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Finite-Time Stabilization of Neutral Hopfield Neural Networks with Mixed Delays

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Abstract In this article, we investigate the problem of finite time stabilization (FTS) of neutral Hopfield neural networks (NHNNs) with mixed delays including infinite distributed time delays. Firstly, general conditions on the control law are established to ensure the FTS of a neutral class of NN investigated here. Then, some specific conditions in the form of linear matrix inequalities which can be numerically checked are derived by constructing different kinds of controllers which include the delay-dependent and delay-free controller. Secondly, for practical applications, based on the Lyapunov–Krasovskii-functional analysis, we design a continuous controller able to stabilize in finite time the NHNNs and overcome the chattering phenomena simultaneously. Thirdly, the restriction of the boundedness of activation functions is removed. Finally, three numerical examples accompanied by graphical illustrations are given to illuminate our main results.

Keywords Hopfield neural networks · Finite time stabilization · Time delay systems · LMI · Neutral systems · Delay-free controller · Lyapunov function

1 Introduction

The neural network (NN) is a model inspired by biological mechanisms. It is currently considered as one of the best methods of sequential treatment and this explains its usefulness in many areas such that memory design, pattern recognition problems, generalized optimization problems, associative memories [4,5,20,27,34,59]. Theoretical considerations have shown that NN should be regarded as an information processing system [49]. Furthermore, Michel et al. reported in [49] that a detailed study of the stability of large-scale dynamical systems is

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required for the design of NN-based computer systems. Therefore, the study of the stability of NNs is of major interest for many applications such as the traveling salesman problem [26], the parallel-operating A/D converter [64] or the security of communications [61]. It is well known that the time delay often occurs in the implementation of NNs and causes a high complexity in their dynamical behaviors. It can create some oscillations and bifurcations which explain the intensity of research around the effect of the delays on the stability of NNs [2,6,7,9,35,37,39,80].

The usual stability analysis of NNs requires the asymptotic convergence. It can imply a large time for obtaining the desired precision which can exceed the scale of human operations. So, it can be interesting that the physical process achieves the convergence in a specific time for real applications. In this context, the concept of finite time stability (FTS) occurs naturally. The FTS means that the solutions of the system reach the equilibrium point in finite time. The time function indicating when the trajectories reach the equilibrium point, variously known as the settling-time, has a great importance in practice. Historically, Haimo was the first to publish an article about the stability in finite time in [23]. It was not until the late 90's that this theory developed by Bhat and Bernstein [12–14] has reached a certain maturity. The above results have been extended to a general class of systems, namely the non-autonomous class of differential equations, in [51]. In practice, this concept of FTS is encountered in control problems such that fixed-time observer [46,47], secure communication [53], finite-time output feedback stabilization of the double integrator [11] or the finite time attitude tracking for a spacecraft [18]. Recently, the FTS of NNs with discrete time delays has been widely investigated in [41–43,56,57,66–69]. On one hand, despite the fact that discrete time delays provide a good approximation for modeling the signal propagation in NNs, this advantage is no longer one when a large number of neurons is taken into account [77]. Indeed, NNs have a spatial extent because of parallel pathways [26]. This behavior of NNs renders the use of discrete time delays irrelevant [77]. Therefore, the concept of continuously distributed delay occurs naturally. On the other hand, the class of high-order NNs is more effective than the lower order class due to its faster convergence rate, higher fault tolerance and greater storage capacity [65]. Therefore, it has been widely used in many applications such as the resolution of optimization problems, identification of dynamical systems, robotics and other fields [22,29,52,55].

It should be pointed out that in practice many delayed NNs can be modeled as dynamical systems, named neutral systems, where the differential expression contains the derivative of the past state [3] such as controlled constrained manipulators [32]. Furthermore, it has been proved that it is difficult to characterize the properties of a neural reaction process without having information about the derivative of the past state that better models the dynamics of complex neural reactions [3]. The above discussion proves that it is significant to study high-order neutral Hopfield NNs with mixed delays.

Despite the design of many controllers for the finite time stabilization of different kinds of NNs, there is no general controller able to guarantee the finite time stabilization of a general class of NHNNs with mixed delays. When NNs are used for the resolution of optimization problems with constraints such that constrained linear programming problems [21], the activation functions which are modeled by diode-like exponential-type functions are unbounded [19] for dealing with the constraints [77]. So far, there is no result available yet to deal with the finite time stabilization problem of NHNNs with unbounded activation functions. Motivated by the above discussion, we investigate in our article the finite time stabilization problem for a general class of high-order NHNNs with infinite distributed delays and NHNNs with unbounded activation functions.

The contributions of this article is as follows:

- inspired by the results in [50] and the FTS theory of differential equations, new sufficient conditions are provided to ensure the finite time stabilization of high-order NHNNs with infinite distributed delays;
- two different kinds of finite time controller are built by using the LMI approach which include a delay-dependent controller for NHNNs with unbounded activation functions and a delay-free controller for high-order NHNNs with bounded activation functions which is more suited for real physical applications because the knowledge of the delays is not necessary.
- for better applications, based on the Lyapunov–Krasovskii-functional (LKF) analysis, a non-chattering controller is designed to stabilize in finite time the NHNNs with unbounded activation functions.

The rest of the article is organized as follows. In Sect. 2, some preliminaries useful for the study of a class of NHNNs are provided. The finite time stabilization of a class of NHNNs is studied in Sect. 3 where different kinds of FTS controller are designed. Then, three examples are given in Sect. 4 to prove the feasibility and the effectiveness of our theoretical results. Finally, some concluding remarks and open problem are addressed in Sect. 5.

2 Preliminaries

Throughout this paper, we use the following notations:

- $\mathbf{C}([a,\ b],\ \mathbb{R}^n)$ stands for the space of the continuous functions $\phi\ :[a,\ b]\rightarrow\mathbb{R}^n$ equipped with the uniform norm $\|\phi\|=\sup_{a\leq s\leq b}\|\phi(s)\|$;
- a function $\nu\ :\mathbb{R}_+\rightarrow\mathbb{R}_+$ belongs to the class $\mathscr{K}$ if $\nu(0)=0$, ν is continuous and strictly increasing;
- $\langle .,.\rangle$ stands for the inner product of Euclidean space;
- $\lambda_{max}(A)$ and $\lambda_{min}(A)$ stand for the maximum eigenvalue of A and the minimum eigenvalue of A respectively;
- $C_c^{\infty}(\mathbb{R}^n)$ stands for the space of bump functions;
- I_n stands for the $n-$dimensional identity matrix.

Now, we consider the following NHNN with mixed delays:

$$\begin{cases}\dot{x}_i(t)=-c_ix_i(t)+\sum\limits_{j=1}^{n}a_{ij}f_j\left(x_j(t)\right)+\sum\limits_{j=1}^{n}b_{ij}f_j\left(x_j(t-\tau(t))\right)\\ \qquad\qquad+\sum\limits_{j=1}^{n}e_{ij}\int\limits_{-\infty}^{t}k_j(t-s)g_j\left(x_j(s)\right)\mathrm{d}s\\ \qquad\qquad+\sum\limits_{j=1}^{n}d_{ij}h_j\left(\dot{x}_j\left(t-\sigma(t)\right)\right)+J_i,\quad t>0\\ x(s)=\phi(s),\quad s\in(-\infty,\ 0]\end{cases}\tag{1}$$

where $x(t)=(x_1(t),\ \ldots,\ x_n(t))^T$ stands for the neuron state; $c_i>0$, a_{ij}, b_{ij}, e_{ij} and d_{ij} stand for the interconnection weight coefficients of the neurons; $K(.)=diag\big(k_1(.),\ \ldots\ ,\ k_n(.)\big)$ and $J=(J_1,..,J_n)^T$ stand for the delay kernel and an external input vector respectively; the continuous activation functions f_j, g_j and h_j satisfies $f_j(0)=g_j(0)=h_j(0)=0$; $\tau(.)$ and $\sigma(.)$ stand for the time-varying transmission delays with $0\leq\tau(t)\leq\overline{\tau}$, $0\leq\sigma(t)\leq\overline{\sigma}$ and $\dot{\sigma}(t)\leq\sigma^*<1$. The initial condition

$\phi \in \mathbf{C}_b^1((-\infty,\ 0], \mathbb{R}^n)$ where $\mathbf{C}_b^1((-\infty,\ 0], \mathbb{R}^n)$ is the space of the continuous and bounded functions equipped with the following norm

$$\|\phi\|_{\overline{\sigma}} = \max\left\{\sup_{s\leq 0}\|\phi(s)\|,\quad \sup_{-\overline{\sigma}\leq s\leq 0}\|\dot{\phi}(s)\|\right\}.$$

Remark 1 Compared with [34,62,70,78,80], system (1) is more general. In fact, the delays considered here contain an infinite distributed delay which causes a problem for the choice of an admissible Banach [6]. Furthermore, the neutral term in the NN investigated here renders the system more realistic because it is difficult to characterize the properties of a neural reaction process without having information about the derivative of the past state [3].

