Netnomics (2009) 10:123–140
DOI 10.1007/s11066-008-9018-x

Pareto-improving and revenue-neutral congestion pricing schemes in two-mode traffic networks

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Accepted: 22 July 2008 / Published online: 16 August 2008

Abstract This paper studies a Pareto-improving and revenue-neutral congestion pricing scheme on a simple two-mode (highway and transit) network: this scheme aims at simultaneously improving system performance, making every individual user better off, and having zero total revenue. Different Pareto-improving situations are explored when a two-mode transportation system serves for travel groups with different value-of-time (VOT) distributions. Since the congestion pricing scheme suggested here charges transit users negative tolls and automobile users positive tolls, it can be considered as a proper way to implement congestion pricing and transit subsidy in one step, while offsetting the inequity for the poor. For a general VOT distribution of commuters, the condition of Pareto-improving is established, and the impact of the VOT distribution on solving the inequity issue is explored. For a uniform VOT distribution, we show that a Pareto-improving and revenue-neutral pricing scheme always exists for any target modal split pattern that reduces the total system travel time.

Keywords Congestion pricing · Revenue-neutral · Pareto-improving · Two-mode network · Traffic equilibrium

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1 Introduction

With the increasing number of automobiles on the roads and lower-than marginal cost pricing of auto use, misallocation and inefficient use of multi-mode transportation resources are often observed. To promote the use of public transportation, ease road congestion and alleviate environmental pollution, subsidies to local transit have become increasingly pervasive and important particularly in the more advanced and congested cities and countries. Currently, congestion pricing is being considered as a measure for restraining auto users and providing revenue for transit subsidy. Most studies considered the determination of both congestion delay and toll mainly on road networks only. Studies for inter-modal competition and pricing have been conducted [3, 6, 10, 12, 14, 18], among many others), largely relying upon the economic theory of utility-maximizing consumers.

Although road pricing is theoretically and technologically easy to implement, it has long been viewed as a political issue. Road pricing can generate revenue and reduce social disutility simultaneously. However, most literature found pricing to be regressive when toll revenue refunding is not considered. Due to the diminishing marginal value of money and increasing value of time (VOT) with individual or household's income, users will benefit or lose in different levels [9]. The inequity effect should be studied at the person-level, and individuals are classified into four categories [16]: *the tolled*, *the tolled-off*, *the tolled-on* and *the un-tolled* to represent those who decide to stay on the tolled system after pricing, those who are priced-out, who are lured from the other unknown system because of the benefits from time saving, and who avoid the tolled facility in any case and are not effected by the pricing. In this respect, several researchers have looked into various revenue distribution strategies. For examples, Small [16] emphasized that the congestion pricing may be progressive if a lump sum refunding is implemented, i.e. an equal travel allowance for all commuters. Goodwin [8] suggested a combination of revenue uses in order to offset several congestion pricing impacts. Poole [13] added that it might be possible to introduce off-peak discounts and peak-hour surcharges on a toll road. DeCorla-Souza [5] proposed a cashing out strategy to induce a shift of peak-period travelers to other modes, thus reducing the need for additional infrastructure. Recently, Kalmanje and Kockelman [11] proposed a credit-based congestion pricing strategy, where the term ‘credit’ refers to a monetary cash-out. The method can help tackle the problem of congestion in an equitable and efficient fashion.

From a theoretical perspective, Bernstein [2] examined the possibility of user-neutral congestion pricing with both positive and negative tolls in a bottleneck congestion model; Adler and Cetin [1] discussed a direct distribution approach to congestion pricing, in which monies collected from users on a more desirable route are directly transferred to users on a less desirable route using a two parallel route example with bottleneck congestion. For a single Origin–Destination (OD) pair connected by a number of parallel routes, Eliasson [7] showed that a tolling and refunding system that reduced aggregate travel time and refunded the toll revenues equally to all users would benefit all users. To realize the "fair access" to major airports under congestion pricing, Daniel [4] evaluated some price-and-rebate programs and proposed several that are self-financing and Pareto-improving, which actually means winners compensate losers. Without direct revenue redistribution among users, Yang and Zhang [21] took explicit consideration of the social and spatial inequity in a bi-level network toll design model with multi-class user equilibrium constraints. More recently, Yang and Guo [19] studied various Pareto-improving congestion pricing and revenue-refunding schemes in general networks with multi-class users. A comprehensive treatment of the various theoretical issues of road pricing is given by Yang and Huang [20]. In most previous studies, pricing and refunding schemes are generally considered in two separate, sequential steps. In this case, users might change their choice if they will receive different refunds by choosing different modes or paths. So refunding may drive traffic demand and flow pattern to a new equilibrium that might be different from that for pricing, thereby making the traffic equilibrium unstable [19]. The UE will fluctuate until the lump sum refunding scheme is introduced that one's choice doesn't change the amount of refund she receives.

This study looks into an alternative method for the Pareto-improving pricing scheme in a simple two-mode traffic network: revenue-neutral pricing scheme imposes direct positive toll on highway and negative toll on transit which actually means subsidy, while maintaining an aggregate zero revenue. Two central issues are addressed. First, whether or not Pareto-improving and revenue-neutral pricing scheme exists for certain target modal split flow pattern? Specifically, an anonymous link toll scheme, i.e. an identical toll across users on the same link, is sought to satisfy three conditions simultaneously: improving system performance, making every individual user better off, and having zero total revenue. Second, what are the impacts of different VOT distributions, including the uniform VOT distributions, on the realization of a Pareto improvement?

The paper is organized as follows. In the next section, we introduce the problem and the basic model. In Section 3, we establish a necessary and sufficient condition for the existence of Pareto-improving and revenue-neutral pricing schemes for a general VOT distribution. We also study the impact of the VOT distribution on the realization of the Pareto-improving goal. In Section 4, specific results are derived for the special case of a uniform VOT distribution, and a simple numerical example is provided. Conclusions are given in Section 5.

2 Problem settings

2.1 Deterministic mode choice and traffic equilibrium

Consider a simple single O–D pair model to present a corridor with a congested highway running in parallel with public transit, as shown in Fig. 1.

