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Adaptive Sampled-Data Observer Design for a Class of Nonlinear Systems with Unknown Hysteresis

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Abstract
In this paper, a novel adaptive sampled-data observer design is studied for a class of nonlinear systems with unknown Prandtl–Ishlinskii hysteresis and unknown unmatched disturbances based on radial basis function neural networks (RBFNNs). To begin with, we investigate a sampled-data nonlinear system and present sufficient conditions such that the sampled-data nonlinear system is ultimately uniformly bounded (UUB). Then, an adaptive sampled-data observer is designed to estimate the unknown states of the nonlinear system. The unknown hysteresis and the unknown disturbances are approximated by RBFNNs. We also give the learning laws of the weights of RBFNNs, and prove that the estimation errors of the states and the weights are UUB, based on the obtained sufficient conditions and a special constructing Lyapunov–Krasovskii function. Finally, the effectiveness of the proposed design method is verified by numerical simulations.

Keywords PI hysteresis · Adaptive sampled-data observer · RBFNNs · Nonlinear systems · UUB

1 Introduction

Due to hysteresis is an important nonlinear phenomenon which exists widely in practical systems, nonlinear systems with hysteresis have been one of the rigorous challenging and worthy research for control [1]. The properties, such as, inaccuracies, oscillations and instability affected by the non-differentiability of hysteresis may gradually deteriorate the system performance [2,3]. Recently, numerous adaptive control schemes have been developed to control uncertain nonlinear systems with unknown backlash-like hysteresis. In [4,5], an adaptive state feedback control and an adaptive fuzzy output feedback control were designed for a class of uncertain nonlinear systems preceded by unknown backlash-like hysteresis, respectively. Note that the controllers designed in [4–6] are based on the backlash-like hysteresis. There are other hysteresis patterns needed to be analyzed. The authors in [7] developed an

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adaptive neural output feedback control scheme for nonlinear systems with unknown Prandtl–Ishlinskii (PI) hysteresis. Additionally, there exist some methods of control and tracking for this hysteresis system, such as adaptive robust output feedback control and robust adaptive backstepping control, etc. [8–11].

In some cases, the states of a control system are usually unmeasurable, whereas the system outputs are measurable at sampling instants. In particular, for a networked control system (NCS), its outputs are usually acquired by data acquisitions at sampling instants. Therefore, the design of sampled-data observers is more significant and challenging than that of traditional observers. For a continuous linear system, observer can be designed on the basis of its accurate discretization model. However, for a continuous nonlinear system, it is usually difficult to obtain its accurate discretization model. Therefore, the design method cannot be extended to continuous nonlinear systems. Recently, researchers have paid great enthusiasm on sampled-data observer design for nonlinear systems, and developed three categories of design method, such as design based on approximate discretization models [12,13], continuous design and corresponding discretization [14,15], and continuous and discrete design [16–21]. Inspired by Chen and Ge [7], we try to extend sampled-data observer design for nonlinear systems with unknown hysteresis by the third method. Because when the sampling-data observer adopts this method, the sampling-data can be directly utilized to update the observer without discretizing the nonlinear system.

In practical engineering applications, nonlinearities and uncertainties are usually contained in an amount of systems. In addition, unmatched time-varying disturbances are unavoidable, with which the whole system will be unstable [22–24]. To this end, RBFNNs and fuzzy logic systems possessing superior approximation and adaptability were employed to compensate the uncertainties and the unmatched time-varying disturbances [25–28].

Following the previous references, few results are combined with sampled-data observers, although researches on control and application of hysteresis have been carried out in recent years. In this paper, we consider sampled-data observer design for a class of nonlinear systems with unknown hysteresis and unknown unmatched disturbance based on RBFNNs. The main contributions of this paper are summarized as follows. (1) We investigate a sampled-data nonlinear system and present sufficient conditions such that the considered system is UUB. (2) Continuous observers are designed for a class of nonlinear systems with unknown hysteresis and unknown disturbances, which are approximated by RBFNNs. The sampled measurements are used to update the observer whenever they are available. (3) By constructing a Lyapunov–Krasovskii function, sufficient conditions are derived to guarantee that the observation errors are UUB. Compared with [7], the ways of process of the hysteresis and the disturbances are different, the restriction on the constant control gain parameter is relaxed, and the problem of parameter selection is solved.

The rest of this paper is organized as follows. In Sect. 2, the problem statement, some assumptions, and the control objective are described. Section 3 describes the design procedure of the adaptive sampled-data observer by using RBFNNs. In Sect. 4, an example is used to illustrate the validity of the proposed design methods. Some conclusions are given in Sect. 5.

2 Problem Formulations and Preliminaries

In this paper, our purpose is to design an adaptive sampled-data observer for the following system

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(\bar{x}_1(t)) + d_1(\bar{x}_1(t), t), \\ \quad \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(\bar{x}_{n-1}(t)) + d_{n-1}(\bar{x}_{n-1}(t), t), \\ \dot{x}_n(t) = b\omega(u(t)) + f_n(\bar{x}_n(t)) + d_n(\bar{x}_n(t), t), \\ y(t) = x_1(t_k), t \in [t_k, t_{k+1}), k \geq 0, \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)] \in R^n$ ($\bar{x}_i(t) = [x_1(t), x_2(t), \ldots, x_i(t)]^T \in R^i$, ($i = 1, 2, \ldots, n$)) is the state vector of the system, the input $u(t) \in R$, the system output $y(t) \in R$ is sampled at time instant t_k, where $\{t_k\}$ is a strictly increasing sequence and satisfies $\lim_{k\to\infty} t_k = \infty$, T is the sampling period, and $T = t_{k+1} - t_k$. $f_i(\bar{x}_i(t))$ ($i = 1, 2, \ldots, n$) are known smooth nonlinear functions, $d_i(\bar{x}_i(t), t) \in R$ ($i = 1, 2, \ldots, n$) denote unknown time-varying unmatched disturbances, $b \in R$ ($b \neq 0$) represents an unknown but bounded constant control gain, $\omega(u(t)) \in R$ represents an unknown PI hysteresis, whose model is given by [29]

$$\begin{aligned} \omega(u(t)) = P[u](t) &= p_0 u(t) - \int_0^{r_1} p(r) F_r[u](t) dr, \\ &= p_0 u(t) + d_0(u(t)), \end{aligned} \tag{2}$$

where r is a threshold, $p(r)$ is a given density function and satisfies $p(r) > 0$ and $\int_0^{\infty} rp(r)dr < \infty$, $p_0 = \int_0^{r_1} p(r)dr$ is a constant and depends on the density function, and r_1 denotes the upper limit of the integration. Let $f_r : R \to R$ be defined by

$$f_r(u(t), \omega(u(t))) = \max\left(u(t) - r, \min(u(t) + r, \omega(u(t)))\right).$$

The play operator $F_r[u](t)$ is given by

$$\begin{aligned} F_r[u](0) &= f_r\left(u(0), 0\right), \\ F_r[u](t) &= f_r\left(u(t), F_r[u](t_i)\right), \end{aligned}$$

with $t_i < t \leq t_{i+1}$ and $0 \leq i \leq N-1$, where $0 = t_0 < t_1 < \cdots < t_N = t_E$ is a partition of $[0, t_E]$ such that the function $u(t)$ is monotone on $(t_i, t_{i+1}]$, and $C[0, t_E]$ is a set of bounded continuous functions on $[0, t_E]$. For any input $u(t) \in C[0, t_E]$, the play operator is Lipschitz continuous [29].

The system (1) can also be expressed as

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(\bar{x}_1(t)) + d_1(\bar{x}_1(t), t), \\ \quad \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(\bar{x}_{n-1}(t)) + d_{n-1}(\bar{x}_{n-1}(t), t), \\ \dot{x}_n(t) = b_0 u(t) + b d_0(u(t)) + \delta(u(t)) + f_n(\bar{x}_n(t)) \\ \qquad + d_n(\bar{x}_n(t), t), \\ y(t) = x_1(t_k), t \in [t_k, t_{k+1}), k \geq 0, \end{cases} \tag{3}$$

where $\delta(u(t)) = (bp_0 - b_0)u(t)$ and b_0 is a design parameter.

We make the following assumptions to facilitate analysis.

