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Intermittent Control Based Exponential Synchronization of Inertial Neural Networks with Mixed Delays

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Abstract

This article is devoted to the synchronization of delayed inertial neural systems by virtue of intermittent control scheme. In the proposed inertial models, a type of mixed delays is introduced which is composed of discrete delays and infinite distributed delays. Particulary, the finite distributed delays can be easily obtained by selecting specific kernel functions in infinite distribute delays. To realize exponential synchronization, different from the previous continuous designs for the first-order systems obtained by suitable substitutions of reduced-order, an intermittent control scheme is directly developed for the response inertial systems. Furthermore, a direct analysis method is proposed to derive the synchronization conditions by constructing a Lyapunov functional formed by the state variables and their derivatives. Lastly, the designed control scheme and established criteria are verified via providing a numerical example.

Keywords Exponential synchronization · Intermittent control · Inertial neural network · Mixed time-varying delay

1 Introduction

Neural network models are regarded as complex nonlinear dynamic learning systems composed of numerous processing elements (called neurons) extensively connected to each other. In recent years, a variety of neural network models, including Cohen-Grossberg types [1], Hopfield types [2], Cellular neural networks and BAM neural systems [3,4], have been successfully proposed in the form of the first-order differential equations. In 1996, Babcock and Westervelt [5] introduced the inductance into the circuit models and proposed a type of new neural models, which are called as inertial neural networks and represented by means of the second-order differential equations. Nowadays, inertial neural systems have been extensively utilized in many practical fields including processing signals, image encryption, secure communication [6–8].

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It is well known that it is inevitable for time delay in the inertial neural system [9–11] because of the inherent delay time in neurons and the finite speeds for information transmission, signals acquisition and collection processing among neurons. So it is great significance for researchers to investigate inertial neural systems with time delay. Nowadays, dynamic features of numerous inertial neural systems with discrete delay have been analyzed in [10,12–14]. The authors of [15–20] introduced the mixed delays for several inertial neural models, where discrete delays and distributed delays are involved, and several stability or synchronization conditions were established.

Currently, the stability [21–24], dissipativity [16,25–28], synchronization [9,19,29,30] of inertial neural models have attracted extensive attention of scholars at home and abroad. It is generally known that synchronization plays an important role in nonlinear systems due to its potential applications in secure communications, optics cryptography, robot control, image processing and optimal combination. At present, to achieve synchronization of neural networks, many control techniques including adaptive control [31], feedback control [32], pinning control [33–36], impulsive control [22,37] and intermittent control [9,19,38–44], have been designed. Particulary, the intermittent control scheme is a kind of discontinuous control strategy, which is only active in the control interval and is off during the rest period. Since the discontinuous control technique can greatly reduce the cost of control, the synchronization problem of the first-order networks under intermittent control has caused extensive research. For example, the the pinning cluster synchronization in [45] for colored community networks was proposed via adaptive aperiodically intermittent control. The problem of exponential synchronization in [46] for delayed dynamical networks with hybrid coupling was discussed via pinning periodically intermittent control. In addition, the exponential stability [47] and exponential synchronization [43] of delayed neural networks are investigated by Lyapunov-Krasovskii functional approach. However, there are few reports on the second-order neural networks based on intermittent control. By combining intermittent control and the reduced-order transforms, the exponential or fixed-time synchronization of inertial neural systems was investigated in [9,19,41].

Actually, the method of the reduced-order transforms has been widely utilized in the current results on inertial neural networks. The main idea of it is that the second-order models are rewritten as the first-order systems by proper variable transforms, and the dynamics is revealed by analyzing the obtained first-order models. For example, under the framework of the reduced-order technique, the exponential synchronization problem of inertial neural system was investigated in [19,48], the global exponential convergence was discussed in [26,49,50] for delay-dependent inertial neural networks by means of matrix measure theory, and the finite time synchronization of delayed inertial neural system was discussed in [51] based on integral inequality technique. Note that the inertial term disappears with the reduction of the order, which means that the important role of inertial term cannot be obtained from the reduced order model. In addition, the dimension of the reduced-order system is twice that of the original second-order system, which makes the theoretical analysis more difficult, and the conditions more complex and conservative. In order to avoid and conquer those problems caused by the reduced-order method, the authors in [52] proposed a new method to discuss the stability and control of inertial neural systems without applying any reduced-order transform. At present, there have been many relevant results on the inertial networks based on non-reduced order idea, such as the globally exponential stability [14,53–55] and exponential synchronization of inertia neural models [32,56]. However, under the intermittent control and the non-reduced order means, it is still challenging and there seems to be no related report on the exponential synchronization of the second-order inertial neural models with discrete and infinitely distributed delays.

Based on the above analysis and discussion, this paper will endeavour to solve the exponential synchronization problem of inertial neural networks with mixed delays under intermittent control. The main innovative contents are listed as follows:

(1) A kind of mixed time delays, composed of discrete-time delays and infinite distributed delays, is proposed in inertial neural networks, which is more general compared with the types of delays given in the previous inertial neural models [14,22,57]. Particulary, the finite distributed delay investigated in [22] can be easily obtained by selecting specific kernel functions in infinite distributed delay.