Let us introduce the following assumptions:

($\mathbf{H_1}$) there exist constants $l_{i_j}^-,\ l_{i_j}^+,\ i = 1, 2, 3$ such that

$$l_{1_j}^- \leq \frac{|f_j(x) - f_j(y)|}{x - y} \leq l_{1_j}^+,\ l_{2_j}^- \leq \frac{|g_j(x) - g_j(y)|}{x - y} \leq l_{2_j}^+,$$
$$l_{3_j}^- \leq \frac{|h_j(x) - h_j(y)|}{x - y} \leq l_{3_j}^+$$

for all $x,\ y \in \mathbb{R}$ and $j = 1, \ldots, n$;

($\mathbf{H_2}$) for $j = 1,\ \ldots,\ n$, the delay kernels $k_j : \mathbb{R}_+ \to \mathbb{R}_+$, are bump functions satisfying

$$\int_0^{+\infty} k_j(s)\mathrm{d}s = \mathbf{k}_j.$$

Remark 2 Under assumptions ($\mathbf{H_1}$) and ($\mathbf{H_2}$), the existence of solutions of the system (1) is ensured as it is explained in [28]. However, the theorems of continuation useful for defining the asymptotic stability and the FTS are not easy for neutral systems, see for instance Theorem 2.4 (page 278) in [24] and [25]. It should be pointed out that, in the assumption (H_1), the constants l_j^+ and l^j can be negative or positive and then (H_1) allows Lipschitz functions if we take $l_j^- = -l_j^+ < 0$. Therefore (H_1) is more general and weaker than the Lipschitz condition.

Some useful lemmas and definitions are given below.

Lemma 1 [66] *If* $a_1, \ldots, a_n,\ r_1,\ r_2 \in \mathbb{R}$ *with* $0 < r_1 < r_2$, *then the following inequality holds*

$$\left[\sum_{i=1}^n |a_i|^{r_2}\right]^{\frac{1}{r_2}} \leq \left[\sum_{i=1}^n |a_i|^{r_1}\right]^{\frac{1}{r_1}}.$$

Let Ω be an open subset of $\mathbf{C}_b^1((-\infty,\ 0], \mathbb{R}^n)$ such that $0 \in \Omega$.

Definition 1 [50] The equilibrium point, if it exists, of system (1) is finite time stable (FTS) if:

(i) the equilibrium of system (1) is Lyapunov stable;
(ii) for any state $\phi(s) \in \Omega$, there exists $0 \leq T(\phi) < +\infty$ such that every solution of system (1) satisfies $x(t,\ \phi) = 0$ for all $t \geq T(\phi)$.

The functional

$$T_0(\phi) = \inf\left\{T(\phi) \geq 0 \;:\; x(t,\ \phi) = 0, \quad \forall t \geq T(\phi)\right\}$$

is called the settling-time of system (1).

Now, we introduce the following notations:

$$\begin{aligned}
&l_{1_i} = \max\left\{|l_{1_i}^-|,\ |l_{1_i}^+|\right\}, \quad l_{2_i} = \max\left\{|l_{2_i}^-|,\ |l_{2_i}^+|\right\}, \quad l_{3_i} = \max\left\{|l_{3_i}^-|,\ |l_{3_i}^+|\right\};\\
&L_1 = diag\left(l_{1_1},\ \ldots,\ l_{1_n}\right), \quad L_2 = diag\left(l_{2_1},\ \ldots,\ l_{2_n}\right), \quad L_3 = diag\left(l_{3_1},\ \ldots,\ l_{3_n}\right);\\
&A = \left(a_{ij}\right)_{n\times n}, \quad B = \left(b_{ij}\right)_{n\times n}, C = diag\left(c_1, \ldots, c_n\right), \quad D = \left(d_{ij}\right)_{n\times n};\\
&E = \left(e_{ij}\right)_{n\times n}, \quad \mathbf{K} = diag(\mathbf{k}_1,\ \ldots,\ \mathbf{k}_n), \quad 1 \leq i, j \leq n;\\
&f(x) = (f_1(x_1), \ldots, f_n(x_n))^T, \ g(x) = (g_1(x_1), \ldots, g_n(x_n))^T,\\
&h(x) = (h_1(x_1), \ldots, h_n(x_n))^T;
\end{aligned}$$

and the operator

$$\mathscr{D}\phi = \phi(0) - D\ h(\phi(-\sigma(.))). \tag{2}$$

The following lemma is an extension of [50, Proposition 4].

Lemma 2 *If there exist three functions ν_1, ν_2 and r of class $\mathscr{K}$ and a continuous functional $V : \Omega \to \mathbb{R}_+$ such that:*

(i) $\nu_1\left(\|\mathscr{D}\phi\|\right) \leq V(\phi) \leq \nu_2(\|\phi\|)$;

(ii) $\dot{V}(\phi) \leq -r\left(V(\phi)\right)$ *with* $\int_0^{\varepsilon} \frac{dz}{r(z)} < \infty, \quad \forall\varepsilon > 0,\ \phi \in \Omega$;

then system (1) is FTS with a settling-time satisfying the inequality $T_0(\phi) \leq \int_0^{V(\phi)} \frac{dz}{r(z)}$. *In particular, if* $r(V) = \lambda V^{\rho}$ *where* $\lambda > 0,\ \rho \in (0,\ 1)$, *then the settling-time satisfies the inequality*

$$T_0(\phi) \leq \int_0^{V(\phi)} \frac{dz}{r(z)} = \frac{V^{1-\rho}(0,\ \phi)}{\lambda(1-\rho)}. \tag{3}$$

Proof Theorem 4.1 (page 287) in [24] implies that the operator $\mathscr{D}\phi = \phi(0) - D\ h(\phi(-\sigma(.)))$ is stable. Then, Theorem 7.1 (page 297) in [24] ensure that the system (1) is asymptotically stable. Moreover, under the conditions of Lemma 2, the second part of the proof of Proposition 4 in [50] remains valid for system (1) which achieves the proof. □

Remark 3 In [40,58,60,71,81], the FTS is studied for NNs with mixed delays but without involving a neutral term. In our work, some results are given for the FTS of a class of neutral NNs with mixed delays. Thus, our results extend and complement the previous works.

In the next lemma, we give sufficient conditions in the form of LMIs that ensure the existence and uniqueness of an equilibrium point of system (1)

Lemma 3 *Under assumptions* $(\mathbf{H_1}) - (\mathbf{H_2})$, *if there exist a matrix* $P > 0$, *two positive scalars* ε_i, $i = 1, 2$ *and two diagonal matrices* R_1 *and* R_2 *such that the following LMI holds:*

$$\Pi = \begin{pmatrix} \Pi_{11} & PA + PB & PE\mathbf{K} \\ * & -\varepsilon_1 R_1 & 0 \\ * & * & -\varepsilon_2 R_2 \end{pmatrix} < 0 \tag{4}$$

where

$$\Pi_{11} = -PC - CP + \varepsilon_1 L_1^T R_1 L_1 + \varepsilon_2 L_2^T R_2 L_2$$

then system (1) *has an unique equilibrium point.*

Proof In order to establish the existence and uniqueness of the equilibrium point of system (1) based on the homomorphism theory, we consider the following map

$$H(x) = -Cx + (A + B) f(x) + E\mathbf{K}g(x) + J.$$

If x^* is an equilibrium point of system (1) then $H(x^*) = 0$. So, it is sufficient to prove that $H(x)$ is a homomorphism on $\mathbb{R}^n$ for proving the existence and uniqueness of the equilibrium point of system (1). If we replace A and B by $A + B$ and $E\mathbf{K}$ respectively in the proof of Theorem 1 in [15] we obtain immediately that system (1) has a unique equilibrium point which achieves the proof. □

Remark 4 It is clear that the condition (4) is not a standard LMI. It is worth noting that if we fix the parameters $\varepsilon_i,\ i = 1, 2$ then the inequality (4) can be transformed into a standard LMI and then can be easily solved by using the Matlab LMI toolbox.

