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A Maximum Consensus Improvement Method for Group Decision Making Under Social Network with Probabilistic Linguistic Information

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Abstract
Group decision-making (GDM) requires consensus building, because an outcome from a consensual decision is indispensable to implement a highly acceptable solution. This paper proposes a novel consensus reaching method for GDM with Probabilistic Linguistic Term Set (PLTS) under a social network environment. First, the preferences and trust evaluations of decision-makers (DMs) are collected using PLTS. Then, two types of centralities are utilized to obtain the significance of DMs, and these centralities are used to derive the group evaluation. Then, a consensus measure is employed to quantify the degree of agreement within the group. To promote further consensus, a novel feedback mechanism that combines the Identification and Direction Rule-based method with an optimization-based approach is developed to achieve maximum consensus improvement in each round of modification. Moreover, DM's bounded rationality is factored into the GDM process for a more reliable result. Finally, illustrative examples and comparison analyses are conducted to demonstrate the effectiveness of the proposed method.

Keywords Probabilistic linguistic term set (PLTS) · Group decision-making (GDM) · Consensus reaching process (CRP)

1 Introduction

Real-life decision-making problems are becoming increasingly complex. A single person cannot consider all the relevant information. Thus, group decision-making (GDM) has attracted much attention from researchers recently [1, 2]. For example, GDM has been widely applied to solve practical problems in the virtual reality industry [3] and for pharmaceutical supplier selection [4]. Generally, four components are necessary to solve GDM problems: preference representation, consensus measure, feedback mechanism, and selection process.

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When considering GDM problems, decision-makers (DMs) often express different evaluations towards the alternatives. Thus, a certain level of consensus must be achieved to guarantee that a group opinion is acceptable to a majority of DMs. Many researchers have modeled the consensus reaching process (CRP). The overarching models can be divided into two categories. The first general model is the Identification and Direction Rule (IDR)-based consensus model. The identification rule finds an opinion that needs to be modified, while the direction rule generates a recommendation for implementing the modification. Liang et al. [5] proposed an IDR-based feedback mechanism that helps the DM with the lowest consensus level to revise his/her opinion. Tan et al. [6] constructed a useful IDR-based model using quantum probability theory, which can guide the DMs to adjust their opinions. Tang et al. [7] developed an IDR-based consensus reaching algorithm that can manage the noncooperative behavior of DMs. Wang et al. [8] recently proposed an IDR-based consensus model to perform a departure audit of natural resources of leading cadres. The second general model is the optimization-based consensus model. Various objective functions are established for different models. Zhang et al. [9] constructed a novel CRP model that could minimize the adjustments between the initial and modified evaluations using multi-granular unbalanced linguistic information. Wu et al. [10] developed a bi-objective programming model to derive a consensual solution under interval type-2 fuzzy environment. Rodríguez et al. [11] recently proposed a comprehensive minimum cost consensus model for large-scale GDM with fuzzy preference relations.

Zhang et al. [12] demonstrated that optimization-based consensus methods can greatly promote consensus efficiency. However, in real-world GDM situations, mathematical modeling and individual participation are essential components of the process. The modified evaluations obtained via the optimization model cannot reflect a DM's interactions. To overcome this limitation, a novel CRP model that combines the IDR-based method with an optimization model is constructed in this paper to achieve a maximum consensus improvement in each round of modification.

In traditional GDM problems, DMs are assumed to be mutually independent. However, this premise rarely holds. In practice, the preference of a DM is liable to change under social influence. Thus, it is important to consider social relationships of DMs when dealing with GDM problems. However, most of the existing studies only consider the in-degree centrality of DMs when analyzing their social influence, ignoring the betweenness centrality that reflects the structural prominence of DMs from a perspective of information control [13–17]. Therefore, in this paper, both centralities are employed to comprehensively determine a DM's importance degree. Personalized feedback is generated based on betweenness centrality, which fully utilizes the social network information.

When a DM verbalizes his/her evaluations, the inherent ambiguity of human cognition is unavoidably introduced into the GDM problem. To better model the preference of a DM, Rodríguez et al. [18] proposed the Hesitant Fuzzy Linguistic Term Set (HFLTS), enabling DMs to express opinions via several linguistic terms. Later, Pang [19] improved HFLTS by associating each linguistic term with a probability and introduced Probabilistic Linguistic Term Set (PLTS). Since PLTS enables DMs to provide more accurate preferences, it is utilized to represent the opinions and trust evaluations of DMs.

While fruitful contributions have been made in CRP for GDM problems, risk preference is rarely considered in existing studies [1, 3, 5, 6, 10]. However, DMs tend to make decisions according to the potential losses and gains relative to a reference point. Thus, prospect theory is employed in this study to characterize the psychological behavior of a DM to ensure a more rational result.

The main contributions of this paper are summarized as follows:

(1) Social network analysis is employed to model the trust relationships between DMs. With in-degree centrality indicating the prestige of a DM within the group, and betweenness centrality representing DM control over information flow, it is more comprehensive to consider both types of centralities when deriving the importance degree of a DM.
(2) A novel consensus reaching model is proposed by combining the IDR-based method with the optimization-based approach, which can ensure the participation of DMs and help the group to achieve a maximum consensus improvement in each round of modification.
(3) During CRP, a referenced opinion is generated based on social network analysis. Besides, the adjustment parameter is customized according to the hesitancy degree and current consensus level to reflect the modification willingness of a DM.
(4) The limited rationality of a DM is considered in the GDM to better model human behavior. Individual opinions and trust evaluations are described using PLTS, which significantly improves the flexibility of the decision information.

The remaining part of this paper is organized as follows: some preliminary concepts are introduced in Sect. 2. In Sect. 3, the maximum consensus improvement-based method for GDM is proposed. In Sect. 4, an illustrative example is provided to verify the effectiveness of the proposed method. In Sect. 5, a comprehensive comparison to the existing CRP methods is conducted. Section 6 completes the paper with some concluding remarks.

2 Preliminaries

In this section, basic concepts of PLTS, Social Network Analysis (SNA), and prospect theory are introduced.

2.1 Probabilistic Linguistic Information

Let $\left\{A_i; i = 1, 2, \dots, m\right\}$ be a set of m alternatives, $\left\{C_j; j = 1, 2, \dots, n\right\}$ be a set of n criteria, and $\left\{e_k; k = 1, 2, \dots, K\right\}$ be a group of DMs. PLTS is utilized to express DMs' evaluations and their trust on the others. Suppose that $(L_{ij}^k(p))_{m\times n}$ is a decision matrix, where $L_{ij}^k(p)$ denotes e_k's preference for alternative A_i over criterion C_j. And $U = (u_{lk})_{K\times K}$ indicates the social trust matrix of DMs, where u_{lk} represents the trust degree of e_l on e_k. The attribute weight can be denoted as $\left\{\delta_j; j = 1, 2, \dots, n\right\}$, with $\sum_{j=1}^n \delta_j = 1$ and $\delta_j \in [0, 1]$.

2.1.1 Linguistic Term Set

The concept of linguistic variable is proposed to approximate human cognition. Generally, the possible value of a linguistic variable can be denoted by a linguistic term set (LTS), which can be denoted as: $S = \left\{S_{-\tau}, \dots, S_{-1}, S_0, S_1, \dots, S_\tau\right\}$, where $2\tau + 1$ is an odd integer representing the granularity of the LTS.S_τ satisfies the following characteristics:

(1) $S_\alpha \geq S_\beta$ iff $\alpha \geq \beta$;
(2) $neg(S_\alpha) = S_{-\alpha}$, where $S_\alpha, S_\beta \in S$ and neg is a negation operator.

2.1.2 Probabilistic Linguistic Term Sets

Definition 1 [19] Let $S = \{S_{-\tau}, \dots, S_{-1}, S_0, S_1, \dots, S_\tau\}$ be a LTS, and the PLTS can be defined as:

$$L(p) = \left\{ L^{(q)}(p^{(q)}) \middle| L^{(q)} \in S, p^{(q)} \geq 0, q = 1, 2, \dots, \#L(p), \sum_{q=1}^{\#L(p)} p^{(q)} \leq 1 \right\}, \tag{1}$$

where $L^{(q)}(p^{(q)})$ denotes the linguistic term $L^{(q)}$ and its corresponding probability $p^{(q)}$, and $\#L(p)$ indicates the number of different linguistic terms in $L(p)$. In this study, we assume $\#L(p) \leq 2$ because PLTS with many linguistic terms could be inaccurate to some extent.

Definition 2 [19] Given $L(p) = \left\{ L^{(q)}(p^{(q)}) | L^{(q)} \in S, p^{(q)} \geq 0, q = 1, 2, \dots, \#L(p), \sum_{q=1}^{\#L(p)} p^{(q)} \leq 1 \right\}$, then the normalized PLTS $\widehat{L}(p)$ can be defined as:

$$\widehat{L}(p) = \left\{ L^{(q)}(\widehat{p}^{(q)}) \middle| L^{(q)} \in S, \widehat{p}^{(q)} \geq 0, q = 1, 2, \dots, \#L(p), \sum_{q=1}^{\#L(p)} \widehat{p}^{(q)} = 1 \right\}, \tag{2}$$

where $\widehat{p}^{(q)} = \frac{p^{(q)}}{\sum_{q=1}^{\#L(p)} p^{(q)}}$ for all $q = 1, 2, \dots, \#L(p)$.

Definition 3 [20] Suppose that $L_k(p) = \left\{ L_k^{(q)}(p_k^{(q)}) \middle| L_k^{(q)} \in S, p_k^{(q)} \geq 0, q = 1, 2, \dots, \#L_k(p), \sum_{q=1}^{\#L_k(p)} p_k^{(q)} = 1 \right\}$ are the normalized PLTS provided by DMs, where $p_k^{(q)}$ is the probability of the linguistic term $L_k^{(q)}$.