(2) Different from the continuous control for inertial neural networks in [48,55], an intermittent control scheme is designed to study the exponential synchronization of the inertial neural systems with mixed delays.

(3) Unlike the traditional reduced-order method used in the most of published results [15,19], a direct analysis is proposed to investigate the synchronization of inertial neural networks by directly constructing a suitable Lyapunov functional in this paper.

The rest structure is organized as follows. In Sect. 2, some necessary preliminaries and the model descriptions are given. In Sect. 3, the exponential synchronization of the addressed inertial models is investigated. In Sect. 4, a numerical example is given to guarantee the validity of the established synchronization criteria.

2 Problem Description and Preliminaries

In this article, a type of inertial neural system with mixed delays is described as

$$\begin{aligned}\ddot{x}_i(t) &= -a_i\dot{x}_i(t) - b_i x_i(t) + \sum_{j\in\Gamma} c_{ij} f_j(x_j(t)) + \sum_{j\in\Gamma} d_{ij} f_j(x_j(t-\nu(t)))\\ &\quad + \sum_{j\in\Gamma} r_{ij}\int_{-\infty}^{t} K_{ij}(t-\eta) f_j(x_j(\eta))d\eta + I_i(t), \quad i\in\Gamma, \end{aligned} \tag{1}$$

where $\Gamma = \{1, 2, \cdots, m\}$, $x_i(t)$ is the state of the ith neuron at time t, the second-order derivative is intituled as an inertial term of (1), $a_i > 0$ and $b_i > 0$, c_{ij}, d_{ij} and r_{ij} represent connection weights, $f_j(\cdot)$ is the activation function of the jth neuron, $\nu(t)$ is time-varying discrete delay, which satisfies $0 < \nu(t) \le \nu$, $\dot{\nu}(t) \le \nu_1 < 1$, the kernel function $K_{ij}(\cdot) : [0, +\infty) \to [0, +\infty)$ is nonnegative and continuous, $I_i(t)$ is an external input.

The initial conditions are provided by

$$x_i(\varsigma) = \varphi_i(\varsigma), \quad \dot{x}_i(\varsigma) = \psi_i(\varsigma), \quad \varsigma \in (-\infty, 0],$$

in which $i \in \Gamma$, $\varphi_i(\varsigma)$, $\psi_i(\varsigma) : (-\infty, 0] \to R$ are continuous and bounded.

Remark 1 Obviously, compared with the models proposed in [10,12,14,15,60], the model of system (1) is more general. For example, when distributed delays are ignored, system (1) is reduced to the inertial neural mondel in [10,12,15], and system (1) is degenerated into the first-order model in [60] if inertial term is not considered and $I_i(t) = I$.

Considering model (1) as the drive system, the response system is given as below:

$$\begin{aligned}\ddot{y}_i(t) &= -a_i\dot{y}_i - b_i y_i(t) + \sum_{j\in\Gamma} c_{ij} f_j(y_j(t)) + \sum_{j\in\Gamma} d_{ij} f_j(y_j(t-\nu(t)))\\ &\quad + \sum_{j\in\Gamma} r_{ij}\int_{-\infty}^{t} K_{ij}(t-\eta) f_j(y_j(\eta))d\eta + I_i(t) + U_i(t), \quad i\in\Gamma, \end{aligned} \tag{2}$$

here $y_i(t)$ indicates the state of the ith neuron in the response system, $U_i(t)$ is a controller, the rest symbols are the same as those of system (1).

The initial condition of system (2) are given as

$$y_i(\varsigma) = \bar{\varphi}_i(\varsigma), \quad \dot{y}_i(\varsigma) = \bar{\psi}_i(\varsigma), \quad \varsigma \in (-\infty, 0],$$

where $i \in \Gamma$, $\bar{\varphi}_i(\varsigma)$ and $\bar{\psi}_i(\varsigma)$ are continuous and bounded.

To accomplish synchronization, $U_i(t)$ is designed as the following intermittent form:

$$U_i(t) = \begin{cases} -\omega_i(\dot{s}_i(t) + s_i(t)), & nT \le t < (n+\sigma)T, \\ 0, & (n+\sigma)T \le t < (n+1)T, \end{cases} \tag{3}$$

in which $T > 0$ is called the control period, σ is called control rate and $0 < \sigma < 1$, $\omega_i > 0$ is called the control gain, $s_i(t) = y_i(t) - x_i(t)$ is the synchronization error.

From systems (1), (2) and the controller (3), the error system can be written as

$$\begin{cases} \ddot{s}_i(t) = -a_i\dot{s}_i(t) - b_i s_i(t) + \sum\limits_{j\in\Gamma} c_{ij}\tilde{f}_j(s_j(t)) + \sum\limits_{j\in\Gamma} d_{ij}\tilde{f}_j(s_j(t-\nu(t))) \\ \qquad + \sum\limits_{j\in\Gamma} r_{ij}\int_{-\infty}^{t} K_{ij}(t-\eta)\tilde{f}_j(s_j(\eta))d\eta - \omega_i(\dot{s}_i(t) + s_i(t)), \\ \qquad nT \le t < (n+\sigma)T, \\ \ddot{s}_i(t) = -a_i\dot{s}_i(t) - b_i s_i(t) + \sum\limits_{j\in\Gamma} c_{ij}\tilde{f}_j(s_j(t)) + \sum\limits_{j\in\Gamma} d_{ij}\tilde{f}_j(s_j(t-\nu(t))) \\ \qquad + \sum\limits_{j\in\Gamma} r_{ij}\int_{-\infty}^{t} K_{ij}(t-\eta)\tilde{f}_j(s_j(\eta))d\eta, \\ \qquad (n+\sigma)T \le t < (n+1)T, \end{cases} \tag{4}$$

where $\tilde{f}_j(s_j(\cdot)) = f_j(y_j(\cdot)) - f_j(x_j(\cdot))$.