3 Main Results

In this section, the finite time stabilization of NHNNs with mixed delays is considered. Assume that $x^* = (x_1^*, \ \ldots, \ x_n^*)^T$ is an equilibrium point of system (1). By a simple transformation

$$z(t) = x(t) - x^*$$

we can shift the equilibrium point x^* to the origin. If in addition we add the control variable $u \in \mathbb{R}^n$, system (1) can be rewritten in this $z-$form as follows

$$\begin{cases} \dot{z}(t) = -Cz(t) + A\ F(z(t)) + B\ F(z(t - \tau(t))) + E \int\limits_{-\infty}^{t} K(t-s)\ G(z(s))\mathrm{d}s \\ \qquad +D\ H(\dot{z}(t - \sigma(t))) + u, \quad t > 0, \\ z(s) = \phi(s) - x^*, \quad s \in (-\infty,\ 0] \end{cases} \tag{5}$$

where $u = (u_1, \ldots, u_n)^T$ and

$$F(z) = f(z + x^*) - f(x^*),\ G(z) = g(z + x^*) - g(x^*);$$
$$H(z) = h(z + x^*) - h(x^*),\ K(.) = diag\,(k_1(.),\ \ldots\ ,\ k_n(.))\,.$$

The state feedback control is supposed to be of the following form

$$u(z) = \overline{u}(z) + \check{u}(z) \tag{6}$$

where

$$\overline{u}(z) = (\overline{u}_1\,(z)\,, \ldots,\ \overline{u}_n\,(z))^T\,, \quad \check{u}(z) = \left(\check{u}_1\,(z)\,, \ldots,\ \check{u}_n\,(z)\right)^T.$$

Now, sufficient general conditions on the state feedback control are established to ensure the finite time stability of the closed-loop system (5)–(6).

Theorem 1 *Under conditions of Lemma 3, if there exist three symmetric positive definite matrices* $P,\ Q_1$ *and* Q_2 *and positive constants* $\varepsilon,\ 0<\mu<1$ *and* δ *such that*

$$-PC - CP + \varepsilon^{-1} PAQ_1^{-1}A^T P + \varepsilon L_1^T Q_1 L_1 - Q_2 < 0; \tag{7}$$

$$\begin{aligned}&\langle PB|F(z(t-\tau(t)))|,\ |z(t)|\rangle + \langle PD|H(\dot z(t-\sigma(t)))|,\ |z(t)|\rangle\\&+\left\langle PE\int_{-\infty}^{t} K(t-s)\,|G(z(s))|\,ds,\ |z(t)|\right\rangle\\&+z(t)^T\ P\ \overline{u}(z(t)) \le -\frac{1}{2} z(t)^T Q_2 z(t);\end{aligned} \tag{8}$$

$$z(t)^T P\ \check{u}(z(t)) \le -\frac{\delta}{2}\sum_{i=1}^{n} |z_i(t)|\mu + 1; \tag{9}$$

then the closed-loop system (5)–(6) is FTS and the settling-time satisfies

$$T_0(\phi) \le \frac{2\lambda_{\max}(P)\|\phi\|^{1-\mu}}{\delta(1-\mu)}.$$

Proof Consider the Lyapunov function

$$V(z(t)) = z(t)^T Pz(t). \tag{10}$$

Calculating the derivative of (10) along the trajectories of the closed-loop system (5)–(6), we obtain

$$\begin{aligned}\dot V(t) &= 2z(t)^T P\dot z(t)\\&\le -z(t)^T(PC+CP)z(t) + 2\langle PD|H(\dot z(t-\sigma(t)))|,\ |z(t)|\rangle\\&\quad+2\langle PA|F(z(t))|,\ |z(t)|\rangle + 2\langle PB|F(z(t-\tau(t)))|,\ |z(t)|\rangle\\&\quad+2\left\langle PE\int_{-\infty}^{t} K(t-s)|G(z(s))|ds,\ |z(t)|\right\rangle + 2z(t)^T P\ u(z(t))\end{aligned} \tag{11}$$

Since

$$2\langle PA|F(z(t))|,\ |z(t)|\rangle \le \varepsilon^{-1} z(t)^T PAQ_1^{-1}A^T P\ z(t) + \varepsilon F(z(t))^T Q_1\ F(z(t)). \tag{12}$$

It follows from (7)–(12) that

$$\begin{aligned}\dot V(t) &\le z(t)^T\left[-PC - CP + \varepsilon^{-1}PAQ_1^{-1}A^T P + \varepsilon L_1^T Q_1 L_1 - Q_2\right] z(t)\\&\quad+2z(t)^T P\ \check{u}(z(t))\\&\le 2z(t)^T P\ \check{u}(z(t)) \le -\delta\sum_{i=1}^{n}|z_i(t)|^{\mu+1}\end{aligned} \tag{13}$$

Since $0<\mu<1$, we get the following inequality

$$-\left[\sum_{i=1}^{n}|z_i(t)|^{\mu+1}\right]^{\frac{1}{\mu+1}} \le -\left[\sum_{i=1}^{n}|z_i(t)|^2\right]^{\frac{1}{2}} \tag{14}$$

from Lemma 1. So, we obtain

$$\dot V(t) \le -r(V(t))$$

where

$$r(s) = \frac{\delta}{\lambda_{max}(P)^{\frac{\mu+1}{2}}} s^{\frac{\mu+1}{2}}.$$

Since

$$\int_0^{\varepsilon} \frac{ds}{r(s)} = \frac{2\varepsilon^{\frac{1-\mu}{2}}}{\delta\lambda_{\max}^{-\frac{(1+\mu)}{2}}(P)(1-\mu)} < +\infty \tag{15}$$

for all $\varepsilon > 0$ and the condition (i) in Lemma 2 is ensured from ($\mathbf{H_1}$), we obtain from Lemma 2 that the closed-loop system (5)–(6) is FTS and $T_0(\phi)$ satisfies

$$T_0(\phi) \leq \frac{2V^{\frac{1-\mu}{2}}(0)}{\delta\lambda_{\max}^{-\frac{(1+\mu)}{2}}(P)(1-\mu)} \leq \frac{2\lambda_{\max}(P)\|\phi\|^{1-\mu}}{\delta(1-\mu)}.$$

□

Remark 5 The results obtained in [40,58,60,71,74,76,79,81] use the L_1-norm and fail for the L_2-norm [76]. As $L_2 \subset L_1$, the settling-time obtained in our work may be smaller than that given in previous works which proves the advantage of our results.

In the following, an explicit state feedback control will be designed.

3.1 Finite Time Stabilization via a Delay-Dependent Controller

In this subsection, we develop some theoretical results of finite time stabilization of system (5) where we design a state feedback control able to ensure the FTS of a class of NHNNs with infinite distributed delay and unbounded activation functions.

Theorem 2 *Under conditions of Lemma 3, if there exist constants $\varepsilon > 0,\ \alpha_1 > 0$ and two symmetric matrices $P > 0,\ Q_1 > 0$, such that*

$$-PC - CP + \varepsilon^{-1}PAQ_1^{-1}A^TP + \varepsilon L_1^TQ_1L_1 - 2\alpha_1P < 0 \tag{16}$$

then the closed-loop system (5)–(17) is FTS where

$$u(z(t)) = -\alpha_1 z(t) - BL_1\, sign(z(t))\, |z(t-\tau(t))| - EL_2\, sign(z(t)) \int_{-\infty}^{t} K(t-s)\, |z(s)| ds$$
$$- DL_3\, sign(z(t))|\dot{z}(t-\sigma(t))| - \alpha_2\, sign(z(t))|z(t)|^{\mu} \tag{17}$$

with $0 \leq \mu < 1,\ \alpha_2 > 0$ and the settling-time satisfies

$$T_0(\phi) \leq \frac{\lambda_{\max}(P)\|\phi\|^{1-\mu}}{\alpha_2\lambda_{\min}(P)(1-\mu)}.$$

Proof Let

$$\overline{u}(z(t)) = -\alpha_1 z(t) - BL_1\, sign(z(t))\, |z(t-\tau(t))| - EL_2\, sign(z(t)) \int_{-\infty}^{t} K(t-s)\, |z(s)| \mathrm{d}s$$
$$- DL_3\, sign(z(t))\, |\dot{z}(t-\sigma(t))|,$$
$$\check{u}(z(t)) = -\alpha_2\, sign(z(t))|z(t)|^{\mu}.$$

From $(\mathbf{H_1})$, we have

$$\langle PB\ |F(z(t-\tau(t)))|\ ,\ |z(t)|\rangle + \langle PD\ |H(\dot{z}(t-\sigma(t)))|\ ,\ |z(t)|\rangle + z(t)^T P\ \overline{u}(z(t)) + \left\langle PE \int_{-\infty}^{t} K(t-s)|\ G(z(s)|ds,\ |z(t)|\right\rangle \leq -\alpha_1 z(t)^T P\ z(t)$$

and

$$2z(t)^T P\ \check{u}(z(t)) = -2\alpha_2 z(t)^T P\ sign(z(t))|z(t)|^{\mu} \leq -2\alpha_2\lambda_{\min}(P)\sum_{i=1}^{n}|z_i(t)|^{\mu+1}.$$

Therefore, by taking $Q_2 = 2\alpha_1 I_n$ and $\delta = 2\lambda_{\min}(P)\alpha_2$, we are in the conditions of application of Theorem 1 and this achieves the proof. □

Remark 6 The controller (17) can only be used if the time-varying delays $\tau(t)$ and $\sigma(t)$ are known which is not an easy task in practice [30]. This is the reason why we develop in Sect. 3.2 a delay-free controller when the activation functions are supposed to be bounded.

