

Pinning Synchronization of Coupled Memristive Recurrent Neural Networks with Mixed Time-Varying Delays and Perturbations

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Abstract In this paper, the pinning synchronization of coupled memristive recurrent neural networks (MNNs) with mixed time-varying delays and perturbations is investigated. Precisely, the considered coupled MNNs include the non-delay, discrete time-varying delays, distributed time delays, impulsive perturbations and stochastic perturbations. Comparing with the existing results, the new and simple feedback controller and adaptive feedback controller are designed to achieve exponential synchronization with pinning schemes. Based on the suitable Lyapunov functional and the definition of pinning control, with the aid of inequality techniques and differential inclusions theory, some effective and novel sufficient conditions are obtained to guarantee the synchronization of our proposed model. Finally, numerical examples are given to illustrate the effectiveness and reasonable of our theoretical results.

Keywords Memristive recurrent neural networks · Pinning control · Mixed time-varying delays · Impulsive perturbations · Stochastic perturbations

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1 Introduction

Memristor as an ideal electronic circuit was predicted by Chua [1]. In 2008, HP researchers had produced the memristor prototype for electronic circuits in their research contributions [2]. Since then, on account of the memristor possesses performances and memory more like biological synapses than the resistor [3], it has been substituted for resistor to construct brain-like computer memory [4–8].

Recently, researchers present that we can obtain a new class of neural networks (NNs) called memristive neural networks (MNNs) as long as replace the resistor to the memristor. Synthesizes each kind of memoristor [9, 10], MNNs will be more rational in the field of emulating the human brain than the traditional NNs. Thus, it will be more meaningful to study the various dynamical behavior of MNNs.

It is well known, synchronization has been an active topic in the area of nonlinear science since Carroll and Pecora introduced chaos synchronization [11]. Recently, many researchers paid their close attention to the study of synchronization of MNNs. Up to now, there are many kinds of synchronization of MNNs such as lag synchronization [12, 13], anti-synchronization [14–17], finite time synchronization [18, 19], exponential synchronization [20–25], and so on. In [12], authors dealt with the exponential lag synchronization of a class of switched NNs with time-varying delays via neural activation function and it can be applied to the image encryption. Zhang et al. [20] investigated synchronization of an array of linearly coupled MNNs with impulses and time-varying delays. In [24], adaptive synchronization of a class of MNNs with time-varying delays was studied by designing a general adaptive controller.

However, the above mentioned synchronization demonstrate that the trajectories of the slave system can catch the trajectories of the master system via all nodes be controlled. In practice, it is extremely important that a small fraction of nodes can realize all nodes synchronized according to the coupling configuration. For the past few years, many researchers have shown their more interest to the investigation of pinning synchronization of MNNs and many better results have been exhibited in the literatures [26, 27]. Wang et al. [26] studied a class of coupled MNNs of neutral type with mixed time varying delays via randomly occurring control in order to achieve anti-synchronization. In [27], authors presented the theoretical results on the master-slave synchronization of two MNNs in the presence adaptive noise. From the above discussions, it is necessary and significant to develop some practical and economical systems for coupled MNNs.

As we all known, during the process of electronic implementation of NNs, distribution between neurons simulated by hardware, spatial and temporal characteristics of signal transmission, various time-varying delays inevitably appear in signal communication [28–30]. Thus, various delays are one of important factors that result in oscillation or instability of the MNNs [31–34]. So the synchronization problem of coupled NNs with time-delays has received much attention [35–37]. Nevertheless, in these results, the type of time delay is relatively simple. In [35], authors studied the problem of synchronization control for directed networks with node balance. In [36], authors investigated the lag synchronization between two coupled NNs via pinning control. In [37], according to the state-dependent Riccati equation (SDRE) technique, authors proposed a suboptimal pinning control scheme in order to synchronize linearly coupled complex networks. To the best of our knowledge, in these exciting papers, there are few researches with regard to the pinning synchronization problem of coupled MNNs with complicated time-varying delays. Thus, it is more applicable to consider the exponential synchronization to the MNNs with mixed time-varying delays via pinning schemes.

Another critical element is perturbation. Impulsive phenomena widely exists in our real world system, a variety of random uncertainties (such as switching phenomenons, frequency changes, etc.) make the instantaneous perturbations on the state of NNs, which lead to the state instantaneous jump in a moment [38–43]. In addition, the stochastic perturbations also should be taken into consideration. The actual communication between subsystems of coupled MNNs is inevitably disturbed by the stochastic perturbations from various uncertainties, which probably causes package loss or influences the signal transmission [44–46]. It should be mentioned, the pinning synchronization results for coupled MNNs with mixed time-varying delays and two kinds of perturbations has not been studied yet, this motivates our present study. The main contributions of this paper are summarized as follows:

- (1) We focus on the study of coupled MNNs models with stochastic perturbations, impulsive perturbations and various time-varying delays, which including non-delayed, discrete time-varying delays and distributed time delays. Many other coupled MNNs models with delays are the special cases of our considered model.
- (2) We first attempt to address the pinning synchronization control problem for a class of our proposed MNNs models. By employing Lyapunov functional and the definition of pinning schemes, exponential synchronization of the considered coupled MNNs is achieved via pinning control, which including the linear and adaptive feedback pinning schemes. We consider and analysis the complex characters of the mixed time-varying delays rather than treat them as constants. Some main results are derived by utilizing the stochastic analysis theory, inequality techniques and differential inclusions theory.
- (3) Finally, we provide the numerical examples to illustrated the effectiveness and rationality of the proposed conclusions.

The rest of this paper is organized as follows. Some definitions, lemmas and assumptions about the proposed model are presented in Sect. 2. In Sect. 3 derives some sufficient conditions of pinning synchronization based our considered coupled MNNs. Numerical simulations are demonstrated to verify the effectiveness of the obtained results in Sect. 4. Finally, the conclusion is given in Sect. 5.

2 Preliminaries

2.1 Model Description

Based on the physical properties of memristor, the mathematical model of coupled MNNs with mixed time-varying delays is introduced as follows

$$\begin{aligned}
 \frac{dx_{iq}(t)}{dt} = & -d_q x_{iq}(t) + \sum_{l=1}^n a_{ql}(x_{iq}(t)) f_l(x_{il}(t)) \\
 & + \sum_{l=1}^n c_{ql}(x_{iq}(t)) \int_{-\infty}^t k_{ql}(\theta) f_l(x_{il}(t - \theta)) d\theta \\
 & + \sum_{l=1}^n b_{ql}(x_{iq}(t)) f_l(x_{il}(t - \tau_{ql}(t))) + \alpha \sum_{j=1}^N \omega_{ij} \Gamma x_{jq}(t) + I_i(t), \quad (1)
 \end{aligned}$$

where $x_i(t) = [x_{i1}(t), \dots, x_{in}(t)]^T$ is the state variable of the i th neural at time t for $q, l = 1, 2, \dots, n$. $W = (\omega_{ij})_{N \times N}$ ($i, j = 1, 2, \dots, N$) represents the coupling matrix of MNNs.

If there is an edge from MNNs j to i , then $\omega_{ij} = 1$, otherwise, $\omega_{ij} = 0(i \neq j)$. And $\omega_{ii} = -\sum_{j=1, j \neq i}^N \omega_{ij}$, α represents the coupling strength.

The positive definite diagonal matrix Γ stands for the inner coupling between two connected MNNs. $a_{ql}(\cdot)$, $b_{ql}(\cdot)$ and $c_{ql}(\cdot)$ denote the inner connection matrix of non-delayed, discrete time-varying delayed and distributively time-delayed, respectively. They can be described by the following functions

$$\begin{aligned}
 a_{ql}(x_{iq}(t)) &= \begin{cases} \hat{a}_{ql}, & |x_{iq}(t)| \leq \Phi_i, \\ \check{a}_{ql}, & |x_{iq}(t)| > \Phi_i, \end{cases} \\
 b_{ql}(x_{iq}(t)) &= \begin{cases} \hat{b}_{ql}, & |x_{iq}(t)| \leq \Phi_i, \\ \check{b}_{ql}, & |x_{iq}(t)| > \Phi_i, \end{cases} \\
 c_{ql}(x_{iq}(t)) &= \begin{cases} \hat{c}_{ql}, & |x_{iq}(t)| \leq \Phi_i, \\ \check{c}_{ql}, & |x_{iq}(t)| > \Phi_i, \end{cases}
 \end{aligned}$$

where the switching jump $\Phi_i > 0$, for $i = 1, 2, \dots, N$. Then $\hat{a}_{ql}, \check{a}_{ql}, \hat{b}_{ql}, \check{b}_{ql}, \hat{c}_{ql}, \check{c}_{ql}$ are known constants relating to memristances.

Remark 1 According to the discussions above, the inner connection matrixes $a_{ql}(x_{iq}(t))$, $b_{ql}(x_{iq}(t))$ and $c_{ql}(x_{iq}(t))$ of system (1) with the change of the memristance. Therefore, the coupled MNNs are considered as the time-varying systems with state-dependent switching. When $a_{ql}(x_{iq}(t))$, $b_{ql}(x_{iq}(t))$ and $c_{ql}(x_{iq}(t))$ are all constants, system (1) becomes a general class of recurrent coupled NNs.

The delay kernel $k_{ql}(\theta) : [0, +\infty) \rightarrow [0, +\infty)$ is bounded, piecewise and satisfies $\int_0^{+\infty} k_{ql}(\theta)e^{\mu\theta} d\theta = 1$, $\int_0^{+\infty} k_{ql}(\theta)e^{\mu\theta} d\theta < +\infty$, where μ is known constant [51]. Self-feedback connection matrix $D = \text{diag}(d_1, d_2, \dots, d_n)$ is a positive definite matrix. $f_i(x_{il}(t))$ is a bounded feedback function without time delay. In addition, $f_i(x_{il}(t - \tau_{ql}(t)))$ and $\int_{-\infty}^t k_{ql}(\theta)f_i(x_{il}(t - \theta))d\theta$ are bounded feedback functions with discrete and distributed time delays.