The weight vector of a group of DMs $\{e_k; k = 1, 2, \dots, K\}$ is $W = (w_1, w_2, \dots, w_K)^T$, with $\sum_{k=1}^{K} w_k = 1$ and $w_k \in [0, 1]$. Then, the aggregated formula to integrate DMs' evaluations can be defined as follows:

$$L_g(p) = \left\{ L_g^{(q)}(p_g^{(q)}) \middle| L_g^{(q)} \in S, p_g^{(q)} = \sum_{k=1}^{K} v_k^{(q)} w_k, q = 1, 2, \dots, \#L_g(p), \sum_{q=1}^{\#L_g(p)} p_g^{(q)} = 1 \right\}, \tag{3}$$

where $v_k^{(q)}$ is the weight of $L_k^{(q)}$ in $L_k(p)$ and

$$v_k^{(q)} = \begin{cases} p_k^{(q)} & if\ L_g^{(q)} \in L_k(p) \\ 0 & if\ L_g^{(q)} \notin L_k(p) \end{cases}. \tag{4}$$

Definition 4 [20] Let $L_1(p)$ and $L_2(p)$ be two PLTS, then the distance between $L_1(p)$ and $L_2(p)$ is defined as follows:

$$\left| L_1(p) - L_2(p) \right| = \sum_{q=1}^{\#L(p)} \left(p_1^{(q_1)} \times p_2^{(q_2)} \left| \frac{\gamma_1^{q_1} - \gamma_2^{q_2}}{2\tau} \right| \right), \tag{5}$$

where $\#L(p) = \#L_1(p) \times \#L_2(p)$, $\gamma_1^{q_1}$ represents the subscript for the q_1 th linguistic term in $L_1(p)$, and $\gamma_2^{q_2}$ is the subscript for the q_2 th linguistic term in $L_2(p)$.

Definition 5 [20] Let $L(p) = \left\{ L^{(q)}(p^{(q)}) \middle| L^{(q)} \in S, p^{(q)} \geq 0, q = 1, 2, \ldots, \#L(p), \sum_{q=1}^{\#L(p)} p^{(q)} \leq 1 \right\}$ be the PLTS with γ^q denoting the subscript of the linguistic term $L^{(q)}$, then the expectation of $L(p)$ can be calculated as:

$$E(L(p)) = \sum_{q=1}^{\#L(p)} \left(\frac{\gamma^q + \tau}{2\tau} p^{(q)} \right). \tag{6}$$

2.2 Social Network Analysis

The structure of a social network can be characterized by a weighted graph $G = \{E, L, U\}$, where $E = \left\{ e_1, \ldots, e_K \right\}$ represents a group of DMs. $L = \left\{ (e_l, e_k) \middle| (e_l, e_k) \in E^2, l \neq k \right\}$ denotes the trust relationship between DMs, and $U = \left\{ u_{lk} \middle| l, k = 1, 2, \ldots, K \right\}$ represents the corresponding trust degree. Traditionally, social relationships can be depicted in three ways: sociometric, graph and algebraic, which are shown in Table 1.

A binary relationship is represented in the above sociometric, which is unrealistic to model the uncertainty of trust relationships. Thus, in this study, the social network is constructed where trust evaluations are expressed by PLTS.

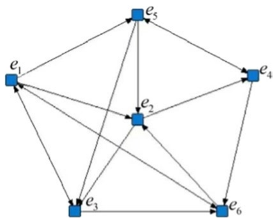
2.3 Prospect Theory

Prospect theory, proposed by Kahneman and Tversky [21], assumes that people rarely behave in an entirely rational way. In a GDM problem, the final decision outcome is derived according to the potential prospect value of gains and losses relative to the reference point. Generally, the higher the prospect value, the better the alternative.

The value function $v(\Delta x)$ is formulated as follows:

$$v(\Delta x) = \begin{cases} \Delta x^{\alpha} & \Delta x \geq 0 \\ -\theta(-\Delta x)^{\beta} & \Delta x < 0 \end{cases}, \tag{7}$$

Table 1 Different representation structures in SNA

Sociometric	Graph	Algebraic
$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$		e_1Re_2 e_2Re_3 e_4Re_5 e_5Re_4 e_1Re_3 e_2Re_4 e_4Re_6 e_6Re_1 e_1Re_5 e_3Re_1 e_5Re_2 e_6Re_2 e_1Re_6 e_3Re_6 e_5Re_3 e_6Re_3

where Δx represents the gain or loss, θ denotes the risk sensitivity of a DM. Besides,α is a risk-seeking parameter while β is a risk-averse parameter, with $0 < \alpha, \beta < 1$. As is depicted in Fig. 1, the value function is concave regarding gains while convex regarding losses. Generally, we set $v(0) = 0$, $\alpha = \beta = 0.88$, $\theta = 2.25$.

3 The Maximum Consensus Improvement-Based Model for GDM Problem

In this section, a novel consensus reaching method is developed. First, the importance degrees of DMs are derived in Sect. 3.1. In Sect. 3.2, the consensus measurement is given to quantify the degree of agreement within the group. In Sect. 3.3, a novel feedback mechanism is constructed to achieve a maximum improvement of group consensus. Then, the selection process is presented in Sect. 3.4 to derive the final ranking. Finally, the framework of the proposed method is summarized in Sect. 3.5.

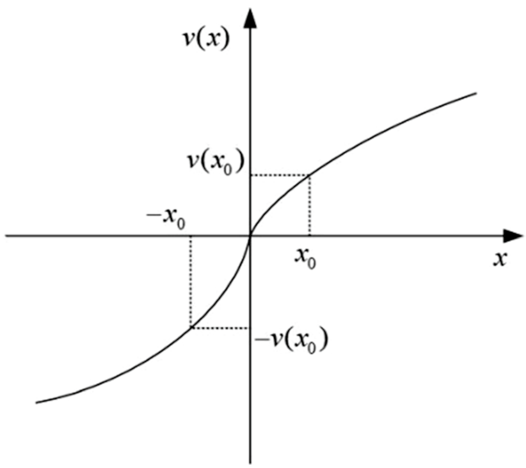
3.1 Determination of DM's Importance Degree

In real-world scenarios, DMs are hardly the same. Different educational levels and social backgrounds affect their trustworthiness. Therefore, it is reasonable to assign different importance degrees to individuals and incorporate such degrees into the generation of group opinion. In Sect. 3.1, a SNA-based method is proposed to derive the weights of DMs.

3.1.1 In-degree Centrality

Since individuals may not express their trust towards others with a simple binary "trust" or "not trust", it may express it in terms of degrees such as "slightly high" or "high" with various probabilities. Thus, PLTS is used to represent the trust evaluation, which will be transformed into trust degrees for later use.

Fig. 1 Value function of prospect theory



As the most used centrality, in-degree centrality measures DM status in terms of incoming links, reflecting how the others trust a DM. The formal definition is given below:

Definition 6 [17] Let u_{lk} be the trust degree $e_l \in E$ has for $e_k \in E$, then the in-degree centrality of e_k can be calculated as:

$$C_D(e_k) = \frac{1}{K-1} \sum_{l=1, l \neq k}^{K} u_{lk}, \tag{8}$$

where $l = 1, 2, \dots, K, k = 1, 2, \dots, K$.

Therefore, in-degree centrality can be used to quantify the prestige of a DM in the social network. The higher the average trust degree aiming at e_k, the larger the $C_D(e_k)$. In other words, e_k is more likely to be the core within the network and should be assigned a larger weight.

3.1.2 Betweenness Centrality

Betweenness centrality, introduced by Freeman [22], serves as an essential indicator to reflect the vertices' structural prominence in a network. When a particular DM is strategically located on the shortest communication path connecting pairs of others, that DM is in a central position. This view is motivated by the notion that DMs with larger betweenness centrality will have access to more information and possess a greater influence on the others. However, the initial measurement [22] is defined only for simple graphs with the binary format, which fails to capture the various trust strengths [23]. Thus, Freeman improved the previous measurement by considering the corresponding trust degrees of the indirect paths, thus linking a pair of DMs [23]. As aforementioned, the value of u_{lk} indicates the trust degree e_l has on e_k, which determines the capacity of maximum information flow from e_l to e_k. Therefore, the initial measurement can be extended as follows:

Definition 7 [23] Let m_{ij} be the maximum flow from $e_i \in E$ to $e_j \in E$, and let $m_{ij}(e_k)$ be the maximum flow from $e_i \in E$ to $e_j \in E$ that passes through e_k. Then, the degree to which the maximum flow between all pairs of DMs depends on e_k is:

$$C_B(e_k) = \sum_{i<j}^{K} \sum_{i \neq j \neq k}^{K} m_{ij}(e_k). \tag{9}$$

In our method, the betweenness centrality is utilized in two ways: first, it is used to derive the weight of DM which will be illustrated in the next subsection; secondly, it is employed to generate modification reference in CRP, which will be demonstrated in Sect. 3.3.

3.1.3 The Comprehensive Weight of DM

The in-degree centrality quantifies a DM's prestige via the average trust degree aiming at him/her, whereas the betweenness centrality indexes the potential for a DM to control information in the social network. Obviously, these two types of centralities reflect the "importance" of the different aspects and may lead to different rankings of DMs within the same network.

However, existing studies on CRP under a social network setting on only employ in-degree centrality to determine DM's social influence [13–17] or simply assume the weights of DMs to be known in advance [24, 25]. Thus, it is more comprehensive to consider these two centralities to derive DMs' importance degrees.

Let $C_D = (C_D(e_1), C_D(e_2), \ldots, C_D(e_K))^T$ and $C_B = (C_B(e_1), C_B(e_2), \ldots, C_B(e_K))^T$ denote the in-degree centrality vector and betweenness centrality vector, respectively. $\zeta \in [0, 1]$ is a predefined parameter representing the relative significance of C_D and C_B. Thus, the importance degree $\widetilde{w_k}$ of DM e_k and its corresponding weight w_k can be derived as follows:

$$\widetilde{w_k} = \zeta \frac{C_D(e_k)}{\max\limits_{k=1,\ldots,K} \left\{C_D(e_k)\right\}} + (1-\zeta)\frac{C_B(e_k)}{\max\limits_{k=1,\ldots,K} \left\{C_B(e_k)\right\}}, \tag{10}$$

$$w_k = \frac{\widetilde{w_k}}{\sum_{k=1}^{K} \widetilde{w_k}}, \tag{11}$$

where $w_k \in [0, 1]$, $\sum_{k=1}^{K} w_k = 1$.