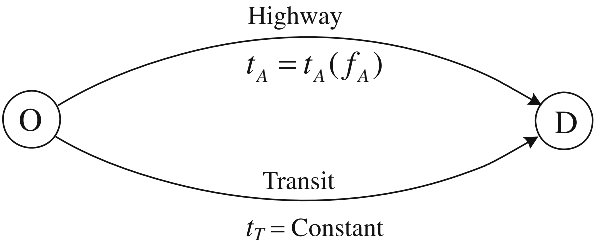


Fig. 1 A simple two-mode network

Several basic assumptions/settings in this article are listed and their meaning and limitations are discussed below.

(a) *A single O–D pair*. The two-mode network with a single O–D pair means that only social inequity is concerned but the spatial inequity among different O–D pairs is ignored. For a network with many origins and destinations, both of the social and spatial inequity issues exist, which have different causes and need different solutions to offset the disadvantage. Then it could be more challenging to devise a combined pricing scheme with toll and subsidization that gives Pareto improvement.
(b) *A fixed total demand*. The total demand is d. Let f_{T} denote the user volume on transit and let $\mathbf{f} = (f_{\mathrm{A}}, f_{\mathrm{T}})$ be the vector of mode flows where $f_{\mathrm{A}} + f_{\mathrm{T}} = d$. The system-wide (external) demand elasticity is ignored but internal elasticity is considered by allowing inter-modal competition. If the assumption were relaxed, i.e. external demand is also elastic, the level of toll would affect the total demand as well as the modal split. Subsidizing transit would induce excessive travel, which would be a disadvantage of scheme proposed here. Those "tolled-on" users, starting to use the system because of time saving or lower transit fare, are not considered here.
(c) *Congested highway and congestion-free & underutilized transit.* Suppose that the travel time on the highway is an increasing, continuous and convex function, $t_{\mathrm{A}} = t_{\mathrm{A}}\ (f_{\mathrm{A}})$, of the volume, f_{A}, of users (vehicles) traveling through the highway. The capacity of transit is supposed to be large enough so that the travel time by the transit mode is a constant denoted by t_{T}. The difference between the free flow travel times of the two modes is negative, $t_{\mathrm{A}}(0) - t_{\mathrm{T}} < 0$. It is realistically assumed that, before introducing a toll scheme $(\tau_{\mathrm{A}}, \tau_{\mathrm{T}})$ to highway and transit respectively, a socially non-optimal modal split pattern emerges in the sense that the transit service with a lower marginal cost is underutilized, and the highway is overused with excessive congestion. It should be noted that the transit link could also be another highway that is longer but congestion-free. So our results can be applied to any single O–D pair network with one congested route and a parallel congestion-free route.
(d) *Constant returns to scale on transit.* We assume the average transit system operation cost per rider is constant and transit users need to pay a

constant basic fee, F_T, to cover the transit operating cost. This, together with constant transit (in-vehicle and out-of-vehicle) travel time in (c), implies the assumption of a constant return to scale in transit service. One should recognize that, with economies of traffic density road pricing can lead to modal shift toward public transport and thus touch off the "virtuous circle" of quicker driving time and higher-quality public transport service. Even without spending any of the road pricing revenues on public transport improvements, the combination of increased traffic speed and increased ridership permits large increases in service and ridership, reductions in user costs, and savings in average agency costs sufficient to pay for the increased service even while reducing fares [17].

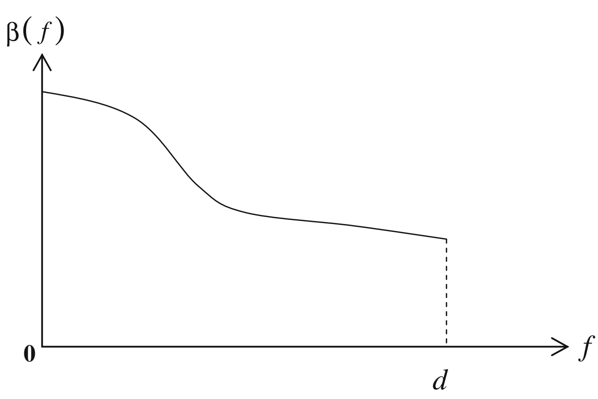
(e) *Constant automobile operating cost.* Automobile users need to pay a constant basic monetary cost, F_A, including fuel, insurance and maintenance fees and so on, which are assumed to be independent of travel time. To mimic real situations, it is assumed that the difference between the basic monetary user costs of the automobile and transit modes is positive, $\Delta F = F_A - F_T > 0$.

(f) *A continuous VOT distribution function.* We consider heterogeneous users with a continuous distribution of VOT. But for each individual user the value of travel time is the same for automobile and transit. Let $\beta(f)$, $\beta(f) \geq 0$, be the VOT distribution function, which gives the VOT of the f-th user. Users are ordered in decreasing VOT, i.e. $\beta(f)$ is decreasing in f, as depicted in Fig. 2.

The above settings allow us to express the generalized travel disutility, which combines the value of travel time and the total monetary travel cost. The travel disutility of the f-th user when choosing automobile or transit can be respectively presented as:

$$\mu_A(f) = t_A(f_A)\beta(f) + c_A, \mu_T(f) = t_T\beta(f) + c_T \tag{1}$$

Fig. 2 Distribution of users in a decreasing order of their VOT



where c_{A} and c_{T} are the total monetary costs on highway and transit respectively. Users choose their travel mode that minimizes their generalized travel disutility in a deterministic manner:

$$\mu(f) = \min\{\mu_{\mathrm{A}}(f), \mu_{\mathrm{T}}(f)\} \tag{2}$$

In the absence of toll, let $\tilde{f}_{\mathrm{A}}$ $(\tilde{f}_{\mathrm{A}} < d)$ be the user indifferent to the two modes. Obviously, users with VOTs larger than $\beta(\tilde{f}_{\mathrm{A}})$ will choose highway and users with VOTs smaller than $\beta(\tilde{f}_{\mathrm{A}})$ will choose transit. Then the untolled user equilibrium (UE) flow is $\tilde{\mathbf{f}} = (\tilde{f}_{\mathrm{A}}, \tilde{f}_{\mathrm{T}}) = (\tilde{f}_{\mathrm{A}}, d - \tilde{f}_{\mathrm{A}})$, which is unique with increasing function $t_A(f_A)$ and decreasing function $\beta(f)$. We consider an interior untolled UE solution $0 < \tilde{f}_{\mathrm{A}} < d$, then, in view of $c_{\mathrm{A}} = F_{\mathrm{A}}$ and $c_{\mathrm{T}} = F_{\mathrm{T}}$ without toll, $\mu_{\mathrm{A}}(\tilde{f}_{\mathrm{A}}) = \mu_{\mathrm{T}}(\tilde{f}_{\mathrm{A}})$ gives the following untolled UE condition

$$\left(t_{\mathrm{T}} - t_{\mathrm{A}}\left(\tilde{f}_{\mathrm{A}}\right)\right)\beta\left(\tilde{f}_{\mathrm{A}}\right) = \Delta F. \tag{3}$$

