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Multi-Period Near-Equilibrium in a Pool-Based Electricity Market Including On/Off Decisions

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Abstract

Based on the well-known concept of single-period equilibrium in an electricity market, this paper defines, analyzes and illustrates the concept of a multi-period equilibrium. Within this equilibrium framework and a multi-period horizon, market participants simultaneously optimize their respective individual and conflicting objectives. Constraints involving prices can be incorporated into the problems of the market participants. To avoid the limitations imposed by the necessary use of binary variables to model on/off decisions, the conditions to attain a multi-period equilibrium are formulated through Benders decomposition, which allows for efficiently solving the resulting equilibrium problem. The proposed procedure is illustrated using a realistic case study based on the IEEE Reliability Test System.

Keywords: electricity market, market equilibrium, multi-objective optimization, quadratic programming, restructured power system

1. Introduction

Based on the well-known concept of (single-period) equilibrium, this paper defines and analyzes the concept of a multi-period equilibrium in a pool-based electricity market.

In a pool-based electricity market, producers submit bids to the market operator consisting of energy blocks and their corresponding minimum selling prices for every hour of the market horizon and every unit (producer bidding stacks), while consumers submit energy blocks and their corresponding maximum buying prices for every hour of the market horizon and every demand (consumer bidding stacks). In turn, the market operator clears the market using an appropriate market-clearing procedure, which results in hourly prices, and production and consumption schedules. The market-clearing procedure may embody network

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constraints or not. We consider that it does, and the resulting prices are therefore nodal or locational marginal prices (LMP) (Schweppe et al., 1988). Background on electricity markets can be found in Ilic et al. (1998), Sheblé (1999) and Shahidehpour et al. (2002).

A single-period equilibrium is defined as the (single-period) producer/consumer energy transaction levels and their associated prices that result in maximum profit for every producer, maximum utility for every consumer, and maximum social welfare overall.

A multi-period equilibrium is defined as the producer/consumer energy transaction levels and their associated prices that result in maximum profit for every producer, maximum utility for every consumer, and maximum social welfare for the whole multi-period framework, while inter-temporal constraints including start-up and shut-down of units and ramping limits are enforced. Additionally, fixed, start-up and shut-down costs are considered (as modeled in Wang and Shahidehpour (1994) or Arroyo and Conejo (2000)).

The single-period equilibrium can be obtained considering the set of continuous optimization problems corresponding to the maximum profit of producers, maximum utility of consumers and maximum social welfare of the independent system operator (ISO), and corresponding optimality conditions (Bazaraa et al., 1993) resulting in a linear complementary problem (Cottle et al., 1992) easy to solve (Cottle et al., 1992; Hobbs, 2001; Júdice et al., 2002).

On the contrary, the multi-period equilibrium embodies binary decisions, i.e., starting up and shutting down units, and therefore optimality conditions cannot be directly applied. To avoid such limitations while retaining the advantages of using optimality conditions for each of the market participants, we define the multi-period equilibrium problem through Benders decomposition, which allows separating binary from continuous decisions.

In some electricity markets, a computationally challenging yet realistic aspect is that producers declare minimum profit requirements for their respective units. The minimum profit requirements are used in some markets, as in the electricity market of mainland Spain, OMEL (2005). We study the effect of imposing minimum profit requirements (or any other price-related nonlinear constraints) on generating units in a multi-period equilibrium model. Note that if constraints involving products of quantities and prices are included, the problem is no longer linear. Once the market is cleared, units not meeting profit constraints should be expelled from the market.

In short, this paper (i) defines the concept of a multi-period equilibrium, (ii) describes a procedure to formulate the conditions needed to attain it, (iii) describes techniques to include minimum profit conditions for generating units in such equilibrium, and (iv) proposes an efficient solution algorithm to compute the multi-period equilibrium considering minimum profit conditions. We believe these are novel contributions.

This paper can be seen as an extension to the previous work on single-period equilibrium by García-Bertrand et al. (2005) towards a multi-period framework through the use of decomposition techniques. This work is related to the body of literature pertaining to equilibrium analysis, which includes, among others, Hobbs (2001), Motto and Galiana (2002), Motto et al. (2002b) and O'Neill et al. (2005). Its contribution consists in the formulation of the conditions for a multi-period market equilibrium and a procedure to compute it, including minimum profit conditions for generating units. If minimum profit requirements for units are not included, the model we present is equivalent in a centralized environment to a multi-period optimal power flow (see, for instance, Baldick (1995), Ma and Shahidehpour (1999) or Alguacil and Conejo (2000)).

A multi-period market equilibrium tool is of interest for producers and consumers because it allows both of these groups to compute their respective market situations in equilibrium once the behavior of competitors has been hypothesized. This tool might also interest regulators for the strategic analysis of market procedures and rules.

The rest of this paper is organized as follows. In Section 2 the multi-period equilibrium problem is formulated using Benders decomposition. Section 3 provides an efficient solution technique to solve the problem formulated in 2. In Section 4 a realistic case study is analyzed and relevant results are reported. Specifically, the differences between a multi-period equilibrium without considering minimum profit conditions and a succession of single-period equilibria, and the effect of imposing minimum profit constraints to the multi-period equilibrium problem are discussed. Section 5 provides several noteworthy conclusions. Finally, the notation used throughout the paper is provided in an Appendix.

2. Formulation using Benders decomposition

The multi-period equilibrium problem includes continuous and binary variables and is formulated using the Benders decomposition technique (Benders, 1962) and (Geoffrion, 1972). In the current context, this technique decomposes the original problem into a mixed-integer linear programming problem and a nonlinear programming problem.

If binary variables are fixed to given values, the multi-period equilibrium problem, corresponding to the status for the generating units defined by binary variables, can be solved through a nonlinear programming problem, the subproblem. In turn, the master problem defines the on/off status for the generating units by solving for the corresponding binary variables.

The solution of the subproblem provides useful information about the quality of the values of the binary variables related to the on/off status of the units, defined at the master problem. In turns, this information is used by the master problem to refine the on/off status for the generating units of the producers. The Benders decomposition applied to the problem at hand is illustrated in figure 1. The formulations of the subproblem and the master problem are stated below.

2.1. Subproblem: Multi-period equilibrium for fixed status (binary) variables

The multi-period equilibrium is defined by the optimality conditions for the problems of the producers, for the problems of the consumers, and for the problem of the ISO, in every period of the market horizon, while satisfying coupling constraints. It should be noted that we use optimality conditions (complementarity theory) to be able to include constraints on prices, i.e., on dual variables; to be able to include minimum profit conditions for generating units. This feature enhances the capabilities of the model we propose.