Assumption 1 ([6]) There exist constants l_{i1} ($i = 1, 2, \ldots, n$) such that the following inequalities

$$\left| f_i(\bar{x}_i(t)) - f_i(\hat{\bar{x}}_i(t)) \right| \leq l_{i1}\left(\left|x_1(t) - \hat{x}_1(t)\right| + \cdots + \left|x_i(t) - \hat{x}_i(t)\right|\right), \tag{4}$$

hold.

Assumption 2 For the nonlinear system (3), the input $u(t) \in C[0, t_E]$, thus, there exists an unknown positive constant σ_0 such that $|\delta(u(t))| \leq \sigma_0$.

Remark 1 Although the parameter b is unknown, we can select the design parameter b_0 to approach bp_0. In other words, the design parameter b_0 should be selected appropriately to achieve better state estimation performance.

The following lemmas are needed, which can be found in [7,30].

Lemma 2.1 ([30]) *Let $M \in R^{n\times n}$ and γ denote a positive definite matrix and a positive real number, respectively. The vector function $\varphi(t)$ is defined on the interval $[0, \gamma]$ and is integrable. Then, we have*

$$\left[\int_0^{\gamma} \varphi(s)ds\right]^T M \left[\int_0^{\gamma} \varphi(s)ds\right] \leq \gamma \left[\int_0^{\gamma} \varphi(s)^T M\varphi(s)ds\right].$$

Lemma 2.2 ([7]) *Let $f(Z)$ be a continuous function on a compact set Ω, which can be approached by a RBFNN, that is,*

$$\hat{f}(Z) = \hat{W}^T S(Z) + \varsigma,$$

where $Z = [z_1, z_2, \ldots, z_m]^T \in \Omega \subset R^m$ and $\hat{W} \subset R^q$ are the input vector and the weight of the RBFNN, respectively, $S(Z) = [S_1(Z), S_2(Z), \ldots, S_q(Z)]^T \in R^q$ is the basis function vector, and $\varsigma > 0$ is the approximation error. The optimal weight W^* of RBFNN is given by

$$W^* = \arg \min_{\hat{W}\in\Omega} \left[\sup_{Z\in R^q} \left|\hat{f}(Z|\hat{W}) - f(Z)\right|\right].$$

By using the optimal weight, we have

$$f(Z) = W^{*T} S(Z) + \varsigma^*, \left|\varsigma^*\right| \leq \bar{\varsigma},$$

where ς^* is the optimal approximation error, and $\bar{\varsigma} > 0$ is the upper bound of the approximation error.

Lemma 2.3 ([31]) *There exist real numbers c_1, c_2 and real-valued function $\Delta(x, y) > 0$ such that the following inequality holds:*

$$|x|^{c_1} |y|^{c_2} \leq \frac{c_1}{c_1 + c_2}\Delta(x, y)\, |x|^{c_1+c_2} + \frac{c_2}{c_1 + c_2}\Delta^{-\frac{c_1}{c_2}}(x, y)\, |y|^{c_1+c_2}.$$

For a sampled-data nonlinear system, we give the definition of UUB and sufficient conditions of UUB.

Definition 1 For the following sampled-data nonlinear system

$$\begin{cases} \dot{x}(t) = g\left(x(t), x(t_k)\right), \\ x(t_{k+1}) = \lim_{t\to t_{k+1}^-} x(t_k), t \in [t_k, t_{k+1}), k \geq 0, \end{cases} \tag{5}$$

where $x(t) \in R^n$ is the state of the system, and $g(\cdot)$ is a continuous function with $g(0) = 0$. Denote the solution to (5) with respect to the initial conditions x_0 as $x(t)$. If there exists a constant $b_1 > 0$ and a constant $T'(x_0, b_1)$ such that

$$|x(t)| < b_1, \forall t > t_0 + T'(x_0, b_1),$$

then, the system (5) is UUB.

Lemma 2.4 *For the nonlinear sampled-data system (5), if there exists a Lyapunov function $V(x(t))$ defined on the interval $[t_0, \infty)$ such that*

$$\frac{dV(x(t))}{dt} \leq -\alpha_1 V(x(t)) + \beta_1 V(x(t_k)) + C, \; t \in [t_k, t_{k+1}), \tag{6}$$

and

$$\alpha_1 > \beta_1,$$

hold, where α_1, β_1 and C are three positive real numbers, then it is UUB. Moreover, we have $\lim_{k\to\infty} V(x(t)) \leq \frac{2-\alpha_2}{1-\alpha_2}\frac{C}{\alpha_1}$.

Proof Multiplying $e^{\alpha_1 t}$ on both sides of the inequality (6), we have

$$\frac{d(e^{\alpha_1 t} V(x(t)))}{dt} \leq e^{\alpha_1 t}\beta_1 V(x(t_k)) + Ce^{\alpha_1 t}, \; t \in [t_k, t_{k+1}).$$

Then,

$$V(x(t)) \leq [(1 - \frac{\beta_1}{\alpha_1})e^{-\alpha_1(t-t_k)} + \frac{\beta_1}{\alpha_1}]V(x(t_k)) + \frac{C}{\alpha_1}, t \in [t_k, t_{k+1}). \tag{7}$$

Let $t = t_{k+1}$, we can obtain

$$V(x(t_{k+1})) \leq \alpha_2 V(x(t_k)) + \frac{C}{\alpha_1} \tag{8}$$

where $\alpha_2 = (1 - \frac{\beta_1}{\alpha_1})e^{-\alpha_1 T} + \frac{\beta_1}{\alpha_1}$. Since $\alpha_1 > \beta_1$, we have $\alpha_2 < 1$. From (8), it follows that

$$V(x(t_k)) \leq {\alpha_2}^k V(x(t_0)) + \frac{1-{\alpha_2}^k}{1-\alpha_2}\frac{C}{\alpha_1}. \tag{9}$$

Substituting (9) into (7) results in

$$V(x(t)) \leq {\alpha_2}^k V(x(t_0)) + \frac{2-\alpha_2-{\alpha_2}^k}{1-\alpha_2}\frac{C}{\alpha_1}, \; t \in [t_k, t_{k+1}).$$

Thus, $\lim_{t\to\infty} V(x(t)) \leq \frac{2-\alpha_2}{1-\alpha_2}\frac{C}{\alpha_1}$, and the sampled-data nonlinear system (5) is UUB. □

3 Adaptive Sampled-Data Observer Design

Since the play operator $F_r[u](t)$ is continuous and the density function is integrable, it is concluded that the PI model is continuous. In order to design the adaptive sampled-data observer, we use RBFNNs to approximate the unknown time-varying unmatched disturbances $\nu_i d_i(\bar{x}_i(t), t)$ $(i = 1, 2, \ldots, n)$ and unknown function $\nu_0 b d_0(u(t))$.

According to Lemma 2.2, we have

$$W^{*T} S\left(u(t)\right) + \varsigma_0^* = \nu_0 b d_0\left(u(t)\right), \tag{10}$$

$$\theta_i^{*T} \varphi_i(\bar{x}_i(t)) + {\varsigma_i}^* = \nu_i d_i(\bar{x}_i(t), t), \tag{11}$$

where $\nu_i > 0$ $(i = 0, 1, \ldots, n)$ are some design parameters.

Then, substituting (10) and (11) into the system (3), we have

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(\bar{x}_1(t)) + \frac{1}{\nu_1}\theta_1^{*T}\varphi_1(\bar{x}_1(t)) + \frac{1}{\nu_1}\varsigma_1{}^*, \\ \quad\vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(\bar{x}_{n-1}(t)) + \frac{1}{\nu_{n-1}}\theta_{n-1}^{*T}\varphi_{n-1}(\bar{x}_{n-1}(t)) \\ \qquad + \frac{1}{\nu_{n-1}}\varsigma_{n-1}{}^*, \\ \dot{x}_n(t) = b_0 u(t) + f_n(\bar{x}_n(t)) + \frac{1}{\nu_n}\theta_n^{*T}\varphi_n(\bar{x}_n(t)) + \frac{1}{\nu_n}\varsigma_n{}^* \\ \qquad + \frac{1}{\nu_0}W^{*T}S(u(t)) + \frac{1}{\nu_0}\varsigma_0{}^* + \delta(u(t)), \\ y(t) = x_1(t_k),\ t \in [t_k, t_{k+1}),\ k \geq 0, \end{cases} \tag{12}$$

where ς_i^* $(i = 0, 1, \ldots, n)$ are the optimal approximation errors, $\varphi_i(\bar{x}_i(t))$ $(i = 1, 2, \ldots, n)$ and $S(u(t))$ are some basis function vectors, and which are selected such that the following conditions

$$\left|\varphi_i(\bar{x}_i(t)) - \varphi_i(\hat{\bar{x}}_i(t))\right| \leq l_{i2}(\left|x_1(t) - \hat{x}_1(t)\right| + \cdots + \left|x_i(t) - \hat{x}_i(t)\right|), \tag{13}$$

hold for some positive real numbers l_{i2} $(i = 1, 2, \ldots, n)$.