3.2 Consensus Measure

The importance degree of a DM in a social network should be incorporated into the generation of the collective opinion. Thus, the group evaluation matrix $(L_{ij}^g(p))_{m\times n}$ can be obtained as follows:

$$L_{ij}^g(p) = \sum_{k=1}^{K} w_k L_{ij}^k(p), \tag{12}$$

where $(L_{ij}^k(p))_{m\times n}$ denotes e_k's decision matrix, and $w_k (k = 1, 2, \ldots, K)$ represents the corresponding weight.

Before deriving the final result, it is necessary to ensure that the group consensus level (GCL) has reached the preset threshold. Generally, consensus could be measured in two ways: one is based on the distance between individual opinion and group evaluation. The other is based on the deviation degree between the individual assessments. The first method is considered in this study due to its relatively low computational complexity.

Definition 8 Let $L_{ij}^k(p)$ be the evaluation of e_k, and $L_{ij}^g(p)$ be the group evaluation. Then, the consensus level CL_{ij}^k of e_k to the group on the evaluation of alternative A_i over attribute C_j is defined as:

$$CL_{ij}^k = 1 - \left|L_{ij}^k(p) - L_{ij}^g(p)\right|, \tag{13}$$

where $CL_{ij}^k \in [0, 1]$.

Definition 9 The overall consensus level CL_k of e_k to the group is defined as:

$$CL_k = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} CL_{ij}^k, \tag{14}$$

where $CL_k \in [0, 1]$. The smaller the deviation between e_k's opinion and the group, the larger the consensus level CL_k.

Definition 10 Let w_k be the weight of e_k and CL_k represents e_k's consensus level, then the current GCL can be calculated as:

$$GCL = \sum_{k=1}^{K} w_k CL_k, \tag{15}$$

clearly, $GCL \in [0, 1]$. A larger GCL indicates a higher consensus level within the group.

Then, GCL will be compared with a preset threshold ξ to verify the acceptability of group opinion. If $GCL \geq \xi$, the selection process will be carried out. Otherwise, the feedback mechanism should be activated to guide the modification of evaluations.

3.3 Feedback Mechanism Guided by Social Relationship

In this subsection, a novel feedback mechanism based on SNA is developed to promote consensus, which mainly includes three parts: first, the generation of referenced opinion is illustrated; then, the process used to obtain the personalized adjustment parameter is discovered. Finally, a nonlinear optimization model is built to maximize the improvement of GCL in each iteration of modification.

3.3.1 Selection of Referenced Evaluation

Generally, consensus improvement could be achieved by two strategies: the IDR-based method and the optimization-based approach. The IDR-based method involves an iterative process, while the optimization-based approach can greatly reduce CRP time. However, the modified evaluations generated merely through an optimization-based approach would be somewhat uninterpretable for DMs and difficult to accept. Therefore, by combining these two approaches, a novel feedback mechanism is constructed to reflect the subjectivity of a DM and ensure the efficiency of CRP.

The existing IDR-based models mainly utilize group evaluation or a collective opinion from trusted DMs as the referenced opinion [13–17, 26]. However, in real-world scenarios, the DM within the group is supposed to be "responsive" to the person with high betweenness centrality because this person is accessible to more information. Thus, the DM is more inclined to refer to that person when modifying their evaluations. Different from existing methods, the DM is assumed to modify his/her preference toward the individual with the highest betweenness centrality among the trusted DMs in this study.

3.3.2 The Adjustment Strategy Based on DM's Subjective Willingness

The personalized update function can be established to guide the adjustment of evaluations. Suppose $L_{ij}^k(p)$ is the value that needs modification, and e_r is the individual with the highest betweenness centrality among the DMs e_k trusts. Then, e_r's corresponding opinion

$L_{ij}^{r(k)}(p)$ will be utilized as the reference for e_k to revise his/her statement. The modified evaluation $L_{ij}^{k}(p)$ can be derived as follows:

$$\overline{L_{ij}^{k}(p)} = (1-\lambda_{ij}^{k})L_{ij}^{k}(p) + \lambda_{ij}^{k}L_{ij}^{r(k)}(p). \tag{16}$$

To obtain an updated evaluation, the different tendency of a DM to change needs to be quantified, which can be denoted as λ_{ij}^{k}. In the previous study, the adjustment parameter is often selected with discretion [16, 27]. However, the willingness of modification varies with each individual. If λ_{ij}^{k} is given randomly, the DM will be forced to accept the recommendation, which is unreasonable. Thus, in the proposed model, λ_{ij}^{k} is customized based on two factors.

The first one is the hesitancy degree. If a DM expresses his/her evaluation via a single linguistic term, he/she shows a high level of confidence in this judgment. Conversely, if the assessment includes several linguistic terms with similar probability, this DM is supposed to exhibit a higher degree of uncertainty and is more inclined to modify his/her opinion when necessary.

Definition 11 Let $L_{ij}^{k}(p) = \left\{ L_{ij}^{k(q)}(p_{ij}^{k(q)}) \middle| L_{ij}^{k(q)} \in S, p_{ij}^{k(q)} \geq 0, q = 1,2,\ldots,\#L_{ij}^{k}(p), \sum_{q=1}^{\#L_{ij}^{k}(p)} p_{ij}^{k(q)} = 1 \right\}$ denotes e_k's normalized evaluation. Then, the hesitancy degree of $L_{ij}^{k}(p)$ is defined as:

$$HD(L_{ij}^{k}(p)) = -\frac{\sum_{q=1}^{\#L_{ij}^{k}(p)} p_{ij}^{k(q)} \log_2 p_{ij}^{k(q)}}{\log_2 \#L_{ij}^{k}(p)}, \tag{17}$$

where $i = 1,2,\ldots,m,\ j = 1,2,\ldots,n,\ k = 1,2,\ldots,K,\ \#L_{ij}^{k}(p)$ is the number of linguistic terms in $L_{ij}^{k}(p)$, and $HD(L_{ij}^{k}(p)) \in [0,1]$. To ensure the accuracy of PLTS, we assume that $\#L_{ij}^{k}(p) \leq 2$. Based on information entropy, the denominator $\log_2 \#L_{ij}^{k}(p)$ in Eq. (17) represents the maximum of $HD(L_{ij}^{k}(p))$, which is utilized to normalize the hesitancy degree. Thus, in this study, $HD(L_{ij}^{k}(p)) = -\sum_{q=1}^{\#L_{ij}^{k}(p)} p_{ij}^{k(q)} \log_2 p_{ij}^{k(q)}$. A larger $HD(L_{ij}^{k}(p))$ indicates that e_k has a stronger willingness to modify $L_{ij}^{k}(p)$ and vice versa.

The second factor relates to the current consensus level CL_{ij}^{k}. We assume that e_k with a larger CL_{ij}^{k} would consider his/her evaluation closer to the consensus. Thus, there is no need to make a major adjustment. Conversely, the DM with a lower CL_{ij}^{k} will be more actively engaged in the opinion modification to promote consensus. Therefore, the adjustment parameter λ_{ij}^{k} of $L_{ij}^{k}(p)$ can be customized as follows:

$$\lambda_{ij}^{k} = \varepsilon HD_{ij}^{k} + (1-\varepsilon)(1-CL_{ij}^{k}), \tag{18}$$

where $\lambda_{ij}^{k} \in [0,1]$, ε is a coefficient with $\varepsilon \in [0,1]$, which denotes the relative importance of the two factors. The larger the λ_{ij}^{k}, the greater the adjustment that e_k is willing to make. Thus, it is easier for the group to reach a consensus.

3.3.3 The Maximum GCL Improvement Optimization Model

In the following subsection, a nonlinear optimization model is constructed to identify the DM whose modification on a specific evaluation could maximize the improvement of group consensus in each iteration

$$
\max : \Delta GCL = \overline{GCL} - GCL
$$

$$
s.t.\begin{cases}
L_{ij}^{g}(p) = \sum_{k=1}^{K} w_k L_{ij}^{k}(p) & \text{(19-1)}\\
\overline{L_{ij}^{g}(p)} = \sum_{k=1}^{K} w_k \overline{L_{ij}^{k}(p)} & \text{(19-2)}\\
CL_{ij}^{k} = 1 - \left| L_{ij}^{k}(p) - L_{ij}^{g}(p) \right| & \text{(19-3)}\\
\overline{CL_{ij}^{k}} = 1 - \left| \overline{L_{ij}^{k}(p)} - \overline{L_{ij}^{g}(p)} \right| & \text{(19-4)}\\
CL_k = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} CL_{ij}^{k} & \text{(19-5)}\\
\overline{CL_k} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{CL_{ij}^{k}} & \text{(19-6)}\\
HD(L_{ij}^{k}(p)) = -\sum_{q=1}^{\#L_{ij}^{k}(p)} p_{ij}^{k(q)} \log_2 p_{ij}^{k(q)} & \text{(19-7)}\\
\lambda_{ij}^{k} = \varepsilon HD(L_{ij}^{k}(p)) + (1-\varepsilon)(1 - CL_{ij}^{k}) & \text{(19-8)}\\
\overline{L_{ij}^{k}(p)} = (1-\lambda_{ij}^{k}) L_{ij}^{k}(p) + \lambda_{ij}^{k} L_{ij}^{r(k)}(p) & \text{(19-9)}\\
GCL = \sum_{k=1}^{K} w_k CL_k & \text{(19-10)}\\
\overline{GCL} = \sum_{k=1}^{K} w_k \overline{CL_k} & \text{(19-11)},
\end{cases}
\tag{19}
$$

where $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, K$. $\overline{CL_{ij}^{k}}$, $\overline{CL_k}$ and $\overline{GCL}$ denote the updated consensus levels after one iteration. The final solution can be computed by LINGO17.0 software.