Suppose a toll scheme, $\tau = (\tau_{\mathrm{A}}, \tau_{\mathrm{T}})$, is introduced to highway and transit, respectively, to drive users' mode choices to a target flow pattern, $\bar{\mathbf{f}} = (\bar{f}_{\mathrm{A}}, \bar{f}_{\mathrm{T}}) = (\bar{f}_{\mathrm{A}}, d - \bar{f}_{\mathrm{A}})$. For the unique tolled UE flow pattern, it holds that:

$$t_{\mathrm{A}}\left(\bar{f}_{\mathrm{A}}\right)\beta\left(\bar{f}_{\mathrm{A}}\right) + \Delta F + \tau_{\mathrm{A}} = t_{\mathrm{T}}\beta\left(\bar{f}_{\mathrm{A}}\right) + \tau_{\mathrm{T}}. \tag{4}$$

Our analysis focuses on exogenous target flow patterns rather than any particular optimal (first-best or second-best) one. Setting an arbitrary modal split target has the advantage of generality, and it includes the "optimal" split as a special case. Also, it should be mentioned that any feasible target flow pattern can be realized by toll schemes satisfying Eq. 4.

Given the untolled and tolled equilibrium flow pattern, the generalized travel disutility of the f-th user, denoted by $\tilde{\mu}(f)$ and $\bar{\mu}(f)$ respectively, can be ascertained by Eq. 2.

2.2 Pareto-improving and revenue-neutral toll scheme

Definition A toll scheme is said to be Pareto-improving if it holds that

$$\bar{\mu}(f) < \tilde{\mu}(f), \forall f \in [0, d] \tag{5}$$

and is said to be revenue-neutral if it holds that

$$\tau_{\mathrm{A}}\bar{f}_{\mathrm{A}} + \tau_{\mathrm{T}}\bar{f}_{\mathrm{T}} = 0 \tag{6}$$

Condition (5) means that the travel disutility of each individual user is reduced, i.e. everyone is made better off. Condition (6) means that the revenue from highway toll charge and the subsidy to transit are break-even.

From UE condition (4), any target flow pattern $\bar{\mathbf{f}}$ can be supported as network equilibrium by a set of toll schemes characterized by the following equation:

$$\Delta\tau = \tau_{\mathrm{A}} - \tau_{\mathrm{T}} = \beta\left(\bar{f}_{\mathrm{A}}\right)\left(t_{\mathrm{T}} - t_{\mathrm{A}}\left(\bar{f}_{\mathrm{A}}\right)\right) - (\Delta F) > 0 \tag{7}$$

From Eqs. 6 and 7, a *unique* revenue-neutral toll scheme τ^{RN} for certain target flow pattern $\bar{\mathbf{f}}$ is given by:

$$\tau^{\mathrm{RN}} = \left(\tau_{\mathrm{A}}^{\mathrm{RN}}, \tau_{\mathrm{T}}^{\mathrm{RN}}\right) = \left(\frac{\Delta\tau\left(d - \bar{f}_{\mathrm{A}}\right)}{d}, -\frac{\Delta\tau\,\bar{f}_{\mathrm{A}}}{d}\right). \tag{8}$$

One major task of this paper is to examine the existence of a Pareto-improving and revenue-neutral toll scheme. If users are identical, a Pareto improvement is always possible by imposing an efficient toll scheme that improves the aggregate system performance and implementing a suitable revenue redistribution scheme [19]. However, when travelers are heterogeneous in VOT, the situation is much more complicated because of the inequity effect of pricing to users with different VOT, which is to be discussed in next subsection.

Here we shall establish a condition for the existence of a Pareto-improving and revenue-neutral toll scheme. The condition is found to be closely related to the system performance improvement measured by the total system travel time T

$$T = t_{\mathrm{A}}\left(f_{\mathrm{A}}\right) f_{\mathrm{A}} + t_{\mathrm{T}} f_{\mathrm{T}}. \tag{9}$$

A reduction in T by a toll scheme is represented by

$$\bar{T} < \tilde{T} \tag{10}$$

where $\bar{T}$ and $\tilde{T}$ are the tolled and untolled total system travel time, respectively. By definition, the total system travel time has no obvious connection with the Pareto-improving goal, because the definition of T takes into account neither the fixed monetary costs of the two modes (F_{A} and F_{T}) nor the users' VOT, both of which are important in determining the utility of individual users. Nevertheless, we shall show later that a reduction in T by a revenue-neutral toll scheme is pertinent to Pareto improvement. Furthermore, reduction in T is by itself an important objective of congestion pricing, because total system travel time is a conventional system performance measure of a transportation system.

From an economic viewpoint, the following aggregate system disutility, C, in monetary unit is also a meaningful system performance measure:

$$C = \int_0^{f_{\mathrm{A}}} \left(t_{\mathrm{A}}\left(f_{\mathrm{A}}\right)\beta\left(\omega\right) + F_{\mathrm{A}}\right)\mathrm{d}\omega + \int_{f_{\mathrm{A}}}^{d} \left(t_{\mathrm{T}}\beta\left(\omega\right) + F_{\mathrm{T}}\right)\mathrm{d}\omega. \tag{11}$$

Note that toll is not included in C (from a central planner's viewpoint). We refer to C as the total system cost. When a revenue-neutral toll scheme is implemented, C is equal to the sum of individual users' disutility. Thus, by definition, a Pareto-improving revenue-neutral toll scheme reduces the total system cost.