Let $D_i = \frac{1}{\nu_i}\varsigma_i{}^*$, $(i = 1, 2, \ldots, n-1)$ and $D_n = \delta(u(t)) + \frac{1}{\nu_n}\varsigma_n{}^* + \frac{1}{\nu_0}\varsigma_0{}^*$. Then, the system (12) can be expressed as

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(\bar{x}_1(t)) + D_1 + \frac{1}{\nu_1}\theta_1^{*T}\varphi_1(\bar{x}_1(t)), \\ \quad\vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(\bar{x}_{n-1}(t)) + D_{n-1} + \frac{1}{\nu_{n-1}}\theta_{n-1}^{*T}\varphi_{n-1}(\bar{x}_{n-1}(t)), \\ \dot{x}_n(t) = b_0 u(t) + f_n(\bar{x}_n(t)) + D_n + \frac{1}{\nu_0}W^{*T}S(u(t)) \\ \qquad + \frac{1}{\nu_n}\theta_n^{*T}\varphi_n(\bar{x}_n(t)), \\ y(t) = x_1(t_k),\ t \in [t_k, t_{k+1}),\ k \geq 0. \end{cases} \tag{14}$$

From Assumption 2 and definitions of D_i and D_n, we can obtain that $|D_i| \leq \sigma_{i1}$ with $\sigma_{i1} > 0$, $(i = 1, 2, \ldots, n)$.

Now, we present the following dynamical system to estimate the unknown states of the nonlinear sampled-data system (14).

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + f_1(\hat{\bar{x}}_1(t)) + \Gamma k_1 e_1(t_k) + \frac{1}{\nu_1}\hat{\theta}_1^T\varphi_1(\hat{\bar{x}}_1(t)), \\ \quad\vdots \\ \dot{\hat{x}}_{n-1}(t) = \hat{x}_n(t) + f_{n-1}(\hat{\bar{x}}_{n-1}(t)) + \Gamma^{n-1}k_{n-1}e_1(t_k) \\ \qquad + \frac{1}{\nu_{n-1}}\hat{\theta}_{n-1}^T\varphi_{n-1}(\hat{\bar{x}}_{n-1}(t)), \\ \dot{\hat{x}}_n(t) = b_0 u(t) + f_n(\hat{\bar{x}}_n(t)) + \Gamma^n k_n e_1(t_k) \\ \qquad + \frac{1}{\nu_0}\hat{W}^T S(u(t)) + \frac{1}{\nu_n}\hat{\theta}_n^T\varphi_n(\hat{\bar{x}}_n(t)), \\ \hat{x}_i(t_{k+1}) = \lim_{t \to t_{k+1}^-}\hat{x}_i(t),\ \ t \in [t_k, t_{k+1}),\ k \geq 0, \end{cases} \tag{15}$$

where $e_1(t_k) = x_1(t_k) - \hat{x}_1(t_k)$, and $\hat{x}_i(t)$, $\hat{\bar{x}}_i(t)$, $\hat{W}$, $\hat{\theta}_i$, $(i = 1, 2, \ldots, n)$ are the estimates of $x_i(t)$, $\bar{x}_i(t)$, W^*, θ_i^*, respectively. $\Gamma \geq 1$, $k_i > 0$ $(k = 1, 2, \ldots, n)$ are the design parameters.

From (14)–(15), the estimation error can be obtained.

$$\begin{cases} \dot{e}_1(t) = e_2(t) + \tilde{f}_1 - \Gamma k_1 e_1(t_k) + D_1 + \frac{1}{\nu_1}\theta_1^{*T}\tilde{\varphi}_1 \\ \qquad + \frac{1}{\nu_1}\tilde{\theta}_1^T\varphi_1(\hat{\bar{x}}_1(t)), \\ \qquad \vdots \\ \dot{e}_{n-1}(t) = e_n(t) + \tilde{f}_{n-1} - \Gamma^{n-1}k_{n-1}e_1(t_k) + D_{n-1} \\ \qquad + \frac{1}{\nu_{n-1}}\theta_{n-1}^{*T}\tilde{\varphi}_{n-1} + \frac{1}{\nu_{n-1}}\tilde{\theta}_{n-1}^T\varphi_{n-1}(\hat{\bar{x}}_{n-1}(t)), \\ \dot{e}_n(t) = \tilde{f}_n - \Gamma^n k_n e_1(t_k) + \frac{1}{\nu_0}\tilde{W}^T S(u(t)) + D_n \\ \qquad + \frac{1}{\nu_n}\theta_n^{*T}\tilde{\varphi}_n + \frac{1}{\nu_n}\tilde{\theta}_n^T\varphi_n(\hat{\bar{x}}_n(t)), \\ t \in [t_k, t_{k+1}), k \geq 0, \end{cases} \tag{16}$$

where $e_i(t) = x_i(t) - \hat{x}_i(t)$, $\tilde{f}_i = f_i(\bar{x}_i(t)) - f_i(\hat{\bar{x}}_i(t))$, $\tilde{\varphi}_i = \varphi_i(\bar{x}_i(t)) - \varphi_i(\hat{\bar{x}}_i(t))$, $\tilde{W} = W^* - \hat{W}$, $\tilde{\theta}_i = \theta_i{}^* - \hat{\theta}_i$. Consider the following coordinate transformation

$$\vartheta_i(t) = \frac{e_i(t)}{\Gamma^{i-1}}, i = 1, 2, \ldots, n.$$

After transformation, the system (16) can be rewritten as

$$\begin{cases} \dot{\vartheta}_1(t) = \Gamma\vartheta_2(t) - \Gamma k_1\vartheta_1(t) + \Gamma k_1\left(\vartheta_1(t) - \vartheta_1(t_k)\right) \\ \qquad + \frac{\tilde{f}_1}{\Gamma^0} + \frac{D_1}{\Gamma^0} + \frac{1}{\nu_1\Gamma^0}\theta_1^{*T}\tilde{\varphi}_1 + \frac{1}{\nu_1\Gamma^0}\tilde{\theta}_1^T\varphi_1\left(\hat{\bar{x}}_1(t)\right), \\ \qquad \vdots \\ \dot{\vartheta}_{n-1}(t) = \Gamma\vartheta_n(t) - \Gamma k_{n-1}\vartheta_1(t) + \Gamma k_{n-1}(\vartheta_1(t) - \\ \qquad \vartheta_1(t_k)) + \frac{\tilde{f}_{n-1}}{\Gamma^{n-2}} + \frac{D_{n-1}}{\Gamma^{n-2}} + \frac{1}{\nu_{n-1}\Gamma^{n-2}}\theta_{n-1}^{*T}\tilde{\varphi}_{n-1} \\ \qquad + \frac{1}{\nu_{n-1}\Gamma^{n-2}}\tilde{\theta}_{n-1}^T\varphi_{n-1}\left(\hat{\bar{x}}_{n-1}(t)\right), \\ \dot{\vartheta}_n(t) = -\Gamma k_n\vartheta_1(t) + \Gamma k_n\left(\vartheta_1(t) - \vartheta_1(t_k)\right) \\ \qquad + \frac{\tilde{f}_n}{\Gamma^{n-1}} + \frac{D_n}{\Gamma^{n-1}} + \frac{1}{\nu_0\Gamma^{n-1}}\tilde{W}^T S(u(t)) \\ \qquad + \frac{1}{\nu_n\Gamma^{n-1}}\theta_n^{*T}\tilde{\varphi}_n + \frac{1}{\nu_n\Gamma^{n-1}}\tilde{\theta}_n^T\varphi_n\left(\hat{\bar{x}}_n(t)\right), \\ t \in [t_k, t_{k+1}), k \geq 0, \end{cases} \tag{17}$$

We can also obtain the following compact form of the system (17).