To preserve the original information as much as possible, only one judgment of a specific DM is modified each time. With model (19), the proposed feedback mechanism can guarantee the interaction of DMs and ensure a maximum improvement of GCL could be achieved in each round of modification. To describe the proposed method more concisely, an algorithm is given as follows:

Algorithm 1 Iterative CRP

Input: The initial normalized decision matrix $(L_{ij}^{k}(p))_{m\times n}$, the weight vector of DMs $W = (w_1, w_2, \ldots, w_K)^T$, with $w_k \in [0, 1]$, $\sum_{k=1}^{K} w_k = 1$ $(k = 1, 2, \ldots, K)$. The predefined consensus threshold $\xi(\xi \in [0, 1])$, and the maximum number of iterations $t_{\max}$ $(t_{\max} \geq 1)$.

Output: The final group decision matrix $(L_{ij}^{g}(p))_{m\times n}$, the group consensus level GCL, and the terminal iteration $t(t \in [0, t_{\max}])$.

Step 1: Let $t = 0$ and $(L_{ij}^{k(0)}(p))_{m\times n} = (L_{ij}^{k}(p))_{m\times n}(k = 1, 2, ..., K)$. The temporary group decision matrix $(L_{ij}^{g(0)}(p))_{m\times n}$ can be computed via Eq. (12).

Step 2: The current consensus levels $CL_{ij}^{k(t)}(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$, $CL_k^{(t)}$, $GCL^{(t)}$ of e_k and the group can be derived via Eqs. (13)-(15). If $GCL^{(t)} \geq \xi$ or $t \geq t_{\max}$, go to the Step 5; otherwise, proceed to Step 3.

Step 3: Assume that $L_{ij}^{k(t)}$ is the evaluation whose modification could maximize the improvement of GCL according to model (19).

Step 4: Generate the modified evaluation $L_{ij}^{k(t+1)}$ and update the decision matrix of e_k; let $t = t + 1$, then return to Step 2.

Step 5: Let $(L_{ij}^{g}(p))_{m\times n} = (L_{ij}^{g(t)}(p))_{m\times n}$, $GCL = GCL^{(t)}$. Output the final group decision matrix $(L_{ij}^{g}(p))_{m\times n}$, ultimate group consensus level GCL and terminal iteration t.

3.4 Selection Process Based on Prospect Theory

Once a group consensus has been reached, the GDM moves on to the selection process to derive the final ranking of alternatives. In this process, the limited rationality of DMs is considered to better model human behavior. The distance between alternatives and the positive ideal point is selected as the reference to measure the deviation degree. Therefore, the prospect value function is formulated as follows:

Definition 12 Suppose $L_1(p)$, $L_2(p)$ are two PLTS, and $d(L_2(p), L_+(p))$ denotes the reference point, with $L_+(p) = \{S_3(1)\}$ representing the positive ideal solution. Then the prospect value function of $L_1(p)$ can be presented as follows:

$$v(L_1(p)) = \begin{cases} \left[d(L_2(p), L_+(p)) - d(L_1(p), L_+(p))\right]^{\alpha} & d(L_1(p), L_+(p)) \le d(L_2(p), L_+(p)) \\ -\theta\left[d(L_1(p), L_+(p)) - d(L_2(p), L_+(p))\right]^{\beta} & d(L_1(p), L_+(p)) > d(L_2(p), L_+(p)) \end{cases}, \tag{20}$$

where $\alpha = \beta = 0.88, \theta = 2.25$.

Then, the positive and negative prospect value matrices can be constructed:

$$v_{ij}^{+} = \left[d_{\max}(L_{ij}^{g}(p), L_+(p)) - d(L_{ij}^{g}(p), L_+(p))\right]^{\alpha}, \tag{21}$$

$$v_{ij}^{-} = -\theta\left[d(L_{ij}^{g}(p), L_+(p)) - d_{\min}(L_{ij}^{g}(p), L_+(p))\right]^{\beta}, \tag{22}$$

when $d_{\max}(L_{ij}^{g}(p), L_+(p))$ is the reference point, then the larger the v_{ij}^{+}, the better the alternative A_i with respect to criterion C_j. When $d_{\min}(L_{ij}^{g}(p), L_+(p))$ is the reference point, then the smaller the v_{ij}^{-}, the worse the alternative A_i regarding C_j. The utilization of these two reference points is beneficial for deriving a more convincing result.

Then, the overall weighted prospect value V_i of the alternatives can be derived to obtain the final ranking.

Definition 13 The weighted prospect value V_i of alternative A_i can be defined as follows:

$$V_i = \sum_{j=1}^{n} \delta_j v_{ij}^{+} - \left|\sum_{j=1}^{n} \delta_j v_{ij}^{-}\right|, \tag{23}$$

where δ_j denotes the weight of criteria C_j, with $\sum_{j=1}^{n} \delta_j = 1$ and $\delta_j \in [0, 1]$. The larger the V_i, the better the alternative A_i.

3.5 Framework for GDM with PLTS Under Social Network

To demonstrate the proposed method, a framework is shown in Fig. 2, and the detailed procedures are illustrated as follows:

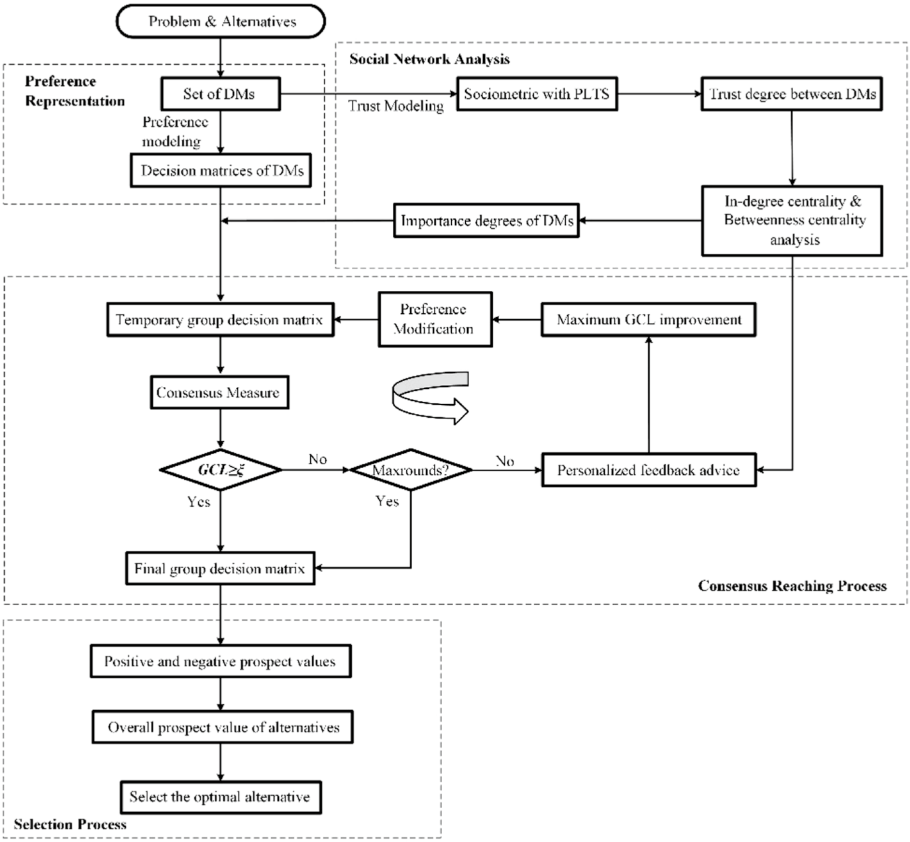


Fig. 2 A general framework of the proposed method

Step 1 The preference of DMs on the alternatives over different criteria is given by PLTS, and a predefined consensus threshold is set.
Step 2 DMs express their trust in the others using PLTS, based on which a social network is established.
Step 3 Calculate $C_D(e_k)$ and $C_B(e_k)$ of each DM by Eqs. (8) and (9), then w_k of each DM is derived by Eq. (11).
Step 4 Then, $L_{ij}^{g}(p)$ can be generated by aggregating each DM's evaluation matrix by Eq. (12).
Step 5 Consensus levels CL_{ij}^{k}, CL_k and $GCL(i = 1, 2, \dots, m, j = 1, 2, \dots, n, k = 1, 2, \dots, K)$ are calculated by Eqs. (13)-(15). If $GCL \geq \xi$, go to Step 7. Otherwise, the CRP should be activated.
Step 6 The personalized feedback advice is generated by model (19) to achieve the maximum GCL improvement, then return to Step 4.
Step 7 The positive and negative prospect value matrices v_{ij}^{+} and v_{ij}^{-} can be established by Eqs. (21) and (22) with the final group evaluation.
Step 8 The overall weighted prospect value V_i can be derived via Eq. (23).
Step 9 The ranking of alternatives is obtained.

4 Case Analysis

In this section, an illustrative example is presented to demonstrate the effectiveness of the proposed model.

4.1 Problem Description

A group of 12 DMs denoted as $\{e_k;k = 1, 2, \dots, 12\}$ are invited by an investment firm to select the most suitable company to invest in. After preliminary analysis, three alternatives are selected: A_1 is an insurance company, A_2 is a technology company, and A_3 is a pharmaceutical company. The following three attributes are chosen to judge the alternatives: C_1 represents the development potential, C_2 represents the business capacity, and C_3 represents social influence. The weight vector of the attributes is given as: $W = (0.3, 0.3, 0.4)^T$.

4.2 The Steps of the Proposed Method for Investment Evaluation

Step 1 $S = \{S_{-3} = \textit{very low},\ S_{-2} = \textit{low},\ S_{-1} = \textit{slightly low},\ S_0 = \textit{fair},\ S_1 = \textit{slightly high},\ S_2 = \textit{high},\ S_3 = \textit{very high}\}$ represents the LTS provided. DMs are invited to provide their evaluations on the alternatives over three attributes by PLTS. For simplicity, the original decision information is omitted here. And the normalized decision matrices are presented in Table 2.