Definition 1 The response system (2) is said to achieve exponential synchronization with the drive system (1) under the intermittent controller (3), if there are two positive constants ε and M which depends on the initial values, such that

$$\|s(t)\| \le Me^{-\varepsilon t}, \quad t \ge 0,$$

where $s(t) = (s_1(t), s_2(t), \cdots, s_n(t))^{\mathsf{T}}$, $\|s(t)\| = \big(\sum\limits_{i\in\Gamma} s_i^2(t)\big)^{\frac{1}{2}}$.

Lemma 1 *[38] Suppose that $V(t)$ is differentiable and positive definite on $[0, +\infty)$ and its derivative satisfies*

$$\begin{cases} \dot{V}(t) \le 0, & nT \le t < (n+\sigma)T, \\ \dot{V}(t) \le \kappa V(t), & (n+\sigma)T \le t < (n+1)T, \end{cases}$$

where $n \in N = \{0, 1, 2, \cdots\}$, $T > 0$, $0 < \sigma < 1$ and $\kappa > 0$, then

$$V(t) \le V(0)e^{\kappa(1-\sigma)t}, \quad t \ge 0.$$

Assumption 1 There exists $l_j > 0$ such that the activation function $f_j(\cdot)$ satisfies

$$|f_j(y) - f_j(x)| \le l_j|y - x|, \quad j \in \Gamma, \quad x, y \in R.$$

Assumption 2 For any $i, j \in \Gamma$, the real-valued kernel function $K_{ij}(\cdot)$ is nonnegative continuous and there exist positive constants k_0, k_1 and λ such that

$$\int_0^{+\infty} K_{ij}(\eta)d\eta = k_0, \quad \int_0^{+\infty} e^{2\lambda\eta} K_{ij}(\eta)d\eta = k_1.$$

Assumption 3 There exist positive constants α_i and β_i such that

$$A_i < 0, \ 4A_iB_i \geq C_i^2, \ \ i \in \Gamma,$$

where

$$\begin{aligned}
A_i &= \alpha_i\big(\lambda + 1 - a_i + \frac{1}{2}\sum_{j\in\Gamma} l_j(|c_{ij}| + |d_{ij}| + k_0|r_{ij}|) - \omega_i\big),\\
B_i &= \alpha_i\big(\lambda - b_i + \frac{1}{2}\sum_{j\in\Gamma} l_j(|c_{ij}| + |d_{ij}| + k_0|r_{ij}|) - \omega_i\big) + \lambda\beta_i + \sum_{j\in\Gamma}\alpha_j l_i|c_{ji}|\\
&\quad + \frac{e^{2\lambda\nu}}{1-\nu_1}\sum_{j\in\Gamma}\alpha_j l_i|d_{ji}| + k_1\sum_{j\in\Gamma}\alpha_j l_i|r_{ji}|,\\
C_i &= \beta_i + \alpha_i\big(2\lambda + 1 - a_i - b_i - 2\omega_i\big).
\end{aligned}$$

3 Main Results

The exponential synchronization between the neural systems (1) and (2) are discussed in this section by directly constructing Lyapunov functionals.

Theorem 1 *Under Assumptions* 1-3*, the exponential synchronization is accomplished between the driving system* (1) *and response system* (2) *under the controller* (3) *if* $\varrho = \lambda - \bar{\omega}(1-\sigma) > 0$*, where* $\bar{\omega} = \max_{i\in\Gamma}\{\omega_i\}$.

Proof Construct a Lyapunov functional as the following form:

$$\begin{aligned}
V(t) &= \frac{1}{2}\sum_{i\in\Gamma}\beta_i s_i^2(t)e^{2\lambda t} + \frac{1}{2}\sum_{i\in\Gamma}\alpha_i(s_i(t) + \dot{s}_i(t))^2 e^{2\lambda t}\\
&\quad + \sum_{i\in\Gamma}\sum_{j\in\Gamma} q_{ij}\int_{-\infty}^{0}\int_{t+h}^{t} K_{ij}(-h)s_j^2(\eta)e^{2\lambda(\eta-h)}d\eta dh\\
&\quad + e^{2\lambda\nu}\sum_{i\in\Gamma}\sum_{j\in\Gamma} p_{ij}\int_{t-\nu(t)}^{t} s_j^2(h)e^{2\lambda h}dh,
\end{aligned}$$

where $p_{ij} = \frac{\alpha_i l_j|d_{ij}|}{1-\nu_1}$, $q_{ij} = \alpha_i l_j|r_{ij}|$.