2.3 Inequity issue in congestion pricing

To highlight the social inequity issue in the absence of revenue-neutral pricing, we consider highway pricing alone without transit subsidy. Such a toll scheme can improve the aggregate system performance as a revenue-neutral pricing scheme does. However, some users are worse off.

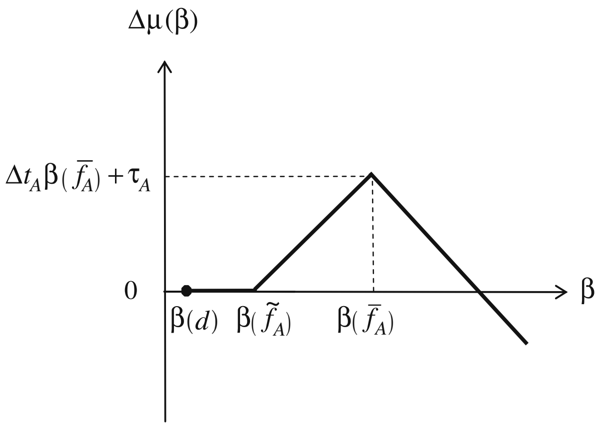
Suppose a toll scheme, $\tau = (\tau_A > 0, \tau_T = 0)$, is introduced to drive users from an untolled flow pattern, $\tilde{\mathbf{f}}$, to a target flow pattern $\bar{\mathbf{f}}$. Let $\Delta\mu(\beta(f))$ be the disutility change of the f-th user due to pricing as given below:

$$\Delta\mu(\beta(f)) = \bar{\mu}(\beta(f)) - \tilde{\mu}(\beta(f)), \forall f \in [0, d] \tag{12}$$

The disutility change as a function of user's VOT is given in Fig. 3, where $\Delta t_A = t_A(\bar{f}_A) - t_A(\tilde{f}_A)$ is the change in travel time on highway after toll.

As seen in Fig. 3, users with $\beta(f) \in [\beta(d), \beta(\tilde{f}_A)]$ (the $\tilde{f}_A$-th to the d-th users) use transit and thus are "un-tolled", their disutility remain unchanged. Users with $\beta(f) \in (\beta(\tilde{f}_A), \beta(\bar{f}_A)]$, are "priced-out", they stop using auto to avoid the toll but are worse off, because they forgo the benefits associated with lower travel time on highway. As a result the change in their travel disutility linearly increases with VOT. Users with $\beta(f) \in [\beta(\bar{f}_A), \beta(0)]$ continue to stay in highway and thus are "tolled". The "tolled" users benefit from time saving but tolerate toll, the change in their disutility linearly decreases with VOT. Thus the "tolled" users with lower VOTs suffer travel disutility increase (time saving does not compensate toll charge) and the "tolled" users with higher VOTs enjoy travel disutility decrease (travel time savings more than compensate toll charge). It is likely that all the "tolled" users will be worse off

Fig. 3 The disutility change of individual users after highway pricing



if their VOTs are all near $\beta(\bar{f}_\mathrm{A})$. In summary, the social inequity arises as a result of highway pricing due to the different changes in disutility among the users with different VOTs.

From Fig. 3, the indifferent user after highway pricing alone suffers the most disutility increase among all the travelers. Small [15] identified this result using a similar model. It should be mentioned that the indifferent user after pricing suffers the most disutility increase is true even when we adopt a pricing scheme with transit subsidy. Because, with different pricing schemes supporting the same target flow pattern, the $\Delta\mu\,(\beta)$ curve only moves up or down as a whole, while the shape of the $\Delta\mu\,(\beta)$ curve does not change. However, when $\tau_\mathrm{T} < 0$, i.e. transit users are subsidized, it is possible to be Pareto-improving and the indifferent user will gain the least disutility decrease instead of suffering the most disutility increase.

Lemma 1 *When a pricing scheme* $\tau = (\tau_\mathrm{A} \geq 0, \tau_\mathrm{T} \leq 0)$ *is introduced to the system, the indifferent user after pricing suffers the most disutility increase or gains the least disutility decrease.*

3 Revenue-neutral pricing with general VOT distributions

3.1 Existence of pareto-improving and revenue-neutral pricing scheme

In this subsection we establish a necessary and sufficient condition for the existence of a Pareto-improving and revenue-neutral pricing scheme.

Proposition 1 *A revenue-neutral pricing scheme is Pareto-improving if and only if the UE flow pattern under the pricing scheme,* $\bar{\mathbf{f}} = \left(\bar{f}_\mathrm{A}, \bar{f}_\mathrm{T}\right)$, *where* $\left(\bar{f}_\mathrm{A} \leq \tilde{f}_\mathrm{A}\right)$, *satisfies that*

$$\bar{T} < \tilde{T} + \left(\frac{\bar{f}_\mathrm{T}}{\beta\left(\bar{f}_\mathrm{A}\right)} - \frac{\tilde{f}_\mathrm{T}}{\beta\left(\tilde{f}_A\right)} \right) \Delta F \tag{13}$$

Proof Note that the indifferent user, $\bar{f}_\mathrm{A}$, in the presence of highway toll, has the highest disutility change, $\Delta\mu\left(\beta\left(\bar{f}_\mathrm{A}\right)\right)$, among all users. Thus $\Delta\mu\left(\beta\left(\bar{f}_\mathrm{A}\right)\right) < 0$ means that every user is made better off, i.e. Pareto-improving. From the two UE conditions (3) and (4), respectively, before and after highway pricing, and the definition of revenue-neutral pricing 6, $\Delta\mu\left(\beta\left(\bar{f}_\mathrm{A}\right)\right)$ is given by

$$\Delta\mu\left(\beta\left(\bar{f}_\mathrm{A}\right)\right) = -\beta\left(\bar{f}_\mathrm{A}\right) \frac{\bar{f}_\mathrm{A}}{d} \left(t_\mathrm{T} - t_\mathrm{A}\left(\bar{f}_\mathrm{A}\right)\right) + \left(\frac{\beta\left(\bar{f}_\mathrm{A}\right)}{\beta\left(\tilde{f}_\mathrm{A}\right)} + \frac{\bar{f}_\mathrm{A}}{d} - 1 \right) \Delta F. \tag{14}$$