Table 2 The decision matrices expressed by PLTS from the group of DMs

		C_1	C_2	C_3
e_1	A_1	$\{S_2(0.286), S_3(0.714)\}$	$\{S_0(1)\}$	$\{S_0(0.1), S_1(0.9)\}$
	A_2	$\{S_{-1}(0.4), S_0(0.6)\}$	$\{S_1(1)\}$	$\{S_0(1)\}$
	A_3	$\{S_2(1)\}$	$\{S_1(0.3), S_2(0.7)\}$	$\{S_1(0.3), S_2(0.7)\}$
e_2	A_1	$\{S_1(0.8), S_2(0.2)\}$	$\{S_3(1)\}$	$\{S_{-1}(0.3), S_0(0.7)\}$
	A_2	$\{S_2(1)\}$	$\{S_0(0.429), S_1(0.571)\}$	$\{S_2(1)\}$
	A_3	$\{S_0(1)\}$	$\{S_0(0.5), S_1(0.5)\}$	$\{S_2(0.222), S_3(0.777)\}$
e_3	A_1	$\{S_1(0.5), S_2(0.5)\}$	$\{S_0(1)\}$	$\{S_0(0.2), S_1(0.8)\}$
	A_2	$\{S_{-1}(0.4), S_0(0.6)\}$	$\{S_1(0.6), S_2(0.4)\}$	$\{S_{-1}(1)\}$
	A_3	$\{S_0(1)\}$	$\{S_0(0.5), S_1(0.5)\}$	$\{S_0(1)\}$
e_4	A_1	$\{S_{-2}(1)\}$	$\{S_3(1)\}$	$\{S_0(0.3), S_1(0.7)\}$
	A_2	$\{S_1(1)\}$	$\{S_2(0.6), S_3(0.4)\}$	$\{S_{-2}(1)\}$
	A_3	$\{S_1(0.7), S_2(0.3)\}$	$\{S_{-1}(0.5), S_0(0.5)\}$	$\{S_1(0.9), S_2(0.1)\}$
e_5	A_1	$\{S_{-1}(1)\}$	$\{S_3(1)\}$	$\{S_3(1)\}$
	A_2	$\{S_0(0.375), S_1(0.625)\}$	$\{S_0(0.6), S_1(0.4)\}$	$\{S_0(1)\}$
	A_3	$\{S_1(0.3), S_2(0.7)\}$	$\{S_{-1}(0.5), S_0(0.5)\}$	$\{S_1(0.286), S_2(0.714)\}$
e_6	A_1	$\{S_1(0.286), S_2(0.714)\}$	$\{S_0(1)\}$	$\{S_1(0.3), S_2(0.7)\}$
	A_2	$\{S_0(0.167), S_1(0.833)\}$	$\{S_{-1}(1)\}$	$\{S_0(1)\}$
	A_3	$\{S_2(0.3), S_3(0.7)\}$	$\{S_{-1}(0.4), S_0(0.6)\}$	$\{S_1(0.4), S_2(0.6)\}$
e_7	A_1	$\{S_{-1}(1)\}$	$\{S_3(1)\}$	$\{S_2(0.8), S_3(0.2)\}$
	A_2	$\{S_1(1)\}$	$\{S_0(0.6), S_1(0.4)\}$	$\{S_{-3}(1)\}$
	A_3	$\{S_{-1}(0.3), S_0(0.7)\}$	$\{S_0(0.5), S_1(0.5)\}$	$\{S_1(0.8), S_2(0.2)\}$
e_8	A_1	$\{S_1(0.3), S_2(0.7)\}$	$\{S_1(0.6), S_2(0.4)\}$	$\{S_2(0.8), S_3(0.2)\}$
	A_2	$\{S_2(1)\}$	$\{S_0(0.6), S_1(0.4)\}$	$\{S_0(0.6), S_1(0.4)\}$
	A_3	$\{S_{-1}(0.2), S_0(0.8)\}$	$\{S_{-2}(1)\}$	$\{S_{-1}(1)\}$
e_9	A_1	$\{S_1(0.25), S_2(0.75)\}$	$\{S_2(0.6), S_3(0.4)\}$	$\{S_2(0.25), S_3(0.75)\}$
	A_2	$\{S_{-2}(1)\}$	$\{S_1(1)\}$	$\{S_0(0.7), S_1(0.3)\}$
	A_3	$\{S_{-2}(0.3), S_{-1}(0.7)\}$	$\{S_0(1)\}$	$\{S_0(1)\}$
e_{10}	A_1	$\{S_0(0.75), S_1(0.25)\}$	$\{S_2(0.6), S_3(0.4)\}$	$\{S_1(0.5), S_2(0.5)\}$
	A_2	$\{S_{-1}(1)\}$	$\{S_{-2}(1)\}$	$\{S_{-1}(0.5), S_0(0.5)\}$
	A_3	$\{S_2(1)\}$	$\{S_2(0.571), S_3(0.429)\}$	$\{S_1(1)\}$
e_{11}	A_1	$\{S_0(1)\}$	$\{S_2(0.5), S_3(0.5)\}$	$\{S_0(0.571), S_1(0.429)\}$
	A_2	$\{S_{-2}(1)\}$	$\{S_1(1)\}$	$\{S_{-1}(0.4), S_0(0.6)\}$
	A_3	$\{S_{-2}(0.4), S_{-1}(0.6)\}$	$\{S_2(0.75), S_3(0.25)\}$	$\{S_3(1)\}$
e_{12}	A_1	$\{S_2(1)\}$	$\{S_0(0.2), S_1(0.8)\}$	$\{S_0(0.5), S_1(0.5)\}$
	A_2	$\{S_2(1)\}$	$\{S_1(1)\}$	$\{S_1(0.4), S_2(0.6)\}$
	A_3	$\{S_1(0.1), S_2(0.9)\}$	$\{S_0(0.5), S_1(0.5)\}$	$\{S_1(1)\}$

Step 2 The linguistic trust evaluations between DMs are presented in the following matrix.

$$T=(t_{lk})_{12\times 12}$$

$$=\begin{bmatrix}
\{S_0(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_1(0.5),S_2(0.5)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_1(1)\} & \{S_{-3}(1)\} & \{S_2(0.5),S_3(0.5)\} & \{S_{-3}(1)\} & \{S_2(1)\} & \{S_2(1)\} \\
\{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_{-3}(1)\} \\
\{S_1(0.5),S_2(0.5)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_{-2}(0.5),S_{-1}(0.5)\} & \{S_{-3}(1)\} & \{S_1(0.25),S_2(0.75)\} & \{S_{-2}(1)\} & \{S_3(1)\} & \{S_{-1}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} \\
\{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_2(1)\} & \{S_3(1)\} \\
\{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_2(1)\} & \{S_{-3}(1)\} & \{S_2(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} \\
\{S_2(0.4),S_3(0.6)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} \\
\{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_1(0.25),S_2(0.75)\} \\
\{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_1(0.5),S_2(0.5)\} & \{S_3(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} \\
\{S_3(1)\} & \{S_3(1)\} & \{S_2(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} & \{S_2(1)\} & \{S_3(1)\} & \{S_0(1)\} & \{S_1(1)\} & \{S_{-3}(1)\} & \{S_3(1)\} \\
\{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_1(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} & \{S_2(0.4),S_3(0.6)\} \\
\left\{S_{-2}(\frac{2}{3}),S_{-1}(\frac{1}{3})\right\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_1(0.75),S_2(0.25)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \left\{S_1(\frac{4}{7}),S_2(\frac{3}{7})\right\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_{-3}(1)\} \\
\{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-3}(1)\} & \{S_{-2}(1)\} & \{S_3(1)\} & \{S_1(0.5),S_2(0.5)\} & \{S_{-3}(1)\} & \{S_0(1)\} & \{S_2(0.4),S_3(0.6)\} & \{S_{-3}(1)\} & \{S_0(1)\}
\end{bmatrix}$$

Then, the linguistic terms are transformed into trust degrees via Eq. (6) to establish the social network.

$$U = (u_{lk})_{12\times 12} = \begin{bmatrix} 0.5 & 0 & 0 & 0.75 & 0 & 0 & 0.67 & 0 & 0.92 & 0 & 0.83 & 0.83 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0.75 & 0 & 0.5 & 0 & 0.25 & 0 & 0.79 & 0.17 & 1 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0.83 & 1 \\ 0 & 0 & 1 & 0 & 0.5 & 0 & 0.83 & 0 & 0.83 & 0 & 0 & 0 \\ 0.93 & 0 & 0 & 1 & 0 & 0.5 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 1 & 0.79 \\ 0 & 0 & 0.75 & 1 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0.83 & 0 & 0 & 1 & 0.83 & 1 & 0.5 & 0.67 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.67 & 0 & 0 & 0.5 & 0 & 0.93 \\ 0.22 & 0 & 0 & 0 & 0.71 & 0 & 0 & 0.74 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.17 & 1 & 0.75 & 0 & 0.5 & 0.93 & 0 & 0.5 \end{bmatrix}$$

The social graph that visualizes the trust relations between DMs is depicted in Fig. 3. The node's color indicates its importance degree in the network, which will be calculated in the following step.

Step 3 $C_D(e_k)$ and $C_B(e_k)$ of DM are calculated via Eqs. (8) and (9), respectively. We assume that two types of centralities are equally important. Thus, the weight of DMs can be obtained by solving Eq. (11) with $\zeta = 0.5$. The results are shown in Table 3.

Table 3 demonstrates that the utilization of different centralities could result in different rankings of DMs. For example, e_7 is the DM with the highest $C_D(e_k)$, but it has a relatively low $C_B(e_k)$. Thus, it is more reasonable and comprehensive to consider both centralities when obtaining the weight of DMs. Overall, e_{12} is the DM with the largest importance degree while e_2 is the one with the least.