$$\begin{cases} \dot{\vartheta} = \Gamma A\vartheta + \tilde{F} + \Gamma\tilde{K} + \tilde{W}_\lambda + \tilde{D} + \tilde{\Phi} + \tilde{D}_\lambda, \\ t \in [t_k, t_{k+1}), k \geq 0, \end{cases} \tag{18}$$

where $\vartheta = [\vartheta_1(t), \vartheta_2(t), \ldots, \vartheta_n(t)]^T$, $\hat{k} = (k_1, k_2, \ldots, k_n)^T$, $\tilde{K} = \hat{k}(\vartheta_1(t) - \vartheta_1(t_k))$, $\tilde{F} = [\frac{\tilde{f}_1}{\Gamma^0}, \frac{\tilde{f}_2}{\Gamma^1}, \ldots, \frac{\tilde{f}_n}{\Gamma^{n-1}}]^T$,

$$\tilde{W}_\lambda = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{1}{\nu_0 \Gamma^{n-1}} \tilde{W}^T S(u(t)) \end{bmatrix}^{n\times 1}, A = \begin{bmatrix} -k_1 & 1 \cdots 0 \\ \vdots & \vdots \ddots \vdots \\ -k_{n-1} & 0 \cdots 1 \\ -k_n & 0 \cdots 0 \end{bmatrix}, \tilde{D} = [\frac{D_1}{\Gamma^0}, \frac{D_2}{\Gamma^1}, \ldots, \frac{D_n}{\Gamma^{n-1}}]^T, \tilde{\Phi} =$$

$[\frac{1}{\nu_1\Gamma^0}\theta_1^{*T}\tilde{\varphi}_1, \frac{1}{\nu_2\Gamma^1}\theta_2^{*T}\tilde{\varphi}_2, \ldots, \frac{1}{\nu_n\Gamma^{n-1}} \times \theta_n^{*T}\tilde{\varphi}_n]^T$, $\tilde{D}_\lambda = [\frac{1}{\nu_1\Gamma^0}\tilde{\theta}_1^T\varphi_1(\hat{\bar{x}}_1(t)), \frac{1}{\nu_2\Gamma^1}\tilde{\theta}_2^T\varphi_2(\hat{\bar{x}}_2(t)),$ $\ldots, \frac{1}{\nu_n\Gamma^{n-1}}\tilde{\theta}_n^T \times \varphi_n(\hat{\bar{x}}_n(t))]^T$, and the gains $k_i > 0$ $(i = 1, 2, \ldots, n)$ are chosen such that the polynomial $H(s) = s^n + k_1 s^{n-1} + \cdots + k_{n-1}s + k_n$ is Hurwitz. Thus, there exists a symmetric positive definite matrix P $(P{=}P^T > 0)$ such that the following matrix inequality holds,

$$A^T P + PA \leq -I. \tag{19}$$

The adaptive laws of the weights $\hat{W}$ and $\hat{\theta}_i$ are designed as follows,

$$\dot{\hat{W}} = \Lambda_0 \left(-S(u(t))e_1(t_k)\chi_0 + \ell_0 \hat{W}\right), \tag{20}$$

$$\dot{\hat{\theta}}_i = \Lambda_i \left(-\varphi_i(\hat{\bar{x}}_i(t))e_1(t_k)\chi_i + \ell_i\hat{\theta}_i\right), i = 1, 2, \ldots, n, \tag{21}$$

where $\Lambda_0 = {\Lambda_0}^T > 0$, $\Lambda_i = {\Lambda_i}^T > 0$ are some constant diagonal design matrices, and $\ell_0 > 0$, $\ell_i > 0$, $\chi_0 > 0$, $\chi_i > 0$ are some parameters to be designed.

Next, we give the definition of adaptive sampled-data observer of the nonlinear system (1).

Definition 2 For the nonlinear system (1), design the system (15), and the adaptive laws of the weights (20) and (21), if there exist two positive real numbers δ_0 and $T_1 > 0$, such that

$$\|e(t)\| < \delta_0, t > T_1,$$

then, the system (15) with the adaptive laws (20) and (21) is called an adaptive sampled-data observer of the system (1).

Theorem 1 *Consider the system* (1) *with conditions* (4) *and* (13). *If* $k_i > 0 (i = 1, 2, \ldots, n)$ *are selected such that the condition* (19) *holds, and the sampling period* T*, the parameters* ϕ, Δ_1, Δ_2, ℓ_0, ℓ_i *satisfy the following conditions*

$$T \leq \min\left\{\frac{\frac{1}{2} - \bar{p}_2\phi}{\Gamma}, \frac{1}{\phi + L_0 + 6L_1}, \sqrt{\frac{\frac{1}{2}\bar{p}_3\phi}{(L_0 + 6L_1)\bar{k}\Gamma^2}}, \sqrt{\frac{1}{4(L_0 + 6L_1)}}, \sqrt{\frac{\frac{\ell_i}{2} - \frac{1}{\Delta_1} - \frac{1}{\Delta_2} - \frac{1}{2}\lambda_{\max}({\Lambda_i}^{-1})\phi}{(L_0 + 6L_1)\frac{{\eta_0}^2}{{\nu_1}^2}}}\right\}, \tag{22}$$

and

$$\phi \leq \min\left\{\frac{1}{2\bar{p}_2}, \frac{\ell_0 - \frac{2}{\Delta_1} - \frac{2}{\Delta_2}}{\lambda_{\max}({\Lambda_0}^{-1})}, \frac{\ell_i - \frac{2}{\Delta_1} - \frac{2}{\Delta_2}}{\lambda_{\max}({\Lambda_i}^{-1})}\right\}, \tag{23}$$

and

$$\ell_0 - \frac{2}{\Delta_1} - \frac{2}{\Delta_2} > 0, \ell_i - \frac{2}{\Delta_1} - \frac{2}{\Delta_2} > 0, \tag{24}$$

then, the state observation error system (18) *is UUB, or, the system* (15)*–*(20)*–*(21) *is an adaptive sampled-data observer of the system* (1)*, where* $L_0 = 24\Gamma\bar{p}_1 n\bar{k}$*,* $L_1 =$

$\frac{\Delta_2}{2}\upsilon_0{}^2\bar{\chi}^2 + \frac{\Delta_2}{2}\eta_0{}^2\bar{\chi}^2$, $\upsilon_0 = \|S(u(t))\|$, $\eta_i = \left\|\varphi(\hat{\bar{x}}_i(t))\right\|$, $\eta_0 = \max(\eta_i)$, $\underline{\nu} = \min(\nu_0, \nu_i)$ $\bar{\chi} = \max(\chi_0, \chi_i)$, $\bar{p}_1 = \lambda_{\max}(P^T P)$, $\bar{p}_2 = \lambda_{\max}(P)$, $\bar{p}_3 = \lambda_{\min}(P)$, $\bar{k} = \max(k_i{}^2)$, $\bar{l} = \max\left(\sqrt{\sum_{i=1}^{n} i l_{i1}{}^2}, \sqrt{\sum_{i=1}^{n} i l_{i2}{}^2}\right)$.

Proof Consider the following Lyapunov–Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + \Gamma^2 V_4(t), \tag{25}$$

where $V_1(t) = \vartheta^T P \vartheta$, $V_2(t) = \frac{1}{2}\tilde{W}^T \Lambda_0{}^{-1}\tilde{W}$, $V_3(t) = \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T \Lambda_i{}^{-1}\tilde{\theta}_i$, $V_4(t) = \int_{t-T}^{t}\int_{\tau}^{t}\left[\vartheta_1(s)^2 + \vartheta_2(s)^2\right]ds d\tau$, $t \in [t_{k0}, \infty)$, and $k_0 = \min\{k : T < t_k\}$.