A producer is the owner of one or more generating units located throughout the network, while a consumer exhibits one or more demands. Any generating unit is located at a node of the network and its production is described using several power blocks with associated

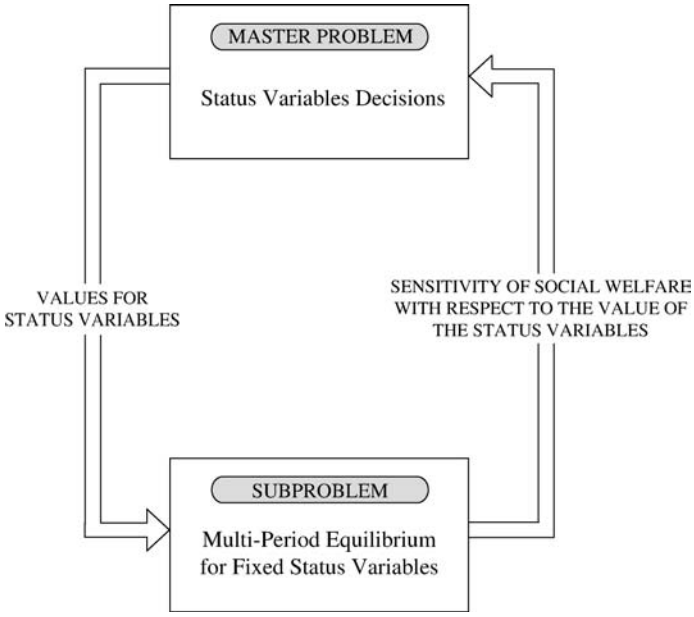

Figure 1. Structure of Benders decomposition.

linear production costs. Analogously, any demand is located at a node and its consumption is described using several power blocks with associated linear utilities. The ISO clears the market maximizing the net social welfare. The problems of the producers, the consumers and the ISO are defined below. Coupling constraints, which link a period with the following and previous ones, are also considered in the problems of the producers. For the sake of clarity, we consider a single unit and a single demand per node in the formulation. To consider more than one unit/demand per node is straightforward but at the cost of obtaining a more obscure formulation. However, in the case study, we consider a varying number of units/demands per node.

2.1.1. The problems of the producers. We first consider producer f that owns units indexed by the set G_f. We assume that the objective for producer f is to maximize its profit, which results in the following linear programming problem for the whole multi-period framework.

Maximize

$$\sum_{t\in T}\sum_{i\in G_f}\sum_{b=1}^{N_{Gi}}\left[\rho_i(t)-\lambda^{\mathrm{C}}_{Gib}(t)\right]P_{Gib}(t) \tag{1a}$$

subject to

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) \le P^{\max}_{Gi} v_i(t) : \alpha_i(t); \quad \forall i \in G_f; \quad \forall t \in T \tag{1b}$$

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) \ge P^{\min}_{Gi} v_i(t) : \beta_i(t); \quad \forall i \in G_f; \quad \forall t \in T \tag{1c}$$

$$P_{Gib}(t) \leq P_{Gib}^{\max}(t) : \phi_{ib}(t); \quad \forall i \in G_f; \quad b = 1, \ldots, N_{Gi} - 1; \quad \forall t \in T \tag{1d}$$

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) \leq R_i^{\mathrm{up}} v_i(t-1) + R_i^{\mathrm{su}}[v_i(t) - v_i(t-1)] + P_{Gi}^{\max}[1 - v_i(t)] : \tau_i(t); \quad \forall i \in G_f; \quad \forall t \in T \tag{1e}$$

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \leq R_i^{\mathrm{dn}} v_i(t) + R_i^{\mathrm{sd}}[v_i(t-1) - v_i(t)] + P_{Gi}^{\max}[1 - v_i(t-1)] : \psi_i(t); \quad \forall i \in G_f; \quad \forall t \in T \tag{1f}$$

$$P_{Gib}(t) \geq 0; \quad \forall i \in G_f; b = 1, \ldots, N_{Gi}; \quad \forall t \in T. \tag{1g}$$

We note that the decision variables of this problem are the amounts of power to be generated by each unit i in each block b, and time t, i.e., $P_{Gib}(t)$; and that the prices $\rho_i(t)$ are fixed values for the producer but variables in the larger overall equilibrium problem. Also, the status variables $v_i(t)$ are constants defined at the master problem level.

The objective function (1a) represents the total profit of producer f which is to be maximized subject to a capacity limit (1b) and a minimum power output (1c) for each unit and each period of time, a capacity limit (1d) for each block of each unit and each period of time, the available maximum and minimum power output of a unit taking into account the start-up and shut-down ramp limits, and the ramp-up and ramp-down limits, (1e) and (1f), respectively, and nonnegative levels of power to be generated by unit i in block b and time t, (1g). Note that constraints (1e) and (1f) link a period with the following and previous ones. Equation (1d) is not imposed on block N_{Gi} to avoid redundancy with Eq. (1b). The dual variables of constraints (1b), (1c), (1d), (1e) and (1f) are given respectively by $\alpha_i(t)$, $\beta_i(t)$, $\phi_{ib}(t)$, $\tau_i(t)$ and $\psi_i(t)$. Note that the dual variables of constraints appear in the right of these constraints following a colon in the problem formulation.

Fixed, start-up and shut-down costs are not incorporated in the objective function because these are constant values in this subproblem.

Due to the fact that binary variables are fixed, the optimality conditions (Bazaraa et al., 1993) for the problem of each producer decompose by unit, and are similar for all producers. Therefore, the conditions below comprise all units of all producers. These conditions can be interpreted as finding generation power block levels $P_{Gib}(t)$ and dual variables $\alpha_i(t)$, $\beta_i(t)$, $\phi_{ib}(t)$, $\tau_i(t)$ and $\psi_i(t)$ such that,

$$0 \leq \lambda_{Gib}^{\mathrm{C}}(t) - \rho_i(t) + \alpha_i(t) - \beta_i(t) + \phi_{ib}(t) + \tau_i(t) - \tau_i(t-1) + \psi_i(t-1) - \psi_i(t) \perp P_{Gib}(t) \geq 0; \quad \forall i \in G; b = 1, \ldots, N_{Gi}; \quad \forall t \in T \tag{2a}$$

$$0 \leq P_{Gi}^{\max} v_i(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \perp \alpha_i(t) \geq 0; \quad \forall i \in G; \quad \forall t \in T \tag{2b}$$

$$0 \leq \sum_{b=1}^{N_{Gi}} P_{Gib}(t) - P_{Gi}^{\min} v_i(t) \perp \beta_i(t) \geq 0; \quad \forall i \in G; \quad \forall t \in T \tag{2c}$$

$$0 \leq P_{Gib}^{\max} - P_{Gib}(t) \perp \phi_{ib}(t) \geq 0; \quad \forall i \in G; \quad b = 1, \ldots, N_{Gi} - 1; \quad \forall t \in T \tag{2d}$$

$$0 \leq R_i^{\mathrm{up}} v_i(t-1) + R_i^{\mathrm{su}}[v_i(t) - v_i(t-1)] + P_{Gi}^{\max}[1 - v_i(t)] - \sum_{b=1}^{N_{Gi}} P_{Gib}(t)$$
$$+ \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) \perp \tau_i(t) \geq 0; \quad \forall i \in G; \quad \forall t \in T \tag{2e}$$
$$0 \leq R_i^{\mathrm{dn}} v_i(t) + R_i^{\mathrm{sd}}[v_i(t-1) - v_i(t)] + P_{Gi}^{\max}[1 - v_i(t-1)] - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1)$$
$$+ \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \perp \psi_i(t) \geq 0; \quad \forall i \in G; \quad \forall t \in T. \tag{2f}$$

By convention, the symbol $\perp$ indicates that the product of the variable used to derive the corresponding optimality condition and the associated equation must be zero, i.e., $0 \leq x \perp y \geq 0$ is equivalent to $x\,y = 0$, $0 \leq x$ and $0 \leq y$.

2.1.2. The problems of the consumers. Next, we consider the problem of consumer q whose set of demands is indexed by D_q. We assume that such a consumer can be modeled as maximizing its economic utility for the whole multi-period framework, resulting in the following linear programming formulation.