For $nT \leq t < (n+\sigma)T$, the derivative of $V(t)$ along the solution of system (4) is estimated as follows:

$$\begin{aligned}
\dot{V}(t) \leq &\sum_{i\in\Gamma}\lambda\beta_i s_i^2(t)e^{2\lambda t} + \sum_{i\in\Gamma}\beta_i s_i(t)\dot{s}_i(t)e^{2\lambda t} + \sum_{i\in\Gamma}\lambda\alpha_i\big(s_i(t) + \dot{s}_i(t)\big)^2 e^{2\lambda t}\\
&+ \sum_{i\in\Gamma}\alpha_i\big(s_i(t) + \dot{s}_i(t)\big)\big(\dot{s}_i(t) + \ddot{s}_i(t)\big)e^{2\lambda t}
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i\in\Gamma}\sum_{j\in\Gamma}q_{ij}\Big(\int_{-\infty}^{0}K_{ij}(-h)s_j^2(t)e^{2\lambda(t-h)}dh-\int_{-\infty}^{0}K_{ij}(-h)s_j^2(t+h)e^{2\lambda t}dh\Big)\\
&+e^{2\lambda\nu}\sum_{i\in\Gamma}\sum_{j\in\Gamma}p_{ij}\Big(s_j^2(t)e^{2\lambda t}-(1-\nu_1)s_j^2(t-\nu(t))e^{2\lambda(t-\nu)}\Big)\\
\le&\sum_{i\in\Gamma}e^{2\lambda t}\Big\{\lambda\beta_i s_i^2(t)+\beta_i s_i(t)\dot{s}_i(t)+\lambda\alpha_i\big(s_i(t)+\dot{s}_i(t)\big)^2+\alpha_i\big(s_i(t)+\dot{s}_i(t)\big)\dot{s}_i(t)\\
&+\alpha_i\big(s_i(t)+\dot{s}_i(t)\big)\Big[-a_i\dot{s}_i(t)-b_is_i(t)+\sum_{j\in\Gamma}c_{ij}\tilde{f}_j(s_j(t))+\sum_{j\in\Gamma}d_{ij}\tilde{f}_j(s_j(t-\nu(t)))\\
&+\sum_{j\in\Gamma}r_{ij}\int_{-\infty}^{t}K_{ij}(t-\eta)\tilde{f}_j(s_j(\eta))d\eta-\omega_i\dot{s}_i(t)-\omega_is_i(t)\Big]\\
&+\sum_{j\in\Gamma}k_1q_{ij}s_j^2(t)-\sum_{j\in\Gamma}q_{ij}\int_{-\infty}^{0}K_{ij}(-h)s_j^2(t+h)dh\\
&+\sum_{j\in\Gamma}e^{2\lambda\nu}p_{ij}s_j^2(t)-\sum_{j\in\Gamma}(1-\nu_1)p_{ij}s_j^2(t-\nu(t))\Big\}. \qquad (5)
\end{aligned}
$$

By using Assumption 1, Assumption 2 and the fundamental inequality,

$$
\begin{aligned}
&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_is_i(t)c_{ij}\tilde{f}_j(s_j(t))\\
\le&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_il_j|c_{ij}||s_i(t)||s_j(t)|\\
\le&\frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_il_j|c_{ij}|\Big(s_i^2(t)+s_j^2(t)\Big), \qquad (6)\\
&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i\dot{s}_i(t)c_{ij}\tilde{f}_j(s_j(t))\\
\le&\frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_il_j|c_{ij}|\Big(\dot{s}_i^2(t)+s_j^2(t)\Big), \qquad (7)\\
&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_is_i(t)d_{ij}\tilde{f}_j(s_j(t-\nu(t)))\\
\le&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_il_j|d_{ij}||s_i(t)||s_j(t-\nu(t))|\\
\le&\frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_il_j|d_{ij}|\Big(s_i^2(t)+s_j^2(t-\nu(t))\Big), \qquad (8)\\
&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i\dot{s}_i(t)d_{ij}\tilde{f}_j(s_j(t-\nu(t)))\\
\le&\frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_il_j|d_{ij}|\Big(\dot{s}_i^2(t)+s_j^2(t-\nu(t))\Big), \qquad (9)\\
&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_is_i(t)\int_{-\infty}^{t}r_{ij}K_{ij}(t-\eta)\tilde{f}_j(s_j(\eta))d\eta
\end{aligned}
$$

$$
\begin{aligned}
&\le \sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i l_j|r_{ij}||s_i(t)|\int_{-\infty}^{t}K_{ij}(t-\eta)|s_j(\eta)|d\eta\\
&\le \frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i l_j|r_{ij}|\int_{-\infty}^{0}K_{ij}(-h)\big(s_i^2(t)+s_j^2(t+h)\big)dh\\
&\le \frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i l_j|r_{ij}|\Big(k_0 s_i^2(t)+\int_{-\infty}^{0}K_{ij}(-h)s_j^2(t+h)dh\Big),
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i\dot{s}_i(t)\int_{-\infty}^{t}r_{ij}K_{ij}(t-\eta)\tilde{f}_j(s_j(\eta))d\eta\\
&\le \frac{1}{2}\sum_{i\in\Gamma}\sum_{j\in\Gamma}\alpha_i l_j|r_{ij}|\Big(k_0\dot{s}_i^2(t)+\int_{-\infty}^{0}K_{ij}(-h)s_j^2(t+h)dh\Big).
\end{aligned}
\tag{11}
$$