Corollary 1 *Under conditions of Lemma 3, if there exist positive constants $\varepsilon,\ \mu < 1,\ \alpha_1$ and p such that*

$$-p(C+C^T) + \varepsilon^{-1}p^2A^TA + \varepsilon L_1^T L_1 - 2\alpha_1 p < 0 \tag{18}$$

then the closed-loop system (5)–(17) is FTS. Moreover, the settling-time satisfies

$$T_0(\phi) \leq \frac{\|\phi\|^{1-\mu}}{\alpha_2(1-\mu)}.$$

If we set $P = pI_n$, the proof of Corollary 1 is straightforward and thus it is omitted.

Remark 7 The conditions of Corollary 1 are less conservative than that established in [63,72–76]. Indeed, the settling-time obtained in our work is independent of p and consequently the same approximation of the settling-time can be conserved equipped with less conservative conditions than the above-mentioned results.

The approach given in [67] for studying the concept of FTS of NNs requires not only the Lipschitz condition of the activation functions but also the boundedness of these functions. For removing these restrictions and improving the results given in [67], we establish the following corollary where the activation functions are not necessary bounded.

Corollary 2 *Under assumptions $(\mathbf{H_1}) - (\mathbf{H_2})$, if there exist a matrix $P > 0$, non negative scalars $\varepsilon,\ \varepsilon_i,\ i = 1, 2,\ p,\ \alpha_1$ and two diagonal positive matrices R_1 and R_2 such that*

$$\Psi = \begin{pmatrix} \Psi_{11} & pA & \varepsilon L_1^T & 0 & 0 & 0 \\ * & -\varepsilon I_n & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon I_n & 0 & 0 & 0 \\ * & * & * & \Pi_{11} & PA+PB & PE\mathbf{K} \\ * & * & * & * & -\varepsilon_1 R_1 & 0 \\ * & * & * & * & * & -\varepsilon_1 R_2 \end{pmatrix} < 0 \tag{19}$$

with

$$\Psi_{11} = -p(C+C^T) - 2p\alpha_1 I_n$$

then system (1) *has an unique equilibrium point and the closed-loop system* (5)–(17) *is FTS and the settling-time satisfies*

$$T_0(\phi) \le \frac{\|\phi\|^{1-\mu}}{\alpha_2(1-\mu)}.$$

Proof Let

$$\Xi = \begin{pmatrix} -p(C+C^T) - 2p\alpha_1 I_n & pA & \varepsilon L_1^T \\ * & -\varepsilon I_n & 0 \\ * & * & -\varepsilon I_n \end{pmatrix} \tag{20}$$

Since $\Psi = diag(\Xi, \ \Pi) < 0$, we have $\Pi < 0$ and consequently system (1) has an unique equilibrium point.

Furthermore, by pre and post multiplying the inequality (18) by $diag(I_n, \frac{1}{\sqrt{\varepsilon}} I_n, \frac{1}{\sqrt{\varepsilon}} I_n)$ we obtain from Schur complement Lemma that $\Xi < 0$ is equivalent to (18) which achieves the proof. □

Remark 8 It should be pointed out that the inequality (16) of Theorem 2 is not linear and consequently difficult to solve. However, Corollary 2 uses the inequality (19) to determine the control gain α_1 which can be turned into a LMI by:

1. letting $\alpha = p\alpha_1$ in (19);
2. finding α and p by solving the LMI with the Matlab LMI Toolbox;
3. deducing the value α_1.

Remark 9 In [78], stabilization of NNs was investigated but the systems are without delay. On the one hand, the class of delayed NNs have more complex dynamic behaviors compared with NNs without delay [35,36,38]. On the other hand, it is delicate to design a Lyapunov functional satisfying the derivative condition for FTS of delayed system [50]. In our article, the stabilization of NNs with mixed delays is investigated which renders the results more general compared with the above-mentioned ones.

3.2 Finite Time Stabilization via a Delay-Free Controller

In this subsection, we apply the theoretical results of Sect. 3.1 for the design of a delay-free controller able to stabilize in finite time system (5) and the following high-order NHNN

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum\limits_{j=1}^n a_{ij} f_j\left(x_j(t)\right) + \sum\limits_{j=1}^n \sum\limits_{k=1}^n T_{ijk} f_k\left(x_k\left(t-\tau(t)\right)\right) f_j\left(x_j\left(t-\tau(t)\right)\right) \\ \qquad + \sum\limits_{j=1}^n b_{ij} f_j\left(x_j\left(t-\tau(t)\right)\right) + \sum\limits_{j=1}^n d_{ij} h_j\left(\dot{x}_j\left(t-\sigma(t)\right)\right) \\ \qquad + \sum\limits_{j=1}^n e_{ij} \int\limits_{-\infty}^t k_j(t-s) g_j\left(x_j(s)\right) ds + u_i, \\ x(s) = \psi(s), \quad s \in (-\infty, \ 0] \end{cases} \tag{21}$$

where T_{ijk} stand for the second-order synaptic weights.

Let use introduce the following assumption:

($\mathbf{H_3}$) there exist constants ω_{1_i}, ω_{2_i} and ω_{3_i} such that

$$|f_i(x)| \le \omega_{1_i}, \ |g_i(x)| \le \omega_{2_i}, |h_i(x)| \le \omega_{3_i}, \quad i = 1, \ldots, n.$$

Let us denote

$$\omega_1 = \max_{1\le i\le n}\{\omega_{1_i}\},\quad \omega_2 = \max_{1\le i\le n}\{\omega_{2_i}\},\quad \omega_3 = \max_{1\le i\le n}\{\omega_{3_i}\}$$
$$\Omega_1 = diag(\omega_1,\ \ldots,\ \omega_1),\quad \Omega_2 = diag(\omega_2,\ \ldots,\ \omega_2),\ \Omega_3 = diag(\omega_3,\ \ldots,\ \omega_3)$$

and $x^* = (x_1^*,\ \ldots,\ x_n^*)^T$ an equilibrium point of system (21) if it exists. By a simple transformation $z(t) = x(t) - x^*$, we can shift the equilibrium point x^* to the origin. Thus, system (21) with $u_i = 0$ leads to