Step 4 The initial group decision matrix $L_{ij}^{g(0)}(p)$ can be obtained by Eq. (12) as follows:

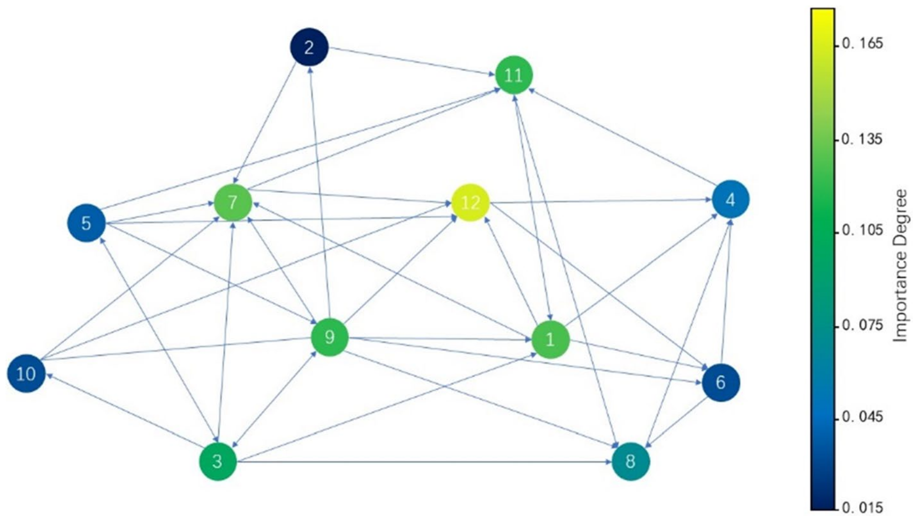


Fig. 3 The social network of 12 DMs

Table 3 Centrality and weight of DMs in Fig. 3

DM	$C_D(e_k)$	$C_B(e_k)$	w_k
e_1	0.523	0.854	0.124
e_2	0.181	0	0.016
e_3	0.466	0.646	0.100
e_4	0.496	0.083	0.052
e_5	0.204	0.250	0.041
e_6	0.361	0.042	0.036
e_7	1.000	0.396	0.126
e_8	0.525	0.250	0.070
e_9	0.587	0.708	0.117
e_{10}	0.348	0.042	0.035
e_{11}	0.661	0.625	0.116
e_{12}	0.821	1.000	0.165

Table 4 The initial consensus level $CL_k^{(0)}$ of each DM to the group ($t = 0$)

$k =$	1	2	3	4	5	6
$CL_k^{(0)}$	0.791	0.760	0.804	0.744	0.774	0.766
$k =$	7	8	9	10	11	12
$CL_k^{(0)}$	0.768	0.756	0.784	0.749	0.756	0.799

$$L_{ij}^{g(0)}(p) = \begin{bmatrix} A & B & C \end{bmatrix},$$

where

$$A = \begin{bmatrix} \{S_{-2}(0.052), S_{-1}(0.167), S_0(0.143), S_1(0.133), S_2(0.416), S_3(0.089)\} \\ \{S_{-2}(0.233), S_{-1}(0.125), S_0(0.156), S_1(0.234), S_2(0.251)\} \\ \{S_{-2}(0.082), S_{-1}(0.203), S_0(0.261), S_1(0.197), S_2(0.231), S_3(0.025)\} \end{bmatrix},$$

$$B = \begin{bmatrix} \{S_0(0.294), S_1(0.174), S_2(0.177), S_3(0.355)\} \\ \{S_{-2}(0.035), S_{-1}(0.036), S_0(0.149), S_1(0.687), S_2(0.072), S_3(0.021)\} \\ \{S_{-2}(0.07), S_{-1}(0.061), S_0(0.389), S_1(0.241), S_2(0.194), S_3(0.044)\} \end{bmatrix},$$

$$C = \begin{bmatrix} \{S_{-1}(0.005), S_0(0.208), S_1(0.389), S_2(0.229), S_3(0.168)\} \\ \{S_{-3}(0.126), S_{-1}(0.052), S_0(0.165), S_1(0.413), S_2(0.129), S_3(0.115)\} \\ \{S_{-1}(0.07), S_0(0.218), S_1(0.411), S_2(0.172), S_3(0.129)\} \end{bmatrix}.$$

Step 5 Then, the consensus degree $CL_{ij}^{k(0)}$ and $CL_k^{(0)}$ of e_k can be derived via Eqs. (13) and (14). For brevity, only $CL_k^{(0)}$ is listed in Table 4.
Thus, the current group consensus level $GCL^{(0)}$ can be obtained by Eq. (15) with $GCL^{(0)} = 0.776$.
Step 6 The consensus threshold is set to $\xi = 0.8$, and the maximum number of iterations is set to $t_{\max} = 30$. Thus, the feedback mechanism should be implemented where personalized modification advice is generated to promote consensus.

Table 5 The consensus level $CL_k^{(1)}$ of each DM to the group $(t=1)$

$k=$	1	2	3	4	5	6
$CL_k^{(1)}$	0.795	0.761	0.806	0.745	0.775	0.777
$k=$	7	8	9	10	11	12
$CL_k^{(1)}$	0.769	0.757	0.785	0.751	0.756	0.800

(i) First-round

According to model (19), the modification of $L_{23}^{12}(p)$ will help the group to achieve a greater improvement of the GCL than any other possible alterations. From the social network information, e_9 is the one with the largest betweenness centrality among the DMs that e_{12} trusts, which indicates that e_9 has access to more information and exerts a stronger influence on e_{12}'s judgement. Due to time pressure in GDM and an individual's limited rationality, the DM is unwilling to refer to too many opinions. Thus, $L_{23}^{9}(p)$ will be the referenced opinion for e_{12} to modify $L_{23}^{12}(p)$.

The hesitancy degree of $L_{23}^{12}(p)$ can be computed via Eq. (17) as:

$$HD(L_{23}^{12}(p)) = -(0.4\log_2 0.4 + 0.6\log_2 0.6) \approx 0.971.$$

The current consensus level of $L_{23}^{12}(p)$ can be obtained by Eq. (13) as $CL_{23}^{12} = 0.669$. Then, by using $\varepsilon = 0.5$, the adjustment parameter λ_{23}^{12} can be derived via Eq. (18) as:

$$\lambda_{23}^{12} = 0.5 \times 0.971 + 0.5 \times (1 - 0.669) = 0.651.$$

Thus, the modified evaluation of $L_{23}^{12}(p)$ can be obtained by Eq. (16) as:

$$\overline{L_{23}^{12}(p)} = (1-0.651)L_{23}^{12}(p) + 0.651L_{23}^{9}(p) = \left\{S_0(0.455), S_1(0.335), S_2(0.209)\right\}.$$

After e_{12} modifies his/her evaluation, the group evaluation will be reaggregated and the updated consensus level $CL_k^{(1)}$ of each DM is given in Table 5.

The group consensus level can be obtained as: $GCL^{(1)} = 0.781$. Obviously, the GCL is improved after one round of modification. However, the group has yet to achieve the predefined threshold. Thus, these steps are repeated until $\xi = 0.8$ is satisfied.

(ii) Round 2-round 12

After 12 iterations, the $GCL = 0.803 > 0.8$. Thus, the CRP is terminated. The detailed iteration process is described in Table 6.

The improvement of GCL during the whole CRP is depicted in Fig. 4.

Figure 4 shows that the GCL is constantly increasing during the process, which illustrates the effectiveness of the proposed method. The consensus levels of the DMs before and after CRP are depicted in Fig. 5, which demonstrates that the consensus degrees of all DMs have been effectively promoted under our proposed strategy.

Table 6 Modification process

t	$L_{ij}^{k(t)}$	$L_{ij}^{r(k(t))}$	$\lambda_{ij}^{k(t)}$	$GCL^{(t)}$
1	L_{23}^{12}	L_{23}^{9}	0.651	0.781
2	L_{32}^{12}	L_{32}^{9}	0.877	0.783
3	L_{31}^{9}	L_{31}^{12}	0.589	0.785
4	L_{31}^{11}	L_{31}^{1}	0.653	0.787
5	L_{31}^{8}	L_{31}^{3}	0.928	0.789
6	L_{11}^{1}	L_{11}^{12}	0.582	0.791
7	L_{31}^{7}	L_{31}^{12}	0.555	0.792
8	L_{31}^{11}	L_{31}^{1}	0.762	0.794
9	L_{31}^{7}	L_{31}^{12}	0.910	0.796
10	L_{22}^{4}	L_{22}^{12}	0.622	0.797
11	L_{23}^{2}	L_{23}^{11}	0.209	0.799
12	L_{23}^{7}	L_{23}^{12}	0.667	0.803

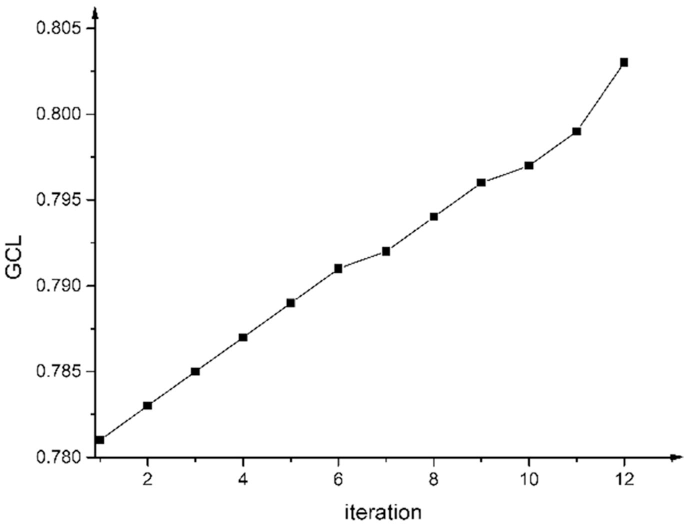


Fig. 4 The improvement of GCL during CRP

Step 7 The final group evaluation $L_{ij}^{g(12)}(p)$ with sufficient consensus level can be determined as follows:

$$L_{ij}^{g(12)}(p) = \left[A' \;\; B' \;\; C' \right],$$

$$A' = \begin{bmatrix} \{S_{-2}(0.052), S_{-1}(0.167), S_0(0.143), S_1(0.133), S_2(0.468), S_3(0.037)\} \\ \{S_{-2}(0.233), S_{-1}(0.125), S_0(0.156), S_1(0.234), S_2(0.251)\} \\ \{S_{-2}(0.005), S_{-1}(0.024), S_0(0.176), S_1(0.408), S_2(0.361), S_3(0.025)\} \end{bmatrix},$$