Submitting (6)–(11) into (5), the following inequalities can be obtained:

$$
\begin{aligned}
\dot{V}(t) \le& \sum_{i\in\Gamma}e^{2\lambda t}\Big\{\Big(\beta_i+\alpha_i(2\lambda+1-a_i-b_i-2\omega_i)\Big)s_i(t)\dot{s}_i(t)\\
&+\alpha_i\Big(\lambda+1-a_i+\frac{1}{2}\sum_{j\in\Gamma}l_j(|c_{ij}|+|d_{ij}|+k_0|r_{ij}|)-\omega_i\Big)\dot{s}_i^2(t)\\
&+\Big(\alpha_i\big(\lambda-b_i+\frac{1}{2}\sum_{j\in\Gamma}l_j(|c_{ij}|+|d_{ij}|+k_0|r_{ij}|)-\omega_i\big)+\lambda\beta_i+\sum_{j\in\Gamma}\alpha_j l_i|c_{ji}|\\
&+k_1\sum_{j\in\Gamma}q_{ji}+e^{2\lambda\nu}\sum_{j\in\Gamma}p_{ji}\Big)s_i^2(t)+\sum_{j\in\Gamma}\big(\alpha_i l_j|r_{ij}|-q_{ij}\big)\int_{-\infty}^{0}K_{ij}(-h)s_j^2(t+h)dh\\
&+\sum_{j\in\Gamma}\Big(\alpha_i l_j|d_{ij}|-(1-\nu_1)p_{ij}\Big)s_j^2(t-\nu(t))\Big\}\\
\le& \sum_{i\in\Gamma}e^{2\lambda t}\Big(A_i\dot{s}_i^2(t)+B_i s_i^2(t)+C_i s_i(t)\dot{s}_i(t)\Big)\\
=& \sum_{i\in\Gamma}e^{2\lambda t}\Big\{A_i\Big(\dot{s}_i(t)+\frac{C_i}{2A_i}s_i(t)\Big)^2+\Big(B_i-\frac{C_i^2}{4A_i}\Big)s_i^2(t)\Big\}\\
\le& 0.
\end{aligned}
\tag{12}
$$

For $(n+\sigma)T \le t < (n+1)T$, similar to the preceding proof, one has

$$
\begin{aligned}
\dot{V}(t) \le& \sum_{i\in\Gamma}e^{2\lambda t}\Big\{\big(\beta_i+\alpha_i(2\lambda+1-a_i-b_i)\big)s_i(t)\dot{s}_i(t)\\
&+\alpha_i\Big(\lambda+1-a_i+\frac{1}{2}\sum_{j\in\Gamma}l_j(|c_{ij}|+|d_{ij}|+k_0|r_{ij}|)\Big)\dot{s}_i^2(t)\\
&+\Big(\alpha_i\big(\lambda-b_i+\frac{1}{2}\sum_{j\in\Gamma}l_j(|c_{ij}|+|d_{ij}|+k_0|r_{ij}|)\big)+\lambda\beta_i\\
&+\sum_{j\in\Gamma}\alpha_j l_i|c_{ji}|+k_1\sum_{j\in\Gamma}q_{ji}+e^{2\lambda\nu}\sum_{j\in\Gamma}p_{ji}\Big)s_i^2(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j\in\Gamma}\left(\alpha_i l_j|r_{ij}|-q_{ij}\right)\int_{-\infty}^{0}K_{ij}(-h)s_j^2(s+h)dh\\
&+\sum_{j\in\Gamma}\Big(\alpha_i l_j|d_{ij}|-(1-\nu_1)p_{ij}\Big)s_j^2(t-\nu(t))\Big\}\\
\le&\sum_{i\in\Gamma}e^{2\lambda t}\Big\{\Big(\beta_i+\alpha_i(2\lambda+1-a_i-b_i-2\omega_i)\Big)s_i(t)\dot{s}_i(t)+2\alpha_i\omega_i s_i(t)\dot{s}_i(t)\\
&+\alpha_i\Big(\lambda+1-a_i+\frac{1}{2}\sum_{j\in\Gamma}l_j(|c_{ij}|+|d_{ij}|+k_0|r_{ij}|)-\omega_i\Big)\dot{s}_i^2(t)+\alpha_i\omega_i\dot{s}_i^2(t)\\
&+\Big(\alpha_i(\lambda-b_i+\frac{1}{2}\sum_{j\in\Gamma}l_j(|c_{ij}|+|d_{ij}|+k_0|r_{ij}|)-\omega_i)+\lambda\beta_i\\
&+\sum_{j\in\Gamma}\alpha_j l_i|c_{ji}|+k_i\sum_{j\in\Gamma}q_{ji}+e^{2\lambda\nu}\sum_{j\in\Gamma}p_{ji}\Big)s_i^2(t)+\alpha_i\omega_i s_i^2(t)\\
\le&\sum_{i\in\Gamma}\omega_i\alpha_i(s_i(t)+\dot{s}_i(t))^2e^{2\lambda t}\\
\le&\,2\bar{\omega}V(t).
\end{aligned}\tag{13}
$$