According to the solution of Fillppov’s and the theory of differential inclusion to this system. Let $\bar{a}_{ql} = \max\{\hat{a}_{ql}, \check{a}_{ql}\}$, $\underline{a}_{ql} = \min\{\hat{a}_{ql}, \check{a}_{ql}\}$, $\bar{b}_{ql} = \max\{\hat{b}_{ql}, \check{b}_{ql}\}$, $\underline{b}_{ql} = \min\{\hat{b}_{ql}, \check{b}_{ql}\}$, $\bar{c}_{ql} = \max\{\hat{c}_{ql}, \check{c}_{ql}\}$, $\underline{c}_{ql} = \min\{\hat{c}_{ql}, \check{c}_{ql}\}$. $co[u, v]$ indicates closure of the convex hull generated by real numbers u and v . In view of system (1), we define the following set-valued maps

$$\begin{aligned}
 co(a_{ql}(x_{iq}(t))) &= \begin{cases} \hat{a}_{ql}, & |x_{iq}(t)| < \Phi_i, \\ co\{\hat{a}_{ql}, \check{a}_{ql}\}, & |x_{iq}(t)| = \Phi_i, \\ \check{a}_{ql}, & |x_{iq}(t)| > \Phi_i, \end{cases} \\
 co(b_{ql}(x_{iq}(t))) &= \begin{cases} \hat{b}_{ql}, & |x_{iq}(t)| < \Phi_i, \\ co\{\hat{b}_{ql}, \check{b}_{ql}\}, & |x_{iq}(t)| = \Phi_i, \\ \check{b}_{ql}, & |x_{iq}(t)| > \Phi_i, \end{cases} \\
 co(c_{ql}(x_{iq}(t))) &= \begin{cases} \hat{c}_{ql}, & |x_{iq}(t)| < \Phi_i, \\ co\{\hat{c}_{ql}, \check{c}_{ql}\}, & |x_{iq}(t)| = \Phi_i, \\ \check{c}_{ql}, & |x_{iq}(t)| > \Phi_i. \end{cases}
 \end{aligned}$$

Clearly, $co\{\hat{a}_{ql}, \check{a}_{ql}\} = [a_{ql}, \bar{a}_{ql}]$, $co\{\hat{b}_{ql}, \check{b}_{ql}\} = [b_{ql}, \bar{b}_{ql}]$ and $co\{\hat{c}_{ql}, \check{c}_{ql}\} = [c_{ql}, \bar{c}_{ql}]$, for $i, j = 1, 2, \dots, N$, $q, l = 1, 2, \dots, n$. By the theory of differential inclusions, the system (1) can be written as follows

$$\begin{aligned} \frac{dx_{iq}(t)}{dt} \in & -d_q x_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t))) f_l(x_{il}(t)) \\ & + \sum_{l=1}^n co(b_{ql}(x_{iq}(t))) f_l(x_{il}(t - \tau_{ql}(t))) \\ & + \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta) f_l(x_{il}(t - \theta)) d\theta \\ & + \alpha \sum_{j=1}^N \omega_{ij} \Gamma x_{jq}(t) + I_i(t), \end{aligned} \tag{2}$$

or equivalently, for $i, j = 1, 2, \dots, N, q, l = 1, 2, \dots, n$, there exist $\check{a}_{ql}(x_{iq}(t)) \in co(a_{ql}(x_{iq}(t)))$, $\check{b}_{ql}(x_{iq}(t)) \in co(b_{ql}(x_{iq}(t)))$, $\check{c}_{ql}(x_{iq}(t)) \in co(c_{ql}(x_{iq}(t)))$, by utilizing the the theories of set-valued maps and differential inclusions above, the system (2) can be regarded as a state-dependent switching system shown by

$$\begin{aligned} \frac{dx_{iq}(t)}{dt} = & -d_q x_{iq}(t) + \sum_{l=1}^n \check{a}_{ql}(x_{iq}(t)) f_l(x_{il}(t)) \\ & + \sum_{l=1}^n \check{b}_{ql}(x_{iq}(t)) f_l(x_{il}(t - \tau_{ql}(t))) + \alpha \sum_{j=1}^N \omega_{ij} \Gamma x_{jq}(t) \\ & + \sum_{l=1}^n \check{c}_{ql}(x_{iq}(t)) \int_0^{+\infty} k_{ql}(\theta) f_l(x_{il}(t - \theta)) d\theta + I_i(t). \end{aligned} \tag{3}$$

Thus, the coupled MNNs with mixed time-varying delays and impulsive perturbations can be written as follows

$$\begin{aligned} \frac{dx_{iq}(t)}{dt} \in & -d_q x_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t))) f_l(x_{il}(t)) \\ & + \sum_{l=1}^n co(b_{ql}(x_{iq}(t))) f_l(x_{il}(t - \tau_{ql}(t))) \\ & + \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta) f_l(x_{il}(t - \theta)) d\theta \\ & + \alpha \sum_{j=1}^N \omega_{ij} \Gamma x_{jq}(t) + I_i(t), \quad t \neq t_k, \\ \Delta x_{iq}(t_k) = & x_{iq}(t_k^+) - x_{iq}(t_k^-) = -r_{ik} x_{iq}(t_k), \quad t = t_k, \\ x_{iq}(t) = & \phi_{iq}(t), \quad t \in (-\infty, 0], \end{aligned} \tag{4}$$

where initial values $x(\theta) = \phi(\theta)$, $\phi(\theta) \in C([-\tau, 0], R^n)$ for $i, j = 1, 2, \dots, N, q, l = 1, 2, \dots, n, k = 1, 2, \dots$. $\phi_{iq}(t)$ is the initial value of $x_{iq}(t)$, r_{ik} is the impulsive gain

constant. Actually, the radio of state variable $\dot{x}_{iq}(t)$ at $t = t_k$ is r_{ik} , so we choose $0 < r_{ik} < 1$. $x_{iq}(t_k^-) = \lim_{t \rightarrow t_k^-} x_{iq}(t_k) = x_{iq}(t_k)$, $x_{iq}(t_k^+) = \lim_{t \rightarrow t_k^+} x_{iq}(t_k)$.

Similarly, the coupled MNNs with mixed time-varying delays and stochastic perturbations can be written as follows

$$\begin{aligned}
 dx_{iq}(t) \in & \left\{ -d_q x_{iq}(t) + \sum_{l=1}^n c_o(a_{ql}(x_{iq}(t))) f_l(x_{il}(t)) \right. \\
 & + \sum_{l=1}^n c_o(b_{ql}(x_{iq}(t))) f_l(x_{il}(t - \tau_{ql}(t))) \\
 & + \sum_{l=1}^n c_o(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta) f_l(x_{il}(t - \theta)) d\theta \\
 & \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma x_{jq}(t) + I_i(t) \right\} dt + \sigma_i(t, x_{il}(t), x_{il}(t - \tau_{ql}(t))) d\omega_i(t). \tag{5}
 \end{aligned}$$

Remark 2 The solution $s_{iq}(t)$ of an isolated node satisfies [25]:

$$\begin{aligned}
 \dot{s}_{iq}(t) &= f(s_{il}(t), t), \\
 \dot{s}_{iq}(t - \tau_{ql}(t)) &= f(s_{il}(t - \tau_{ql}(t)), t - \tau_{ql}(t)), \\
 \dot{s}_{iq}(t - \theta) &= f(s_{il}(t - \theta), t - \theta), \tag{6}
 \end{aligned}$$

where $s_{iq}(t)$ may be an equilibrium point or an orbit of a chaotic attractor. This paper aims to find some appropriate systems such that the solutions of networks (4) and (5) synchronize with the solution of system (6), in the sense that for $i = 1, \dots, N, q = 1, \dots, n$.

Definition 1 (see [46]) Suppose $E \subset \mathbb{R}^n$. Then $x \mapsto F(x)$ is called as a *set-valued* map defined on E , if for each point x of E , there exists a corresponding nonempty set $F(x) \subset \mathbb{R}^n$. A set-valued map F with nonempty values is said to be *upper-semicontinuous* at $x_0 \in E$, if for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$. Then $F(x)$ is said to have a *closed image* if for each $x \in E$, $F(x)$ is closed.

Definition 2 (see [9]) The equilibrium point x^* or an orbit of a chaotic attractor of system (5), which is said to be globally exponentially stable, for any $t \geq 0$ and initial values $x(\theta) = \phi(\theta)$, $\phi(\theta) \in C([- \tau, 0], \mathbb{R}^n)$. Such that $\|x(t; \phi) - x^*\| \leq \beta \| \phi - x^* \| e^{-\alpha t}$, where constants $\alpha > 0$ and $\beta > 0$ represent the decay coefficient and decay rate.

Assumption 1 The activation function $f_l(\cdot)$ is globally Lipschitz continuous in \mathbb{R} , i.e there exist constant $z_l > 0$ for $x, y \in \mathbb{R}$, such that

$$|f_l(x) - f_l(y)| \leq z_l |x - y|, \quad l = 1, 2, \dots, n.$$

Assumption 2 The time-varying delay $\tau_{ql}(t)$ in this paper is a differential function, where $0 < \tau_{ql}(t) < \tau_{ql}$, for all $t \geq 0$, and $q, l \in 1, 2, \dots, n$.