Then, along the trajectory of the system (18), the derivative of $V_1(t)$ is given as follows:

$$\begin{aligned}\dot{V}_1(t) =& \Gamma\vartheta^T(A^T P + PA)\vartheta + 2\vartheta^T P(\tilde{F} + \Gamma\tilde{K} + \tilde{D} + \tilde{\Phi} + \tilde{D}_\lambda + \tilde{W}_\lambda)\\ \leq& -\Gamma\vartheta^T\vartheta + 2\vartheta^T P\tilde{F} + 2\Gamma\vartheta^T P\tilde{K} + 2\vartheta^T P\tilde{D} + 2\vartheta^T P\tilde{\Phi}\\ &+ 2\vartheta^T P\tilde{D}_\lambda + 2\vartheta^T P\tilde{W}_\lambda.\end{aligned} \tag{26}$$

Based on Assumption 1 and Lemma 2.3, the following inequalities hold.

$$\begin{aligned}2\vartheta^T P\tilde{F} &\leq 2\left\|\vartheta^T\right\|\|P\|\bar{l}\sqrt{\sum_{i=1}^{n}\frac{\left|x_i(t) - \hat{x}_i(t)\right|^2}{\Gamma^{2(i-1)}}}\\ &\leq 2\bar{l}\sqrt{\bar{p}_1}\|\vartheta\|^2,\end{aligned} \tag{27}$$

$$2\vartheta^T P\tilde{\Phi} \leq 2\left\|\vartheta^T\right\|\|P\|\left\|\tilde{\Phi}\right\| \leq 2\bar{l}\sqrt{\bar{p}_1}\frac{\varpi_0}{\underline{\nu}}\|\vartheta\|^2, \tag{28}$$

$$\begin{aligned}2\vartheta^T P\tilde{D}_\lambda &\leq 2\left\|\vartheta^T\right\|\|P\|\left\|\tilde{D}_\lambda\right\|\\ &\leq \frac{\Delta_1\eta_0{}^2\bar{p}_1}{\underline{\nu}^2}\|\vartheta\|^2 + \frac{1}{\Delta_1}\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2,\end{aligned} \tag{29}$$

$$\begin{aligned}2\vartheta^T P\tilde{W}_\lambda &\leq 2\left\|\vartheta^T\right\|\|P\|\left\|\tilde{W}_\lambda\right\|\\ &\leq \frac{\Delta_1\upsilon_0{}^2\bar{p}_1}{\underline{\nu}^2}\|\vartheta\|^2 + \frac{1}{\Delta_1}\left\|\tilde{W}\right\|^2,\end{aligned} \tag{30}$$

$$2\vartheta^T P\tilde{D} \leq 2\left\|\vartheta^T\right\|\|P\|\left\|\tilde{D}\right\| \leq 4\bar{p}_1\|\vartheta\|^2 + \frac{1}{4}\sum_{i=1}^{n}\sigma_{i1}^2, \tag{31}$$

$$\begin{aligned}2\Gamma\vartheta^T P\tilde{K} &= 2\Gamma\vartheta^T P\hat{k}(\vartheta_1(t) - \vartheta_1(t_k))\\ &\leq 4\Gamma\bar{p}_1 n\bar{k}(\vartheta_1(t) - \vartheta_1(t_k))^2 + \frac{\Gamma}{4}\|\vartheta\|^2,\end{aligned} \tag{32}$$

where $\varpi_i = \left\|\theta_i^{*T}\right\|$, $\varpi_0 = \max(\varpi_i)$.

According to Lemma 2.1, we can obtain

$$|\vartheta_1(t)-\vartheta_1(t_k)|^2=\left|\int_{t_k}^{t}\dot{\vartheta}_1(s)ds\right|^2\le(t-t_k)\int_{t_k}^{t}\left|\dot{\vartheta}_1(s)\right|^2ds$$
$$\le(t-t_k)\int_{t_k}^{t}\Bigg[\Gamma\vartheta_2(s)-\Gamma k_1\vartheta_1(t_k)+\frac{\tilde{f}_1}{\Gamma^0}+\frac{D_1}{\Gamma^0}+\frac{1}{\nu_1\Gamma^0}\theta_1^{*T}\tilde{\varphi}_1$$
$$+\frac{1}{\nu_1\Gamma^0}\tilde{\theta}_1^T\varphi_1\left(\hat{\tilde{x}}_1(s)\right)\Bigg]^2ds$$
$$\le 6\Gamma^2(t-t_k)\int_{t_k}^{t}\Bigg[\vartheta_2(s)^2+\frac{(1+\frac{\varpi_0{}^2}{\nu_1{}^2})l_1{}^2}{\Gamma^2}\vartheta_1(s)^2+\bar{k}\vartheta_1(t_k)^2$$
$$+\frac{1}{\Gamma^2}\sum_{i=1}^{n}\sigma_{i1}^2+\frac{\eta_0{}^2}{\nu_1{}^2\Gamma^2}\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2\Bigg]ds,\ t\in[t_k,t_{k+1}),\ k\ge k_0, \tag{33}$$

where $l_1=\max(l_{11},l_{12})$, $\Gamma\ge\sqrt{\left(1+\frac{\varpi_0{}^2}{\nu_1{}^2}\right)l_1{}^2}$.

It follows from (32) and (33) that,

$$2\Gamma\vartheta^T P\tilde{K}\le\frac{\Gamma}{4}\|\vartheta\|^2+L_0\Gamma^2\bar{k}(t-t_k)^2\vartheta_1(t_k)^2$$
$$+L_0\Gamma^2(t-t_k)\int_{t_k}^{t}\left[\vartheta_1(s)^2+\vartheta_2(s)^2\right]ds$$
$$+L_0\frac{\eta_0{}^2}{\nu_1{}^2}(t-t_k)^2\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2+L_0(t-t_k)^2\sum_{i=1}^{n}\sigma_{i1}{}^2,$$
$$t\in[t_k,t_{k+1}),\ k\ge k_0. \tag{34}$$

From (26)–(31), and (34), we have

$$\dot{V}_1(t)\le-\frac{5}{8}\Gamma\|\vartheta\|^2+L_0\Gamma^2\bar{k}(t-t_k)^2\vartheta_1(t_k)^2$$
$$+L_0\Gamma^2(t-t_k)\int_{t_k}^{t}\left[\vartheta_1(s)^2+\vartheta_2(s)^2\right]ds$$
$$+\left(L_0\frac{\eta_0{}^2}{\nu_1{}^2}(t-t_k)^2+\frac{1}{\Delta_1}\right)\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2$$
$$+\left(L_0(t-t_k)^2+\frac{1}{4}\right)\sum_{i=1}^{n}\sigma_{i1}{}^2+\frac{1}{\Delta_1}\left\|\tilde{W}\right\|^2,$$
$$t\in[t_k,t_{k+1}),\ k\ge k_0, \tag{35}$$

where $\Gamma\ge 8(2\bar{l}\sqrt{\bar{p}_1}(1+\frac{\varpi_0}{\nu_1})+4\bar{p}_1+\frac{\Delta_1\upsilon_0{}^2\bar{p}_1}{\underline{\nu}^2}+\frac{\Delta_1\eta_0{}^2\bar{p}_1}{\underline{\nu}^2})$.

In order to deal with $\tilde{W}$ and $\tilde{\theta}_i$, the derivatives of $V_2(t)$ and $V_3(t)$ are given as follows.

$$\dot{V}_2(t)=\tilde{W}^T\Lambda_0{}^{-1}\dot{\tilde{W}}=-\tilde{W}^T\Lambda_0{}^{-1}\dot{\hat{W}}, \tag{36}$$
$$\dot{V}_3(t)=\sum_{i=1}^{n}\tilde{\theta}_i^T\Lambda_i{}^{-1}\dot{\tilde{\theta}}_i=-\sum_{i=1}^{n}\tilde{\theta}_i^T\Lambda_i{}^{-1}\dot{\hat{\theta}}_i. \tag{37}$$

Substituting (20)–(21) into (36)–(37) results in

$$\begin{aligned}
\dot{V}_2(t) + \dot{V}_3(t) = &-\tilde{W}^T \left(-S(u(t))\vartheta_1(t_k)\chi_0 + \ell_0\hat{W}\right) \\
&- \sum_{i=1}^{n} \tilde{\theta}_i^T \left(-\varphi(\hat{\hat{x}}_i(t))\vartheta_1(t_k)\chi_i + \ell_i\hat{\theta}_i\right) \\
= &-\tilde{W}^T \left(-S(u(t))\vartheta_1(t)\chi_0 + S(u(t))(\vartheta_1(t) - \vartheta_1(t_k))\chi_0\right. \\
&\left.+\ell_0\hat{W}\right) - \sum_{i=1}^{n} \tilde{\theta}_i^T \left(-\varphi(\hat{\hat{x}}_i(t))\vartheta_1(t)\chi_i + \varphi(\hat{\hat{x}}_i(t))(\vartheta_1(t)-\right. \\
&\left.\vartheta_1(t_k))\chi_i + \ell_i\hat{\theta}_i\right),
\end{aligned} \tag{38}$$