$$\begin{aligned}
\dot{z}_i(t) = &-c_i z_i(t) + \sum_{j=1}^n a_{ij} F_j\left(z_j(t)\right) + \sum_{j=1}^n b_{ij} F_j\left(z_j(t-\tau(t))\right)\\
&+\sum_{j=1}^n\sum_{k=1}^n T_{ijk}\Big[\big[f_j\left(x_j(t-\tau(t))\right)\\
&- f_j(x_j^*)\big] f_k\left(x_k(t-\tau(t))\right) + f_j(x_j^*)\big[f_k\left(x_k(t-\tau(t))\right) - f_k(x_k^*)\big]\Big]\\
&+\sum_{j=1}^n d_{ij} H_j(\dot{z}_j(t-h(t))) + \sum_{j=1}^n e_{ij}\int_{-\infty}^t k_j(t-s) H_j\left(z_j(s)\right)\mathrm{d}s\\
= &-c_i z_i(t) + \sum_{j=1}^n a_{ij} F_j\left(z_j(t)\right) + \sum_{j=1}^n b_{ij} F_j\left(z_j(t-\tau(t))\right)\\
&+\sum_{j=1}^n\sum_{k=1}^n T_{ijk}\Big[F_j\left(z_j(t-\tau(t))\right) f_k\left(x_k(t-\tau(t))\right) + f_j(x_j^*) F_k\left(z_k(t-\tau(t))\right)\Big]\\
&+\sum_{j=1}^n d_{ij} H_j(\dot{z}_j(t-h(t))) + \sum_{j=1}^n e_{ij}\int_{-\infty}^t k_j(t-s) G_j\left(z_j(s)\right)\mathrm{d}s\\
= &-c_i z_i(t) + \sum_{j=1}^n a_{ij} F_j\left(z_j(t)\right) + \sum_{j=1}^n \Big[b_{ij} + \sum_{k=1}^n \Big(T_{ijk} f_k\left(x_k(t-\tau(t))\right)\\
&+ T_{ikj} f_k(x_k^*)\Big)\Big] F_j\left(z_j(t-\tau(t))\right)\\
&+\sum_{j=1}^n d_{ij}\dot{z}_j(t-h(t)) + \sum_{j=1}^n e_{ij}\int_{-\infty}^t k_j(t-s) F_j\left(z_j(s)\right)\mathrm{d}s\\
= &-c_i z_i(t-\sigma(t)) + \sum_{j=1}^n a_{ij} F_j\left(z_j(t)\right) + \sum_{j=1}^n \Big[b_{ij}\\
&+\sum_{k=1}^n\Big(T_{ijk} + T_{ikj}\Big)\xi_{ijk}\Big] F_j\left(z_j(t-\tau(t))\right)\\
&+\sum_{j=1}^n d_{ij} H_j(\dot{z}_j(t-h(t))) + \sum_{j=1}^n e_{ij}\int_{-\infty}^t k_j(t-s) G_j\left(z_j(s)\right)\mathrm{d}s
\end{aligned}$$

where

$$\xi_{ijk} = \begin{cases}\frac{T_{ijk}}{T_{ijk}+T_{ikj}} f_k\left(x_k(t-\tau(t)\right) + \frac{T_{ikj}}{T_{ijk}+T_{ikj}} f_k(x_k^*) & \text{if } T_{ijk}+T_{ikj}\neq 0\\ 0 & \text{if } T_{ijk}+T_{ikj}\neq 0\end{cases}$$

Therefore, the z-form of system (21) can be written as follows

$$\begin{cases} \dot{z}(t) = -Cz(t) + A\ F(z(t)) + \Gamma^T T^*\ F(z(t-\tau(t))) + B\ F(z(t-\tau(t)) \\ \qquad\qquad +D\ H(\dot{z}(t-\sigma(t))) + E \int\limits_{-\infty}^{t} K(t-s)\ G(z(s))ds + u \\ z(s) = \psi(s) - x^*, \quad s \in (-\infty,\ 0] \end{cases} \tag{22}$$

where $u = (u_1, \ldots, u_n)^T$ and

$$T_i = [T_{ijk}]_{n\times n},\ T^* = [T_1 + T_1^T,\ \ldots,\ T_n + T_n^T]^T,\ \xi_{ij} = [\xi_{ij1},\ \ldots,\ \xi_{ijn}]^T;$$
$$\xi_i = [\xi_{i1}^T,\ \ldots,\ \xi_{in}^T]^T,\ \Gamma = [\xi_1,\ \ldots,\ \xi_n]^T.$$

We can now state the main result of this subsection.

Theorem 3 *Under assumptions* $(\mathbf{H_1}) - (\mathbf{H_2}) - (\mathbf{H_3})$*, if there exist positive scalars* $\varepsilon,\ p,\ \alpha_1$ *such that the following LMI holds:*

$$\Omega = \begin{pmatrix} -p(C+C^T) - 2p\alpha_1 I_n & pA & \varepsilon L_1^T \\ * & -\varepsilon I_n & 0 \\ * & * & -\varepsilon I_n \end{pmatrix} < 0 \tag{23}$$

then the closed-loop system (22)–(24) *is FTS where*

$$u(z(t)) = -\alpha_1 z(t) - (B + T^*\Omega_1)\Omega_1\ sign(z(t)) - E\mathbf{K}\Omega_2\ sign(z(t)) - D\Omega_3\ sign(z(t)) \\ - \alpha_2\ sign(z(t))|z(t)|^{\mu} \tag{24}$$

with the settling-time satisfies

$$T_0(\phi) \le \frac{\|\phi\|^{1-\mu}}{\alpha_2(1-\mu)}.$$

Proof Let

$$\overline{u}(z(t)) = -\alpha_1 z(t) - (B + T^*\Omega_1)\Omega_1\ sign(z(t)) - E\mathbf{K}\Omega_2\ sign(z(t)) - D\Omega_3\ sign(z(t))$$
$$\check{u}(z(t)) = -\alpha_2\ sign(z(t))|z(t)|^{\mu}$$

From $(\mathbf{H_3})$, we have

$$\Big\langle P(B + \Gamma^T T^*)\ |F(z(t-\tau(t)))|,\ |z(t)|\Big\rangle \\ + \langle PD\ |H(\dot{z}(t-\sigma(t)))|,\ |z(t)|\rangle + z(t)^T P\ \overline{u}(z(t)) \\ + \left\langle PE \int\limits_{-\infty}^{t} K(t-s)\ G(z(s))ds,\ |z(t)| \right\rangle \le -\alpha_1 z(t)^T P\ z(t)$$

and

$$2z(t)^T P\ \check{u}(z(t)) = -2\alpha_2 z(t)^T P\ sign(z(t))|z(t)|^{\mu} \\ \le -2\alpha_2 \lambda_{\min}(P) \sum_{i=1}^{n} |z_i(t)|^{\mu+1}.$$

The rest of the proof is similar to the proof of Theorem 2. □

Remark 10 The criterion given in [40,58,60,71–74,76,79,81] that ensures the FTS of NNs requires the boundedness of the derivative of the time-varying delay and fails when the time-varying delay is not differentiable even without neutral delay. The results given in our article overcome these difficulties and remove this restriction because the NNs studied are subjected to non differentiable time-varying delays which proves the advantage of our approach.

Remark 11 Thanks to their ability to solve optimization problems, many results around the stability of lower order class of NNs are established [8,48]. However, the authors of [17] have proved that this class of NNs can lead to the poorest quality of solution with a large complexity as determined by the order of the NNs. Thus, Theorem 3 can also be considered as a basis for the construction of neutral high-order NNs with infinite distributed delays more effective in the resolution of optimization problems thanks to the second order synaptic terms T_{ijk} [6].

Now, based on 1-norm analytical approach, the assumption $(H3)$ is removed and a new delay-free controller is designed to ensure the FTS of system (5) for the unbounded case where we impose the following assumption:

($\mathbf{H_4}$) There are positive constants $\bar{\tau}_1,\ \bar{\tau}_2,\ \bar{\sigma}_1$ and $\bar{\sigma}_2$ such that $\tau(.) \leq \bar{\tau}_1,\ \dot{\tau}(.) \leq \bar{\tau}_2 < 1$ and $\sigma(.) \leq \bar{\sigma}_1,\ \dot{\sigma}(.) \leq \bar{\sigma}_2 < 1$

The delay-free controller is constructed as follows:

$$u_i(z(t)) = -\lambda_{1_i} z_i(t) - \lambda_{2_i}\ \dot{z_i}(t) - \lambda_{3_i}\ sign(z_i(t)) \tag{25}$$

where $\lambda_{k_i},\ k = 1, 2, 3,\ i = 1, \ldots n$ stand for the control strength to be determined.