Maximize

$$\sum_{t \in T} \sum_{i \in D_q} \sum_{k=1}^{N_{Di}} \left[\lambda_{Dik}^{\mathrm{U}}(t) - \rho_i(t)\right] P_{Dik}(t) \tag{3a}$$

subject to

$$\sum_{k=1}^{N_{Di}} P_{Dik}(t) \geq P_{Di}^{\min}(t) : \sigma_i(t); \quad \forall i \in D_q; \quad \forall t \in T \tag{3b}$$
$$P_{Dik}(t) \leq P_{Dik}^{\max}(t) : \varphi_{ik}(t); \quad \forall i \in D_q; k = 1, \ldots, N_{Di}; \quad \forall t \in T \tag{3c}$$
$$P_{Dik}(t) \geq 0; \quad \forall i \in D_q; \quad k = 1, \ldots, N_{Di}; \quad \forall t \in T. \tag{3d}$$

The objective function (3a) represents the economic utility for consumer q. Equation (3b) represents the minimum load that must be supplied, with the dual variable of this equation being $\sigma_i(t)$. Equation (3c) represents the maximum power in each block of each demand and each period of time, and its dual variable is $\varphi_{ik}(t)$. Equation (3d) imposes the constraint that the power to be consumed by demand i in block k in time t is nonnegative.

The optimality conditions (Bazaraa et al., 1993) for this optimization problem decompose by demand and by time, and are similar for each consumer. Therefore, the conditions below comprise all demands of all consumers. They can be interpreted as finding demand power blocks $P_{Dik}(t)$ and dual variables $\sigma_i(t)$ and $\varphi_{ik}(t)$ such that,

$$0 \leq \rho_i(t) - \lambda_{Dik}^{\mathrm{U}}(t) - \sigma_i(t) + \varphi_{ik}(t) \ \perp\ P_{Dik}(t) \geq 0;$$
$$\forall i \in D; k = 1, \ldots, N_{Di}; \quad \forall t \in T \tag{4a}$$

$$0 \le \sum_{k=1}^{N_{Di}} P_{Dik}(t) - P_{Di}^{\min}(t) \ \perp \sigma_i(t) \ge 0; \quad \forall i \in D; \quad \forall t \in T \tag{4b}$$

$$0 \le P_{Dik}^{\max}(t) - P_{Dik}(t) \ \perp \varphi_{ik}(t) \ge 0; \quad \forall i \in D; \ k = 1, \ldots, N_{Di}; \quad \forall t \in T. \tag{4c}$$

2.1.3. The problem of the ISO. Lastly, the ISO clears the market by seeking maximum social welfare for all time periods considered and enforcing transmission capacity limits. The problem of the ISO is formulated as the following linear program.

Maximize

$$\sum_{t\in T}\sum_{i\in D}\sum_{k=1}^{N_{Di}} \lambda_{Dik}^{\mathrm{B}}(t)\tilde{P}_{Dik}(t) - \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{B}}(t)\tilde{P}_{Gib}(t) \tag{5a}$$

subject to

$$-\sum_{b=1}^{N_{Gi}} \tilde{P}_{Gib}(t) + \sum_{k=1}^{N_{Di}} \tilde{P}_{Dik}(t) + \sum_{j\in\Omega_i} B_{ij}[\delta_i(t) - \delta_j(t)] = 0 : \rho_i(t);$$
$$\forall i \in N; \quad \forall t \in T \tag{5b}$$

$$B_{ij}[\delta_i(t) - \delta_j(t)] \le P_{ij}^{\max} : \gamma_{ij}(t); \quad \forall i \in N; \quad \forall j \in \Omega_i; \quad \forall t \in T \tag{5c}$$

$$\tilde{P}_{Gib}(t) - P_{Gib}(t) = 0 : \mu_{Gib}(t); \quad \forall i \in G; \ b = 1, \ldots, N_{Gi}; \quad \forall t \in T \tag{5d}$$

$$\tilde{P}_{Dik}(t) - P_{Dik}(t) = 0 : \upsilon_{Dik}(t); \quad \forall i \in D; \ k = 1, \ldots, N_{Di}; \quad \forall t \in T \tag{5e}$$

$$\delta_i(t) \le 2\pi : \Gamma_i(t); \quad \forall i \in N; \quad \forall t \in T \tag{5f}$$

$$\delta_i(t) \ge 0; \quad \forall i \in N; \quad \forall t \in T. \tag{5g}$$

The objective function (5a) is the net social welfare (Takayama and Judge, 1971). It is subject to enforcing power balance at every node (5b), line capacity limits (5c), that the power generated and demanded in the problem of the ISO are equal to the power generated and demanded in the problems of the producers and consumers, (5d) and (5e), respectively, and bounds on voltage angles, (5f) and (5g).

Note that power generation and demand variables are replicated to make the problems of the producers and consumers compatible with the problem of the ISO.

Note that security constraints, either deterministic (Motto et al., 2002a) or probabilistic (Bouffard and Galiana, 2004), can be incorporated into the formulation of the problem of the ISO. Moreover, transmission losses can also be incorporated (Yuandong and Hobbs, 1998; Motto et al., 2002a; de la Torre et al., 2003). Based on numerical experience, adding losses does not significantly change results. Losses are considered in the case study in Section 4. However, losses are not taken into account in the formulation presented above for the sake of clarity.

The optimality conditions (Bazaraa et al., 1993) for problem (5) are to find the generation power blocks levels $\tilde{P}_{Gib}(t)$, the demand power blocks levels $\tilde{P}_{Dik}(t)$, voltage angle $\delta_i(t)$ and dual variables $\rho_i(t)$, $\gamma_{ij}(t)$, $\mu_{Gib}(t)$, $\upsilon_{Dik}(t)$ and $\Gamma_i(t)$ such that

$$0 = \lambda_{Gib}^{\mathrm{B}}(t) - \rho_i(t) + \mu_{Gib}(t); \quad \forall i \in G; \ b = 1, \ldots, N_{Gi}; \quad \forall t \in T \tag{6a}$$

$$0 = \rho_i(t) - \lambda_{Dik}^{\mathrm{B}}(t) + \upsilon_{Dik}; \quad \forall i \in D; \ k = 1, \ldots, N_{Di}; \quad \forall t \in T \tag{6b}$$

$$0 \leq \sum_{j \in \Omega_i} [B_{ij}(\rho_i(t) - \rho_j(t)) + B_{ij}(\gamma_{ij}(t) - \gamma_{ji}(t))] + \Gamma_i(t) \perp \delta_i(t) \geq 0;$$
$$\forall i \in N; \quad \forall t \in T \tag{6c}$$

$$-\sum_{b=1}^{N_{Gi}} \tilde{P}_{Gib}(t) + \sum_{k=1}^{N_{Di}} \tilde{P}_{Dik}(t) + \sum_{j \in \Omega_i} B_{ij}[\delta_i(t) - \delta_j(t)] = 0; \quad \forall i \in N; \quad \forall t \in T \tag{6d}$$

$$0 \leq P_{ij}^{\max} - B_{ij}[\delta_i(t) - \delta_j(t)] \perp \gamma_{ij}(t) \geq 0; \quad \forall i \in N; \quad \forall j \in \Omega_i; \quad \forall t \in T \tag{6e}$$

$$\tilde{P}_{Gib}(t) - P_{Gib}(t) = 0; \quad \forall i \in G; b = 1, \ldots, N_{Gi}; \quad \forall t \in T \tag{6f}$$

$$\tilde{P}_{Dik}(t) - P_{Dik}(t) = 0; \quad \forall i \in D; k = 1, \ldots, N_{Di}; \quad \forall t \in T \tag{6g}$$

$$2\pi - \delta_i(t) \geq 0 \perp \Gamma_i(t) \geq 0; \quad \forall i \in N; \quad \forall t \in T. \tag{6h}$$

Free dual variables, $\tilde{P}_{Gib}(t)$, $\tilde{P}_{Dik}(t)$, $\rho_i(t)$, $\mu_{Gib}(t)$ and $\upsilon_{Dik}(t)$, are associated with Eqs. (6a), (b), (d), (f) and (g), respectively.

