

Impulsive Synchronization of Stochastic Neural Networks via Controlling Partial States

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Abstract In the paper, synchronization problem for stochastic neural networks are studied by impulsively controlling partial states. At each impulsive instant, only part of the states are controlled to realize the synchronization of impulsively coupled stochastic neural networks. By using the method of average impulsive interval, less conservative synchronization criteria are derived. The derived sufficient conditions are closely related to the parameters of system dynamics, impulsive gain, impulsive interval and the proportion of the controlled components. Finally, numerical example is given to illustrate the effectiveness of our theoretical results.

Keywords Synchronization · Neural networks · Impulsive control · Partial states

1 Introduction

Synchronization of coupled chaotic dynamical systems has recently received increasing interest in the control and physical community due to its wide potential applications including the secure communication and chemical reaction [7,27,35]. Some recent results concerning chaotic synchronization and a review of relevant experimental applications of these techniques and schemes have been reported in [2,25,28]. Synchronization can be much beneficial in many practical applications [13]. Synchronization of coupled systems can be utilized to the coordination of simultaneous threads to complete a task of obtaining a correct runtime order while avoiding unexpected rate conditions for parallel computing [5].

Recently, dynamical behaviors of neural networks have been extensively investigated in [3,32,37], and many applications have been found in different areas. Much attention has been paid to the study of the stability and periodicity for neural networks [1,4,11,36], and also the synchronization of coupled neural networks have been recently investigated [33,40] since some neural networks need synchronous behavior for information transmission, pattern

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recognition et al. [10,29]. Many different kinds of control methods have been utilized to control or synchronize chaotic dynamical systems, such as PC method [27], adaptive control [20,31], impulsive control [15,26,38,39], delayed feedback control [9], and so on. For most of the obtained results, the full states' information is necessary for the control or synchronization of chaotic systems, and this is sometimes not easy to implement. Hence, in this paper we will investigate the synchronization of chaotic neural networks by only controlling part of the states.

Many practical systems can be well described by impulsive control systems. Examples include the population control system of a kind of insects with the number of insects and the natural enemies as state variables [15,17,38]. In recent years, impulsive systems and impulsive control theory have been widely studied [8,19,21,23,24,34]. Some sufficient conditions were derived to guarantee the asymptotic stability of impulsive control systems with fixed time impulses in [15]. Impulsive synchronization of chaotic systems with time-varying impulsive intervals was investigated in [30], and the results can be used to realize synchronization of chaotic systems by only using small impulses generated by samples of the state variables of the driving system at discrete time instants. In [12], Ji et al. studied the problem of robust adaptive-impulsive synchronization in chaotic delayed neural networks with uncertainties. In [41], the synchronization of coupled switched neural networks with mode-dependent impulsive effects was studied. By proposing a novel approach named "average impulsive interval", a unified synchronization criterion was obtained for impulsive dynamical networks in [22], and the result was simultaneously suitable for impulses with synchronizing effects and desynchronizing effects. In [18], the exponential stability and L_2 -gain problem was studied for a class of non-linear switched impulsive systems with time-varying disturbances. In [14], an impulsive controller is used to achieve the exponential synchronization of chaotic delayed neural networks with stochastic perturbation.

Motivated by the above discussions, this paper intend to study the synchronization of stochastic neural networks by only impulsively controlling partial states. The principle of choosing the controlled states is obtained by referring to the sorting of the norms of the synchronization errors. By using Lyapunov stability method for stochastic systems and comparison methods, an efficient criterion is derived for stochastic synchronization of coupled chaotic neural networks. Instead of choosing the impulsive interval to be constant, we will utilize the concept of average impulsive interval to describe the sequence of impulses, hence further to make the obtained results less conservative. In the synchronization criterion, the detailed relationship among the system dynamics, proportion of controlled components, impulsive strength and impulsive interval will be presented. The proposed method can also be extended to study the control of chaotic systems, and the impulsive strength can be different at different impulsive instants. Numerical example is finally given to verify the effectiveness of the obtained theoretical results.

