$

and, given a constant discount rate $\rho$, the monopolist must choose $y(t)$ and $k(t)$ so as to maximise:[4]

$$\Pi \equiv \int_0^\infty \left\{ p(t)\, y(t) - c[k(t)]^2 \right\} e^{-\rho t} dt$$
$$s.t. : \dot{q} = bk(t) - \delta q(t)\ . \tag{8}$$

If instead the firm is run by a benevolent social planner, the scale of production and the intensity of R&D efforts are chosen to maximise the discounted flow of social welfare, defined as the sum of profits and consumer surplus. The latter, at any time $t$, corresponds to:

$$cs(t) \equiv \int_{\widehat{\theta}}^{\overline{\theta}} U(t)\, d\theta. \tag{9}$$

Therefore, the discounted stream of consumer surplus is:

$$CS \equiv \int_0^\infty cs(t)\, e^{-\rho t} dt. \tag{10}$$

Accordingly, the planner's problem is

$$\max_{y(t),k(t)} SW \equiv \Pi + CS \tag{11}$$

subject to Eq. (6).

## 3 Monopoly optimum

The Hamiltonian of the firm is:[5]

$$\mathcal{H}_M = e^{-\rho t} \Big\{ \left[\overline{\theta} q(t) + (\alpha - q(t))\, y(t)\right] y(t) - c[k(t)]^2 + \lambda(t)\left[bk(t) - \delta q(t)\right] \Big\} \tag{12}$$

[4]Since we are considering a monopolistic industry, the alternative between choosing price or quantity is of course immaterial. Additionally, note that the integral in Eq. (8) surely converges as $p(t)$ and $y(t)$ are both bounded above.

[5]An alternative but equivalent approach to solving the profit-seeking monopoly's problem is in the Appendix.

where $\lambda(t) = \mu(t)e^{\rho t}$, $\mu(t)$ being the co-state variable associated to quality. The initial and transversality conditions are $q(0) = q_0$ and

$$\lim_{t\to\infty} \mu(t)q(t) = 0. \tag{13}$$

The FOCs (first order condition) are (henceforth we omit the indication of time and discounting):[6]

$$\frac{\partial \mathcal{H}_M}{\partial k} = -2ck + b\lambda = 0 \tag{14}$$

$$\frac{\partial \mathcal{H}_M}{\partial y} = \overline{\theta} q + 2y(\alpha - q) = 0 \tag{15}$$

$$-\frac{\partial \mathcal{H}_M}{\partial q} = \dot{\lambda} - \rho\lambda \Rightarrow$$

$$\dot{\lambda} = \lambda(\rho + \delta) - y\left(\overline{\theta} - y\right). \tag{16}$$

FOC Eq. (14) yields:

$$\lambda = \frac{2ck}{b}\,;\ \dot{k} = \frac{b\dot{\lambda}}{2c}. \tag{17}$$

From Eq. (15), we have $y_M^* = \overline{\theta} q / \left[2(q - \alpha)\right]$ which is surely positive since $q > \alpha$. Moreover, it is immediate to check that $\partial y_M^* / \partial q \leq 0$ for all $\alpha \geq 0$.[7] On this basis, we can claim:

**Lemma 1** *The monopolist trades off quantity and quality along the equilibrium path, provided any positive network effect operates.*

The above Lemma illustrates what is by now a well known result in the static models on the interplay between network effects and product quality, according to which the presence of the externality, while inducing the monopolist to expand output, brings also about an otherwise undesirable reduction of the quality level (see, e.g., Lambertini and Orsini 2001, 2003). Here, we extend this conclusion to a dynamic setting.

Now we are in a position to characterise the steady state equilibrium. Using $y_M^*$, we may write the dynamics of the R&D investment as follows:

$$\dot{k} = \frac{8c(\rho + \delta)(\alpha - q)^2 k - \overline{\theta}^2 bq(q - 2\alpha)}{8c(q - \alpha)} \tag{18}$$

[6]Throughout the paper, we also omit the analysis of second order (concavity) condition, which are always satisfied at saddle point equilibria.