Then the following relation is true:

$$\Delta\mu\left(\beta\left(\bar{f}_{\mathrm{A}}\right)\right)<0\Leftrightarrow-\bar{f}_{\mathrm{A}}\left(t_{\mathrm{T}}-t_{\mathrm{A}}\left(\bar{f}_{\mathrm{A}}\right)\right)<-\frac{d}{\beta\left(\bar{f}_{\mathrm{A}}\right)}\left(\frac{\beta\left(\bar{f}_{\mathrm{A}}\right)}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}-\frac{\bar{f}_{\mathrm{T}}}{d}\right)\Delta F$$

$$\Leftrightarrow t_{\mathrm{T}}d-\left(t_{\mathrm{T}}-t_{\mathrm{A}}\left(\bar{f}_{\mathrm{A}}\right)\right)\bar{f}_{\mathrm{A}}<t_{\mathrm{T}}d-\frac{\tilde{f}_{\mathrm{A}}}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}\Delta F+\left[\frac{\bar{f}_{\mathrm{T}}}{\beta\left(\bar{f}_{\mathrm{A}}\right)}-\frac{\tilde{f}_{\mathrm{T}}}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}\right]\Delta F. \tag{15}$$

From the untolled UE condition (3), we have

$$\frac{\Delta F}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}=t_{\mathrm{T}}-t_{\mathrm{A}}\left(\tilde{f}_{\mathrm{A}}\right) \tag{16}$$

Substitute Eq. 16 into the second term of the right-hand side of Eq. 15, we have

$$\Delta\mu\left(\beta\left(\bar{f}_{\mathrm{A}}\right)\right)<0\Leftrightarrow t_{\mathrm{T}}d-\left(t_{\mathrm{T}}-t_{\mathrm{A}}\left(\bar{f}_{\mathrm{A}}\right)\right)\bar{f}_{\mathrm{A}}<t_{\mathrm{T}}d-\left(t_{\mathrm{T}}-t_{\mathrm{A}}\left(\tilde{f}_{\mathrm{A}}\right)\right)\tilde{f}_{\mathrm{A}}$$

$$+\left[\frac{\bar{f}_{\mathrm{T}}}{\beta\left(\bar{f}_{\mathrm{A}}\right)}-\frac{\tilde{f}_{\mathrm{T}}}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}\right]\Delta F$$

$$\Leftrightarrow,\ \bar{T}<\tilde{T}+\left(\frac{\bar{f}_{\mathrm{T}}}{\beta\left(\bar{f}_{\mathrm{A}}\right)}-\frac{\tilde{f}_{\mathrm{T}}}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}\right)\Delta F,\ \text{from } T=t_{\mathrm{T}}d-\left(t_{\mathrm{T}}-t_{\mathrm{A}}\left(f_{\mathrm{A}}\right)\right)f_{\mathrm{A}}$$

The proof is completed. □

Proposition 1 states that condition (13) is a necessary and sufficient condition for a revenue-neutral pricing scheme to be Pareto-improving. In particular, condition (13) is related to condition (10) of total system travel time reduction. If the second term of its right-hand side is positive, then condition (13) becomes a relaxation of condition (10). This means that even if the total system travel time is increased by an amount less than the second term, everyone can still be better off. It shows an interesting result that the system time is increased but the system cost is decreased. If the second term of its right-hand side is negative, then condition (13) is stronger than condition (10), which means that only when the total system travel time is decreased to a certain value, a revenue-neutral Pareto improvement can be achieved.

It should be noted that the target modal split pattern $\bar{\mathbf{f}}=\left(\bar{f}_{\mathrm{A}},\ \bar{f}_{\mathrm{T}}\right)$ could be regarded as being endogenously chosen. In condition (13), $\bar{T}, \bar{f}_{\mathrm{T}}$ and $\beta\left(\bar{f}_{\mathrm{A}}\right)$ are all functions of the target highway flow $\bar{f}_{\mathrm{A}}$. If the VOT function is synthesized out of data collected from survey, and the highway travel time function is also known, then it is easy to solve inequality (13) to obtain (or check the existence of) a feasible range in terms of target highway flow such that any target highway flow within that range corresponds to a Pareto-improving

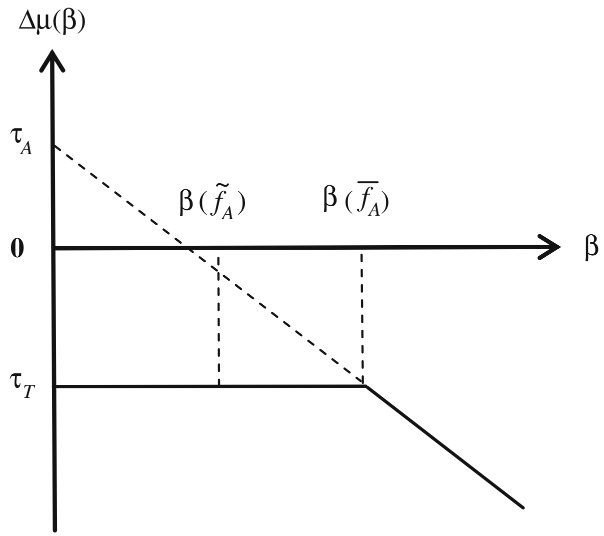


Fig. 4 Disutility change after revenue-neutral pricing when $\Delta F = 0$

and revenue-neutral pricing scheme. Nevertheless, it is difficult to deal with this problem analytically without any assumption on the form of the VOT function. A numerical example based on uniform VOT distribution is provided in Section 4.

It is worthy to further note that when $\Delta F = 0$, which means that the basic monetary costs of the two modes are equal, condition (13) simply becomes condition (10), and thus everyone will be better off through revenue-neutral pricing as long as the total system travel time is reduced. This is shown in Fig. 4. Note that $\Delta F = 0$ means $t_{\mathrm{T}} = t_{\mathrm{A}}\left(\tilde{f}_{\mathrm{A}}\right) > t_{\mathrm{A}}\left(\bar{f}_{\mathrm{A}}\right)$, those who move from highway to transit do not suffer travel time increase, all of the transit users after pricing benefit $(-\tau_{\mathrm{T}})$ which is the lump sum subsidy. The highway users after pricing will benefit more than$(-\tau_{\mathrm{T}})$ because their benefits from time saving increase linearly with their VOTs.