2.1.4. Minimum profit condition. Next, we consider the minimum profit condition below for each generating unit that declares such a condition. This minimum profit condition can be used to internalize fixed and other costs that do not directly appear in the bidding stack. Minimum profit conditions have the form,

$$\sum_{t \in T} \sum_{b=1}^{N_{Gi}} \left[\rho_i(t) - \lambda_{Gib}^{\mathrm{C}}(t)\right] P_{Gib}(t) - \sum_{t \in T} \left(C_{\mathrm{Gi}}^{\mathrm{fx}} v_i(t) + c_{\mathrm{Gi}}^{\mathrm{su}}(t) + c_{\mathrm{Gi}}^{\mathrm{sd}}(t)\right) \geq K_i; \quad \forall i \in G^{\mathrm{on}} \tag{7}$$

where K_i is a positive constant that represents the minimum profit for generating unit i for the whole multi-period framework.

Note that condition (7) only enforces minimum profit when the unit is online. If the minimum profit condition is very restrictive, the generating unit might be expelled from the market.

Note also that Eq. (7) is nonlinear because the left-hand side is the sum of bilinear terms. Minimum profit constraints such as (7) are used in some actual markets (OMEL, 2005) to ensure peaker profitability and to promote generation capacity investment.

2.1.5. Formulation of the subproblem. The market equilibrium, if binary variables are fixed to given values, is determined by the mixed-linear complementarity problem (LCP) defined by the optimality conditions for the problems of the producers, the consumers and the ISO, conditions (2), (4) and (6), respectively. This mixed-linear complementarity problem can be solved using an equivalent quadratic programming problem (Cottle et al., 1992). This quadratic problem is extended to include the minimum profit constraints for the units that declare such condition, turning the subproblem into a nonlinearly constrained nonlinear programming problem. The above is done considering all time periods. Moreover, to facilitate the decomposition and to improve computational behavior, social welfare is subtracted from the objective function. Note that subtracting this term does not alter the solution of the problem. That is, minimize

$$Z_{\mathrm{ME}} - \sum_{t\in T}\sum_{i\in D}\sum_{k=1}^{N_{Di}} \lambda^{\mathrm{B}}_{Dik}(t)P_{Dik}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib}(t)P_{Gib}(t) \tag{8a}$$

subject to

1. Optimality conditions of all problems of producers: constraints (2).
2. Optimality conditions of all problems of consumers: constraints (4).
3. Optimality conditions of the problem of the ISO: constraints (6).
4. Minimum profit conditions

$$\sum_{t\in T}\sum_{b=1}^{N_{Gi}} \left[\rho_i(t) - \lambda^{\mathrm{C}}_{Gib}(t)\right]P_{Gib}(t) - \sum_{t\in T}\left(C^{\mathrm{fx}}_{\mathrm{Gi}}\bar{v}_i(t) + c^{\mathrm{su}}_{\mathrm{Gi}}(t) + c^{\mathrm{sd}}_{\mathrm{Gi}}(t)\right) \geq K_i; \quad \forall i \in G^{\mathrm{on}}. \tag{8b}$$

5. Fixed binary variables:

$$\bar{v}_i(t) = v_i^{(\nu)}(t) \ : \kappa_{vi}(t); \quad \forall i \in G; \quad \forall t \in T. \tag{8c}$$

The first term of the objective function (8a), Z_{ME}, corresponds to the summation of all the inequality constraints of (2), (4) and (6), multiplied by their respective dual variables. Note that Z_{ME} should be zero at an optimal solution, corresponding to an actual LCP solution.

The last Eq. (8c), enforces the on/off status of the units to the values obtained in the master problem at the present iteration. Note that superscript ν is the iteration counter for the overall problem.

It should be noted that the term that expressed the social welfare

$$\sum_{t\in T}\sum_{i\in D}\sum_{k=1}^{N_{Di}} \lambda^{\mathrm{B}}_{Dik}(t)P_{Dik}(t) - \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib}(t)P_{Gib}(t)$$

is subtracted from the objective function to avoid a null objective function value that might provoke convergence problems in the Benders master problem. Recall that the Benders master problem reconstructs from below the objective function of the problem expressed as a function of the complicating variables. Numerically, we have experienced a better behavior subtracting the social welfare term shown above.

2.2. *Master problem*

The master problem provides the on/off status for the associated generating units by giving values to the binary variables. This problem refines the on/off status for the generating units of the producers using information provided by the solutions of the subproblem. Additionally, fixed, start-up and shut-down costs are incorporated into the objective function of this problem. The master problem at iteration ν is defined by the following mixed-integer linear programming problem.

Minimize

$$\alpha + \sum_{t \in T} \sum_{i \in G} \left(C_{Gi}^{\mathrm{fx}} v_i(t) + c_{Gi}^{\mathrm{su}}(t) + c_{Gi}^{\mathrm{sd}}(t) \right) \tag{9a}$$

subject to

1. Benders cuts:

$$\alpha \geq Z_{\mathrm{SUB}}^{(\ell)} + \sum_{t \in T} \sum_{i \in G} \left(\kappa_{vi}^{(\ell)}(t) \left[v_i(t) - v_i^{(\ell)}(t) \right] \right); \quad \ell = 1, \ldots, \nu - 1. \tag{9b}$$

2. Start-up and shut-down cost constraints for generating units:

$$c_{Gi}^{\mathrm{su}}(t) \geq C_{Gi}^{\mathrm{su}} [v_i(t) - v_i(t-1)]; \quad \forall i \in G; \quad \forall t \in T \tag{9c}$$

$$c_{Gi}^{\mathrm{su}}(t) \geq 0; \quad \forall i \in G; \quad \forall t \in T \tag{9d}$$

$$c_{Gi}^{\mathrm{sd}}(t) \geq C_{Gi}^{\mathrm{sd}} [v_i(t-1) - v_i(t)]; \quad \forall i \in G; \quad \forall t \in T \tag{9e}$$

$$c_{Gi}^{\mathrm{sd}}(t) \geq 0; \quad \forall i \in G; \quad \forall t \in T. \tag{9f}$$

3. Feasibility conditions:

$$\sum_{i \in G} v_i(t) P_{Gi}^{\max} \geq \sum_{i \in D} P_{Di}^{\min}(t); \quad \forall t \in T. \tag{9g}$$

4. Lower limits for α:

$$\alpha \geq \alpha^{\min}. \tag{9h}$$

The objective function (9a) includes α, which is a lower bound approximation of the objective function of the multi-period equilibrium problem, and fixed, start-up and shut-down costs. The set of constraints (9b) are called the Benders cuts. These cuts provide information to the master problem to improve the on/off status decisions. Note that $Z_{\mathrm{SUB}}^{(\ell)}$ represents the objective function value of the subproblem at iteration ℓ. Constraints (9c)–(9f) state start-up and shut-down cost constraints of the generating units in each time period. Constraints (9g) force the master problem to generate solutions that satisfy the minimum demand requirements. These constraints ensure the feasibility of the subproblem. Finally, constraint (9h) states a lower bound for α. The solution of this problem defines the on/off status of each unit of each producer in each time period, $v_i(t)$.