Combining (12), (13) and Lemma 1, for any $t\ge 0$, one gets

$$
V(t)\le V(0)e^{2\bar{\omega}(1-\sigma)t}.\tag{14}
$$

Hence,

$$
\|s(t)\|^2=\sum_{i\in\Gamma}s_i^2(t)\le\frac{2}{\check{\beta}}e^{-2\lambda t}V(t)\le\frac{2}{\check{\beta}}V(0)e^{-2(\lambda-\bar{\omega}(1-\sigma))t},
$$

where $\check{\beta}=\min_{i\in\Gamma}\{\beta_i\}$. Therefore,

$$
\|s(t)\|\le\sqrt{\frac{2V(0)}{\check{\beta}}}e^{\lambda-\bar{\omega}(1-\sigma)t},
$$

which means that the exponential synchronization is realized. □

In the following, $K_{ij}(\cdot)$ is considered as the following special form:

$$
K_{ij}(\eta)=\begin{cases}0, & \eta\ge\mu,\\ 1, & \eta<\mu,\end{cases}\tag{15}
$$

where $\mu>0$. In this case, it is obvious that $k_0=\mu$, $k_1=\frac{1}{2\lambda}(e^{2\lambda\mu}-1)$ in Assumption 2. Moreover, the driving and response systems (1) and (2) are reduced to the following form in this case:

$$
\begin{aligned}
\ddot{x}_i(t)=&-a_i\dot{x}_i(t)-b_ix_i(t)+\sum_{j\in\Gamma}c_{ij}f_j(x_j(t))+\sum_{j\in\Gamma}d_{ij}f_j(x_j(t-\nu(t)))\\
&+\sum_{j\in\Gamma}r_{ij}\int_{t-\mu}^{t}f_j(x_j(\eta))d\eta+I_i(t),\quad i\in\Gamma,
\end{aligned}\tag{16}
$$

$$
\ddot{y}_i(t)=-a_i\dot{y}_i(t)-b_iy_i(t)+\sum_{j\in\Gamma}c_{ij}f_j(y_j(t))+\sum_{j\in\Gamma}d_{ij}f_j(y_j(t-\nu(t)))
$$

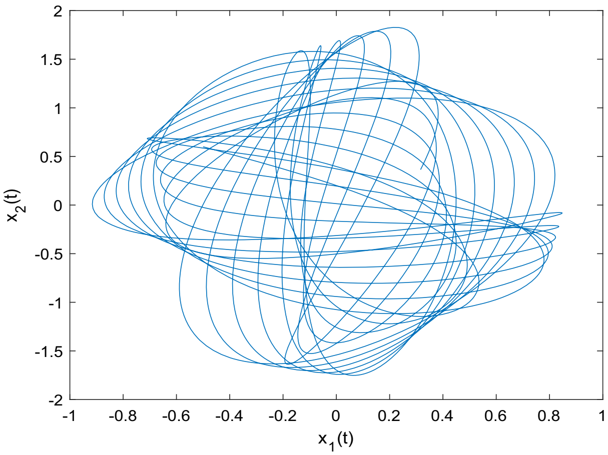

Fig. 1 The phase trajectory of system (18) without control

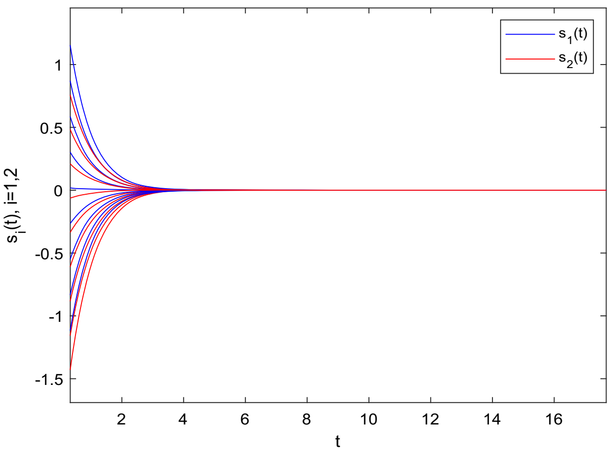

Fig. 2 The evolution of synchronized error between systems (18) and (19)

$$+\sum_{j\in\Gamma} r_{ij}\int_{t-\mu}^{t} f_j(y_j(\eta))d\eta + I_i(t) + U_i(t), \quad i\in\Gamma, \tag{17}$$

which implies that the infinite distributed delays are transformed into bounded distributed delays.