[7]Throughout the paper, we use stars to indicate optimal controls and states along the path to the steady state, and superscript *ss* to identify steady state levels.

and letting $\dot{k} = 0$ identify steady states, we get

$$k_M^{ss}(q) = \frac{\overline{\theta}^2 bq(q - 2\alpha)}{8c(\rho + \delta)(\alpha - q)^2} > 0 \,\forall\, q > 2\alpha. \tag{19}$$

From Eq. (6), $\dot{q} = 0$ in $q_M^* = bk/\delta$. Plugging it into Eq. (19), we have three steady state levels of the R&D effort: $k_{M1}^{ss} = 0$, which is not acceptable as it cannot counterbalance the effects of depreciation, and

$$k_{M2,3}^{ss} = \frac{16\alpha c\delta(\delta + \rho) + b^2\overline{\theta}^2 \mp b\overline{\theta}\sqrt{\Psi}}{16bc(\delta + \rho)} \tag{20}$$

where

$$\Psi \equiv b^2\overline{\theta}^2 - 32\alpha c\delta(\delta + \rho) \geq 0 \tag{21}$$

for all $\overline{\theta} \geq \sqrt{32\alpha c\delta(\delta + \rho)}/b$, which implies that the steady state solution is admissible provided that the market is sufficiently rich, i.e., the marginal willingness to pay for quality is high enough. On the basis of Eqs. (6) and (18), we can write the Jacobian matrix:

$$J_M \equiv \begin{bmatrix} \dfrac{\partial \dot{q}}{\partial q} & \dfrac{\partial \dot{q}}{\partial k} \\ \dfrac{\partial \dot{k}}{\partial q} & \dfrac{\partial \dot{k}}{\partial k} \end{bmatrix} \tag{22}$$

where:

$$\frac{\partial \dot{q}}{\partial q} = -\delta\,; \; \frac{\partial \dot{q}}{\partial k} = b \tag{23}$$

$$\frac{\partial \dot{k}}{\partial q} = -\frac{\alpha^2\overline{\theta}^2 b}{4c(q - \alpha)^3}\,; \; \frac{\partial \dot{k}}{\partial k} = \rho + \delta. \tag{24}$$

Hence, the trace and determinant of the Jacobian matrix $J_M$ are:

$$T(J_M) = \rho > 0$$

$$\Delta(J_M) = \frac{\alpha^2\overline{\theta}^2 b^2}{4c(q - \alpha)^3} - \delta(\rho + \delta) < 0. \tag{25}$$

Using $\Delta(J_M)$, one finds that $\left(q_M^*\left(k_{M3}^{ss}\right), k_{M3}^{ss}\right)$ is a saddle point, while the other steady state is an unstable focus.[8]

[8]The details are omitted for brevity.

The discussion carried so far establishes:

**Proposition 2** *Provided* $\Psi \geq 0$, *the monopolist reaches a unique saddle point equilibrium at*

$$k_M^{ss} = \frac{16\alpha c\delta\left(\delta+\rho\right) + b^2\overline{\theta}^2 + b\overline{\theta}\sqrt{\Psi}}{16bc\left(\delta+\rho\right)}$$

$$q_M^{ss} = \frac{b}{\delta}k_M^{ss}.$$

Note that, since $k_M^{ss} > 0$, then surely $q_M^{ss} > 2\alpha$. The associated price and output are:

$$y_M^{ss} = \frac{3b\overline{\theta} - \sqrt{\Psi}}{4b}\,;\ p_M^{ss} = \frac{b\overline{\theta}}{2\delta}k_M^{ss}. \tag{26}$$

On the basis of Eq. (26), without further proof, we can state:

**Corollary 3** *The steady state output of the profit-seeking monopolist is smaller than* $\overline{\theta}$ *in the whole admissible range of parameters.*

In other words, the monopolist always prices some consumers in the lower part of the income distribution out of consumption.

Now we consider the issue of introductory price offers, which has been largely discussed in the existing literature on network externalities.[9] The price dynamics obtains by differentiating the inverse demand function w.r.t. time:

$$\dot{p} = \frac{\dot{q}\left[2q\left(\overline{\theta}-y\right) - \alpha\left(\overline{\theta}-2y\right)\right]}{2\left(q-\alpha\right)} \tag{27}$$

which, using $y_M^*$, rewrites as $\dot{p} = \overline{\theta}\dot{q}/2 > 0$ as long as $\dot{q} > 0$. This entails the following corollary to Proposition 2:

**Corollary 4** *As long as the monopolist invests in R&D to increase quality, he also monotonically increases the price over time. That is, the firm makes an introductory price offer.*

Note that the initial offer also involves a relatively low quality, both price and quality being bound to increase over time up to the steady state. If instead $q_0 > q_M^{ss}$, the monopolist would have set $k = 0$ and let quality depreciate at rate $\delta$ towards $q_M^{ss}$; once $q$ had reached exactly $q_M^{ss}$, then the firm would just have to make up for depreciation by investing $k = \delta q_M^{ss}/b$. During the transition to this equilibrium, $\dot{q}$ and therefore also $\dot{p}$ would be negative,

[9]For an overview, see Shy (2000). For static and dynamic analyses of this aspect in spatial monopoly models, see Rohlfs (1974) and Lambertini and Orsini (2007), respectively.

whereby the monopolist would practice intertemporal price discrimination over the population of consumers.