3. Solution technique

The proposed solution technique is described in this section.

3.1. Solution algorithm

The multi-period equilibrium problem as stated in the previous section is a large-scale problem that includes continuous and binary variables and is defined and solved using the

Benders decomposition method (Benders, 1962; Geoffrion, 1972). The master problem defines the on/off status for the generating units fixing the corresponding binary variables. The subproblem is a multi-period equilibrium problem with the binary variables fixed to given values by the master problem. In turn, the master problem refines the on/off status for the generating units using the sensitivity of social welfare with respect to the value of the status variables defined at the master problem in the previous iteration. This iterative procedure continues until some cost tolerance is reached. The steps of the algorithm are:

Step 1. Once binary variables are fixed to specified feasible values, the resulting continuous multi-period equilibrium problem, that is, the subproblem, is solved for its continuous variables. The subproblem is a nonlinear program and is solved using one of the three procedures indicated in 3.2.

Step 2. Using marginal information obtained in Step 1, the master problem finds improved values for the binary variables fixed in Step 1.

Step 3. The coordinated iteration of Steps 1 and 2 (Benders decomposition algorithm), allows attaining an optimum in both continuous and binary variables within the whole multi-period market horizon.

The master problem and the subproblem are iteratively solved until a convergence tolerance ε is met, i.e., until

$$\left| Z_{\mathrm{ME}}^{(\nu)} - \sum_{t \in T} \sum_{i \in D} \sum_{k=1}^{N_{Di}} \lambda_{Dik}^{\mathrm{B}}(t) P_{Dik}^{(\nu)}(t) - \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{B}}(t) P_{Gib}^{(\nu)}(t) - \alpha^{(\nu)} \right| \leq \varepsilon. \quad (10)$$

3.2. *Solution of the subproblem*

The subproblem is a nonlinearly constrained nonlinear problem difficult to solve. The main difficulty lies in the nonlinearity of the minimum profit conditions. Three alternative procedures can be used to solve this problem.

(a) To directly solve the subproblem using an appropriate nonlinear solver.
(b) To linearize the nonlinear minimum profit constraints using the Schur's decomposition and binary variables as stated in García-Bertrand et al. (2005), and to solve the resulting mixed-integer quadratic problem.
(c) To use the inner algorithm stated below, which is based on a successive over-relaxation. The description of this method is as follows.

Step 1.1. Without considering the minimum profit conditions, the subproblem is solved. The solution of this problem is used to compute an initial estimate of the generating power. That is,

$$\hat{P}_{Gib}^{(1)}(t) = a \bar{P}_{Gib}^{(1)}(t); \quad \forall i \in G^{\mathrm{on}}; \quad \forall t \in T \quad (11)$$

where $\hat{P}_{Gib}^{(1)}(t)$ represents the initial estimate of the generating power of block b of unit i in hour t; $\bar{P}_{Gib}^{(1)}(t)$ is the optimal generating power value of block b of generating unit i in hour t for the subproblem; and $a \geq 1$ is a constant. This constant does not change with each iteration.

Step 1.2. The generating power values appearing in the minimum profit conditions are fixed to the corresponding estimated values. Therefore, the minimum profit conditions turn into linear expressions and the subproblem becomes a quadratic program which is solved. The minimum profit conditions considered in the subproblem have the form,

$$\sum_{t \in T} \sum_{b=1}^{N_{Gi}} \left(\rho_{n(i)}(t) - \lambda^{\mathrm{C}}_{Gib}(t)\right) \hat{P}^{(\eta)}_{Gib}(t) - \sum_{t \in T} \left(C^{\mathrm{fx}}_{\mathrm{Gi}} \bar{v}_i(t) + c^{\mathrm{su}}_{\mathrm{Gi}}(t) + c^{\mathrm{sd}}_{\mathrm{Gi}}(t)\right) \geq K_i; \quad \forall i \in G^{\mathrm{on}} \qquad (12)$$

where η represents the iteration counter of this inner iterative procedure. Counter η is initialized to 1 at the start of the algorithm. The optimal generating power values for this problem are $\bar{P}^{(\eta+1)}_{Gib}(t)$.

Step 1.3. Update the estimate of the generating power through the equation,

$$\hat{P}^{(\eta+1)}_{Gib}(t) = d\,\bar{P}^{(\eta+1)}_{Gib}(t) + (1-d)\hat{P}^{(\eta)}_{Gib}(t); \quad \forall i \in G^{\mathrm{on}}; \quad \forall t \in T \qquad (13)$$

where the constant $d \in (0, 1)$. Note that constant d does not change with each iteration. If for all $i \in G^{\mathrm{on}}$, $\sum_{t \in T} |\frac{\hat{P}^{(\eta+1)}_{Gib}(t) - \hat{P}^{(\eta)}_{Gib}(t)}{\hat{P}^{(\eta)}_{Gib}(t)}| \leq \epsilon$, stop, the solution has been found and corresponds to the solution of Step 1.2; the inner algorithm concludes and the procedure continues in Step 2 of the outer algorithm. If this is not the case, the iteration counter is updated, $\eta \leftarrow \eta + 1$ and the algorithm continues in Step 1.2.

Note that ϵ is an appropriate tolerance.

From an experimental point of view, this successive over-relaxation algorithm presents good convergence behavior. A characterization of its convergence characteristic can be constructed based on results reported in Golub and Loan (1996), Saad (1996), Conejo et al. (2002) or Nogales et al. (2003).

3.3. Problem size

The master problem is a mixed-integer linear programming problem whose number of variables and constraints are indicated in Table 1. Analogously, the subproblem is a nonlinear programming problem but it is solved through an iterative method, the inner algorithm, that requires the solution of a quadratic programming problem in each iteration. The number of variables and constraints of this quadratic programming problem are also shown in Table 1.

In Table 1, N_G and N_D represent the total number of units and demands in the system, respectively, $N_{G^{\mathrm{on}}}$ represents the number of generating units that impose minimum profit conditions and remain on-line during at least a period of time along the market horizon, N_{GB} and N_{DB} represent the total number of blocks offered by all units and demanded by all demands, respectively, N and N_L represent the total number of nodes and lines of the system, respectively, and N_T represents the number of time periods considered.

Table 1. Size of problems.

	Number of continuous variables	Number of binary variables	Number of constraints
Master problem	$1 + 2N_G N_T$	$N_G N_T$	$4N_G N_T + N_T + \nu$
Subproblem	$4N_T(N_{GB} + N_{DB})$ $+N_T(4N_G + N_D)$ $+N_T(3N + N_L)$	—	$4N_T(N_{GB} + N_{DB})$ $+N_T(5N_G + N_D)$ $+N_T(3N + N_L) + N_{G^{\text{on}}}$

3.4. *Feasibility*

In what follows, we discuss the effect of imposing minimum profit conditions on the multi-period equilibrium problem. There are two cases that are treated below.