2 Model Formulation and Some Preliminaries

Consider a stochastic neural network as the drive system, described by

$$dx(t) = [-Cx(t) + B\bar{f}(x(t)) + I]dt + \bar{g}(t, x(t))dw(t), \quad (1)$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in R^n$ is the state vector of the drive system associated with the neurons, $C = \text{diag}(c_1, c_2, \dots, c_n) > 0$ is a positive diagonal matrix, $B = (b_{ij})_{n \times n}$ is the connection weight matrix, $I = (I_1, I_2, \dots, I_n)^T \in R^n$ is a constant

external input vector, $\bar{f}(x(t)) = (\bar{f}_1(x_1(t)), \bar{f}_2(x_2(t)), \dots, \bar{f}_n(x_n(t)))^T \in R^n$ denotes the activation functions of the neurons, $\bar{g} : R^+ \times R^n \rightarrow R^{n \times m}$ is the noise intensity function matrix satisfying $\bar{g}(t, \mathbf{0}) = \mathbf{0}^{n \times m}$ with $\mathbf{0}$ being a vector or matrix with compatible dimension, and $w(t) \in R^m$ is an m -dimensional Brownian motion.

The response system corresponding to the master system (1) is constructed as follows

$$dy(t) = [-Cy(t) + B\bar{f}(y(t)) + I]dt + \bar{g}(t, y(t))dw(t) + u(t)dt, \tag{2}$$

where $u(t) \in R^n$ is the impulsive controller to be determined for the purpose of synchronizing drive and response systems, $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in R^n$ is the state vector of the response system associated with the neurons, and the rest of the notations are the same with that of the drive system.

Throughout this paper, we have the following two assumptions:

Assumption 1 The neuron activation function $\bar{f}(\cdot)$ satisfies the following the Lipschitz condition: $\|\bar{f}(y) - \bar{f}(x)\| \leq L\|y - x\|$ for any $x, y \in R^n$, where $L > 0$ is called the Lipschitz constant.

Assumption 2 The noise intensity function matrix $\bar{g} : R^+ \times R^n \rightarrow R^{n \times m}$ is assumed to be uniformly Lipschitz continuous in terms of the norm as follows:

$$\text{trace} \left[(\bar{g}(t, y) - \bar{g}(t, x))^T \cdot (\bar{g}(t, y) - \bar{g}(t, x)) \right] \leq M\|y - x\|^2, \quad \forall x, y \in R^n, \tag{3}$$

where M is a constant matrix with compatible dimensions.

Let $e(t) = y(t) - x(t)$ be the error state of the drive-response systems (1) and (2). Then subtracting (1) from (2) yields the following error dynamical system:

$$de(t) = [-Ce(t) + Bf(e(t))]dt + g(t, e(t))dw(t) + u(t)dt, \tag{4}$$

where $f(e(t)) = \bar{f}(y(t)) - \bar{f}(x(t))$ and $g(t, e(t)) = \bar{g}(t, y(t)) - \bar{g}(t, x(t))$.

Definition 1 The master-slave systems (1) and (2) are said to be globally exponentially synchronized if there exist $\gamma > 0, T > 0$ and $M_0 > 0$, such that for any initial values $x(0)$ and $y(0)$,

$$\|y(t) - x(t)\| \leq M_0e^{-\gamma t}$$

holds for all $t > T$.

To achieve the exponential synchronization of the drive-response systems (1) and (2), we design the impulsive controllers $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T \in R^n$ as follows:

$$u(t) = \mu \sum_{k=1}^{+\infty} D(t) \cdot e(t) \cdot \delta(t - t_k), \tag{5}$$

where $\mu \in (-2, 0)$ is the impulsive strength, $\delta(\cdot)$ is the Dirac delta function, the discrete impulsive instant set $\{t_k\}$ satisfies $0 < t_1 < \dots < t_k < t_{k+1} < \dots$ and $\lim_{k \rightarrow \infty} t_k = +\infty$, and $D(t) = \text{diag}\{d_1(t), d_2(t), \dots, d_n(t)\}$ with $d_i(t)$ ($i = 1, 2, \dots, n$) being

$$d_i(t) = \begin{cases} 1, & i \in \mathcal{J}(t_k), \\ 0, & i \notin \mathcal{J}(t_k) \end{cases} \tag{6}$$

Here $\mathcal{J}(t_k)$ is the index set of the impulsively controlled states to be determined. Let $\#\mathcal{J}(t_k) := \rho$ denote the number of the elements of the index set $\mathcal{J}(t_k)$.