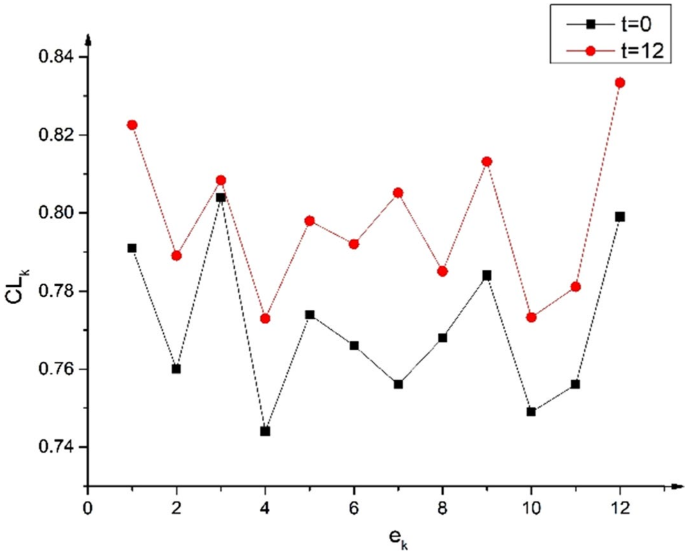


Fig. 5 The consensus levels of DMs before and after CRP

$$B' = \begin{bmatrix} \{S_0(0.294), S_1(0.174), S_2(0.177), S_3(0.355)\} \\ \{S_{-2}(0.035), S_{-1}(0.036), S_0(0.149), S_1(0.719), S_2(0.052), S_3(0.008)\} \\ \{S_{-2}(0.07), S_{-1}(0.061), S_0(0.389), S_1(0.241), S_2(0.194), S_3(0.044)\} \end{bmatrix},$$

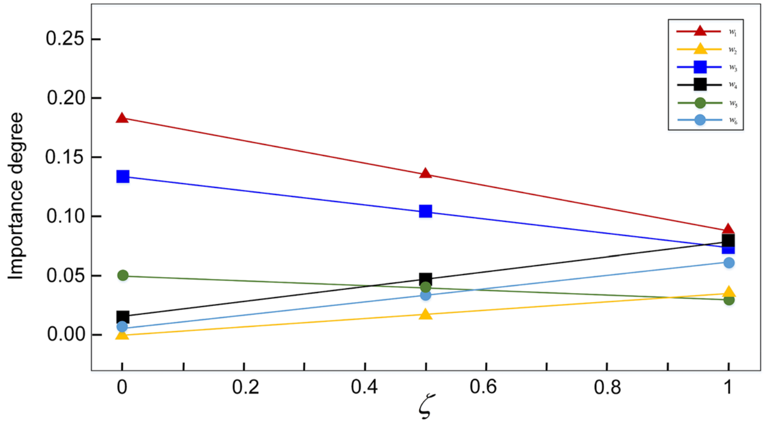
$$C' = \begin{bmatrix} \{S_{-1}(0.005), S_0(0.208), S_1(0.389), S_2(0.229), S_3(0.168)\} \\ \{S_{-3}(0.033), S_{-2}(0.052), S_{-1}(0.165), S_0(0.585), S_1(0.141), S_2(0.023)\} \\ \{S_{-1}(0.07), S_0(0.218), S_1(0.411), S_2(0.172), S_3(0.129)\} \end{bmatrix}.$$

Step 8 The positive and negative prospect value matrices can be computed via Eqs. (21) and (22), respectively:

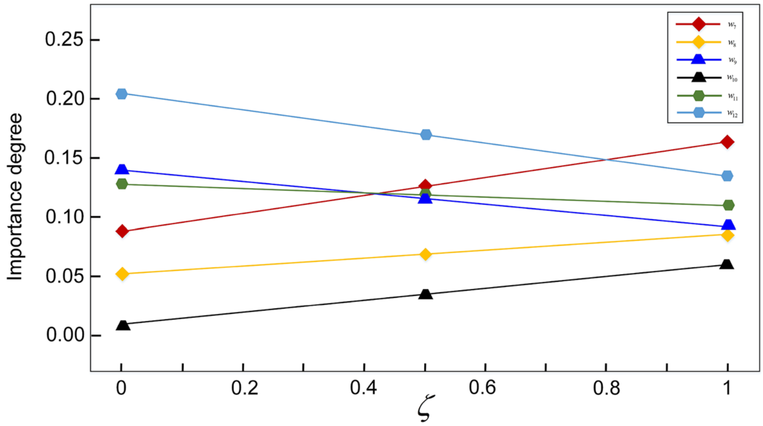
$$v_{ij}^{+} = \begin{bmatrix} 0.223 & 0.342 & 0.3 \\ 0.077 & 0.192 & 0 \\ 0.27 & 0.159 & 0.252 \end{bmatrix}$$

$$v_{ij}^{-} = \begin{bmatrix} -0.333 & 0 & -0.135 \\ -0.644 & -0.404 & -0.77 \\ -0.216 & -0.478 & -0.262 \end{bmatrix}$$

Step 9 With the weight vector of attributes, the overall weighted prospect value V_i of alternative A_i $(i = 1, 2, 3)$ can be calculated by Eq. (23) to arrive at: $V_1 = 0.136$,$V_2 = -0.542$, $V_3 = -0.083$.Therefore, the final ranking of the alternatives is acquired: $A_1 \succ A_3 \succ A_2$. In other words, the insurance company would be the optimal investment.



(A) The importance degree of $w_k (k = 1,2,...,6)$ under different ζ .



(B) The importance degree of $w_k (k = 7,8,...,12)$ under different ζ .

Fig. 6 **a** The importance degree of $w_k(k = 1, 2, \ldots, 6)$ under different ζ. **b** The importance degree of $w_k(k = 7, 8, \ldots, 12)$ under different ζ

4.3 Analysis on the Effect of Parameter

Section 3.1 demonstrates how to obtain the importance degree of a DM, and there exists a parameter ζ indicating the relative significance between in-degree centrality and betweenness centrality. Therefore, it is necessary to demonstrate the influence of ζ on the weight of DM and decision result.

Based on the data in Sect. 4.2, the weight of DMs is investigated for a ζ that caries in the interval $[0, 1]$. The results are shown in Fig. 6. From Fig. 6a, b, the values of w_1, w_3 w_5, w_9, w_{11}, w_{12} decrease gradually as ζ increase, whereas the values of w_2, w_4, w_6, w_7, w_8, w_{10} increase steadily. Therefore, the selection of ζ depends on practical situations and we should constantly optimize the parameters to derive the best result.

The robustness of the proposed strategy could be verified by adjusting ζ from 0.1 to 0.9, with a step size of 0.2. The ranking results $V_i(i = 1, 2, 3)$ of alternatives under different ζ

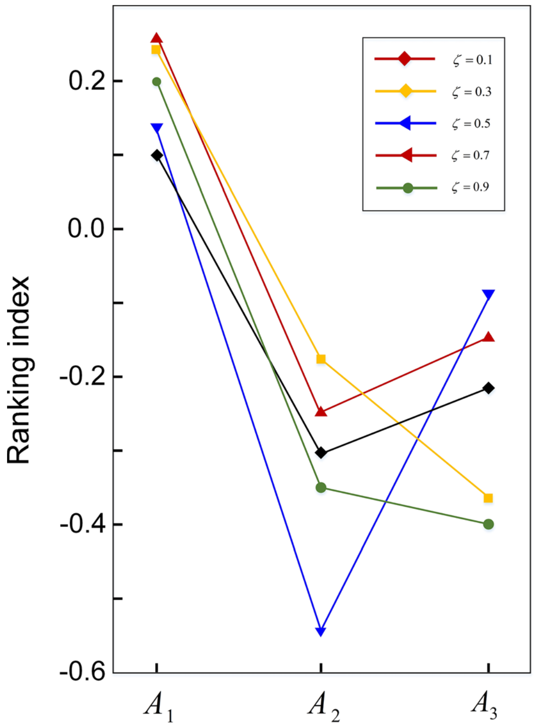


Fig. 7 The variation of ranking results with different values of ζ

are shown in Fig. 7. Although the ranking index is different when ζ takes on different values, the optimal alternative is consistent. Therefore, the stability of the optimal result verifies the relative robustness of the proposed method.

5 Comparative Analyses

In this section, a comprehensive comparison is made with present studies to further illustrate the reliability of the proposed method.

5.1 Some Qualitative Comparisons

A qualitative comparison of the proposed CRP method is conducted against existing ones. The following aspects are investigated: the representation structure of evaluation and trust statement, weight allocation method, CRP strategy and the consideration of psychological behavior. Table 7 compared the performances of these method to different items.

First, comparisons are made from the perspective of a social network. The weight of DM is assumed to be known in [3, 9, 11, 24], which cannot reflect an individual's objective importance. Since social network information can serve as a reliable source to obtain weight allocation, this allocation has been utilized in various studies. However, methods in [5, 16, 28] only employ in-degree centrality to derive the importance of a DM. In this paper, the weights of DMs are distributed more reasonably by considering both in-degree centrality and betweenness centrality.