3.4.1. Degenerate case. The multi-period equilibrium problem might be degenerate and therefore have multiple prices (dual solutions) for any given time period. This case is illustrated in figure 2(a) for a given time period. Note that there is a range of prices at which the power supplied is equal to the power demanded. In this situation, minimum

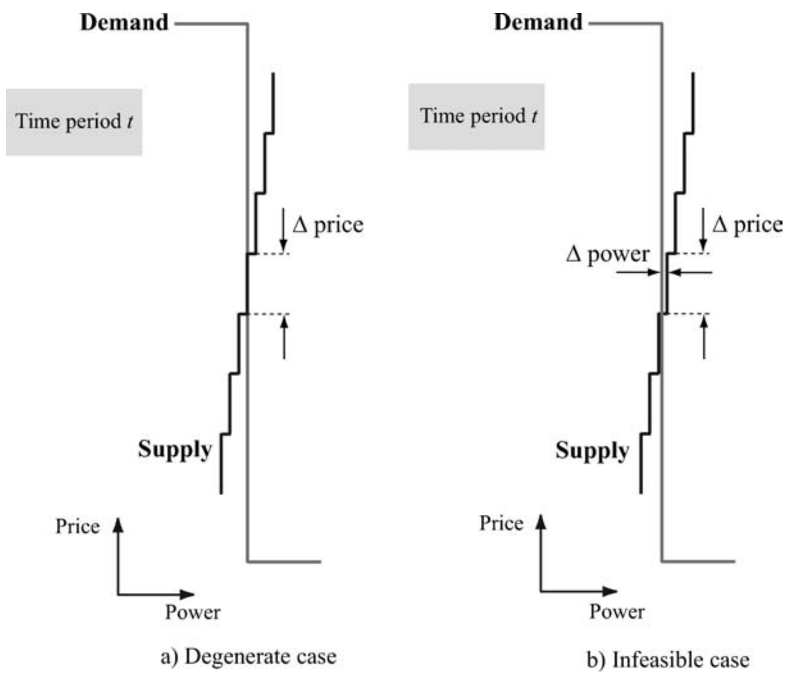

Figure 2. Degenerate and infeasible cases.

profit-constrained, multi-period equilibrium problem generally results in a feasible problem whose optimal solution meets minimum profit conditions.

3.4.2. Infeasible case. The multi-period equilibrium problem has a unique solution in prices, generations, demands and flows. By adding minimum profit constraints to this problem, we simply create infeasibilities, assuming that the minimum profit condition was not attained for this unique solution beforehand. However, it should be noted that for practical applications, these infeasibilities are generally negligible. The reason for that is as follows. Power supply curves tend to be "hockey-stick" shaped around the market-clearing price in most practical markets, while demand curves are rather inelastic around the market-clearing price. The supply curve has a hockey-stick shape because many bids are made at zero price to ensure acceptance. The demand curve is rather inelastic due to the nature of electricity consumption. Moreover, the number of steps in the supply curve is usually large. The conclusion is that the multi-period equilibrium satisfying minimum profit conditions is often in the vicinity of these "near-degeneracy" regions, in which there is a unique multi-period equilibrium albeit with steep supply and demand curves as illustrated in figure 2(b). In this figure, it can be observed that small increments in power (which create small infeasibilities) result in significant price differences (which allow minimum profit conditions to be easily met).

Small infeasibilities cause an optimal objective function value, Z_{ME}, to be slightly different from zero in the subproblem. These small infeasibilities are related to prices because power balance is enforced at every node and in every time period. The cost incurred due to price infeasibilities can be allocated pro-rata among market participants (see García-Bertrand et al., 2005).

Considering the discussion above, we define a near-equilibrium as an optimal solution of a minimum profit-constrained, multi-period equilibrium problem that results in an optimal objective function value, Z_{ME}, slightly different from zero. This implies that one or more of the complementarity conditions of the equilibrium problem are slightly not satisfied.

Thus, in general, the optimal solution of the multi-period equilibrium problem represents a near-equilibrium that may include small complementary infeasibilities of negligible practical significance. An appropriate metric for the importance of such infeasibilities is the optimal value of the objective function, Z_{ME}. The closer to zero this value is, the closer to solving the minimum profit-constrained, multi-period equilibrium problem.

4. Case study

A case study based on the 24-node IEEE Reliability Test System (RTS) is presented in this section. The transmission network consists of 24 nodes connected by 38 lines and transformers. The transmission lines are at two voltages, 138 and 230 kV. There are 32 generating units connected throughout the network, with two nuclear units, six hydraulic units and the rest thermal units. The maximum power generated by the total of the generating units is 3405 MW. There is an electricity demand in 17 nodes of the network. Also, we consider that every generating company only owns one generating unit, and every consumer has one demand; therefore there are 32 generating companies and 17 consumers. The market

Table 2. Size of problems of the case study.

	Number of continuous variables	Number of binary variables	Number of constraints
Master problem	1537	768	3097
Subproblem	21576	—	22376

horizon considered comprises 24 time periods. Topology, line and generating unit data can be found in Reliability Test System Task Force (1999) (figure 1 and Tables 12 and 9, respectively, of this reference). The transmission limit of line 14–16 is reduced to 380 MW in our study (instead of 500 MW) so that congestion occurs. The size and price of each block of each generating unit and of each demand, and the minimum demand requirement are provided in García-Bertrand et al. (2005) (Tables 1 and 3, respectively, of this reference). For simplicity, in this case study, price bids and costs do not change throughout the time periods. A similar consideration applies to consumers. Ramp rates and start-up costs can be found in Reliability Test System Task Force (1999) (Tables 8 and 10 of this reference, respectively). Shut-down costs are considered to be zero. Fixed costs are 5.25 $/h for units with 12 MW of capacity, 5 $/h for 20 MW units, 7.5 $/h for 76 MW units, 8.5 $/h for 100 MW units, 6.25 $/h for 155 MW units, 15 $/h for 197 MW units, 20 $/h for 350 MW units, and 0 $/h for the rest.

The size of the master problem and the subproblem for this case are illustrated in Table 2.

We obtain the multi-period equilibrium of the 24-node IEEE RTS considering that generating units 3 and 4 each imposes a minimum profit requirement of $9000 and generating units 9, 10 and 11 of $200. The rest of the generating units of the system impose a minimum profit requirement of $0. These values are represented in column 2 of Table 3. This multi-period equilibrium has been obtained through the solution algorithm explained in Section 3.1. The subproblem has been solved using the successive over-relaxation algorithm stated in Section 3.2 because it converges faster than the other two algorithms reported in that subsection.

Table 3 provides results of the multi-period equilibrium problem concerning generating unit production and profits for the whole time horizon for two cases. In the first case, no generating unit is allowed to impose minimum profit conditions (MPC), corresponding to columns 3 and 4 in the table. In the second case, generating units impose the minimum profit conditions specified above, corresponding to columns 5 and 6 of the table. Note that in the first case, the profit for unit 4 is lower than $9000, and the profit for unit 10 is lower than $200. In the second case, we force the condition that if these generating units are running in any period of time of the market horizon, they must have profits at least equal to the minimum value they have imposed, that is, $9000 and $200 respectively. Also, note that in this second case, generating power is redistributed so that the minimum profit requirements are satisfied. In fact, generating unit 4 increases its production and as a consequence its profit increases. Generating unit 10 decreases its production but changes the periods of time when it is producing; that is, this unit now produces power in periods of time with higher prices.

Table 3. Results for generating units without and with minimum profit conditions.