Corollary 1 *Under Assumptions* 1-3*, the neural models* (16) *and* (17) *are exponentially synchronized if* $\varrho = \lambda - \bar{\omega}(1-\sigma) > 0$*,* $\bar{\omega} = \max_{i\in\Gamma}\{\omega_i\}$*.*

Apparently, $C_i = 0$ *if* $\beta_i = \alpha_i\big(a_i + b_i + 2\omega_i - 2\lambda - 1\big) > 0$*. Assumption* 3 *in this situation can be rewritten as follows.*

Assumption 4 There exist positive constants α_i such that

$$a_i + b_i + 2\omega_i - 2\lambda - 1 > 0,$$
$$A_i < 0,\ B_i < 0,\ i\in\Gamma.$$

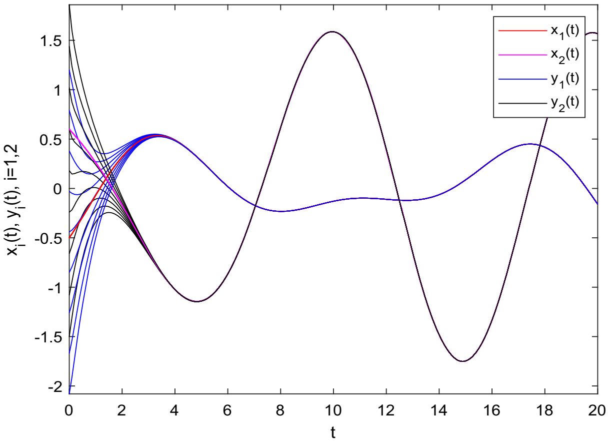

Fig. 3 The evolution of synchronization between system (18) and (19)

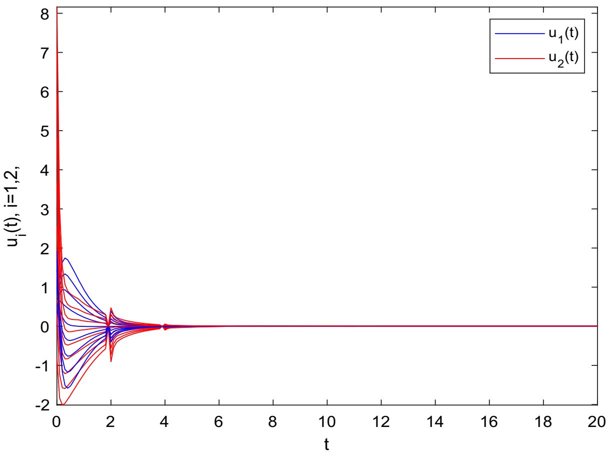

Fig. 4 The evolution of the intermittent controller

Corollary 2 *Based on Assumptions 1–2 and 4, if* $\varrho = \lambda - \bar{\omega}(1 - \sigma) > 0$ *is satisfied, then systems* (1) *and* (2) *can achieve exponential synchronization.*

Remark 2 Unlike the traditional variable transformation method in the reports of [19,44] based on intermittent control scheme, a non-reduced order technique is developed by directly establishing a Lyapunov functional formed by both the state variables and their derivatives to discuss the synchronization problem of inertial neural networks.

Remark 3 The dynamic behaviors of delayed neural networks with discrete and infinitely distributed delays have been sufficiently studied in [58,59]. Compared with the first-order differential systems, the inertial neural systems with mixed delays in this paper are more general and practical.

Remark 4 In [17,48,60], the inertial system was converted into two first-order differential equations by appropriate variable transformation. It is noted that the dimensions of the

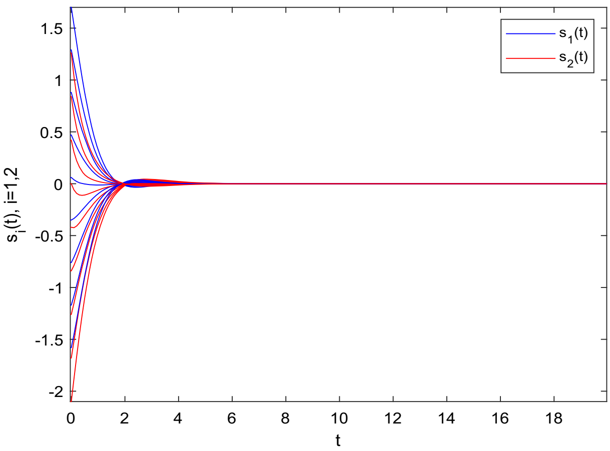

Fig. 5 The evolution of synchronized error between system (18) and (19)

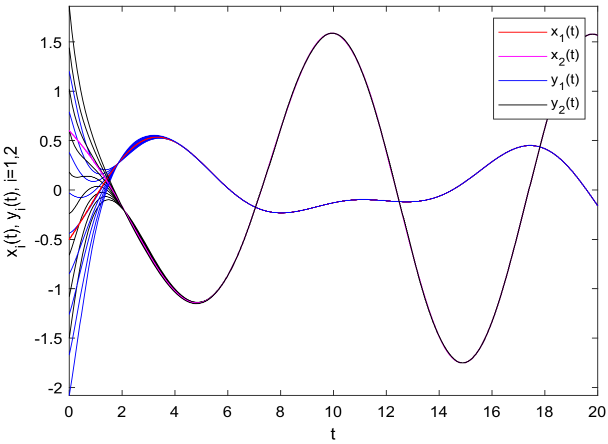

Fig. 6 The evolution of synchronization between system (18) and (19)

reduced-order system are twice as much as the original second-order system, which makes the theoretical analysis more difficult and the obtained conditions more complex and conservative. In order to conquer these difficulties, in this paper, some novel criteria are derived based a direct method of the order non-reduction, which are simpler and less conservative compared with those conditions given in [17,48,60].