3.2 The impact of VOT distributions

We now examine the impacts of VOT distributions on the realization of the Pareto-improving and revenue-neutral pricing on the two-mode network. As shown in Subsection 2.3, the indifferent user after pricing will suffer the most disutility change among all users, thus, a lower value of $\Delta\mu\left(\beta\left(\bar{f}_{\mathrm{A}}\right)\right)$ means a better situation for the most adversely affected user after pricing, which is a more favorable situation for mitigating the inequity concern. For this reason, we focus on a comparison of the indifferent or critical user's disutility change, $\Delta\mu\left(\beta\left(\bar{f}_{\mathrm{A}}\right)\right)$, when user groups with different VOT distributions are shifted from a common untolled flow pattern $\tilde{\mathbf{f}}$ to the same target flow pattern $\bar{\mathbf{f}}$. To keep the untolled flow pattern $\tilde{\mathbf{f}}$ identical for each distribution, we assume that the untolled indifferent user's VOTs $\beta\left(\tilde{f}_{\mathrm{A}}\right)$ are identical. Then, as seen from Eq. 14, the indifferent user's disutility change, $\Delta\mu(\beta(\bar{f}_A))$, depends on her VOT $\beta(\bar{f}_A)$ only. We shall see whether $\Delta\mu(\beta(\bar{f}_A))$ decreases or increases with $\beta(\bar{f}_A)$ by considering its derivative in $\beta(\bar{f}_A)$. From Eq. 14, we have

$$\frac{\mathrm{d}\left[\Delta\mu\left(\beta\left(\bar{f}_A\right)\right)\right]}{\mathrm{d}\left[\beta\left(\bar{f}_A\right)\right]} = -\frac{\bar{f}_A}{d}\left(t_T - t_A\left(\bar{f}_A\right)\right) + \frac{\Delta F}{\beta\left(\tilde{f}_A\right)} \tag{17}$$

In view of UE condition (3), we have $\Delta F \big/ \beta(\tilde{f}_A) = t_T - t_A(\tilde{f}_A)$, then Eq. 17 becomes

$$\begin{aligned}\frac{\mathrm{d}\left[\Delta\mu\left(\beta\left(\bar{f}_A\right)\right)\right]}{\mathrm{d}\left[\beta\left(\bar{f}_A\right)\right]} &= -\frac{\bar{f}_A}{d}\left(t_T - t_A\left(\bar{f}_A\right)\right) + t_T - t_A\left(\tilde{f}_A\right). \\ &= \frac{\mathrm{d}t_T - \bar{f}_A\left(t_T - t_A\left(\bar{f}_A\right)\right)}{\mathrm{d}} - t_A\left(\tilde{f}_A\right) \end{aligned} \tag{18}$$

In view of $\bar{T} = \bar{f}_A t_A(\bar{f}_A) + (d - \bar{f}_A)t_T = dt_T - \bar{f}_A(t_T - t_A(\bar{f}_A))$, we finally have

$$\frac{\mathrm{d}\left[\Delta\mu\left(\beta\left(\bar{f}_A\right)\right)\right]}{\mathrm{d}\left[\beta\left(\bar{f}_A\right)\right]} = \frac{\bar{T}}{d} - t_A\left(\tilde{f}_A\right). \tag{19}$$

Then we readily have the following proposition.

Proposition 2 *With the same untolled flow pattern* $\tilde{\mathbf{f}}$*, the same untolled indifferent user's VOT* $\beta\left(\tilde{f}_A\right)$*, and the same target flow pattern* $\bar{\mathbf{f}}$*, (a) if*

$$\frac{\bar{T}}{d} < t_A\left(\tilde{f}_A\right) \tag{20}$$

then, the larger $\beta(\tilde{f}_A)$ *is, the more benefit (strict positive gain) the indifferent user (the* $\tilde{f}_A$*-th user) will receive after imposing a revenue-neutral pricing scheme, and the more favorable situation will emerge for solving the inequity issue; (b) if*

$$\frac{\bar{T}}{d} > t_A\left(\tilde{f}_A\right) \tag{21}$$

then, the larger $\beta(\tilde{f}_A)$ *is, the less benefit (or more loss) the indifferent user (the* $\tilde{f}_A$*-th user) will receive after imposing a revenue-neutral pricing scheme, and the more unfavorable situation will emerge for solving the inequity issue.*

Here a simple criteria is provided to discern the favorable and unfavorable situation of inequity resolution according to the average travel time of all users after pricing being less or larger than the untolled highway travel time

$t_{\mathrm{A}}(\tilde{f}_{\mathrm{A}})$. In view of UE condition (3), the condition (13) in Proposition 1 can be rewritten as:

$$\bar{T} < t_{\mathrm{A}}\left(\tilde{f}_{\mathrm{A}}\right)d + \frac{\bar{f}_{\mathrm{T}}}{\beta\left(\bar{f}_{\mathrm{A}}\right)}\Delta F \tag{22}$$

which is a relaxation of condition (20) in Proposition 2. Therefore, when a larger $\beta(\bar{f}_{\mathrm{A}})$ gives a more favorable situation for resolving the inequity issue, everyone will be made better off; when a larger $\beta(\bar{f}_{\mathrm{A}})$ gives a more unfavorable situation for resolving the inequity issue, users may or may not be made better off.

Based on a common expectation that VOT is increasing with individual or household's income, for given $\tilde{\mathbf{f}}$ and $\bar{\mathbf{f}}$, a larger difference between the two VOT values $\beta(\bar{f}_{\mathrm{A}})$ and $\beta(\tilde{f}_{\mathrm{A}})$ generally represents a larger rich-poor gap among users, especially for uniform VOT distributions. Actually, Proposition 2 tells us whether or not a large rich-poor gap among users is favorable or unfavorable for solving the inequity issue. A large rich-poor gap does not always mean an unfavorable, inequitable situation after revenue-neutral pricing; their relationship depends on the reduction of total system travel time after pricing rather than the VOT distribution. When the reduction of system time is large enough so that the average travel time after pricing is less than the untolled highway travel time, a large rich-poor gap will give a favorable inequity-resolving situation. Otherwise, the opposite situation emerges. More specific discussions are given for the case of uniform VOT distributions in next section.