Table 7 Qualitative comparison of different CRP methods in GDM

Methods	Preference structure	Trust statement	Weight allocation method	Feedback mechanism of CRP	Source of recommendation	Psychological behavior of DM
Method in [5]	APR	Numerical values	In-degree centrality	IDR-based method	The DM who has a minimum preference similarity with the identified one	Not considered
Method in [16]	HFPR	Numerical values	In-degree centrality	IDR-based method	Group evaluation	Not considered
Method in [26]	IVF	IVF	Consensus degree	IDR-based method	The collective evaluation of the trusted DMs	Not considered
Method in [7]	FPR	Not considered	Determined by the size of subgroup	IDR-based method	Generated according to a given direction	Noncooperative behavior is considered
Method in [1]	Numerical values	Not considered	Consensus degree	IDR-based method	The average collective preferences of the DMs except the modified one	Not considered
Method in [11]	FPR	Not considered	Given in advance	Minimum cost model	Not considered	Not considered
Method in [9]	MGULI	Not considered	Given in advance	Minimum adjustment model	Not considered	Bounded confidence is considered
Method in [24]	HFPR	Not considered	Assumed to be equal	IDR-based method	Randomly generated according to a given direction	Not considered
Method in [28]	Numerical values	Numerical values	In-degree centrality	IDR-based method	Randomly generated from a certain interval	Non-cooperative behavior is considered
Method in [3]	PULTS	Not considered	Given in advance	IDR-based method	Replace the identified evaluation with group opinion	Not considered
The purposed method	PLTS	PLTS	In-degree centrality and betweenness centrality	The integrated method	The DM with the largest betweenness centrality among the trusted individuals	Considered through prospect theory

APR Additive preference relations, *HFPR* Hesitant fuzzy preference relations, *IVF* Interval-valued function, *FPR* Fuzzy preference relations, *MGULI* Multi-granular unbalanced linguistic information, *PULTS* Probabilistic uncertain linguistic term set, *PLTS* Probabilistic linguistic term set

Social network information is only used to determine weighting factors for DMs in [16, 28], whereas in our proposed method, a referenced opinion is generated based on social relationship that fully uses social network information. Thus, the recommendation could be more readily accepted by a DM, which would further promote the implementation of the decision result.

Second, comparisons are made regarding CRP strategy. An optimization-based consensus model is employed in [9, 11] to derive the modified evaluations. However, real-world GDM situations involve the field of mathematics and the participation among DMs. Although more iterations are needed than that in [9, 11], the proposed method can better reflect real-world CRP.

Group evaluations are usually used as modification references [16]. However, it is unreasonable to force DMs to implement a recommendation based on group preference because this strategy ignores the actual relationships among DMs. The method proposed in [24, 28] can only provide implicit suggestions without specifying the degree of adjustment to be made. In this study, a DM is assumed to refer to the individual who is accessible to more information among trusted DMs, which is more in line with reality.

As for the identification rule: (1) in [3], when GCL does not meet the given threshold, the preference with the lowest consensus level will be determined and replaced by the corresponding group evaluation. However, modifying an evaluation that deviates most from the group does not necessarily guarantee a maximum group consensus improvement. Thus, a more effective approach is to employ an optimization model that can identify the evaluation that needs modification. (2) in [3, 16, 17], all the evaluations with inadequate consensus level are identified. Thus, multiple DMs are suggested to adjust these evaluations simultaneously. However, group opinion changes dynamically within the CRP. Therefore, it is unreasonable to require that several DMs make modifications at the same time.

Third, the adjustment parameter exerts a great influence on CRP because different parameters can result in different evaluation results in the new round. In previous studies, the adjustment parameter is predefined and remains the same for all DMs, regardless of their differences [16, 27]. In our proposed method, the adjustment parameter is customized for different DMs, which can comprehensively depict the willingness of a DM to change and avoid unreasonable results brought by limited subjectivity of the moderator.

Finally, our proposed method also characterizes the psychological behavior of DMs. The existing studies on CRP usually follow the principle of expected utility theory to derive the final ranking [1, 5, 11, 16, 26]. However, DMs tend to perceive gains and losses from a reference point. To the best of our knowledge, our work is the first to consider DM risk preferences in CRP for GDM with PLTS under social network.

5.2 Some Quantitative Comparisons

After qualitatively analyzing the novelty of our proposed method, a quantitative comparison is conducted to further illustrate the effectiveness of the proposed method (denoted as M_1) within the same GDM scenario.

Two typical methods with feedback mechanism are selected to perform the validation: the CRP model proposed in [1] and [28], denoted as M_2 and M_3, respectively. However, a numerical value is utilized to express the decision information in [1, 28]. Thus, the expectation function provided in Definition 5 can be employed to transform PLTS into crisp numbers. Two indicators are introduced to assist the comparative analysis. The first one is information deviation, which can be calculated as follows.

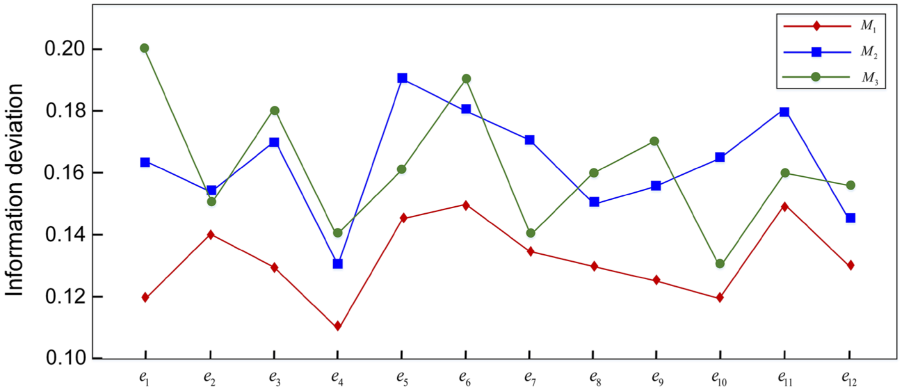


Fig. 8 The information deviation of different methods

$$D^k = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left| L_{ij}^{k}(p) - \overline{L_{ij}^{g}}(p) \right| \tag{24}$$

where $L_{ij}^{k}(p)(k = 1, 2, \ldots, K)$ and $\overline{L_{ij}^{g}}(p)$ represent the original decision information and the modified group evaluation, respectively.

For simplicity, the detailed calculation process is omitted, but the results are shown in Fig. 8. From Fig. 8, the proposed method M_1 can preserve the individual evaluation to the largest extent in the CRP with less information loss. Thus, the proposed method can retain original information from DMs more effectively compared with other strategies.

The second indicator is the overall number of adjusted evaluations, which is denoted as N. Let $x_{ij}^{k(t)}$ be a 0 or 1 variable. If the evaluation $L_{ij}^{k(t-1)}$ is adjusted in the tth iteration, then $x_{ij}^{k(t)} = 1$; otherwise $x_{ij}^{k(t)} = 0$. The formulation is given as follows:

$$x_{ij}^{k(t)} = \begin{cases} 0 & L_{ij}^{k(t-1)} = L_{ij}^{k(t)} \\ 1 & L_{ij}^{k(t-1)} \neq L_{ij}^{k(t)} \end{cases}. \tag{25}$$

The overall number of the modified evaluations can be derived as:

$$N = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{t=2}^{T} x_{ij}^{k(t)} \tag{26}$$

where T indicates the total iteration number in CRP.

The results are shown in Fig. 9. Figure 9 illustrates that $M_1(N_1 = 10)$ has the minimum overall number of individual evaluations that need modification in the CRP. The reason for this is that the proposed strategy selects a specific preference that could maximize the improvement of a group's consensus in each iteration, which can increase the efficiency and reduce the complexity of CRP. However, $M_2(N_2 = 14)$ and $M_3(N_3 = 17)$ identify all the judgements that don't meet the preset threshold at a time, making the number of modified evaluations relatively high.

Based on the analysis, the proposed method can better preserve the original decision information of DMs and reduce the complexity of a CRP more effectively.

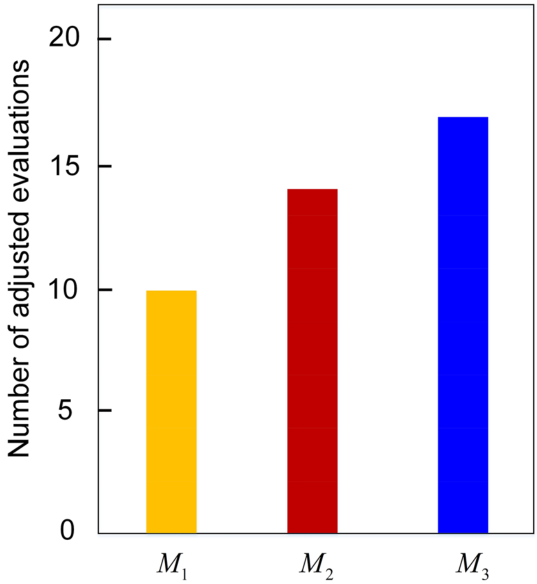


Fig. 9 Total number of adjusted evaluations under different methods

6 Conclusion

This paper develops a novel CRP model based on SNA for GDM with probabilistic linguistic information. The primary contributions can be outlined as follows:

(1) Unlike existing studies, the importance degree of DM is comprehensively quantified by considering both in-degree centrality and betweenness centrality. The utilization of betweenness centrality is necessary for CRP, where the potential for control over communication is substantively significant.
(2) By combining the IDR-based method with an optimization-based approach, a novel CRP model is further established to achieve maximum GCL improvement in each round of adjustment. Thus, the proposed method can consider the participation of DMs and ensure the efficiency of CRP.
(3) In the feedback mechanism of CRP, the recommendation is generated based on the idea that a DM is more willing to seek advice from the individual with access to more information. Besides, DM's modification willingness is comprehensively determined by hesitancy degree and current consensus level.
(4) By utilizing positive and negative prospect values, the limited rationality of DMs is factored into GDM process. And the overall prospect values of alternatives are obtained to derive a more convincing result.

The proposed method can be slightly adjusted to address other GDM problems under a social network environment. Considering the relationship among DMs can be altered with interactions in CRP, this study can be further extended by investigating the non-stationary social network in GDM.

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