		Multi-period equilibrium without MPC		Multi-period equilibrium with MPC	
Unit	Minimum profit requirement (k$/h)	Total energy (MWh)	Profit (k$/h)	Total energy (MWh)	Profit (k$/h)
1, 2	0.00	0.00	0.00	0.00	0.00
3	9.00	1379.64	9.27	1511.14	9.67
4	9.00	1314.80	8.86	1478.80	9.67
5, 6	0.00	0.00	0.00	0.00	0.00
7	0.00	1395.81	8.89	1470.90	9.72
8	0.00	1466.69	9.34	1530.20	9.72
9	0.20	691.10	0.53	0.00	0.00
10	0.20	1111.80	0.17	1040.00	0.98
11	0.20	1094.43	1.13	1271.47	1.86
12, 13	0.00	0.00	0.00	0.00	0.00
14	0.00	1700.29	0.74	1955.29	1.49
15–19	0.00	0.00	0.00	0.00	0.00
20	0.00	3385.76	23.31	3513.17	24.13
21	0.00	3472.54	23.66	3534.00	24.50
22	0.00	9600.00	104.01	9600.00	106.15
23	0.00	9600.00	102.54	9600.00	104.66
24–29	0.00	1200.00	18.73	1200.00	18.99
30	0.00	3565.00	23.62	2883.00	22.59
31	0.00	3565.00	23.62	3541.19	24.58
32	0.00	7699.60	51.09	7880.12	53.20

Table 4 provides results of the multi-period equilibrium when minimum profit conditions are considered including generating unit profits and revenues, demand costs and minimum and maximum locational marginal prices for each period of time. These results are discussed below. Observe that locational marginal prices are different at different nodes due to congestion during hours 1–24 in line 14–16 (at 380 MW), which split the system into two areas, one with an excess of inexpensive generation and another one with expensive generation. There is a low and approximately constant demand during hours 1–6 that causes low locational marginal prices throughout the system because the on-line generating units are the more inexpensive. In hours 7, 8 and 9, demand increases sharply forcing more expensive generating units to start-up, increasing locational marginal prices in the system. In hours 10–21, demand is high and approximately constant, and extra units with high costs start up to supply the demand. Finally, demand in hours 22–24 decreases sharply, forcing the more expensive generating units to shut down and/or to decrease the generation, producing a decrease in the locational marginal prices of the system. Note that unit profits are higher

Table 4. Multi-period equilibrium with minimum profit conditions.

Hour	Unit profit (k$/h)	Unit revenue (k$/h)	Demand cost (k$/h)	Minimum LMP ($/MWh)	Maximum LMP ($/MWh)
1	8.17	23.87	24.74	10.53	14.20
2	7.61	21.55	22.40	10.15	13.70
3	7.45	20.44	21.17	10.06	13.13
4	7.39	20.03	20.76	10.04	13.09
5	7.39	20.03	20.76	10.04	13.09
6	7.45	20.44	21.17	10.06	13.13
7	22.25	39.31	40.62	16.59	22.28
8	24.94	48.78	49.84	17.97	22.65
9	29.61	53.54	54.64	18.97	23.84
10	31.61	56.04	57.14	19.68	24.67
11	28.70	53.67	54.76	18.82	23.07
12	28.17	52.44	53.50	18.44	22.60
13	28.17	52.44	53.50	18.44	22.60
14	28.17	52.44	53.50	18.44	22.60
15	27.49	51.76	52.71	18.22	22.22
16	28.04	52.31	53.26	18.41	22.45
17	31.81	57.94	58.85	19.78	23.24
18	32.24	58.87	59.77	19.94	23.24
19	32.24	58.87	59.77	19.94	23.24
20	29.28	53.67	54.76	18.82	23.07
21	27.18	51.30	52.34	18.09	22.17
22	23.24	43.87	44.85	16.59	20.92
23	10.98	28.90	29.88	11.66	15.20
24	7.29	21.52	22.25	10.14	13.08

in hours with high demand. The same behavior is observed in unit revenues and demand costs.

These results have been obtained solving the problem formulated in Section 2 using the technique described in Section 3 employing commercial solvers GAMS/CPLEX 9.0 (master problem) and GAMS/MINOS 5.51 (subproblems)(see Brooke et al. (1998)). The solution has been achieved in 18 iterations and approximately 2 hours of CPU time within a relative tolerance lower than 1%. The computer used is a Linux-based Dell PowerEdge 6600 with 4 processors at 1.60 GHz and 2 GB of RAM memory. Note that the objective function optimal value, Z_{ME}, is equal to zero, and therefore minimum profit conditions have not generated complementarity infeasibilities.

The convergence behavior of Benders decomposition is illustrated in figure 3. Observe the appropriate convergence of this decomposition and the smooth increase of the lower

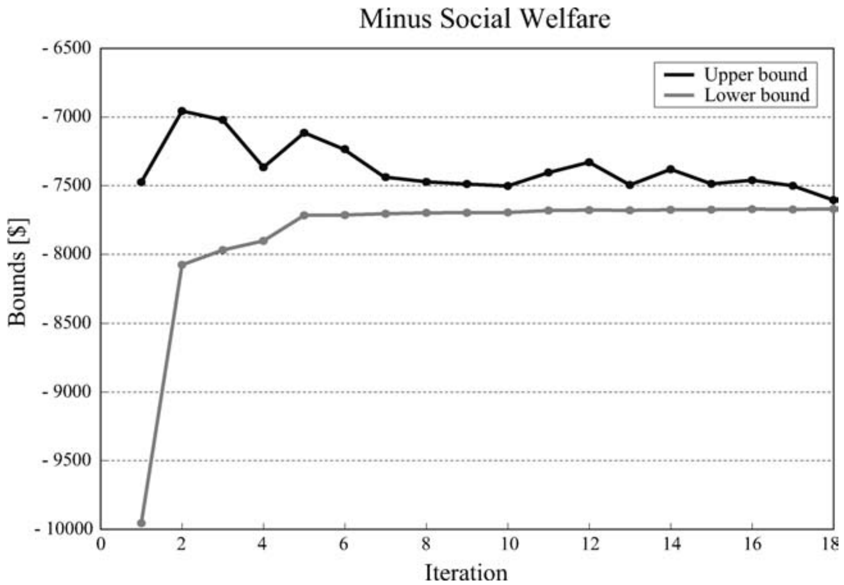

Figure 3. Evolution of bounds of the Benders decomposition.

bound of the optimal objective function value. Nevertheless, the lower bound progresses slowly once a reasonably small gap between the bounds is achieved. This is typical behavior of Benders decomposition for large-scale problems.

Finally, Table 5 provides a comparison between the multi-period equilibrium without imposing minimum profit conditions for the units and its corresponding sequence of single-period equilibria (not considering inter-temporal constraints). The on/off status of the generating units in any single-period equilibrium is decided considering unit status only in the previous period. Each single-period equilibrium is equivalent to an optimal power flow (Hobbs, 2001). The very fact that the multi-period equilibrium problem considers on/off status changes along the time horizon as optimization variables, causes start-up costs to be lower in the multi-period equilibrium case. It can be observed that social welfare is almost 2% higher in the multi-period equilibrium case than in the case of a sequence of single-period equilibria, and total energy produced and consumed in the market is lower in the multi-period case. This fact implies that the multi-period equilibrium problem provides more efficient results than a succession of single-period equilibria. As compared with the more realistic multi-period equilibrium, a sequence of single-period equilibria overestimates the consumer surplus while underestimates the producer surplus. Note that the merchandising surplus is defined as the total demand costs minus the total production revenues.

Table 6 provides a comparison between the multi-period equilibrium imposing minimum profit conditions for the units and the multi-period equilibrium without imposing such conditions. We observe that the consideration of minimum profit conditions implies an increase of producer revenues and, therefore, an increase of the demand costs, whose consequence is

Table 5. Multi-period equilibrium without MPC versus the corresponding sequence of single-period equilibria.