Assumption 3 The activation function $f_l(\cdot)$ is bounded, i.e. there exists a constant $m_l > 0$, such that $|f_l(x)| \leq m_l, \quad \forall x \in \mathbb{R}, l = 1, 2, \dots, n$.

Lemma 1 *There exist constants $R_1 \geq 0$ and $R_2 \geq 0$, such that*

$$\begin{aligned}
 \text{Trace}[\sigma^T(t, x(t), x(t - \xi(t))) \sigma(t, x(t), x(t - \xi(t)))] &\leq x^T(t) R_1 x(t) \\
 + x^T(t - \xi(t)) R_2 x(t - \xi(t)).
 \end{aligned}$$

For the stochastic system [47]:

$$dy(t) = g(t, y(t))dt + \sigma(t, y(t))d\omega(t), \tag{7}$$

where $\omega(t)$ is the Brownian motion and it is truly $\mathbb{E}\omega(t) = 0$. \mathcal{L} is the operator designed as following:

$$\begin{aligned} \mathcal{L}V(t, y) &= V_t(t, y) + V_y g(t, y) \\ &\quad + \frac{1}{2} \text{Trace}[\sigma^T(t, y(t))V_{yy}\sigma(t, y(t))], \end{aligned} \tag{8}$$

where

$$\begin{aligned} V_t(t, y) &= \frac{\partial V_t(t, y)}{\partial t}, \\ V_{yy}(t, y) &= \left(\frac{\partial^2 V_t(t, y)}{\partial y_i \partial y_j} \right) \\ V_y(t, y) &= \left(\frac{\partial V_t(t, y)}{\partial y_1}, \frac{\partial V_t(t, y)}{\partial y_2}, \dots, \frac{\partial V_t(t, y)}{\partial y_n} \right)^T. \end{aligned}$$

Let $e_{iq}(t) = x_{iq}(t) - s_{iq}(t)$ denotes the error variable. From the Definitions 1–2, theory of set-valued maps and Assumption 1, thus the error dynamics of the systems (4) and (5) can be expressed as follows

$$\begin{aligned} \frac{de_{iq}(t)}{dt} &\in -d_q e_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t)))F_l(e_{il}(t)) \\ &\quad + \sum_{l=1}^n co(b_{ql}(x_{iq}(t)))F_l(e_{il}(t - \tau_{ql}(t))) \\ &\quad + \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta)F_l(e_{il}(t - \theta))d\theta \\ &\quad + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) + U_{iq}(t), \quad t \neq t_k, \\ \Delta e_{iq}(t_k) &= e_{iq}(t_k^+) - e_{iq}(t_k^-) = -r_{ik} e_{iq}(t_k), \quad t = t_k, \\ e_{iq}(t) &= \psi_{iq}(t) \quad t \in (-\infty, 0], \end{aligned} \tag{9}$$

and

$$\begin{aligned} de_{iq}(t) &\in \left\{ -d_q e_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t)))F_l(e_{il}(t)) \right. \\ &\quad + \sum_{l=1}^n co(b_{ql}(x_{iq}(t)))F_l(e_{il}(t - \tau_{ql}(t))) \\ &\quad + \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta)F_l(e_{il}(t - \theta))d\theta \\ &\quad \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) + U_{iq}(t) \right\} dt \end{aligned}$$

$$+ \sigma(t, e_{il}(t), e_{il}(t - \tau_{ql}(t)))d\omega_i(t), \tag{10}$$

or equivalently, there exist $\check{a}_{ql}(x_{iq}(t)) \in co(a_{ql}(x_{iq}(t)))$, $\check{b}_{ql}(x_{iq}(t)) \in co(b_{ql}(x_{iq}(t)))$, $\check{c}_{ql}(x_{iq}(t)) \in co(c_{ql}(x_{iq}(t)))$, with the similar process of system (2), we get the following equalities

$$\begin{aligned} \frac{de_{iq}(t)}{dt} &= -d_q e_{iq}(t) + \sum_{l=1}^n \check{a}_{ql}(x_{iq}(t)) F_l(e_{il}(t)) \\ &\quad + \sum_{l=1}^n \check{b}_{ql}(x_{iq}(t)) F_l(e_{il}(t - \tau_{ql}(t))) \\ &\quad + \sum_{l=1}^n \check{c}_{ql}(x_{iq}(t)) \int_0^{+\infty} k_{ql}(\theta) F_l(e_{il}(t - \theta)) d\theta \\ &\quad + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) + U_{iq}(t), \quad t \neq t_k, \\ \Delta e_{iq}(t_k) &= e_{iq}(t_k^+) - e_{iq}(t_k^-) = -r_{ik} e_{iq}(t_k), \quad t = t_k, \\ e_{iq}(t) &= \psi_{iq}(t) \quad t \in (-\infty, 0], \end{aligned} \tag{11}$$

and

$$\begin{aligned} de_{iq}(t) &= \left\{ -d_q e_{iq}(t) + \sum_{l=1}^n \check{a}_{ql}(x_{iq}(t)) F_l(e_{il}(t)) \right. \\ &\quad + \sum_{l=1}^n \check{b}_{ql}(x_{iq}(t)) F_l(e_{il}(t - \tau_{ql}(t))) \\ &\quad + \sum_{l=1}^n \check{c}_{ql}(x_{iq}(t)) \int_0^{+\infty} k_{ql}(\theta) F_l(e_{il}(t - \theta)) d\theta \\ &\quad \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) + U_{iq}(t) \right\} dt \\ &\quad + \sigma(t, e_{il}(t), e_{il}(t - \tau_{ql}(t)))d\omega_i(t), \end{aligned} \tag{12}$$

where $\psi_{iq}(t) = \phi_{iq}(t) - s_{iq}(t)$ is the initial conditions, $F_l(e_{ij}(t)) = f_l(x_{il}(t)) - f_l(s_{il}(t))$, $F_l(e_{ij}(t - \tau_{ql}(t))) = f_l(x_{il}(t - \tau_{ql}(t))) - f_l(s_{il}(t - \tau_{ql}(t)))$, $F_l(e_{ij}(t - \theta)) = f_l(x_{il}(t - \theta)) - f_l(s_{il}(t - \theta))$. r_{ik} is the ratio of the error state variable $\dot{e}_{iq}(t)$ at $t = t_k$, $U_{iq}(t)$ is the controller to be designed.

Remark 3 Compare with the exciting literatures for researching the exceptional synchronization of coupled MNNs [35–37], the proposed system contains not only non-delayed, discrete time-varying delay $\tau_{ql}(t)$ but also distributed delay. Therefore, the obtained results are more reasonable and practical.

3 Main Results

In this section, we obtain some sufficient conditions to achieve the exceptional synchronization of the coupled MNNs with mixed time-varying delays, impulsive perturbations and

stochastic perturbations, respectively. Then two kinds of feedback and adaptive feedback controller with pinning schemes are designed.

We investigate pinning synchronization of the impulsive coupled MNNs with mixed time-varying delays under the feedback controller U_{iq} . Suppose there exist $m(1 \leq m \leq N)$ nodes of system (4) are controlled. Then the appropriate feedback control input U_{iq} with pinning schemes is designed as

$$\begin{aligned} U_{iq} &= -p_i e_{iq}(t), \quad i = 1, 2, \dots, m, \quad q = i = 1, 2, \dots, n, \\ U_{iq} &= 0, \quad i = m + 1 \dots, N, \end{aligned} \tag{13}$$

where the synchronization error $e_{iq}(t)$ is defined as $e_{iq}(t) = x_{iq}(t) - s_{iq}(t)$. And $p_i > 0(i = 1, 2, \dots, m)$ is feedback gains.

Notations Before starting the main results, some annotations should be given. Let $\tilde{a}_{ql} = \max\{|\hat{a}_{ql}|, |\check{a}_{ql}|\}$, $\tilde{b}_{ql} = \max\{|\hat{b}_{ql}|, |\check{b}_{ql}|\}$ and $\tilde{c}_{ql} = \max\{|\hat{c}_{ql}|, |\check{c}_{ql}|\}$, for $i, j = 1, 2, \dots, N, q, l = 1, 2, \dots, n, k = 1, 2, \dots$

According to (9), we get the following synchronization errors of system (5). When $t \neq t_k$, one finds that when $i = 1, 2, \dots, m$

$$\begin{aligned} \frac{de_{iq}(t)}{dt} &\in -d_q e_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t))) F_l(e_{il}(t)) \\ &+ \sum_{l=1}^n co(b_{ql}(x_{iq}(t))) F_l(e_{il}(t - \tau_{ql}(t))) \\ &+ \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta) F_l(e_{il}(t - \theta)) d\theta \\ &+ \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) - \sum_{i=1}^m p_i e_{iq}(t). \end{aligned} \tag{14}$$

When $i = m + 1 \dots, N$.

$$\begin{aligned} \frac{de_{iq}(t)}{dt} &\in -d_q e_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t))) F_l(e_{il}(t)) \\ &+ \sum_{l=1}^n co(b_{ql}(x_{iq}(t))) F_l(e_{il}(t - \tau_{ql}(t))) \\ &+ \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta) F_l(e_{il}(t - \theta)) d\theta \\ &+ \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t). \end{aligned} \tag{15}$$

When $t = t_k$,

$$\Delta e_{iq}(t_k) = e_{iq}(t_k^+) - e_{iq}(t_k^-) = -r_{ik} e_{iq}(t_k). \tag{16}$$

Theorem 1 Suppose that Assumption 1 holds and if there exists a constant ζ , such that

$$\begin{aligned}
 H_i(\zeta) = & \sum_{i=1}^N \sum_{q=1}^n \left[d_q - \zeta - \sum_{l=1}^n z_l |\tilde{a}_{ql}| - \sum_{l=1}^n z_l |\tilde{b}_{ql}| e^{\zeta \tau_{ql}} \right. \\
 & \left. - \sum_{l=1}^n |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql}(\theta) e^{\zeta \theta} d\theta + \alpha \sum_{j=1}^N \Gamma |\omega_{ij}| \right] + \sum_{i=1}^m \sum_{q=1}^n p_i \\
 > 0.
 \end{aligned}
 \tag{17}$$

Then, the error system (11) will be converged to zero by means of pinning schemes.