Theorem 4 *Under assumptions* $(\mathbf{H_1}) - (\mathbf{H_2})$ *and* $(\mathbf{H_4})$*, if* $\lambda_{3_i} > 0$ *and* $\lambda_{1_i},\ \lambda_{2_i}$ *satisfy the following inequalities*

$$\lambda_{1_i} > -c_i + \sum_{j=1}^{n} |a_{ji}||l_1^i| + \sum_{j=1}^{n} \frac{1}{1-\bar{\tau}_2} |b_{ji}||l_1^i| + \sum_{j=1}^{n} \mathbf{k}_j |e_{ji}||l_2^i|; \tag{26}$$

$$\lambda_{2_i} > \sum_{j=1}^{n} \frac{1}{1-\bar{\sigma}_2} |d_{ji}||l_3^i| \tag{27}$$

then the closed-loop system (5)–(25) *is FTS.*

The proof of Theorem 4 is inspired by the proof of Theorem 1 in [71,74]

Proof Consider the Lyapunov–Krasovskii functional as follows:

$$V(z_t) = \sum_{i=1}^{4} V_i(z_t). \tag{28}$$

where

$$V_1(z_t) = \sum_{i=1}^{n} |z_i(t)|;$$

$$V_2(z_t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{1-\bar{\tau}_2} |b_{ij}| l_1^j \int_{t-\tau(t)}^{t} |z_j(s)| \mathrm{d}s;$$

$$V_3(z_t) = \sum_{i=1}^{n}\sum_{j=1}^{n} |e_{ij}| l_2^j \int_{-\infty}^{0}\int_{t+s}^{t} k_j(-s)|z_j(u)|\mathrm{d}u\mathrm{d}s;$$

$$V_4(z_t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\bar{\sigma}_2}|d_{ij}|l_3^j| \int_{t-\sigma(t)}^{t} |\dot{z}_j(s)|\mathrm{d}s;$$

Calculating the derivative of (28) along the trajectories of the closed-loop system (5)–(25), we obtain

$$\begin{aligned}\dot{V}_1(z_t) = \sum_{i=1}^{n} sign(z_i(t))\Big[&- c_i z_i(t) + \sum_{j=1}^{n} a_{ij} f_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij} f_j\left(z_j(t-\tau(t))\right) \\ &+ \sum_{j=1}^{n} d_{ij} h_j\left(\dot{z}_j(t-\sigma(t))\right) + \sum_{j=1}^{n} e_{ij} \int_{-\infty}^{t} k_j(t-s) g_j\left(z_j(s)\right)\mathrm{d}s \\ &- \lambda_{1_i} z_i(t) - \lambda_{2_i}\dot{z}_i(t) - \lambda_{3_i}\, sign(z_i(t))\Big] \end{aligned} \tag{29}$$

It is obtained from $(\mathbf{H_1})$ and the approach used in [76] that

$$\begin{aligned}\dot{V}_1(z_t) \le \sum_{i=1}^{n}\Big[&-\left(c_i+\lambda_{1_i}\right)|z_i(t)| + \sum_{j=1}^{n} |a_{ij} l_1^j||z_j(t)| + \sum_{j=1}^{n} |b_{ij} l_1^j||z_j(t-\tau(t)))| \\ &+ \sum_{j=1}^{n} |d_{ij} l_3^j||\dot{z}_j(t-\sigma(t))| \\ &+ \sum_{j=1}^{n} |e_{ij} l_2^j| \int_{-\infty}^{t} k_j(t-s)|z_j(s)|\mathrm{d}s - \lambda_{2_i}|\dot{z}_i(t)| - \lambda_{3_i}\bar{\lambda}_i\Big] \end{aligned} \tag{30}$$

where

$$\bar{\lambda}_i = \begin{cases} 1 & \text{if } \; z_i(t) \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

From $(\mathbf{H_1})$, $(\mathbf{H_2})$ and $(\mathbf{H_4})$, one has

$$\begin{aligned}\dot{V}_2(z_t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\bar{\tau}_2}|b_{ij} l_1^j||z_j(t)| - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1-\dot{\tau}(t)}{1-\bar{\tau}_2}|b_{ij} l_1^j||z_j(t-\tau(t))| \\ &\le \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\bar{\tau}_2}|b_{ij} l_1^j||z_j(t)| - \sum_{i=1}^{n}\sum_{j=1}^{n} |b_{ij} l_1^j||z_j(t-\tau(t))| \end{aligned} \tag{32}$$

$$\dot{V}_3(z_t) = \sum_{i=1}^{n}\sum_{j=1}^{n} |e_{ij} l_2^j|\mathbf{k}_j|z_j(t)| - \sum_{i=1}^{n}\sum_{j=1}^{n} |e_{ij} l_2^j| \int_{-\infty}^{t} k_j(t-s)|z_j(s)|\mathrm{d}s \tag{33}$$

$$\dot{V}_4(z_t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\bar{\sigma}_2}|d_{ij} l_3^j||\dot{z}_j(t)| - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1-\dot{\sigma}(t)}{1-\bar{\sigma}_2}|d_{ij} l_1^j||\dot{z}_j(t-\sigma(t))|$$

$$\leq \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\bar{\sigma}_2}|d_{ij}l_3^j||\dot{z}_j(t)| - \sum_{i=1}^{n}\sum_{j=1}^{n}|d_{ij}l_3^j||\dot{z}_j(t-\sigma(t))| \tag{34}$$

It follows from (29)–(34) that

$$\dot{V}(z_t) \leq \sum_{i=1}^{n}\Bigg[\Big(-\big(c_i+\lambda_{1_i}\big)+\sum_{j=1}^{n}|a_{ji}||l_1^i| + \sum_{j=1}^{n}\frac{1}{1-\bar{\tau}_2}|b_{ji}||l_1^i| + \sum_{j=1}^{n}\mathbf{k}_j|e_{ji}||l_2^i|\Big)|z_i(t)| + \Big(\sum_{j=1}^{n}\frac{1}{1-\bar{\sigma}_2}|d_{ji}||l_3^i| - \lambda_{2_i}\Big)|\dot{z}_i(t)| - \lambda_{3_i}\bar{\lambda}_i\Bigg] \tag{35}$$

When $\|z(t)\|_1 \neq 0$, we deduce that

$$\dot{V}(z_t) \leq -\sum_{i=1}^{n}\lambda^- < 0 \tag{36}$$

where $\lambda^- = \min_{1\leq i\leq n}\{\lambda_{3_i}\}$. Therefore, from the proof of Theorem 1 in [76], system (5) is FTS via (25) which achieves the proof. □

Remark 12 On the one hand, the exact values of the delay is often poorly known in practice because it is difficult to assess the delays and most of the time, only approximate values are available [30]. On the other hand even the real time operating system can only guarantee a maximum values for the time-varying delay [30]. For this, the delay-free controllers (24) and (25) does not use the knowledge of the time-varying delays are more suitable for real applications.

The controllers (24) and (25) are without delay which make them more suitable in practice. However, these controllers contain the sign function and then the chattering phenomena will be appears [73]. For this, based on the results obtained in[74,79], we design the delay-free non chattering control as follows:

$$u_i(z(t)) = -\lambda_{1_i}z_i(t) - \lambda_{2_i}\ \dot{z}_i(t) - \lambda_{3_i}\ sat(z_i(t), \Delta) \tag{37}$$

where $\lambda_{k_i},\ k = 1, 2, 3$ stand for the control strength to be determined and

$$sat(z_i(t)) = \begin{cases} 1 & \text{if } \frac{z_j(t)}{\Delta} \geq 1 \\ -1 & \text{if } \frac{z_j(t)}{\Delta} \leq -1 \\ \frac{z_j(t)}{\Delta} & \text{otherwise} \end{cases} \tag{38}$$

with $\Delta > 0$.

Corollary 3 *Under assumptions* $(\mathbf{H_1}) - (\mathbf{H_2})$ *and* $(\mathbf{H_4})$, *if* $\lambda_{3_i} > 0$ *and* $\lambda_{1_i},\ \lambda_{2_i}$ *satisfy the following inequalities*

$$\lambda_{1_i} > -c_i + \sum_{j=1}^{n}|a_{ji}||l_1^i| + \sum_{j=1}^{n}\frac{1}{1-\bar{\tau}_2}|b_{ji}||l_1^i| + \sum_{j=1}^{n}\mathbf{k}_j|e_{ji}||l_2^i|; \tag{39}$$

$$\lambda_{2_i} > \sum_{j=1}^{n}\frac{1}{1-\bar{\sigma}_2}|d_{ji}||l_3^i| \tag{40}$$

then the closed-loop system (5)–(37) *is FTS.*

Proof The proof is similar to the one of Theorem 4 so it is omitted her . □

Remark 13 If the activation functions are discontinuous, system (5) becomes a differential equation with discontinuous right-hand side. Thus, based on the Filipov theory, the study of the obtained differential inclusion can be transformed into the study of an uncertain differential equation. It should be pointed out that the existence of solutions is the most fundamental and a strict mathematical proof about the existence of solution should be presented. For this, we can be use the similar approach used in [73,76,79] combined with the method of exchanging integral order presented in [72] to deal with the infinite distributed delay.

Remark 14 The non-linear discontinuous part of the control law (25) can be circumvented by using the controller designed in [34] as follows:

$$u_i(z(t)) = -\lambda_{1_i} z_i(t) - \lambda_{2_i}\ \dot{z}_i(t) - \lambda_{3_i} \frac{z_i(t)}{\|z(t)\|_1 + \nu} \tag{41}$$

where $\lambda_{k_i},\ k = 1, 2, 3$ stand for the control strength to be determined and ν a small positive constant.