	Multi-period equilibrium without MPC	Sequence of single-period equilibria	Difference (%)
Total energy produced (MWh)	58242.44	59177.48	−1.58
Total energy consumed (MWh)	56566.85	57792.85	−2.12
Generating unit revenues (k$)	1003.81	1013.43	−0.95
Demand costs (k$)	1025.51	1033.98	−0.82
Start-up costs (k$)	3.06	11.19	−72.65
Producer surplus (k$)	503.20	473.84	6.20
Consumer surplus (k$)	237.44	253.93	−6.49
Merchandising surplus (k$)	21.70	20.56	5.54
Social welfare (k$)	762.34	748.36	1.87

Table 6. Multi-period equilibrium with MPC versus multi-period equilibrium without MPC.

	Multi-period equilibrium with MPC	Multi-period equilibrium without MPC	Difference (%)
Total energy produced (MWh)	58009.30	58242.44	−0.40
Total energy consumed (MWh)	56306.04	56566.85	−0.46
Producer revenues (k$)	1014.04	1003.81	1.02
Demand costs (k$)	1036.96	1025.51	1.12
Star-up costs (k$)	2.79	3.06	−8.82
Producer surplus (k$)	516.88	503.20	2.72
Consumer surplus (k$)	220.74	237.44	−7.03
Merchandising surplus (k$)	22.92	21.70	5.62
Social welfare (k$)	760.54	762.34	−0.24

a higher producer surplus and a lower consumer surplus. Social welfare is lower if minimum profit conditions are considered than if they are not.

5. Conclusions

This paper describes, analyzes and illustrates a multi-period equilibrium in a pool-based electricity market that may include minimum profit constraints for on-line generating units. To be able to use optimality conditions, we formulate the multi-period equilibrium problem using Benders decomposition in such a manner that it can be easily solved.

A procedure to identify multi-period equilibria in an electricity market is of interest for the market regulator that may use it for market monitoring. It is also of interest for producers and consumers to analyze their most appropriate strategies.

A multi-period market equilibrium approach reproduces a real-world functioning of the market in a better manner than a succession of single period equilibria since coupling conditions are properly taken into account. Unlike a succession of single-period equilibria, the multi-period approach allows incorporating minimum profit constraints for on-line generating units comprising the whole market horizon, which are relevant constraints in actual markets and represents an important modeling advantage.

Appendix: Notation

We list the main notation used in this paper.

Primal variables

$c_{Gi}^{\mathrm{sd}}(t)$ is the shut-down cost of generating unit i in hour t.
$c_{Gi}^{\mathrm{su}}(t)$ is the start-up cost of generating unit i in hour t.
$P_{Dik}(t)$ is the power block k that demand i is consuming in hour t.
$\tilde{P}_{Dik}(t)$ is the power block k that demand i is consuming in hour t. This variable is equal to $P_{Dik}(t)$ and it is used in the problem of the ISO.
$P_{Gib}(t)$ is the power block b that generating unit i is producing in hour t.
$\tilde{P}_{Gib}(t)$ is the power block b that generating unit i is producing in hour t. This variable is equal to $P_{Gib}(t)$ and it is used in the problem of the ISO.
$v_i(t)$ on/off status of generating unit i in hour t (1 if generating unit i is on in hour t and 0 otherwise).
$\delta_i(t)$ is the voltage angle of node i in hour t.
$\rho_i(t)$ is the locational marginal price in node i in hour t.

Dual variables

$\alpha_i(t)$ is the dual variable associated with the maximum capacity constraint of generating unit i in hour t.
$\beta_i(t)$ is the dual variable associated with the minimum power output of generating unit i in hour t.
$\gamma_{ij}(t)$ is the dual variable associated with the transmission capacity constraint of line i-j in hour t.
$\Gamma_i(t)$ is the dual variable associated with the upper bound of the voltage angle of node i in hour t.
$\kappa_{vi}(t)$ is the dual variable associated with the constraint that fixes the variable $v_i(t)$ in the subproblem.
$\mu_{Gib}(t)$ is the dual variable associated with the equality equation of the power generated (unit i, block b) in the problem of the ISO and in the corresponding problem of the producer.

$\upsilon_{Dik}(t)$ is the dual variable associated with the equality equation of the power demanded (demand i, block k) in the problem of the ISO and in the corresponding problem of the consumer.

$\sigma_i(t)$ is the dual variable associated with the minimum demand constraint of demand i in hour t.

$\tau_i(t)$ is the dual variable associated with the available maximum power output constraint of unit i in hour t.

$\phi_{ib}(t)$ is the dual variable associated with the maximum capacity limit for the block b of the generating unit i in hour t.

$\varphi_{ik}(t)$ is the dual variable associated with the maximum capacity limit for the block k of the demand i in hour t.

$\psi_i(t)$ is the dual variable associated with the available minimum power output constraint of unit i in hour t.

Constants

B_{ij} is the susceptance of line i-j.

C_{Gi}^{fx} is the fixed cost coefficient of generating unit i.

C_{Gi}^{sd} is the shut-down cost coefficient of generating unit i.

C_{Gi}^{su} is the start-up cost coefficient of generating unit i.

K_i is a positive constant that represents minimum profit for generating unit i.

$P_{Di}^{\min}(t)$ is the minimum power supplied to demand i in hour t.

$P_{Dik}^{\max}(t)$ is the maximum power block k that demand i is willing to buy at price $\lambda_{Dik}^{\mathrm{B}}$ in hour t.

$P_{Gi}^{\max}$ is the maximum power output of generating unit i.

$P_{Gi}^{\min}$ is the minimum power output of generating unit i.

$P_{Gib}^{\max}(t)$ is the maximum power block b that generating unit i is willing to sell at price $\lambda_{Gib}^{\mathrm{B}}$ in hour t.

$\hat{P}_{Gib}^{(\eta)}(t)$ is the estimate of the generating power in block b of unit i in hour t at iteration η.

$\bar{P}_{Gib}^{(\eta)}(t)$ is the optimal value of the generating power in block b of unit i in hour t at iteration η.

$P_{ij}^{\max}$ is the transmission capacity limit of line i-j.

R_i^{dn} is the ramp-down limit for generating unit i.

R_i^{sd} is the shut-down ramp limit for generating unit i.

R_i^{su} is the start-up ramp limit for generating unit i.

R_i^{up} is the ramp-up limit for generating unit i.

Z_{ME} is the summation of all the inequality constraints of the subproblem multiplied by their respective dual variables.

$\lambda_{Dik}^{\mathrm{B}}(t)$ is the price offered by demand i to buy power block k in hour t.

$\lambda_{Dik}^{\mathrm{U}}(t)$ is the utility ($/MWh) associated to power block k of demand i in hour t.

$\lambda_{Gib}^{\mathrm{B}}(t)$ is the price offered by generating unit i to sell power block b in hour t.

$\lambda_{Gib}^{\mathrm{C}}(t)$ is the production cost of power block b of generating unit i in hour t.

Sets

D is the set of indices of demands.
D_q is the set of indices of demands owned by consumer q.
G is the set of indices of generating units.
G^{on} is the set of indices of generating units that impose minimum profit requirements.
G_f is the set of indices of generating units owned by producer f.
N is the set of nodes.
T is the set of time periods considered.
Ω_i is the set of nodes adjacent to node i.

Numbers

N_{Di} is the number of blocks demanded by demand i.
N_{Gi} is the number of blocks offered by generating unit i.

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