where $e_1(t_k) = \vartheta_1(t_k)$. Then, according to $\tilde{W} = W^* - \hat{W}$, $\tilde{\theta}_i = \theta_i{}^* - \hat{\theta}_i$ and Lemma 2.3, we have

$$\begin{aligned}
&2\tilde{W}^T\hat{W} + 2\tilde{\theta}_i^T\hat{\theta}_i \\
&\quad = \left\|\tilde{W}\right\|^2 + \left\|\hat{W}\right\|^2 - \|W^*\|^2 + \left\|\tilde{\theta}_i\right\|^2 + \left\|\hat{\theta}_i\right\|^2 - \|\theta_i{}^*\|^2 \\
&\quad \geq \left\|\tilde{W}\right\|^2 - \|W^*\|^2 + \left\|\tilde{\theta}_i\right\|^2 - \|\theta_i{}^*\|^2,
\end{aligned} \tag{39}$$

$$\begin{aligned}
&\tilde{W}^T S(u(t))\vartheta_1(t)\chi_0 + \sum_{i=1}^{n} \tilde{\theta}_i^T \varphi(\hat{\hat{x}}_i(t))\vartheta_1(t)\chi_i \\
&\quad \leq L_1\|\vartheta\|^2 + \frac{1}{2\Delta_2}\left\|\tilde{W}\right\|^2 + \frac{1}{2\Delta_2}\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2,
\end{aligned} \tag{40}$$

$$\begin{aligned}
&- \tilde{W}^T S(u(t))\,(\vartheta_1(t) - \vartheta_1(t_k))\,\chi_0 \\
&\quad \leq \frac{\Delta_2}{2}\upsilon_0{}^2\bar{\chi}^2(\vartheta_1(t) - \vartheta_1(t_k))^2 + \frac{1}{2\Delta_2}\left\|\tilde{W}\right\|^2,
\end{aligned} \tag{41}$$

$$\begin{aligned}
&- \sum_{i=1}^{n} \tilde{\theta}_i^T \varphi(\hat{\hat{x}}_i(t))(\vartheta_1(t) - \vartheta_1(t_k))\chi_i \\
&\quad \leq \frac{\Delta_2}{2}\eta_0{}^2\bar{\chi}^2(\vartheta_1(t) - \vartheta_1(t_k))^2 + \frac{1}{2\Delta_2}\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2,
\end{aligned} \tag{42}$$

Considering (33), (41), and (42), we have

$$\begin{aligned}
&- \tilde{W}^T S(u(t))\,(\vartheta_1(t) - \vartheta_1(t_k))\chi_0 - \sum_{i=1}^{n} \tilde{\theta}_i^T \varphi(\hat{\hat{x}}_i(t))\times \\
&\quad (\vartheta_1(t) - \vartheta_1(t_k))\chi_i \\
&\quad \leq 6L_1\Gamma^2\bar{k}(t - t_k)^2\vartheta_1(t_k)^2 + 6L_1(t - t_k)^2\sum_{i=1}^{n}\sigma_{i1}{}^2 \\
&\qquad + 6L_1\Gamma^2(t - t_k)\int_{t_k}^{t}\left[\vartheta_1(s)^2 + \vartheta_2(s)^2\right]ds + \frac{1}{2\Delta_2}\left\|\tilde{W}\right\|^2
\end{aligned}$$

$$+\left(6L_1\frac{{\eta_0}^2}{{\nu_1}^2}(t-t_k)^2+\frac{1}{2\Delta_2}\right)\sum_{i=1}^{n}\left\|\tilde{\theta}_i\right\|^2,$$
$$t\in[t_k,t_{k+1}),k\geq k_0. \tag{43}$$

Based on (38)–(43), we have

$$\begin{aligned}
&\dot{V}_2(t)+\dot{V}_3(t)\\
&\quad\leq\frac{1}{8}\Gamma\|\vartheta\|^2+6L_1\Gamma^2\bar{k}(t-t_k)^2\vartheta_1(t_k)^2+6L_1\Gamma^2(t-t_k)\\
&\qquad\times\int_{t_k}^{t}\left[\vartheta_1(s)^2+\vartheta_2(s)^2\right]ds-\sum_{i=1}^{n}\left(\frac{\ell_i}{2}-\frac{1}{\Delta_2}-6L_1\frac{{\eta_0}^2}{{\nu_1}^2}\right.\\
&\qquad\left.\times(t-t_k)^2\right)\left\|\tilde{\theta}_i\right\|^2-\left(\frac{\ell_0}{2}-\frac{1}{\Delta_2}\right)\left\|\tilde{W}\right\|^2+6L_1(t-t_k)^2\times\\
&\sum_{i=1}^{n}{\sigma_{i1}}^2+\frac{\ell_0}{2}\|W^*\|^2+\sum_{i=1}^{n}\frac{\ell_i}{2}\|{\theta_i}^*\|^2,\\
&t\in[t_k,t_{k+1}),k\geq k_0,
\end{aligned} \tag{44}$$

where $\Gamma\geq 8L_1$.

Note that when $t\in[t_k,t_{k+1})$, we have $t-T<t_k$. Thus, from (35) and (44), it follows that

$$\begin{aligned}
&\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)\\
&\quad\leq-\frac{1}{2}\Gamma\|\vartheta\|^2+(L_0+6L_1)\,\bar{k}\Gamma^2T^2\vartheta_1(t_k)^2\\
&\qquad+(L_0+6L_1)\Gamma^2T\int_{t-T}^{t}[\vartheta_1(s)^2+\vartheta_2(s)^2]ds\\
&\qquad-\sum_{i=1}^{n}\left(\frac{\ell_i}{2}-\frac{1}{\Delta_1}-\frac{1}{\Delta_2}-(L_0+6L_1)\frac{{\eta_0}^2}{{\nu_1}^2}T^2\right)\left\|\tilde{\theta}_i\right\|^2\\
&\qquad-\left(\frac{\ell_j}{2}-\frac{1}{\Delta_1}-\frac{1}{\Delta_2}\right)\left\|\tilde{W}\right\|^2+\left((L_0+6L_1)T^2+\frac{1}{4}\right)\sum_{i=1}^{n}{\sigma_{i1}}^2\\
&\qquad+\frac{\ell_0}{2}\|W^*\|^2+\sum_{i=1}^{n}\frac{\ell_i}{2}\|{\theta_i}^*\|^2,\ t\in[t_k,t_{k+1}),k\geq k_0.
\end{aligned} \tag{45}$$

Further, the derivative of $V_4(t)$ is given by

$$\begin{aligned}
\dot{V}_4(t)&=T(\vartheta_1(t)^2+\vartheta_2(t)^2)-\int_{t-T}^{t}(\vartheta_1(s)^2+\vartheta_2(s)^2)ds,\\
&\quad t\in[t_k,t_{k+1}),k\geq k_0.
\end{aligned} \tag{46}$$

Next, for $n\geq 2$, we have

$$\begin{aligned}
\dot{V}_4(t)\leq T\|\vartheta\|^2-\int_{t-T}^{t}\left[\vartheta_1(s)^2+\vartheta_2(s)^2\right]ds,\\
t\in[t_k,t_{k+1}),k\geq k_0.
\end{aligned} \tag{47}$$

Substituting (45) and (47) into (25), we have

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \Gamma^2 \dot{V}_4(t) \\ &\le -(\frac{1}{2} - T\Gamma)\Gamma\vartheta^T(t)\vartheta(t) + (L_0 + 6L_1)\bar{k}\Gamma^2 T^2 \vartheta_1(t_k)^2 \\ &\quad + \left((L_0 + 6L_1)\Gamma^2 T - \Gamma^2\right)\int_{t-T}^{t} [\vartheta_1(s)^2 + \vartheta_2(s)^2]ds \\ &\quad - \sum_{i=1}^{n}\left(\frac{\ell_i}{2} - \frac{1}{\Delta_1} - \frac{1}{\Delta_2} - (L_0 + 6L_1)\frac{\eta_0{}^2}{\nu_1{}^2}T^2\right)\left\|\tilde{\theta}_i\right\|^2 \\ &\quad - \left(\frac{\ell_0}{2} - \frac{1}{\Delta_1} - \frac{1}{\Delta_2}\right)\left\|\tilde{W}\right\|^2 + \left((L_0 + 6L_1)T^2 + \frac{1}{4}\right)\sum_{i=1}^{n}\sigma_{i1}{}^2 \\ &\quad + \frac{\ell_0}{2}\|W^*\|^2 + \sum_{i=1}^{n}\frac{\ell_i}{2}\|\theta_i{}^*\|^2, t \in [t_k, t_{k+1}), k \ge k_0, \end{aligned} \tag{48}$$