4 Revenue-neutral pricing with a uniform VOT distribution

Now we examine the existence of Pareto-improving and revenue-neutral pricing scheme and the impact of VOT distribution on its realization in the case of a uniform VOT distribution. For a uniform VOT distribution in the interval $\left[\beta, \bar{\beta}\right]$, where $\bar{\beta} = \beta\,(0)$ and $\beta = \beta\,(d)$ are the highest and lowest VOT among the users, $\beta(f)$ is a linear function. Denote $\varepsilon = \left(\bar{\beta} - \beta\right)/d$, then $-\varepsilon$ is the slope of $\beta(f)$, and we have

$$\beta\,(f) = \beta + \varepsilon\,(d - f)\,,\ f \in \left[0, d\right] \tag{23}$$

With the specific $\beta(f)$ function given by Eq. 23, we can check condition (13) in Proposition 1, and see whether it is stronger or weaker than condition (10) of total system travel time reduction. To this end, we need to know the sign of the second term of the right-hand side of condition (13). We define the following function $Y(\bar{f}_{\mathrm{A}})$ for any given untolled flow $\tilde{\mathbf{f}}$

$$Y\left(\bar{f}_{\mathrm{A}}\right) = \left(\frac{\bar{f}_{\mathrm{T}}}{\beta\left(\bar{f}_{\mathrm{A}}\right)} - \frac{\tilde{f}_{\mathrm{T}}}{\beta\left(\tilde{f}_{\mathrm{A}}\right)}\right)\Delta F, 0 \le \bar{f}_{\mathrm{A}} \le \tilde{f}_{\mathrm{A}} \tag{24}$$

$Y(\bar{f}_A)$ is just the second term of the right-hand side of condition (13). Here for a uniform VOT distribution, in view of $\beta(f)$ given by Eq. 23, and $\bar{f}_T = d - \bar{f}_A$, we have

$$\frac{\mathrm{d}Y\left(\bar{f}_A\right)}{\mathrm{d}\bar{f}_A} = -\frac{\Delta F\beta}{\left(\beta\left(\bar{f}_A\right)\right)^2}, 0 \leq \bar{f}_A \leq \tilde{f}_A \tag{25}$$

Clearly, $\mathrm{d}Y(\bar{f}_A)/\mathrm{d}\bar{f}_A \leq 0$ for $0 \leq \bar{f}_A \leq \tilde{f}_A$, which means that $Y(\bar{f}_A)$ is a non-increasing function of $\bar{f}_A$ for $0 \leq \bar{f}_A \leq \tilde{f}_A$. Then, in view of $Y(\bar{f}_A) = 0$ when $\bar{f}_A = \tilde{f}_A$, we have $Y(\bar{f}_A) \geq 0$ for $0 \leq \bar{f}_A \leq \tilde{f}_A$. Therefore, for a uniform VOT distribution, the condition (13) for every user being better off is a relaxation of the condition (10) for total system travel time reduction. In other words, total system travel time reduction is sufficient for realizing Pareto-improving goal.

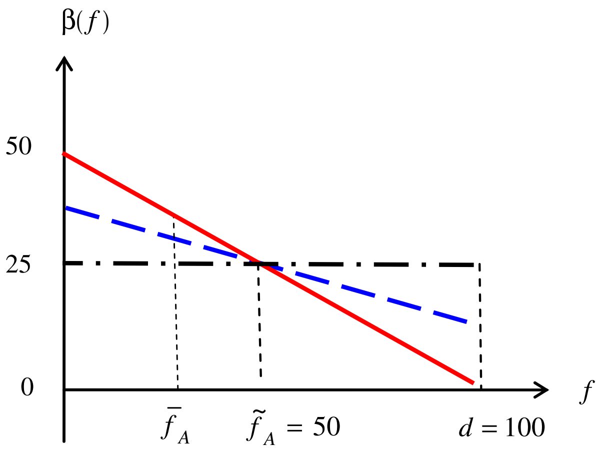
To sum up, we have the following proposition.

Proposition 3 *For a traveler group with a uniform VOT distribution, a revenue-neutral pricing scheme is Pareto-improving if it induces a UE flow pattern that reduces the total system travel time.*

Proposition 3 states that, for a uniform VOT distribution, a revenue-neutral pricing scheme is always Pareto-improving as long as it reduces the total system time. This implies that any flow pattern near the time-based system optimal flow can be selected to be the target flow to be supported by a Pareto-improving and revenue-neutral pricing scheme. Observe that total system travel time reduction as well as the time-based system optimal flow depends on the travel time function of the highway, but is independent of the specific range of the uniform VOT distribution. Therefore, it is safe to say that, provided that the VOT distribution is uniform, then the range of the VOT distribution does not affect the existence of a Pareto-improving and revenue neutral pricing scheme. Nevertheless, we know from Proposition 2 that the VOT range, representing the rich-poor gap of the demand, does have impact on the disutility change of the indifferent user. We shall see this by a numerical example.

Consider a single O–D pair two-mode transportation system with a total travel demand $d = 100$, the travel time on highway is $t_A(f_A) = 26 + (f_A)^2/50$, the travel time by transit is $t_T = 80$, and the difference between the basic monetary costs of the two modes is $(F_A - F_T) = 100$. The user better off situation is examined for three groups of users with different uniform VOT distributions of $\beta \in [0,50]$, $[15,35]$ and $[25,25]$. Here $\beta \in [25,25]$ represents a homogenous user case, i.e. all users have the same VOT $\beta = 25$. As shown in Fig. 5, the three groups have the same average VOT (the VOT of the middle user, also made intentionally to be the untolled indifferent user in this example).