Proof Construct the following Lyapunov functional

$$V(t) = \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t} |e_{iq}(t)|.
 \tag{18}$$

Then we have

$$\dot{V}(t) = \sum_{i=1}^N \sum_{q=1}^n \lambda e^{\lambda t} |e_{iq}(t)| + \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t} |\dot{e}_{iq}(t)|.
 \tag{19}$$

According to Eq. (14), we obtain the following inequality among Eq. (19)

$$\begin{aligned}
 \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t} |\dot{e}_{iq}(t)| &= \sum_{i=1}^N \sum_{q=1}^n \operatorname{sgn}(e_{iq}(t)) \dot{e}_{iq}(t) \\
 &\leq \operatorname{sgn}(e_i(t)) \sum_{q=1}^n \left\{ \sum_{i=1}^N \left[-d_q e_{iq}(t) + \sum_{l=1}^n \tilde{a}_{ql} F_l(e_{il}(t)) \right. \right. \\
 &\quad \left. \left. + \sum_{l=1}^n \tilde{b}_{ql} F_l(e_{il}(t - \tau_{ql}(t))) + \sum_{l=1}^n \tilde{c}_{ql} \int_0^{+\infty} k_{ql}(\theta) F_l(e_{il}(t - \theta)) d\theta \right. \right. \\
 &\quad \left. \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) \right] - \sum_{i=1}^m p_i e_{iq}(t) \right\}.
 \end{aligned}
 \tag{20}$$

Under Assumptions 1-2, we obtain that

$$\begin{aligned}
 F_l(e_{il}(t)) &\leq z_l e_{il}(t) \\
 F_l(e_{il}(t - \tau_{ql}(t))) &\leq z_l e_{il}(t - \tau_{ql}(t)) \\
 F_l(e_{il}(t - \theta)) &\leq z_l e_{il}(t - \theta)
 \end{aligned}
 \tag{21}$$

Combining with Eq. (21), we calculate the upper right derivation of the error system (11)

$$\begin{aligned}
 \sum_{i=1}^N \sum_{q=1}^n |\dot{e}_{iq}(t)| &\leq \operatorname{sgn}(e_{iq}(t)) \sum_{i=1}^N \sum_{q=1}^n \left[-d_q e_{iq}(t) + \sum_{l=1}^n z_l \tilde{a}_{ql} e_{il}(t) \right. \\
 &\quad \left. + \sum_{l=1}^n z_l \tilde{b}_{ql} e_{il}(t - \tau_{ql}(t)) + \sum_{l=1}^n \tilde{c}_{ql} \int_0^{+\infty} z_l k_{ql}(\theta) e_{il}(t - \theta) d\theta \right. \\
 &\quad \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) \right] - \operatorname{sgn}(e_{iq}(t)) \sum_{i=1}^m \sum_{q=1}^n p_i e_{iq}(t).
 \end{aligned}
 \tag{22}$$

Then we deduce

$$\begin{aligned} \sum_{i=1}^N \sum_{q=1}^n |\dot{e}_{iq}(t)| &\leq \sum_{i=1}^N \sum_{q=1}^n \left[-d_q |e_{iq}(t)| + \sum_{l=1}^n z_l |\tilde{a}_{ql}| |e_{il}(t)| \right. \\ &\quad + \sum_{l=1}^n z_l |\tilde{b}_{ql}| |e_{il}(t - \tau_{ql}(t))| \\ &\quad + \sum_{l=1}^n |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql}(\theta) |e_{il}(t - \theta)| d\theta \\ &\quad \left. + \alpha \sum_{j=1}^N |\omega_{ij}| \Gamma |e_{jq}(t)| \right] - \sum_{i=1}^m \sum_{q=1}^n p_i |e_{iq}(t)|. \end{aligned} \tag{23}$$

Then

$$\begin{aligned} \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t} |\dot{e}_{iq}(t)| &\leq \sum_{i=1}^N \sum_{q=1}^n \left[-d_q e^{\lambda t} |e_{iq}(t)| + \sum_{l=1}^n z_l |\tilde{a}_{ql}| e^{\lambda t} |e_{il}(t)| \right. \\ &\quad + \sum_{l=1}^n z_l |\tilde{b}_{ql}| e^{\lambda t - \tau_{ql}} |e_{il}(t - \tau_{ql})| e^{\lambda \tau_{ql}} \\ &\quad + \sum_{l=1}^n |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql} e^{\lambda(t-\theta)}(\theta) |e_{il}(t - \theta)| e^{\lambda \theta} d\theta \\ &\quad \left. + \alpha \sum_{j=1}^N |\omega_{ij}| \Gamma e^{\lambda t} |e_{jq}(t)| \right] - \sum_{i=1}^m \sum_{q=1}^n p_i e^{\lambda t} |e_{iq}(t)|. \end{aligned} \tag{24}$$

Consider $v_{iq}(t) = e^{\lambda t} |e_{iq}(t)|$ ([9]). Let $\delta > 1$ and $P = \max_{1 \leq i \leq N} \sup_{\theta \in (-\infty, 0]} |\phi_{iq}(\theta) - s_{iq}(t)| > 0$. Therefore, $v_{iq}(t) < P\delta$ for $t \in [0, +\infty)$. Hence

$$\begin{aligned} \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t} |\dot{e}_{iq}(t)| &\leq \sum_{i=1}^N \sum_{q=1}^n \left[|\tilde{a}_{ql}| |z_l| \sum_{l=1}^n V(t) + d_q V(t) \right. \\ &\quad + |\tilde{b}_{ql}| |z_l| \sum_{l=1}^n e^{\lambda(t-\tau_{ql})} V(t - \tau_{ql}) \\ &\quad + |\tilde{c}_{ql}| \sum_{l=1}^n \int_0^{+\infty} |z_l| k_{ql}(\theta) V(t - \theta) e^{\lambda \theta} d\theta \\ &\quad \left. + \alpha |\omega_{ij}| \sum_{j=1}^N \Gamma V(t) \right] - p_i \sum_{i=1}^m \sum_{q=1}^n V(t). \end{aligned} \tag{25}$$

From all above the calculations, we conclude that

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N \sum_{q=1}^n \lambda e^{\lambda t} |e_i(t)| + \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t} |\dot{e}_{iq}(t)| \\ &\leq \left\{ \sum_{i=1}^N \sum_{q=1}^n \left[-d_q + \lambda + \sum_{l=1}^n |\tilde{a}_{ql}| |z_l| + \sum_{l=1}^n |\tilde{b}_{ql}| |z_l| e^{\lambda(t-\tau_{ql})} \right. \right. \end{aligned}$$

$$\begin{aligned}
 & +\alpha \sum_{j=1}^N \Gamma |\omega_{ij}| + |\tilde{c}_{ql}| \sum_{l=1}^n \int_0^{+\infty} |z_l| k_{ql}(\theta) e^{\lambda\theta} d\theta \Big] \\
 & - \sum_{i=1}^m \sum_{q=1}^n p_i \Big\} P\delta \\
 & < 0.
 \end{aligned} \tag{26}$$

We get $v_{iq}(t) < P\delta$ which leads to

$$|x_{iq}(t) - s_{iq}(t)| \leq e^{\lambda t} \max_{1 \leq i \leq N} \left\{ \sup_{\theta \in (-\infty, 0]} |\phi_{iq}(\theta) - s_{iq}(t)| \right\},$$

for any $t > 0$.

Therefore, $x_{iq}(t)$ converges to $s_{iq}(t)$.

When $t = t_k$, one finds that

$$\begin{aligned}
 V(t_k^+) &= \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t_k^+} |e_{iq}(t_k^+)| = \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t_k} [(1 - r_{ik}) |e_{iq}(t_k)|] \\
 &\leq \sum_{i=1}^N \sum_{q=1}^n e^{\lambda t_k} |e_{iq}(t_k)| = V(t_k).
 \end{aligned} \tag{27}$$

We can select ζ to satisfy

$$\begin{aligned}
 H_i(\zeta) &= \sum_{i=1}^N \sum_{q=1}^n \left[d_q - \zeta - \sum_{l=1}^n z_l |\tilde{a}_{ql}| - \sum_{l=1}^n z_l |\tilde{b}_{ql}| e^{\zeta \tau_{ql}} \right. \\
 &\quad \left. - \sum_{l=1}^n |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql}(\theta) e^{\zeta \theta} d\theta + \alpha \sum_{j=1}^N \Gamma |\omega_{ij}| \right] + \sum_{i=1}^m \sum_{q=1}^n p_i \\
 &> 0.
 \end{aligned} \tag{28}$$

Then $\dot{V}(t) \leq 0$, The proof is completed. □

Corollary 1 Under Assumptions 1–2, for given constant $\zeta > 0$. If the following inequality holds, the coupled MNNs without impulsive perturbation will achieve exponential synchronization with pinning rules.

$$\begin{aligned}
 H_i(\zeta) &= \sum_{i=1}^N \sum_{q=1}^n \left[d_q - \zeta - \sum_{l=1}^n |z_l| |\tilde{a}_{ql}| - \sum_{l=1}^n |z_l| |\tilde{b}_{ql}| e^{\zeta \tau_{ql}} \right. \\
 &\quad \left. - \sum_{l=1}^n |\tilde{c}_{ql}| \int_0^{+\infty} |z_l| k_{ql}(\theta) e^{\zeta \theta} d\theta + \alpha \sum_{j=1}^N \Gamma |\omega_{ij}| \right] \\
 &\quad + \sum_{i=1}^m \sum_{q=1}^n |p_i| \\
 &> 0.
 \end{aligned} \tag{29}$$

Proof Let the error system (11) without impulsive perturbation in Theorem 1. The proof can be followed, thus it is omitted here. □

Remark 4 Usually, the linear feedback controller is indispensable in a synchronization of the system. We observe that the actual communication between subsystems of coupled MNNs is inevitably disturbed by the stochastic perturbations from various uncertainties. Thus, using an appropriate Lyapunov method, a simple adaptive controller is designed for the exponential synchronization of the system (5).