4 Numerical Examples

In this section, three numerical examples are provided to show the effectiveness of our main results. As all the equilibrium points are at the origin, we use the z-form for the systems instead of the x-form because they are equivalent.

4.1 Example 1: FTS via a Delay-Dependent Controller

Consider the following delayed Hopfield neural network with unbounded activation functions

$$\dot{z}(t) = -Cz(t) + A\ F(z(t)) + B\ G(z(t-\tau)) + u \tag{42}$$

where $n = 2$, $F_i(z_i) = 0.2[z_i - sin(z_i)]$ and $G_i(z_i) = 0.4z_i$ for $i = 1, 2$, $\tau = 2$,

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0 \\ 1 & 1.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0.5 \end{bmatrix}$$

and the initial condition $z_1(s) = \phi_1(s) = -1.6$, $z_2(s) = \phi_2(s) = 1.2$ for all $s \in [-2, 0]$. System (42) has been studied in [15] where only the global exponential stability is ensured. By using Matlab LMI toolbox [45] for solving (19) with $\varepsilon_1 = 3,\ \varepsilon_2 = 1$ and $\alpha = p\alpha_1$ we obtain the following solution

$$P = \begin{bmatrix} 9.0006 & -0.6194 \\ -0.6194 & 7.5220 \end{bmatrix},\ R_1 = \begin{bmatrix} 10.228 & 0 \\ 0 & 9.6053 \end{bmatrix},\ R_2 = \begin{bmatrix} 25.5043 & 0 \\ 0 & 24.4666 \end{bmatrix}$$

and

$$p = 0.02, \quad \varepsilon = 0.8972, \quad \alpha_1 = 26.555.$$

From Corollary 2, we deduce that system (42) has a unique equilibrium, the origin, which is FTS with the following delay-dependent controller

$$u\,(z(t)) = -26.555z(t) - B\ sign(z(t))\,|z(t-\tau(t))| - sign(z(t))|z(t)|^{\frac{1}{2}} \tag{43}$$

We plot the state trajectories of the closed-loop system (42)–(43) in Fig. 1.

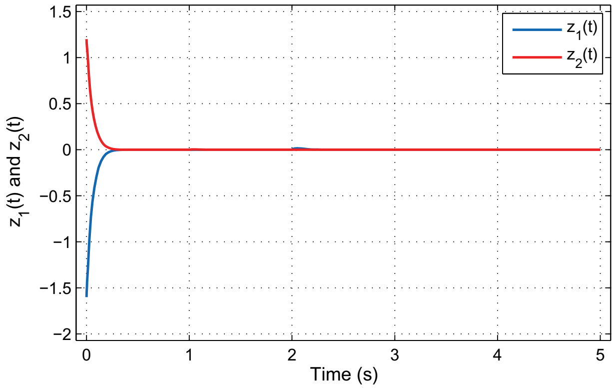

Fig. 1 State trajectories of system (42) with controller (43)

Corollary 2 guarantees the FTS of the closed-loop system (42)–(43) but also the following inequality for the settling-time functional

$$T_0(\phi) \leq \frac{\|\phi\|^{\mu}}{1-\mu} = 2.88$$

with $\mu = 0.5$.

Remark 15 It should be pointed out that the results given in [67] fail for system (42) because the above activation functions are unbounded.

4.2 Example 2: FTS via a Delay-Free Controller

Consider the following NHNN with mixed delays

$$\begin{cases} \dot{z}(t) &= -Cz(t) + A\ F(z(t)) + B\ F(z(t-\tau)) + E \int\limits_{-\infty}^{t} K(t-s)\ G(z(s))\mathrm{d}s \\ & +D\ H(\dot{z}(t-\sigma)) + u \end{cases} \tag{44}$$

with $n = 2$, $\tau = 1$, $\sigma = 0.1$, $k_1(x) = k_2(x) = e^{-x}$, the initial condition $x_1(s) = \phi_1(s) = -0.7$, $x_2(s) = \phi_2(s) = 0.5$ for all $s \in (-\infty, 0]$ and parameters $C,\ A,\ B,\ E$ and D as follows

$$C = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix},\ A = B = \begin{bmatrix} 0.3 & 0.15 \\ -0.25 & -0.4 \end{bmatrix},\ E = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{bmatrix},\ D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

System (44) has been studied in [32] where only the asymptotic stability is ensured. The Matlab LMI toolbox [45] for solving (23) when we fix $\alpha_2 = 1$ and we let $\alpha = p\alpha_1$ leads to the solution

$$p = 0.1911, \quad \varepsilon = 1.1221, \quad \alpha = 0.7632, \quad \alpha_1 \simeq 3.993721.$$

4.2.1 Bounded Activation Function Case

Firstly, we take $F_i(z_i) = G_i(z_i) = H_i(z_i) = \tanh(z_i)$ for $i = 1, 2$.

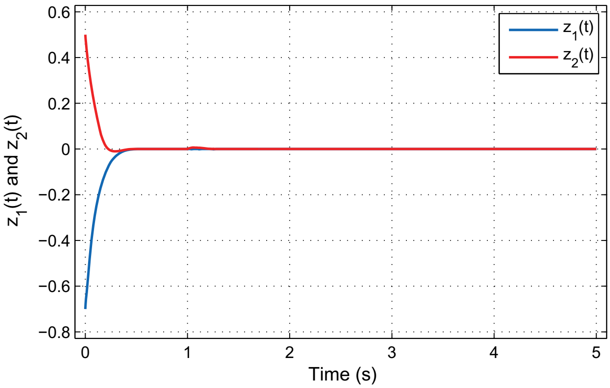

Fig. 2 State trajectories of system (44) with controller (45)

Therefore, since $\Omega_i = diag(1, 1),\ i = 1, 2, 3$, Theorem 3 implies that the equilibrium point of system (44), which is the origin, is FTS with the following delay-free controller

$$u(z(t)) = -3.993721z(t) - B\ sign(z(t)) - E\mathbf{K}\ signz(t)) - D\ sign(z(t)) \\ - sign(z(t))|z(t)|^{\frac{1}{2}} \quad (45)$$

where $\mathbf{K} = diag(1, 1)$ and $T^* = 0$. The state trajectories of the closed-loop system (44)–(45) is depicted in Fig. 2.

4.2.2 *Unbounded Activation Function Case*

Now, we choose

$$F_i(z_i) = G_i(z_i) = H_i(z_i) = |z_i + 1| + |z_i - 1| \quad (46)$$

and other parameters similar to Example 4.2.1. Obviously, the above activation functions are unbounded. According to Remark 14, system (44) is FTS with the following controller:

$$u_1(z(t)) = -2z_1(t) - 0.2\ \dot{z_1}(t) - \frac{z_1(t)}{\|z(t)\|_1 + \nu}; \quad (47)$$

$$u_2(z(t)) = -2z_2(t) - 0.2\ \dot{z_2}(t) - 2\frac{z_2(t)}{\|z(t)\|_1 + \nu} \quad (48)$$

when we fix $\nu = 0.001$. The state trajectories of the closed-loop system (44)–(48) with unbounded activation functions (46) is depicted in Fig. 3.

Remark 16 It should be pointed out that from Theorem 2, the following controller

$$u(z(t)) = -3.993721z(t) - B\ sign(z(t))\,|z(t - \tau(t))| \\ - E\ sign(z(t)) \int_{-\infty}^{t} K(t-s)\ |z(s)|ds \\ - D\ sign(z(t))\ |\dot{z}(t - \sigma(t))|)) - |z(t)|^{\frac{1}{2}} \quad (49)$$

can be stabilize in finite time system (44) under activation functions (46) which is illustrated in Fig. 4.