Moreover, we can investigate the bearings of network effects on the steady state levels of controls, state and price:

$$\frac{\partial k_M^{ss}}{\partial \alpha} < 0; \ \frac{\partial y_M^{ss}}{\partial \alpha} > 0; \ \frac{\partial q_M^{ss}}{\partial \alpha} < 0; \ \frac{\partial p_M^{ss}}{\partial \alpha} < 0. \tag{28}$$

As the weight of the network externality increases, the steady state levels of R&D effort and quality shrink, since expanding the output is more convenient than increasing quality. To allow for a larger output, the price must be lower. In balance, the effects of a change in $\alpha$ on equilibrium price, output and quality entail that social welfare increases as the weight attached to network effects becomes larger.

## 4 Social optimum

While the problem of the private monopolist consists in choosing how many consumers to serve in order to maximise profits, given the cost structure we are considering, the planner surely covers the entire market provided that, by doing so, the resulting welfare level is positive. This fact can be easily shown using an intuitive argument, that runs as follows.

Observe that the planner can choose any price ensuring that the poorest consumer in the market is indeed able to buy. This requires:

$$U(t)|_{\theta=0} = -p(t) + \alpha\overline{\theta} \geq 0 \Leftrightarrow p(t) \in \left[0, \alpha\overline{\theta}\right]. \tag{29}$$

Conversely, any $p(t) > \alpha\overline{\theta}$ would entail partial market coverage. For all $p(t) \in \left[0, \alpha\overline{\theta}\right]$, i.e., under full coverage, instantaneous welfare results from the sum of profits $\pi(t) \equiv p(t)\overline{\theta} - c\left[k(t)\right]^2$ and consumer surplus

$$cs(t) \equiv \int_0^{\overline{\theta}} \left[\theta q(t) - p(t) + \alpha\overline{\theta}\right] d\theta = \frac{\overline{\theta}}{2}\left[\overline{\theta}\left(q(t) + 2\alpha\right) - 2p(t)\right]. \tag{30}$$

Therefore, we can write it as:

$$sw(t) = \frac{\overline{\theta}^2}{2}\left[q(t) + 2\alpha\right] - c\left[k(t)\right]^2. \tag{31}$$

The above expression, which is independent of instantaneous price, for any given pair $(q(t), k(t))$ is surely larger than the corresponding expression under partial coverage. This is true, independently of $\alpha$, as long as the production of the final output takes place at constant or increasing returns. Therefore, if Eq. (31) is positive, the planner surely covers the entire market from the very outset. Alternatively, under decreasing returns, the planner could in fact opt not to serve all consumers.

Accordingly, the planner's relevant Hamiltonian function is:[10]

$$\mathcal{H}_{SP} = e^{-\rho t}\left\{\frac{\overline{\theta}^2}{2}\left[q+2\alpha\right] - ck^2 + \beta\left(bk - \delta q\right)\right\} \tag{32}$$

From the above Hamiltonian we derive the following dynamics of the R&D effort:

$$\dot{k} = k\left(\rho+\delta\right) - \frac{b\overline{\theta}^2}{4c}. \tag{33}$$

Equation (33) shows that the planner's instantaneous investment along the equilibrium path is independent of quality. This stems from the fact that, all consumers being served at all times, the planner finds it convenient to fully smooth investment costs over time. Additionally, it is worth stressing that, in the present setting, the state and control equations are independent of $\alpha$ (and therefore also of the relative size of $q$ and $\alpha$) precisely because of full coverage.

Solving the system $\left\{\dot{q}=0, \dot{k}=0\right\}$, one finds the steady state levels of quality and R&D effort:

$$q_{SP}^{ss} = \frac{b^2\overline{\theta}^2}{4c\delta\left(\rho+\delta\right)},\; k_{SP}^{ss} = \frac{b\overline{\theta}^2}{4c\left(\rho+\delta\right)}. \tag{34}$$

Using the Jacobian matrix of the dynamic system, which is defined as in Eq. (22), we can calculate the trace and determinant, $T\left(J_{SP}\right) = \rho > 0$ and $\Delta\left(J_{SP}\right) = -\delta\left(\rho+\delta\right) < 0$ for all $\delta \in (0, 1]$. Therefore, the steady state Eq. (34) is stable in the saddle point sense. The foregoing discussion leads to

**Proposition 5** *The pair* $\left(q_{SP}^{ss}, k_{SP}^{ss}\right)$ *is a saddle point, unaffected by network externalities.*

Given that (i) the price level is not univocally defined, (ii) quality improvements hinge upon fixed costs only without interacting with the output level, and, above all, (iii) the planner operates under full coverage, the equilibrium R&D effort and quality are exactly the same that the planner would have chosen without network externalities.