Note that

$$V_4(t) \le T\int_{t-T}^{t} (\vartheta_1(s)^2 + \vartheta_2(s)^2)ds, t \in [t_k, t_{k+1}), k \ge k_0. \tag{49}$$

Then, from (48) and (49), we can obtain

$$\begin{aligned}\dot{V}(t) &\le -\frac{1 - 2T\Gamma}{2\bar{p}_2}V_1(t) - \frac{2\left(\frac{\ell_0}{2} - \frac{1}{\Delta_1} - \frac{1}{\Delta_2}\right)}{\lambda_{\max}(\Lambda_0{}^{-1})}V_2(t) \\ &\quad - \frac{2\left(\frac{\ell_i}{2} - \frac{1}{\Delta_1} - \frac{1}{\Delta_2} - (L_0 + 6L_1)\frac{\eta_0{}^2}{\nu_1{}^2}T^2\right)}{\lambda_{\max}(\Lambda_i{}^{-1})}V_3(t) \\ &\quad - \frac{(1 - (L_0 + 6L_1)T)}{T}\Gamma^2 V_4(t) + \frac{(L_0 + 6L_1)\bar{k}\Gamma^2 T^2}{\bar{p}_3}V_1(t_k) \\ &\quad + \left((L_0 + 6L_1)T^2 + \frac{1}{4}\right)\sum_{i=1}^{n}\sigma_{i1}{}^2 + \frac{\ell_0}{2}\|W^*\|^2 + \sum_{i=1}^{n}\frac{\ell_i}{2}\|\theta_i{}^*\|^2, \\ &t \in [t_k, t_{k+1}), k \ge k_0. \end{aligned} \tag{50}$$

Since the sampling period T, and the parameters ϕ, Δ_1, Δ_2, ℓ_0, ℓ_i satisfy the conditions (22)–(24), then,

$$\frac{d}{dt}V(t) \le -\phi V(t) + \frac{\phi}{2}V(t_k) + C_1, t \in [t_k, t_{k+1}), k \ge k_0, \tag{51}$$

where $C_1 = \frac{1}{2}\sum_{i=1}^{n}\sigma_{i1}{}^2 + \frac{\ell_0}{2}\|W^*\|^2 + \sum_{i=1}^{n}\frac{\ell_i}{2}\|\theta_i{}^*\|^2$. In order to ensure the error system is UUB, the corresponding high-gain design parameter Γ should be chosen such that

$$\begin{aligned}\Gamma \ge \max\Bigg\{&1, \sqrt{\left(1 + \frac{\varpi_0{}^2}{\nu_1{}^2}\right)l_1{}^2}, 8L_1, \\ &8\left(2\bar{l}\sqrt{\bar{p}_1}(1 + \frac{\varpi_0}{\nu_1}) + 4\bar{p}_1 + \frac{\Delta_1 \upsilon_0{}^2\bar{p}_1}{\underline{\nu}^2} + \frac{\Delta_1\eta_0{}^2\bar{p}_1}{\underline{\nu}^2}\right)\Bigg\}.\end{aligned}$$

Since $\phi_1 = (1 - \frac{1}{2})e^{-\phi T} + \frac{1}{2} < 1$. From the differential inequality (51) and Lemma 2.4, we have

$$V(t) \le {\phi_1}^k V(t_0) + \frac{2 - \phi_1 - {\phi_1}^k}{1 - \phi_1}\frac{C_1}{\phi}.$$

Thus, we obtain that the system (18) is UUB, i.e., $\lim_{t\to\infty} V(t) \le \frac{(2-\phi_1)C_1}{(1-\phi_1)\phi}$. On the one hand, $\lim\limits_{t\to\infty} V_1(t) \le \lim\limits_{t\to\infty} V(t) \le \frac{(2-\phi_1)C_1}{(1-\phi_1)\phi}$, on the other hand, $V_1(t) \ge \bar{p}_3 \vartheta^T \vartheta \ge \frac{\bar{p}_3 e^T e}{\Gamma^{2(n-1)}}$. Thus, we have

$$\lim_{t\to\infty} e^T e \le \frac{(2 - \phi_1)C_1\Gamma^{2(n-1)}}{(1 - \phi_1)\phi \bar{p}_3},$$

This completes the proof. □

Remark 2 The design method can be extended to nonlinear systems with other hysteresis inputs and may not necessarily limited to the system described by (1). On the one hand, C_1 in (3) is determined by the parameters σ_{i1}, ℓ_0, ℓ_i. Note that the value of the parameters σ_{i1}, ℓ_0, ℓ_i can be adjusted. Thus, a small value of C_1 can be guaranteed. On the other hand, we can properly select the design parameters Γ, k_i, l_{i1}, l_{i2}, υ_0, η_i, ν_0, ν_i, ℓ_0, ℓ_i, χ_0, χ_i, Δ_1 and Δ_2. Then, based on these parameters, the sampling period T and ϕ can be found such that the error system converges to a relatively small neighborhood of the origin.

Remark 3 In [7], the estimation state $\hat{x}(t)$ is introduced into the RBFNNs to approximate the hysteresis and the uncertainties. Therefore, the considered nonlinear system not only has the unknown state $x(t)$ but also the estimation state $\hat{x}(t)$. Whereas, in this paper, we only use the system state $x(t)$ to approximate the hysteresis and the disturbances. Thus, the considered nonlinear system (12) only has $x(t)$ but not $\hat{x}(t)$. Moreover, compared with [7], we relaxed the restriction on the constant control gain parameter b by using the approximation formulation (10), and solved the problem of parameter selection by introducing the high gain parameter Γ.

4 Simulation Example

In this section, a simulation example will be demonstrated the effectiveness of the proposed scheme.

Example 1 Consider the nonlinear system with unknown hysteresis and unknown unmatched disturbance

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = b\omega(u(t)) + f(\bar{x}(t)) + d(\bar{x}(t), t), \\ y(t) = x_1(t_k), t \in [t_k, t_{k+1}), k \ge 0, \end{cases}$$

where $f(\bar{x}(t)) = -3\sin(x_1(t))$ and $d(\bar{x}(t), t) = 0.1\sin(x_1)e^{-0.1x_2}$. The PI hysteresis out $\omega(u(t))$ is determined by (2) and the density function is chosen as $p(r) = 0.8e^{-0.067(r-1)^2}$. We choose R=100 as the upper limit of integration.

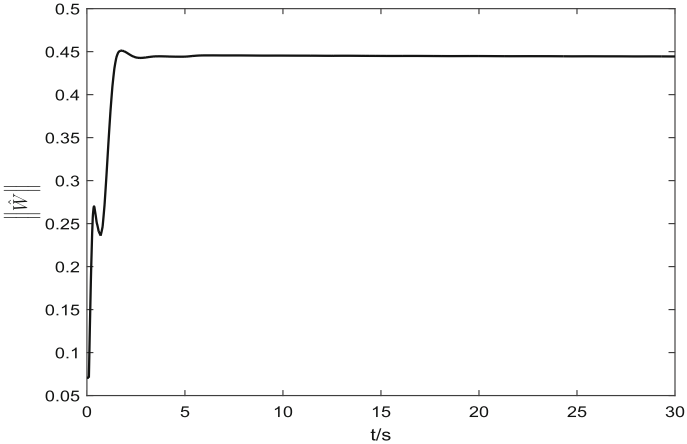

Fig. 1 Trajectory of $\left\|\hat{W}\right\|$

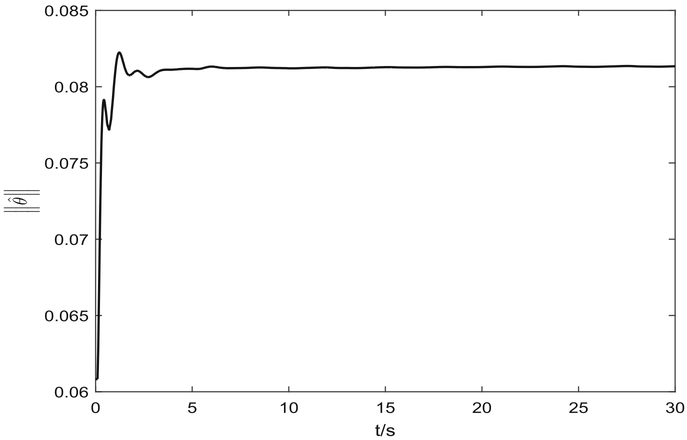

Fig. 2 Trajectory of $\left\|\hat{\theta}\right\|$

Considering (15), the adaptive sampled-data observer is constructed as

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + \Gamma k_1 e_1(t_k), \\ \dot{\hat{x}}_2(t) = b_0 u(t) - 3\sin(\hat{x}_1) + \Gamma^2 k_2 e_1(t_k) + \frac{1}{\nu_0}\hat{W}^T S(u(t)) \\ \qquad\qquad + \frac{1}{\nu_1}\hat{\theta}^T \varphi(\hat{\bar{x}}(t)), \\ \hat{y}(t) = \hat{x}_1(t_k), t \in [t_k, t_{k+1}), k \geq 0, \end{cases}$$