In this subsection, we investigate pinning synchronization of the coupled MNNs with mixed time-varying delay and stochastic perturbations under the considered adaptive feedback controller $U_{iq}(t)$. Suppose there exist m ($1 \leq m < N$) nodes of system (5) will be controlled. Then the adaptive feedback controller U_{iq} with pinning laws is designed as follows

$$\begin{aligned}
 U_{iq}(t) &= -p_i(t)e_{iq}(t), \\
 \dot{p}_i(t) &= h_i e_{iq}^T(t)e_{iq}(t), \quad i = 1, 2, \dots, m, \quad q = 1, 2, \dots, n, \\
 U_{iq}(t) &= 0, \quad i = m + 1 \dots, N,
 \end{aligned}
 \tag{30}$$

where h_i is a positive constant.

According to the definition of pinning scheme and the system (10), we get the following synchronization error system.

When $i = 1, 2, \dots, m$.

$$\begin{aligned}
 de_{iq}(t) \in & \left\{ -d_q e_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t)))F_l(e_{il}(t)) \right. \\
 & + \sum_{l=1}^n co(b_{ql}(x_{iq}(t)))F_l(e_{il}(t - \tau_{ql}(t))) \\
 & + \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta)F_l(e_{il}(t - \theta))d\theta \\
 & + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) - \sum_{i=1}^m p_i(t)e_{iq}(t) \left. \right\} dt \\
 & + \sigma(t, e_{il}(t), e_{il}(t - \tau_{ql}(t)))d\omega_i(t).
 \end{aligned}
 \tag{31}$$

When $i = m + 1 \dots N$.

$$\begin{aligned}
 de_{iq}(t) \in & \left\{ -d_q e_{iq}(t) + \sum_{l=1}^n co(a_{ql}(x_{iq}(t)))F_l(e_{il}(t)) \right. \\
 & + \sum_{l=1}^n co(b_{ql}(x_{iq}(t)))F_l(e_{il}(t - \tau_{ql}(t))) \\
 & + \sum_{l=1}^n co(c_{ql}(x_{iq}(t))) \int_0^{+\infty} k_{ql}(\theta)F_l(e_{il}(t - \theta))d\theta \\
 & + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) \left. \right\} dt \\
 & + \sigma(t, e_{il}(t), e_{il}(t - \tau_{ql}(t)))d\omega_i(t).
 \end{aligned}
 \tag{32}$$

Theorem 2 *Suppose Assumptions 1–2 are satisfied, then the error systems (31) and (32) are exponentially stable via the adaptive controller (30), if there exists a gain constant ζ , such that*

$$\begin{aligned}
 H_i(\zeta) &= \sum_{i=1}^N \sum_{q=1, l=1}^n \left[d_q - \zeta - \rho_1 R_1 - \rho_2 R_2 e^{\zeta \tau_{ql}} - z_l |\tilde{a}_{ql}| - z_l |\tilde{b}_{ql}| e^{\zeta \tau_{ql}} \right. \\
 &\quad \left. - |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql}(\theta) e^{\zeta \theta} d\theta + \alpha \sum_{j=1}^N \Gamma |\omega_{ij}| \right] + \sum_{i=1}^m \sum_{q=1}^n L \\
 &> 0.
 \end{aligned}
 \tag{33}$$

Then, the pinning synchronization of system (5) is achieved.

Proof Construct a Lyapunov functional as follows

$$V(t) = \frac{1}{2} e^{2\lambda t} \sum_{i=1}^N \sum_{q=1}^n \left| e_{iq}^T(t) e_{iq}(t) - \frac{(p_i(t) + L)^2}{h_i} \right|,
 \tag{34}$$

where L is a positive constant to be determined below.

Then, the upper right derivative of $V(t)$ along the trajectories of (5) gives

$$\begin{aligned}
 \dot{V}(t) &= \sum_{i=1}^N \sum_{q=1}^n \lambda e^{2\lambda t} \left| e_{iq}^T(t) e_{iq}(t) - \frac{(p_i(t) + L)^2}{h_i} \right| \\
 &\quad + \sum_{i=1}^N \sum_{q=1}^n e^{2\lambda t} \operatorname{sgn}(e_{iq}(t)) \left[e_{iq}^T(t) \dot{e}_{iq}(t) - \frac{(p_i(t) + L)^2}{h_i} \dot{p}_i(t) \right].
 \end{aligned}
 \tag{35}$$

After that, the system (35) is decomposed into two parts as follows

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t).
 \tag{36}$$

Combining systems (35) and (36), we get

$$\dot{V}_1(t) = \sum_{i=1}^N \sum_{q=1}^n e^{2\lambda t} \operatorname{sgn}(e_{iq}(t)) \left[e_{iq}^T(t) \dot{e}_{iq}(t) - \frac{(p_i(t) + L)^2}{h_i} \dot{p}_i(t) \right],
 \tag{37}$$

and

$$\dot{V}_2(t) = \sum_{i=1}^N \sum_{q=1}^n \lambda e^{2\lambda t} \left| e_{iq}^T(t) e_{iq}(t) - \frac{(p_i(t) + L)^2}{h_i} \right|.
 \tag{38}$$

Integrated the error systems (31) and (32) into account, we get the following equality

$$\begin{aligned}
 \dot{V}_1(t) &\leq \sum_{i=1}^N \sum_{q=1}^n e^{2\lambda t} \operatorname{sgn}(e_{iq}(t)) e_{iq}^T(t) \left\{ -d_q e_{iq}(t) + \sum_{l=1}^n \tilde{a}_{ql} F_l(e_{il}(t)) \right. \\
 &\quad + \sum_{l=1}^n \tilde{b}_{ql} F_l(e_{il}(t - \tau_{ql}(t))) + \sum_{l=1}^n \tilde{c}_{ql} \int_0^{+\infty} k_{ql}(\theta) F_l(e_{il}(t - \theta)) d\theta \\
 &\quad \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) \right\} + \sum_{i=1}^N \sum_{q=1}^n e^{2\lambda t} \operatorname{sgn}(e_{iq}(t))
 \end{aligned}$$

$$\begin{aligned} & \times \text{Trace} \left[\sigma^T(t, x_{il}(t), x_{il}(t - \tau_{ql}(t))) P \sigma(t, x_{il}(t), x_{il}(t - \tau_{ql}(t))) \right] \\ & - \sum_{i=1}^m \sum_{q=1}^n e^{2\lambda t} \text{sgn}(e_{iq}(t)) L e_{iq}^T(t) e_{iq}(t). \end{aligned} \tag{39}$$

Taken Eq. (21) and Lemma 1 into consideration, by calculating the upper right derivation of $V_1(t)$ along with the solution of system (6), we obtain

$$\begin{aligned} \dot{V}_1(t) \leq & \sum_{i=1}^N \sum_{q=1}^n e^{2\lambda t} \text{sgn}(e_{iq}(t)) e_{iq}^T(t) \left\{ -d_q e_{iq}(t) + \sum_{l=1}^n \tilde{a}_{ql} z_l e_{il}(t) \right. \\ & + \sum_{l=1}^n \tilde{b}_{ql} e_{il} z_l((t - \tau_{ql}(t))) + \sum_{l=1}^n \tilde{c}_{ql} \int_0^{+\infty} z_l k_{ql}(\theta) e_{il}(t - \theta) d\theta \\ & \left. + \alpha \sum_{j=1}^N \omega_{ij} \Gamma e_{jq}(t) \right\} + \sum_{i=1}^N \sum_{q=1}^n e^{2\lambda t} \text{sgn}(e_{iq}(t)) \\ & \times \left[\rho_1 e_{il}^T(t) R_1 e_{il}(t) + \rho_2 e_{il}^T(t - \tau_{ql}(t)) R_1 e_{il}(t - \tau_{ql}(t)) \right] \\ & - \sum_{i=1}^m \sum_{q=1}^n e^{2\lambda t} \text{sgn}(e_{iq}(t)) L e_{iq}^T(t) e_{iq}(t). \end{aligned} \tag{40}$$

According to the above discussions, we get

$$\begin{aligned} \dot{V}_1(t) \leq & e^{\lambda t} |e_{iq}^T(t)| \left\{ \sum_{i=1}^N \sum_{q=1}^n \left[-d_q e^{\lambda t} |e_{iq}(t)| + \sum_{l=1}^n z_l |\tilde{a}_{ql}| e^{\lambda t} |e_{il}(t)| \right. \right. \\ & + \sum_{l=1}^n z_l |\tilde{b}_{ql}| e^{\lambda(t-\tau_{ql})} |e_{il}((t - \tau_{ql}))| e^{\lambda \tau_{ql}} \\ & + \sum_{l=1}^n z_l |\tilde{c}_{ql}| \int_0^{+\infty} k_{ql}(\theta) e^{\lambda(t-\theta)} |e_{il}(t - \theta)| e^{\lambda \theta} d\theta \\ & \left. + \alpha \sum_{j=1}^N |\omega_{ij}| \Gamma |e_{jq}(t)| \right] - \sum_{i=1}^m \sum_{q=1}^n e^{\lambda t} L |e_{iq}(t)| \left\{ \right. \\ & + \sum_{i=1}^N \sum_{q=1}^n \left[\rho_1 e^{\lambda t} |e_{il}^T(t)| R_1 e^{\lambda t} |e_{il}(t)| \right. \\ & \left. \left. + \rho_2 e^{\lambda(t-\tau_{ql})} |e_{il}^T(t - \tau_{ql})| e^{\lambda(t-\tau_{ql})} R_2 |e_{il}(t - \tau_{ql})| e^{2\lambda \tau_{ql}} \right]. \end{aligned} \tag{41}$$

Consider $v_{iq}(t) = e^{\lambda t} |e_{iq}(t)|$ ([9]). Let $\delta > 1$, $K = \max_{1 \leq i \leq N} \sup_{\theta \in (-\infty, 0]} |\phi_{iq}(\theta) - s_{iq}(t)| > 0$. Therefore, $v_{iq}(t) < K\delta$ for $t \in [0, +\infty)$.