A brief comparative assessment of the two regimes (profit-seeking monopoly and social planning) is now in order. Given that partial coverage prevails under monopoly while full coverage obtains from the outset under planning, obviously monopoly power gives rise to output distortion, as usual.

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[10] Again, the indication of time and the list of FOCs are omitted for brevity.

As to the supply of quality, a straightforward comparison of $q_{SP}^{ss}$ versus $q_M^{ss}$ yields:

$$q_{SP}^{ss} - q_M^{ss} \propto 3b^2\overline{\theta}^2 - 16\alpha c\delta\left(\delta + \rho\right) - b\overline{\theta}\sqrt{\Psi} \tag{35}$$

After simple manipulations, the r.h.s. of the above expression simplifies to

$$b^2\overline{\theta}^2 - 8\alpha c\delta\left(\delta + \rho\right)$$

Provided $\Psi \geq 0$ (so that the monopoly solution is acceptable), then it is easily checked that $q_{SP}^{ss} > q_M^{ss}$. Accordingly, the profit-seeking monopolist distorts quality downwards as compared to the social optimum. Moreover,

$$\frac{\partial\left(q_{SP}^{ss} - q_M^{ss}\right)}{\partial\alpha} = \frac{b\overline{\theta} - \sqrt{\Psi}}{\sqrt{\Psi}} > 0\,\forall\,\Psi \geq 0, \tag{36}$$

i.e., any increase in the weight attached to network externalities brings about an increase in quality distortion.

## 5 Conclusions

We have assessed the bearings of network effects on the incentive to improve product quality through costly R&D efforts in a monopoly market where consumers have different marginal willingness to pay for quality and the firm may alternatively maximise profits or social welfare.

The analysis has been carried out in a dynamic model where the firm chooses the extent of market coverage together with the quality-improving investment.

Our results can be summarised in the following terms. While confirming much of the existing wisdom from the static analysis of network externalities, the dynamic setting reveals some additional features that have remained neglected so far. Contrary to the result obtained in the static model (Lambertini and Orsini 2003), the monopolist never finds it profitable to cover the entire market, no matter how high the network externality can be. Provided that the viability condition for the monopoly optimum does hold, the profit-seeking firm's investment in quality improvement takes into account both the current quality level and the extent of the externality, while the planner's investment plan takes into account intertemporal parameters and hedonic preferences, being independent of the current quality level and the extent of network externality. The latter feature of the planner's behaviour becomes indeed intuitive by observing that it is socially optimal to serve all consumers in the market at every instant.

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## Appendix

Here we illustrate an alternative approach to the characterisation of the optimal behaviour of the profit-maximising monopolist.[11] Since the quantity choice of the firm is independent of the dynamics of product quality, one can (i) solve the quantity-setting choice at each given date and (ii) find the optimal investment rule using the appropriate Euler-Lagrange condition.

As the production of the final good takes place at zero costs, using the demand function we have that

$$p = \begin{cases} \alpha\overline{\theta} \text{ for all } q \in [\alpha, 2\alpha] \\ \dfrac{q\overline{\theta}}{2} \text{ for all } q > 2\alpha \end{cases} \tag{37}$$

and

$$\pi(q) = \begin{cases} \alpha\overline{\theta}^2 \text{ for all } q \in [\alpha, 2\alpha] \\ \dfrac{q^2\overline{\theta}^2}{4(q-\alpha)} \text{ for all } q > 2\alpha \end{cases} \tag{38}$$

Then, examining the system at the steady state, we have:

$$\dot{\lambda} = \lambda(\rho+\delta) - \frac{\partial\pi(q)}{\partial q}a3 \tag{39}$$

$$\dot{q} = \frac{b^2\lambda}{2c} - \delta q a4 \tag{40}$$

whereby

$$\frac{\partial\pi(q)}{\partial q} = \frac{2\delta c}{b^2}(\rho+\delta)q\,;\ \lambda = \frac{2\delta c}{b^2}q. \tag{41}$$

Now, since

$$\frac{\partial\pi(q)}{\partial q} = \frac{q\overline{\theta}^2(q-2\alpha)}{4(q-\alpha)^2}, \tag{42}$$

clearly any steady state where $q > 0$ satisfies $q > 2\alpha$. Moreover, noting that $\partial^2\pi(q)/\partial q^2 < 0$, it follows that this is locally a saddle point.

The same method can be also applied to solve the planner's problem.

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[11] This alternative was proposed by one of the referees, whom we warmly thank for the insightful suggestion.

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