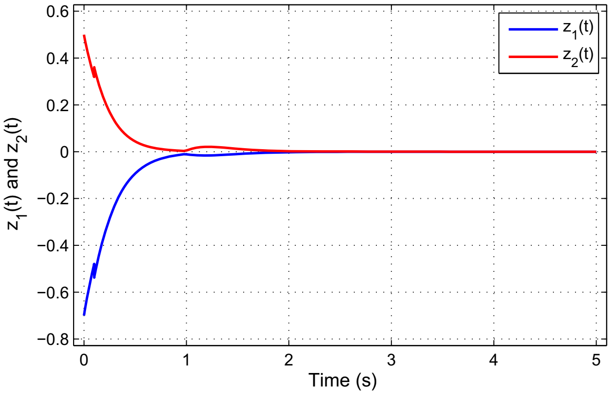

Fig. 3 State trajectories of system (44) under activation functions (46) with controller (47)–(48)

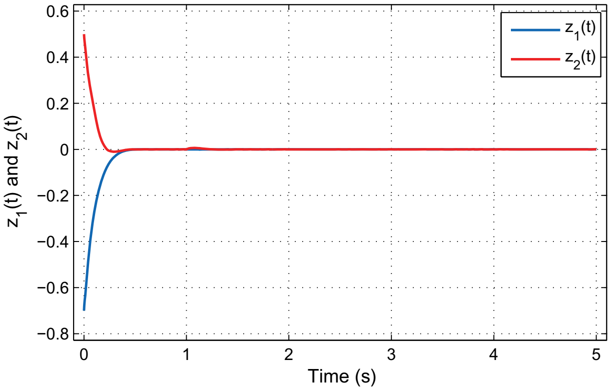

Fig. 4 State trajectories of system (44) under activation functions (46) with controller (49)

Moreover, despite the controller (47)–(48)is more suitable in practice, the delay dependent-controller (49) provide a settling time more accurate than that founding from (47)–(48).

Now, if the time- varying delay $\tau(.)$ given by the following non-differentiable function

$$\tau(t) = 0.5|sin(t)| \tag{50}$$

System (44) stays FTS which is illustrated in Fig. 5.

4.2.3 FTS via Non-chattering Control

Now, we choose A, B and E as follows

$$A = B = \begin{bmatrix} 1.3 & 1.15 \\ -1.25 & -1.4 \end{bmatrix}, \ E = \begin{bmatrix} 1.7 & -1.2 \\ -1.2 & 1.5 \end{bmatrix},$$

and others parameters similar to 4.2.1. From (39), if we taking $\lambda_{1_1} = 0.8$, $\lambda_{1_2} = 1.5$ and $\lambda_{2_1} = \lambda_{2_2} = 0.2$, Corollary 3 implies that the following controller which is more suitable in practice

$$u_1(z(t)) = -0.8z_1(t) - 0.2\,\dot{z_1}(t) - \ sat(z_1(t), \Delta); \tag{51}$$

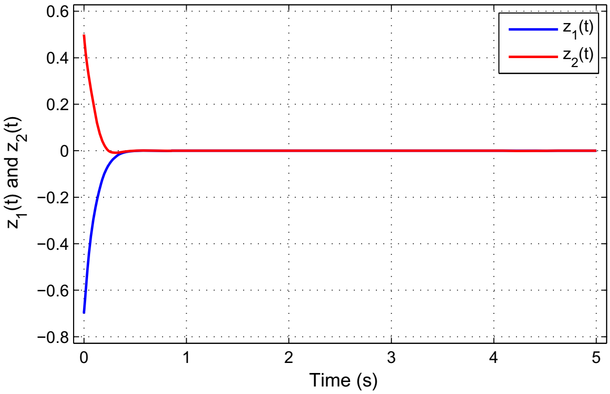

Fig. 5 State trajectories of system (44) with controller (45) under time varying delay (50)

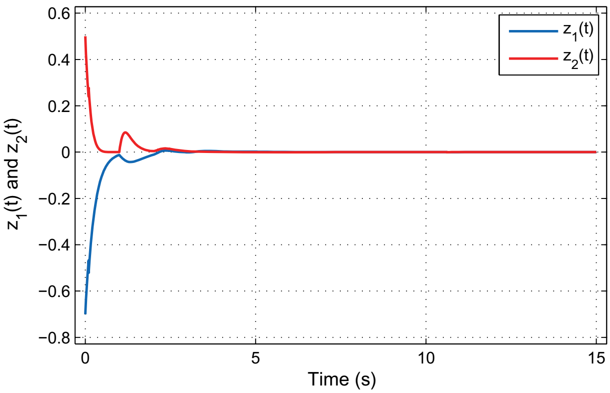

Fig. 6 Trajectories of system (44) with controller (51)–(52)

$$u_2(z(t)) = -1.5z_2(t) - 0.2\,\dot{z_2}(t) - 1.1\,sat(z_2(t), \Delta) \tag{52}$$

can be stabilize in finite time system (44) when we fix $\Delta = 0.01$. The state trajectories of system (44) with controller (51)–(52) is illustrated in Fig. 6.

4.3 Example 3: Resistance-Capacitance Network Circuit

A two dimensional resistance capacitance network circuit (RCNC) studied in [1, Example 4.4.] can be modeled by the following nonlinear NN

$$\dot{z}(t) = -Az(t) + W_1\ F(z(t)) + W_2\ u(z(t)) \tag{53}$$

with

$$A = \begin{bmatrix} \frac{1}{R_1C_1} & 0 \\ 0 & \frac{1}{R_2C_2} \end{bmatrix}, \quad W_1 = \begin{bmatrix} \frac{\omega_{11}}{C_1} & \frac{\omega_{12}}{C_1} \\ \frac{\omega_{21}}{C_2} & \frac{\omega_{22}}{C_2} \end{bmatrix}, \quad W_2 = \begin{bmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{bmatrix}$$

where all constants are positive. We consider system (53) with the following values

$$R_i = C_i = \omega_{11} = 1,\ F_i(z_i) = \tanh(z_i),\ i = 1, 2,\ \omega_{12} = 1.5,\ \omega_{21} = -1.5,\ \omega_{22} = -1.$$

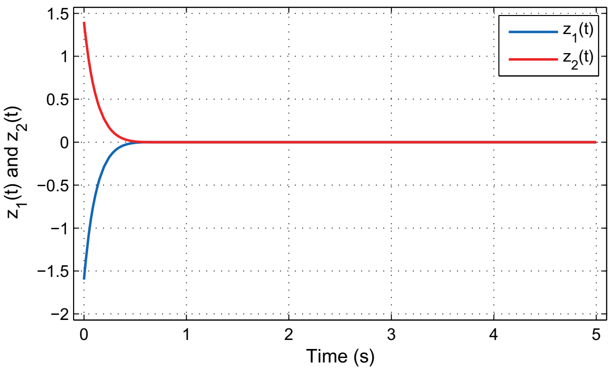

Fig. 7 State trajectories of system (53) with initial condition $(-1.6, 1.4)^T$ and controller (54)

proposed in [1, Example 4.4.]. We have $\Omega_1 = diag(1, 1)$ and consequently the Matlab LMI toolbox [45] for solving (23) with $\alpha_2 = 1$, $\mu = 0.5$ and $\alpha = p\alpha_1$ leads to the following solution

$$p = 0.1409, \quad \varepsilon = 0.8063, \quad \alpha = 0.9321, \; \alpha_1 \simeq 6.6153302.$$

Therefore, Theorem 3 implies that the origin of system (53) is FTS via the following controller

$$u(z(t)) = -6.6153302z(t) - sign(z(t))|z(t)|^{\frac{1}{2}}. \tag{54}$$

and the settling-time functional satisfies

$$T_0(z(0)) \leq \frac{\|z(0)\|^{\mu}}{1-\mu} < 3.$$

The state trajectories of the closed-loop system (53)–(54) is depicted in Fig. 7. In [1], only asymptotic stability of system (53) is ensured.

5 Conclusion and Open Problem

The problem of finite time stabilization of a class of neutral Hopfield neural networks with mixed time delays is investigated. First, theoretical results are established around the stabilization in finite time. Then, based on LMI techniques, these results are used to design different kinds of feedback controls which overcome the chattering phenomena and provides a favourable situation for real applications. On one hand, our results extend the results given in [40,41,58,62,63,66,67] where the neutral class, infinite distributed delay and unbounded activation functions are not taken into account simultaneously and offers a fast settling time compared with [40,58,60] . On the other hand, our study offers an improvement compared with [1,31,33,36,38,54,70,78] where only asymptotic and exponential stability of NNs are considered. Finally, the effectiveness of our proposed approach has been shown in simulation on three examples.

In future work, we would like to extend our results to quaternion-valued NNs (QVNNs). On the one hand, the Hamilton rules about quaternion multiplication renders the famous inequalities such as given in [10] and Lemma 1 are not applicable for the study of the stability of QVNNs [16]. To solve this problem, Chen et al. are established in [16] the modulus inequalities for QVNNs. Based on the obtained results in [16], we can be used a direct method to study the stability of system (1) by imposing the Lipschitz conditions entries.

On the other hand, a decomposition method such as presented in [44] can be used to solve this problem. This method gives a wider class of the quaternion-valued activation functions. However, the dimensions grow four times for the QVNNs which complicated the calculus for a large number of neurones. The corresponding results will appear in the near future.

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