Then we conclude

$$\begin{aligned} \dot{V}_1(t) \leq & K^2\delta^2 \left\{ \sum_{i=1}^N \sum_{q=1, l=1}^n \left[-d_q + z_l|\tilde{a}_{ql}| + z_l|\tilde{b}_{ql}|e^{\lambda\tau_{ql}} \right. \right. \\ & + z_l|\tilde{c}_{ql}| \int_0^{+\infty} k_{ql}(\theta)e^{\lambda\theta}d\theta + \alpha \sum_{j=1}^N |\omega_{ij}|\Gamma \\ & \left. \left. + \rho_1 R_1 + \rho_2 R_2 e^{2\lambda\tau_{ql}} \right] - \sum_{i=1}^m \sum_{q=1}^n L \right\}. \end{aligned} \tag{42}$$

From (38) we deduce the equality as follows

$$\dot{V}_2(t) = \sum_{i=1}^N \sum_{q=1}^n \lambda e^{2\lambda t} \left| e_{iq}^T(t) e_{iq}(t) - \frac{(p_i(t) + L)^2}{h_i} \right|. \tag{43}$$

Then we get

$$\begin{aligned} e^{\lambda t} |e_{iq}(t)| & < K\delta \\ \dot{V}_2(t) \leq & \sum_{i=1}^N \sum_{q=1}^n \lambda \left[e^{\lambda t} |e_{iq}(t)| e^{\lambda t} |e_{iq}^T(t)| \right] < \sum_{i=1}^N \sum_{q=1}^n \lambda K^2 \delta^2. \end{aligned} \tag{44}$$

According to (42) and (44), we obtain the following theorem to guarantee the synchronization of system (5).

$$\begin{aligned} \dot{V}(t) \leq & K^2\delta^2 \left\{ \sum_{i=1}^N \sum_{q=1, l=1}^n \left[\lambda - d_q + z_l|\tilde{a}_{ql}| + z_l|\tilde{b}_{ql}|e^{\lambda\tau_{ql}} \right. \right. \\ & + z_l|\tilde{c}_{ql}| \int_0^{+\infty} k_{ql}(\theta)e^{\lambda\theta}d\theta + \alpha \sum_{j=1}^N |\omega_{ij}|\Gamma \\ & \left. \left. + \rho_1 R_1 + \rho_2 R_2 e^{2\lambda\tau_{ql}} \right] - \sum_{i=1}^m \sum_{q=1}^n L \right\}. \end{aligned} \tag{45}$$

We can select ζ to satisfy

$$\begin{aligned} H_i(\zeta) = & \sum_{i=1}^N \sum_{q=1, l=1}^n \left[d_q - \zeta - \rho_1 R_1 - \rho_2 R_2 e^{\zeta\tau_{ql}} - z_l|\tilde{a}_{ql}| \right. \\ & - z_l|\tilde{b}_{ql}|e^{\zeta\tau_{ql}} - |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql}(\theta)e^{\zeta\theta}d\theta \\ & \left. + \alpha \sum_{j=1}^N \Gamma |\omega_{ij}| \right] + \sum_{i=1}^m \sum_{q=1}^n L \\ & > 0. \end{aligned} \tag{46}$$

Then $\dot{V}(t) \leq 0$, The proof is completed. □

Corollary 2 *Due to Assumptions 1–2, for given constant $\zeta > 0$, if the following inequality holds, the considered coupled MNNs without stochastic perturbations will achieve exponential synchronization under pinning control.*

$$\begin{aligned}
 H_i(\zeta) &= \sum_{i=1}^N \sum_{q=1, l=1}^n \left[d_q - \zeta - z_l |\tilde{a}_{ql}| - z_l |\tilde{b}_{ql}| e^{\zeta \tau_{ql}} \right. \\
 &\quad \left. - |\tilde{c}_{ql}| \int_0^{+\infty} z_l k_{ql}(\theta) e^{\zeta \theta} d\theta + \alpha \sum_{j=1}^N \Gamma |\omega_{ij}| \right] + \sum_{i=1}^m \sum_{q=1}^n L \\
 &> 0.
 \end{aligned}
 \tag{47}$$

Proof Let the error system (12) without impulsive perturbations in Theorem 2. The proof can be followed, thus it is omitted here.

Remark 5 Due to the condition of time-varying delay $\tau_{ql}(t)$ and the property of stochastic perturbations, Theorem 2 provides a suitable adaptive controller. It’s worth pointing out that no redundant numerical calculation such as computing complex algebraic conditions ([48]) or solving linear matrix inequality (LMIs) ([49, 50]) are needed in the synchronization conditions. Thus, our synchronization consequences have a stronger adaptive capability and more powerful application.

Remark 6 There is no extra restraint on activation functions but demanding they are bounded and the time-varying delays are mixed. Furthermore, overall consideration of our obtained results with pinning schemes, which can be expected to have a powerful potential application in areas such as associative memory, image encryption, digital processing, and so on.

4 Illustrative Example

In this section, we will give numerical examples to verify the effectiveness of our conclusions. Consider a coupled two-dimensional MNNs as follows

$$\begin{aligned}
 \frac{dx_1(t)}{dt} &= -d_1 x_1(t) + a_{11}(x_1(t)) f(x_1(t)) + a_{12}(x_1(t)) f(x_2(t)) \\
 &\quad + b_{11}(x_1(t)) f(x_1(t - \tau_{11}(t))) + b_{12}(x_1(t)) f(x_2(t - \tau_{12}(t))) \\
 &\quad + c_{11}(x_1(t)) \int_0^{+\infty} k_{11}(\theta) f(x_1(t - \theta)) d\theta \\
 &\quad + c_{12}(x_1(t)) \int_0^{+\infty} k_{12}(\theta) f(x_2(t - \theta)) d\theta \\
 &\quad + \alpha \omega_{12} \Gamma x_2(t) + \alpha \omega_{13} \Gamma x_3(t) + I_1(t),
 \end{aligned}
 \tag{48}$$

$$\begin{aligned}
 \frac{dx_2(t)}{dt} &= -d_2 x_2(t) + a_{21}(x_2(t)) f(x_1(t)) + a_{22}(x_2(t)) f(x_2(t)) \\
 &\quad + b_{21}(x_2(t)) f(x_2(t - \tau_{21}(t))) + b_{22}(x_2(t)) f(x_2(t - \tau_{22}(t))) \\
 &\quad + c_{21}(x_2(t)) \int_0^{+\infty} k_{21}(\theta) f(x_1(t - \theta)) d\theta \\
 &\quad + c_{22}(x_2(t)) \int_0^{+\infty} k_{22}(\theta) f(x_2(t - \theta)) d\theta \\
 &\quad + \alpha \omega_{21} \Gamma x_1(t) + \alpha \omega_{23} \Gamma x_3(t) + I_2(t),
 \end{aligned}
 \tag{49}$$

$$\begin{aligned} \frac{dx_3(t)}{dt} = & -d_3x_3(t) + a_{31}(x_3(t))f(x_1(t)) + a_{32}(x_3(t))f(x_2(t)) \\ & + b_{31}(x_3(t))f(x_1(t - \tau_{31}(t))) + b_{32}(x_3(t))f(x_2(t - \tau_{32}(t))) \\ & + c_{31}(x_3(t)) \int_0^{+\infty} k_{31}(\theta)f(x_1(t - \theta))d\theta \\ & + c_{32}(x_3(t)) \int_0^{+\infty} k_{32}(\theta)f(x_2(t - \theta))d\theta \\ & + \alpha\omega_{31}\Gamma x_1(t) + \alpha\omega_{32}\Gamma x_2(t) + I_3(t), \end{aligned} \tag{50}$$