Fig. 5 Three uniform VOT distributions



For this numerical example, the total system travel time as a function of the highway flow is given by

$$T\left(f_{\rm A}\right)=f_{\rm A}\left(26+\frac{f_{\rm A}^{2}}{50}\right)+\left(100-f_{\rm A}\right)80,\quad 0\leq f_{\rm A}\leq 100.$$

The time-based system optimum is given by

$$f_{\rm A}^{\rm SO}=30,\ \text{and}\, T_{\min}=T\left(30\right)=6{,}920.$$

A critical total system travel time level implied by Proposition 2 is

$$T_{\rm C}=t_{\rm A}\left(\tilde{f}_{\rm A}\right)d=7{,}600.$$

Since $T_{\min}<T_{\rm C}$ and $T(f_{\rm A})$ is convex, there are two critical highway flow levels $\bar{f}_{\rm A}=f_{\rm C1}$ and $\bar{f}_{\rm A}=f_{\rm C2}$ such that $\bar{T}=T_{\rm C}$. Let $f_{\rm C1}<f_{\rm C2}$, $f_{\rm C1}$ and $f_{\rm C2}$ are marked in Fig. 6. For any target highway flow $\bar{f}_{\rm A}\in(f_{\rm C1},f_{\rm C2})$, we have $\bar{T}<T_{\rm C}=t_{\rm A}\left(\tilde{f}_{\rm A}\right)d$, then, according to Proposition 2, the indifferent user's benefit (negative disutility change) increases with her VOT, which means that a larger rich-poor gap gives a better result for the indifferent user (the most adversely affected user after a revenue-neutral pricing). This is true for the numerical example here. As seen in Fig. 6, for $\bar{f}_{\rm A}\in\left(f_{\rm C1},f_{\rm C2}\right)$, the case with the largest VOT range $\beta\in[0{,}50]$ gives the most negative disutility change of the indifferent user, while the homogeneous user case with constant $\beta=25$ gives the least negative disutility change of the indifferent user. Figure 6 also shows that, as predicted by Proposition 2, for $\bar{f}_{\rm A}\in\left(0,f_{\rm C1}\right)\cup\left(f_{\rm C2},d\right)$, because $\bar{T}>T_{\rm C}=t_{\rm A}\left(\bar{f}_{\rm A}\right)d$, a larger VOT range gives a worse result for the indifferent user.

We have marked a flow level $f_{\rm B}$ for each of the three cases in Fig. 6. As seen from the figure, $f_{\rm B}$ is a critical target highway flow level such that the disutility change of the indifferent user is zero. For each case, if the target highway flow $\bar{f}_{\rm A}\in\left(f_{\rm B},\tilde{f}_{\rm A}\right)$, then the indifferent user has benefit (negative disutility change),

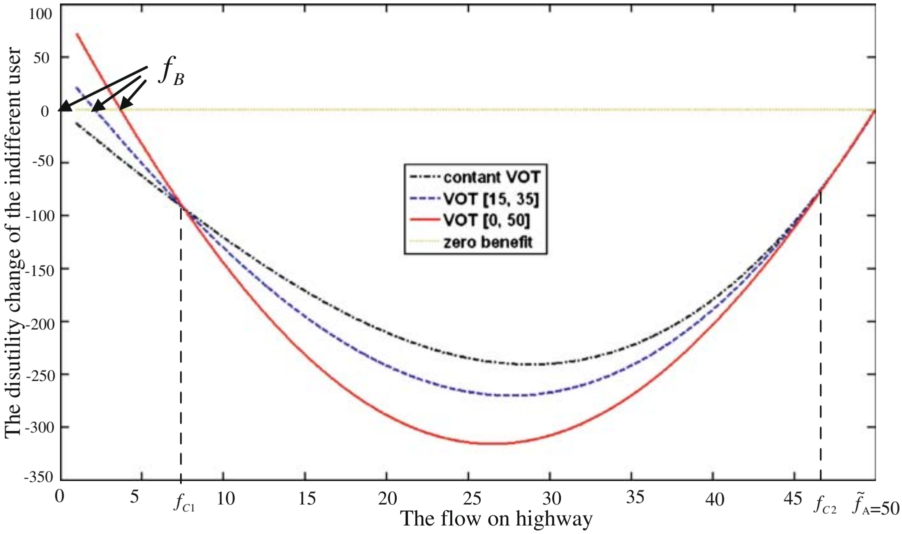


Fig. 6 The disutility change of the indifferent user for three VOT distributions

and thus every user is made better off by the revenue-neutral pricing scheme. As shown in Fig. 6, f_B is a very low flow level (f_B is zero for the homogeneous user case). We mention here that f_B is an over-depressed highway flow in the sense that the total system travel time is increased when $\bar{f}_A = f_B$. Therefore, there is typically a small target highway flow interval within $\left(f_B, \tilde{f}_A\right)$ and close to f_B, such that, every user is made better off by the revenue-neutral pricing, but the total system travel time is increased. In other words, a conflicting situation emerges in the sense that the total system travel cost is reduced but the total system travel time is increased. This conflicting situation is not a surprise, because we have shown that, for a uniform VOT distribution, the condition (13) for every user being better off is a relaxation of the condition (10) for total system travel time reduction. That is, there is room for such a situation that condition (13) holds (every user better off) while condition (10) does not hold (total system travel time is not reduced). This attributes to the difference of the basic monetary cost of the two modes and the user heterogeneity.

5 Conclusions

Starting with a general VOT distribution for users in a simple two-mode network, we investigated the existence of a Pareto-improving and revenue-neutral pricing scheme. We found that the user indifferent to the two modes after introducing a revenue-neutral pricing is the most critical (adversely affected) user in terms of disutility change. The VOT of the indifferent user after pricing is a critical factor for resolving the social inequity issue. When the reduction in total system travel time is large enough, a larger VOT of the indifferent user after pricing, which represents a larger rich-poor gap of the users, will give a more advantageous situation to solve the social inequity issue. We made more detailed investigation on traveler group with a uniform VOT distribution. It is found that the condition for every user being better off is weaker than the condition for total system travel time reduction. Thus any revenue-neutral pricing scheme is Pareto-improving as long as it reduces the total system travel time.

Acknowledgements The research described here was supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. HKUST6215/06E). The authors are grateful to three anonymous referees for helpful comments.

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