and

$$\begin{cases} S(u(t)) = [\frac{4}{1+e^{-u(t)}} - 2.5, \frac{5}{1+e^{-u(t)}} - 3]^T \\ \varphi(\hat{\bar{x}}(t)) = \exp\left[-\frac{(\hat{x}_1-6+2l)^2}{2}\right] \times \exp\left[-\frac{(\hat{x}_2-3+l)^2}{4}\right], l = 1, \ldots, 5. \end{cases}$$

where $e_1(t_k) = x_1(t_k) - \hat{x}_1(t_k)$, and the update laws of the weights are given by (20) and (21). In the following simulation, we choose $\Gamma = 2$, $k_1 = k_2 = 1.5$, $u(t) =$

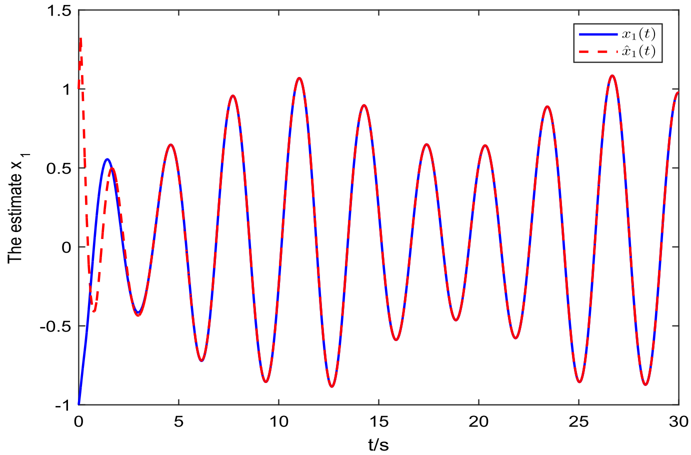

Fig. 3 Estimation of x_1 of the nonlinear system

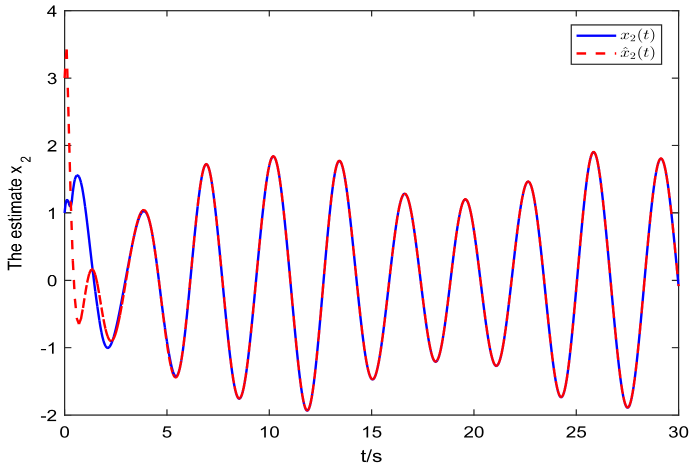

Fig. 4 Estimation of x_2 of the nonlinear system

$(12\cos(3t) - 4)/(1 + 6t) + \cos(2t)$, $b_0 = 1$, $\nu_0 = 0.8$, $\nu_1 = 1.5$, $\Lambda_0^1 = \Lambda_0^2 = \Lambda_l = 0.004$, $\ell_0^1 = \ell_0^2 = \ell_l = 0.05$, $\chi_0^1 = 60$, $\chi_0^2 = 55$, $\chi_l = (0.5, 40, 0.5, 60, 0.5)$ and $\Delta_1 = \Delta_2 = 100$. The initial conditions $(x_1(0), x_2(0)) = (-1, 1)$, $(\hat{x}_1(0), \hat{x}_2(0)) = (1, 3)$, $(\hat{W}_0^1(0), \hat{W}_0^2(0)) = (0.05, 0.05)$ and $\hat{\theta}_l(0) = (0, 0.01, 0, -0.05, 0)$. By simple computation, we have $P = [0.8482, -0.5093; -0.5093, 1.0689]$, $\lambda_{\max}(P) = 1.4797$ and $\lambda_{\min}(P) = 0.4374$. The sampling period T is given as $T = 0.1s$. Figures 1 and 2 illustrate the trajectories of $\left\|\hat{W}\right\|$ and $\left\|\hat{\theta}\right\|$, respectively. In Figs. 3 and 4, the state estimation results of two unmeasurable states are presented, respectively. The trajectories of the state estimate errors and Lyapunov function $V(t)$ are depicted in Figs. 5 and 6, respectively.

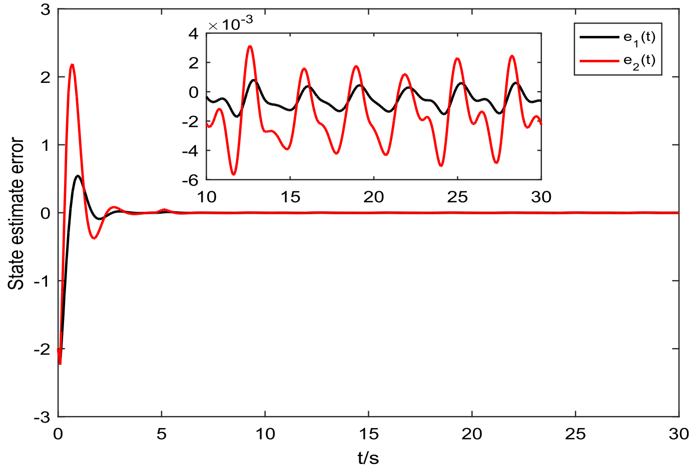

Fig. 5 Trajectories of the errors $e_1(t)$ and $e_2(t)$

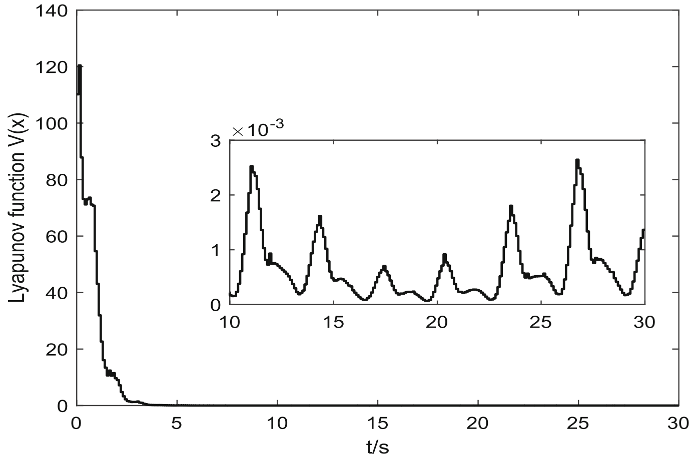

Fig. 6 Trajectory of the Lyapunov function $V(t)$

5 Conclusion

In this paper, a novel adaptive sampled-data observer design based on RBFNNs was proposed for nonlinear systems with unknown PI hysteresis and unknown unmatched disturbances. Firstly, RBFNNs were designed to approximate the unknown time-varying unmatched disturbances and unknown hysteresis of the systems. Then, a sampled-data observer was constructed to estimate the unmeasured states, and the learning laws of the weights of RBFNNs were also given. Based on a Lyapunov function and the corresponding sufficient conditions, we demonstrated that the observer errors were UUB. Finally, the effectiveness of the design scheme was verified by the illustrative simulation case. In the future, the developed adaptive sampled-data observer design method will be extended to the MIMO nonlinear systems with hysteresis and multiple uncertainties.

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