where

$$\begin{aligned} a_{11}(x_1(t)) &= \begin{cases} -0.9, & \dot{f}(x_1(t)) - \dot{x}_1(t) \leq 0, \\ 0.9, & \dot{f}(x_1(t)) - \dot{x}_1(t) > 0, \end{cases} \\ a_{12}(x_1(t)) &= \begin{cases} 0.4, & \dot{f}(x_2(t)) - \dot{x}_1(t) \leq 0, \\ -0.4, & \dot{f}(x_2(t)) - \dot{x}_1(t) > 0, \end{cases} \\ a_{21}(x_2(t)) &= \begin{cases} -1.2, & \dot{f}(x_1(t)) - \dot{x}_2(t) \leq 0, \\ 1.2, & \dot{f}(x_1(t)) - \dot{x}_2(t) > 0, \end{cases} \\ a_{22}(x_2(t)) &= \begin{cases} 0.8, & -\dot{f}(x_2(t)) - \dot{x}_2(t) \leq 0, \\ -0.8, & -\dot{f}(x_2(t)) - \dot{x}_2(t) > 0, \end{cases} \\ a_{31}(x_3(t)) &= \begin{cases} 0.3, & -\dot{f}(x_1(t)) - \dot{x}_3(t) \leq 0, \\ -0.3, & -\dot{f}(x_1(t)) - \dot{x}_3(t) > 0, \end{cases} \\ a_{32}(x_3(t)) &= \begin{cases} 0.4, & -\dot{f}(x_2(t)) - \dot{x}_3(t) \leq 0, \\ -0.4, & -\dot{f}(x_2(t)) - \dot{x}_3(t) > 0, \end{cases} \\ b_{11}(x_1(t)) &= \begin{cases} 1, & -\dot{f}(x_1(t - \tau_{11}(t))) - \dot{x}_1(t) \leq 0, \\ -1, & -\dot{f}(x_1(t - \tau_{11}(t))) - \dot{x}_1(t) > 0, \end{cases} \\ b_{12}(x_1(t)) &= \begin{cases} -5, & -\dot{f}(x_2(t - \tau_{12}(t))) - \dot{x}_1(t) \leq 0, \\ 5, & -\dot{f}(x_2(t - \tau_{12}(t))) - \dot{x}_1(t) > 0, \end{cases} \\ b_{21}(x_2(t)) &= \begin{cases} -1.2, & \dot{f}(x_1(t - \tau_{21}(t))) - \dot{x}_2(t) \leq 0, \\ 1.2, & \dot{f}(x_1(t - \tau_{21}(t))) - \dot{x}_2(t) > 0, \end{cases} \\ b_{22}(x_2(t)) &= \begin{cases} -1, & \dot{f}(x_2(t - \tau_{22}(t))) - \dot{x}_2(t) \leq 0, \\ 1, & \dot{f}(x_2(t - \tau_{22}(t))) - \dot{x}_2(t) > 0, \end{cases} \\ b_{31}(x_3(t)) &= \begin{cases} 0.9, & \dot{f}(x_1(t - \tau_{31}(t))) - \dot{x}_3(t) \leq 0, \\ -0.9, & \dot{f}(x_1(t - \tau_{31}(t))) - \dot{x}_3(t) > 0, \end{cases} \\ b_{32}(x_3(t)) &= \begin{cases} 0.7, & \dot{f}(x_2(t - \tau_{32}(t))) - \dot{x}_3(t) \leq 0, \\ -0.7, & \dot{f}(x_2(t - \tau_{32}(t))) - \dot{x}_3(t) > 0, \end{cases} \\ c_{11}(x_1(t)) &= \begin{cases} 3, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_1(t - \theta))d\theta) - \dot{x}_1(t) \leq 0, \\ -3, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_1(t - \theta))d\theta) - \dot{x}_1(t) > 0, \end{cases} \\ c_{12}(x_1(t)) &= \begin{cases} 1, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_2(t - \theta))d\theta) - \dot{x}_1(t) \leq 0, \\ -1, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_2(t - \theta))d\theta) - \dot{x}_1(t) > 0, \end{cases} \\ c_{21}(x_2(t)) &= \begin{cases} -2, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_1(t - \theta))d\theta) - \dot{x}_2(t) \leq 0, \\ 2, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_1(t - \theta))d\theta) - \dot{x}_2(t) > 0, \end{cases} \\ c_{22}(x_2(t)) &= \begin{cases} 2, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_2(t - \theta))d\theta) - \dot{x}_2(t) \leq 0, \\ -2, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_2(t - \theta))d\theta) - \dot{x}_2(t) > 0, \end{cases} \end{aligned}$$

$$c_{31}(x_3(t)) = \begin{cases} -1, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_1(t-\theta)d\theta)) - \dot{x}_3(t) \leq 0, \\ 1, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_1(t-\theta)d\theta)) - \dot{x}_3(t) > 0, \end{cases}$$

$$c_{32}(x_3(t)) = \begin{cases} -1, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_2(t-\theta)d\theta)) - \dot{x}_3(t) \leq 0, \\ 1, & \dot{f}(\int_0^{+\infty} e^{-\theta} f(x_2(t-\theta)d\theta)) - \dot{x}_3(t) > 0. \end{cases}$$

Let $J(t) = [J_1(t), J_2(t), J_3(t)]^T = [0, 0, 0]^T$, $\tau_{11}(t) = \tau_{12}(t) = \tau_{21}(t) = \tau_{22}(t) = \tau_{31}(t) = \tau_{32}(t) = 2 \sin(t)$, let $f(x) = \frac{1}{2}(|1+x| + |1-x|)$ be the activation function. Obviously, we have $d_1 = 8, d_2 = 6, d_3 = 5, \bar{a}_{11} = 0.9, \bar{a}_{12} = 0.4, \bar{a}_{21} = 1.2, \bar{a}_{22} = 0.8, \bar{a}_{31} = 0.3, \bar{a}_{32} = 0.4, \bar{b}_{11} = 1, \bar{b}_{12} = 5, \bar{b}_{21} = 2, \bar{b}_{22} = 1, \bar{b}_{31} = 0.9, \bar{b}_{32} = 0.7, \bar{c}_{11} = 3, \bar{c}_{12} = 1, \bar{c}_{21} = 2, \bar{c}_{22} = 2, \bar{c}_{31} = 1, \bar{c}_{32} = 1, k_{qi}(\theta) = e^{-2\theta}, \mu = -1$.

The phase trajectories $x_1(t), x_2(t)$ and $x_3(t)$ of system (1) are shown in Fig. 1. The state trajectories $x_{i1}(t), x_{i2}(t)$ and $x_{i3}(t)$ with 15 initial values of such system without the effective control are shown in Fig. 2, it indicates the state of system (1) cannot keep stable without controller. And the following simulations conduct on the basis of this situation.

The parameters are selected as $\alpha = 1, P = 1$ and $\zeta = 1$. We assume that the inner matrix $\Gamma = I$ and these nodes are connected with weighted zero-row-sum. For example, the weighed outer-coupling configuration matrix is given by

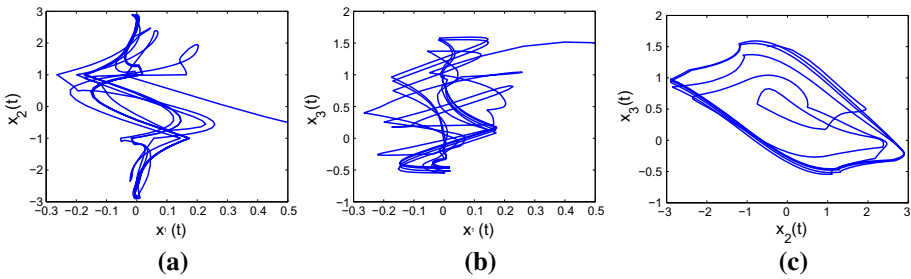


Fig. 1 **a** Phase trajectories of the $x_1(t)$ of system (1); **b** Phase trajectories of the $x_2(t)$ of system (1); **c** Phase trajectories of the $x_3(t)$ of system (1)

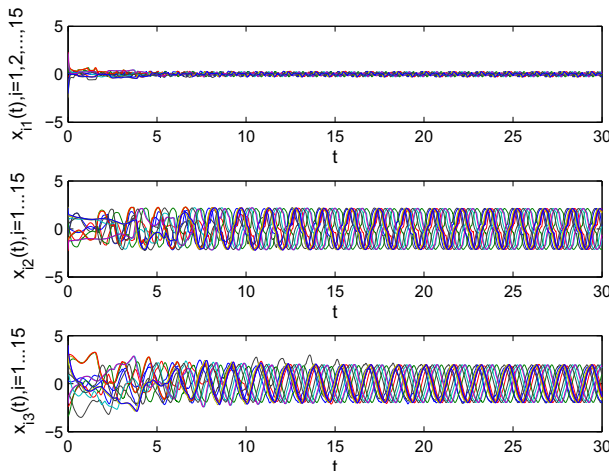


Fig. 2 State trajectories $x_{i1}(t), x_{i2}(t)$ and $x_{i3}(t)$ of system (1) with 15 initial values

$$W = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

According to Theorem 1, we add impulsive perturbations on such system, it is easily to see that the states are quickly converged to stable from Fig. 3. It is clear that the error system (11) is converged to zero by means of pinning schemes. Hence, it can be concluded that, the results shown the feedback control inputs contribute to the chaos exceptional synchronization under the pinning control mechanism.