4 Numerical Simulations

To illustrate the theoretical work, a numerical example and some detailed simulations are given in this part.

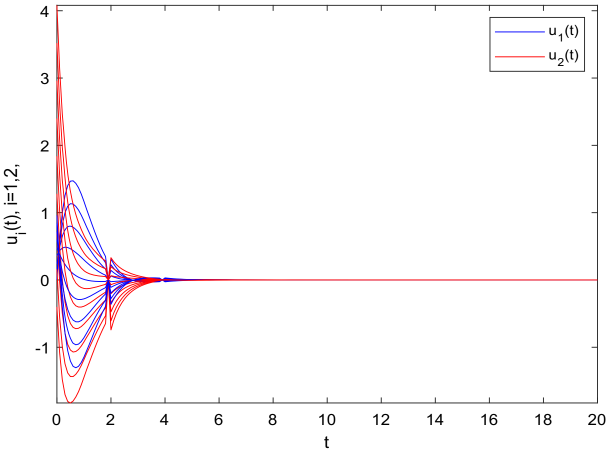

Fig. 7 The evolution of the intermittent controller

Consider the following a type of inertial systems composed of two neurons:

$$\ddot{x}_i(t) = -a_i\dot{x}_i(t) - b_i x_i(t) + \sum_{j=1}^{2} c_{ij} f_j(x_j(t)) + \sum_{j=1}^{2} d_{ij} f_j(x_j(t-\nu(t))) + \sum_{j=1}^{2} r_{ij} \int_{-\infty}^{t} K_{ij}(t-\eta) f_j(x_j(\eta))d\eta + I_i(t), \quad (18)$$

$$\ddot{y}_i(t) = -a_i\dot{y}_i - b_i y_i(t) + \sum_{j=1}^{2} c_{ij} f_j(y_j(t)) + \sum_{j=1}^{2} d_{ij} f_j(y_j(t-\nu(t))) + \sum_{j=1}^{2} r_{ij} \int_{-\infty}^{t} K_{ij}(t-\eta) f_j(y_j(\eta))d\eta + I_i(t) + U_i(t), \quad (19)$$

where $f_j(x) = \tanh(0.1x)$ for $j = 1, 2$, $\nu(t) = \frac{e^t}{1+e^t}$, $K_{ij}(\eta) = e^{-4\lambda\eta}$, $a_1 = a_2 = 0.3$, $b_1 = 0.4$, $b_2 = 0.2$, and

$$C = (c_{ij})_{2\times 2} = \begin{bmatrix} 0.3 & -0.4 \\ -3.4 & 1.1 \end{bmatrix}, D = (d_{ij})_{2\times 2} = \begin{bmatrix} -2.0 & -0.1 \\ -0.4 & -1.6 \end{bmatrix},$$

$$R = (c_{ij})_{2\times 2} = \begin{bmatrix} 0.1 & -0.3 \\ 0.5 & -2.8 \end{bmatrix}.$$

The dynamic behavior of driving system (18) without control can be shown in Fig.1, where the initial values are given as $\varphi_1(\varsigma) = -0.5$, $\psi_1(\varsigma) = 0.6$, $\varphi_2(\varsigma) = 0.4$, $\psi_2(\varsigma) = -0.3$ with $\varsigma \in (-\infty, 0]$.

Next, consider the exponential synchronization between driving system (18) and response system (19) under intermittent control (3). Note that $0 < \nu(t) < \nu = 1, 0 < \dot{\nu}(t) < \nu_1 = \frac{1}{4}$, $l_1 = l_2 = 0.1$. Choose $\lambda = 0.5$, $\alpha_1 = 0.5$, $\alpha_2 = 0.4$, $\beta_1 = 4$, $\beta_2 = 4.5$, then, $k_0 = 0.5$, $k_1 = 1$, $A_1 = -2.325$, $B_1 = -0.225075$, $C_1 = -1.35$, $A_2 = -2.557$, $B_2 = -0.236542$, $C_2 = -1.3$, $\bar{\omega} = \max\{6, 8\}$ and $\sigma = 0.95$ by calculation, which means that all conditions of

Theorem 1 are satisfied and the exponential synchronization is realized by Theorem 1. The synchronization is shown in Figs.2–4.

Next, Let's consider the special case $C_i = 0$ in Corollary 2. Choose $\lambda = 0.2, \alpha_1 = \alpha_2 = 1$, $\beta_1 = 5.3$, $\beta_2 = 7.1$, then, $k_0 = 1.25$, $k_1 = 2.5$, $A_1 = -1.935$, $B_1 = -0.585150$, $A_2 = -2.56875$, $B_2 = -0.507606$, $C_1 = C_2 = 0$, $\bar{\omega} = \max\{3, 4\}$ and $\sigma = 0.95$. From Corollary 2, the synchronization between system (18) and (19) is achieved, which is revealed in Fig.5-Fig.7.

5 Conclusion

In this paper, a type of inertial neural systems with mixed delays is proposed. In order to reduce the control cost, an intermittent control scheme is designed for the second-order inertial models. Meanwhile, some novel sufficient conditions are established to ensure the exponential synchronization of drive-response neural models by directly constructing an appropriate Lyapunov functional. Some numerical simulations are provided to support the theoretical analysis in the end.

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