The system (4) has chaotic attractors with the initial values which can be seen in Fig. 1. It follows from Corollary 1, the system (4) without impulsive perturbations has reached exceptionally synchronized by means of pinning control. Figure 4 depict the synchronization error of the state variables $e_{i1}(t)$, $e_{i2}(t)$, and $e_{i3}(t)$ without impulsive perturbations, respectively. Figure 4a illustrated that the synchronization error under controller (13) with unsuitable parameters, and Fig. 4b shown that the synchronization error under controller (13) with suit-

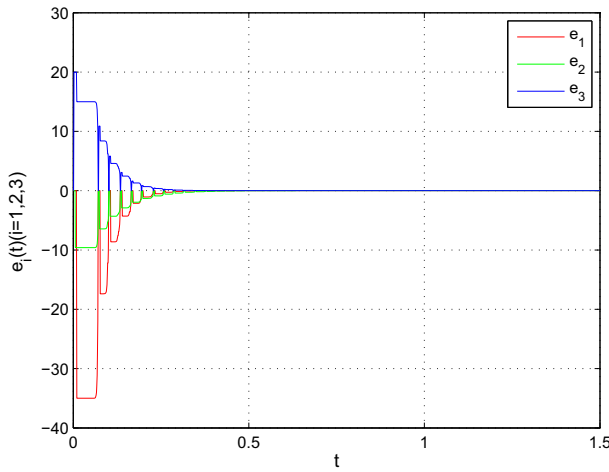


Fig. 3 Synchronization errors $e_{i1}(t)$, $e_{i2}(t)$, and $e_{i3}(t)$ of system (4) under controller (13)

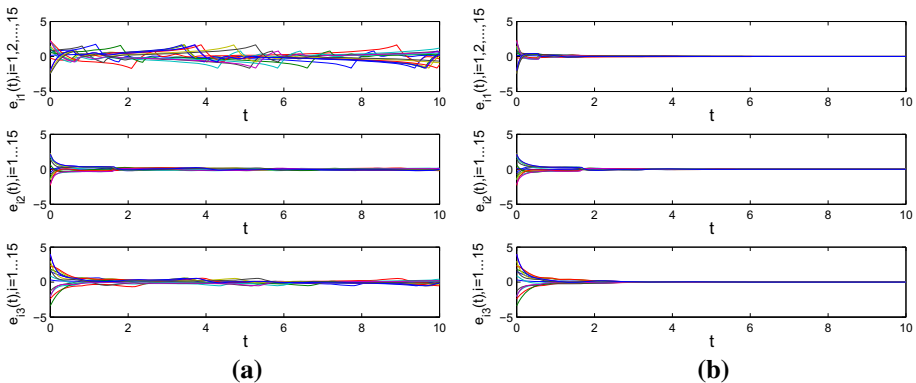


Fig. 4 **a** The synchronization error of system (4) without impulsive perturbations and control; **b** The synchronization error of system (4) without impulsive perturbations but under control

able parameters. The results indicate the suitable parameters of controller (13) is critical to the synchronization. Thus, the synchronization conditions from Theorem 2 is reasonable and resultful.

Based on Theorem 2, we select stochastic system (5) as an example. The Brownian motion satisfies $E\omega(t) = 0, D\omega(t) = 1$. And

$$\begin{aligned} \sigma_1(t, x_{11}(t), x_{11}(t - \tau_{12}(t))) &= 0.2 \times x_{11}(t) + 0.4 \times x_{11}(t - \tau_{12}(t)), \\ \sigma_1(t, x_{12}(t), x_{12}(t - \tau_{21}(t))) &= 0.2 \times x_{12}(t) + 0.4 \times x_{12}(t - \tau_{21}(t)), \\ \sigma_2(t, x_{21}(t), x_{21}(t - \tau_{12}(t))) &= -0.5 \times x_{21}(t) + 0.3 \times x_{21}(t - \tau_{12}(t)), \\ \sigma_2(t, x_{22}(t), x_{22}(t - \tau_{21}(t))) &= -0.5 \times x_{22}(t) + 0.3 \times x_{22}(t - \tau_{21}(t)), \\ \sigma_3(t, x_{31}(t), x_{31}(t - \tau_{12}(t))) &= 0.1 \times x_{31}(t) + 0.1 \times x_{31}(t - \tau_{12}(t)), \\ \sigma_3(t, x_{32}(t), x_{32}(t - \tau_{21}(t))) &= 0.1 \times x_{32}(t) + 0.3 \times x_{32}(t - \tau_{21}(t)). \end{aligned}$$

It is shown that the system (5) is exponentially synchronized with the pinning schemes. Figure 5 illustrated the synchronization error of the state variables of system (12). It is obviously that the states are quickly converged to stable according to controller (30). Figure 5a indicated that the synchronization error under controller (13) with unsuitable parameters. Figure 5b illustrated that the synchronization error under controller (13) with suitable parameters. It can be seen, the parameters of the controller which unsatisfied the rule of synchronization will lead to instability and oscillation. Thus, it can be resulted that, the considered coupled MNNs system (5) can be exceptionally synchronized based on the pinning control due to Theorem 2.

In order to verify Corollary 2, we choose system (5) without Brownian motion as an example. We add the adaptive controller (30) on such system. Figure 6 demonstrated the synchronization errors of system (5) without stochastic perturbations under different parameters of controller. Figure 6a presented the synchronization error under controller (30) with unsuitable parameters. Figure 6b demonstrated the synchronization error under controller (30) with suitable parameters. The simulation experiments demonstrate that the proposed control laws are effective.

In order to show the validity of the pinning method, we choose system (5) as an example. Figure 7a illustrated the synchronization error of system (5) which selected only one node to be controlled under controller (30), Fig. 7b demonstrate the synchronization error of system

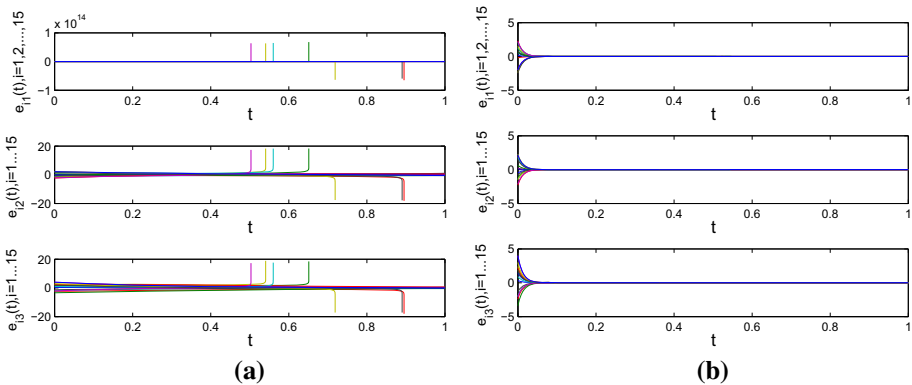


Fig. 5 a The synchronization error of system (5) without control; b The synchronization error of system (5) under control

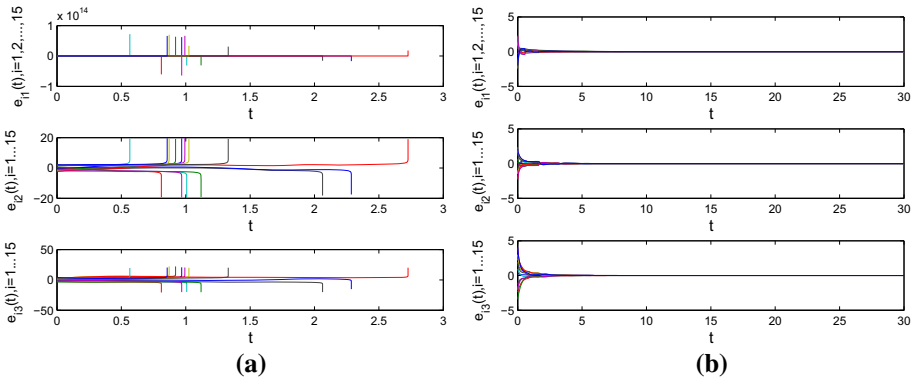


Fig. 6 **a** The synchronization error of system (5) without stochastic perturbations and control; **b** The synchronization error of system (5) without stochastic perturbations but under control

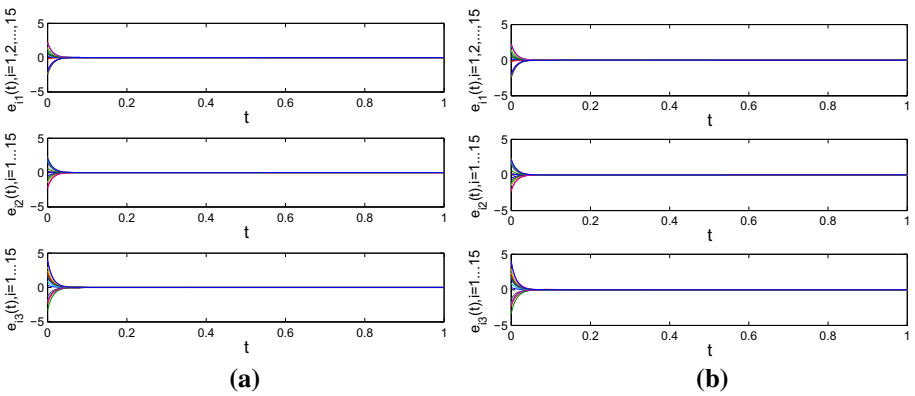


Fig. 7 **a** The synchronization error of system (5) with one controller node; **b** The synchronization error of system (5) with three controller nodes

(5) which selected all nodes to be controlled under controller (30). It can be seen that the rate of convergence and the accuracy between Fig. 7a, b are basically the same. Thus, the results proved that our mechanism is effective and reasonable. Especially for the practical application [52,53] our proposed scheme can maximize energy savings.

5 Conclusion

We have committed to research the pinning synchronization of coupled MNNs. The proposed coupled MNNs models including the stochastic perturbations, impulsive perturbations, non-delay, discrete time-varying delays and distributed time delays. Based on the suitable Lyapunov functional and the definition of pinning control, with the aid of inequality techniques and differential inclusions theory, two kinds of controllers are designed. And sufficient conditions which depend on the mixed time-varying delays are derived to guarantee the coupled MNNs achieve pinning synchronization. Numerical examples are provided to demonstrate the usefulness and effectiveness of the proposed control strategy.

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