To determine the index set of $\mathfrak{D}(t_k)$, at time instant $t = t_k$, we arrange the components $\{e_1(t_k), e_2(t_k), \dots, e_n(t_k)\}$ of the error state vector $e(t_k)$ as follows:

$$\|e_{p_1(t_k)}(t_k)\| \geq \dots \geq \|e_{p_\rho(t_k)}(t_k)\| \geq \|e_{p_{\rho+1}(t_k)}(t_k)\| \geq \dots \geq \|e_{p_n(t_k)}(t_k)\|, \tag{7}$$

where $p_i(t_k) \in \{1, 2, \dots, n\}$, $i = 1, 2, \dots, n$. If $\|e_{p_\rho(t_k)}(t_k)\| = \|e_{p_{\rho+1}(t_k)}(t_k)\|$, then we simply let $p_\rho(t_k) < p_{\rho+1}(t_k)$. Now, we can determine the index set of the controlled states at time instant $t = t_k$ as $\mathfrak{I}(t_k) = \{p_1(t_k), p_2(t_k), \dots, p_\rho(t_k)\}$.

After these discussions, we can obtain the following error dynamical system with impulsive controller:

$$\begin{cases} de(t) = [-Ce(t) + Bf(e(t))]dt + g(t, e(t))dw(t) + u(t)dt, & t \neq t_k, k = 1, 2, \dots \\ e(t_k^+) = e(t_k^-) + \mu D(t_k)e(t_k^-), & t = t_k. \end{cases} \tag{8}$$

where the elements of matrix $D(t_k)$ is defined as in (6), $e(t_k^+) = \lim_{t \rightarrow t_k^+} e(t)$, and $e(t_k^-) = \lim_{t \rightarrow t_k^-} e(t)$. Here, we assume that $e(t)$ is left-hand continuous at $t = t_k$, i.e., $e(t_k) = e(t_k^-)$. Then, all solutions of error system (8) are left-hand continuous at $t = t_k$.

With respect to the impulsive sequence, the method of average impulsive interval will be used to describe our impulsive sequence to make the obtained results less conservative.

Definition 2 [22] (*average impulsive interval*) The *average impulsive interval* of the impulsive sequence $\zeta = \{t_1, t_2, \dots\}$ is less than T_a , if there exist a positive integer N_0 and a positive number T_a , such that

$$N_\zeta(T, t) \geq \frac{T - t}{T_a} - N_0, \quad \forall T \geq t \geq 0, \tag{9}$$

where $N_\zeta(T, t)$ denotes the number of impulsive times of the impulsive sequence ζ in the time interval (t, T) .

In order to derive our main result about the synchronization of stochastic neural networks, we need to present the following lemma [16].

Lemma 1 [16] Consider the following stochastic system with impulses:

$$\begin{cases} dx(t) = \phi(t, x(t))dt + \eta(t, x(t)) dw(t), & t \geq t_0, t \neq t_k, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k \in \mathbb{N}. \end{cases} \tag{10}$$

Assume that there exist a Lyapunov function $V(t, x(t))$, and functions φ, ψ_k with $\varphi(t, 0) = \psi_k(0) = 0$ for any $t \geq 0, k \in \mathbb{N}$, such that:

- (i) There exist positive constants c_1 and c_2 such that for all $t \geq t_0, c_1 \|x(t)\| \leq V(t, x(t)) \leq c_2 \|x(t)\|$;
- (ii) There exists continuous function $\varphi : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$, and $\varphi(t, s)$ is concave on s for each $t \in \mathbb{R}^+$, such that $\mathcal{L}V(t, x) \leq \varphi(t, V(t, x))$, where the operator \mathcal{L} is defined as $\mathcal{L}V(t, x) = V_t(t, x) + V_x(t, x)\phi(t, x) + \frac{1}{2}\text{trace}[\eta^T(t, x)V_{xx}\eta(t, x)]$;
- (iii) There exist continuous and concave functions $\psi_k : \mathbb{R}^+ \rightarrow \mathbb{R}^+, k \in \mathbb{N}$, such that $V(t_k^+, x(t_k^+)) \leq \psi_k(V(t_k^-, x(t_k^-)))$;

then exponential stability of the trivial solution of comparison system (11)

$$\begin{cases} \dot{\omega}(t) = \varphi(t, \omega(t)), & t \geq t_0, t \neq t_k; \\ \omega(t_k^+) = \psi_k(\omega(t_k^-)), & k \in \mathbb{N}; \\ \omega(t_0) = E(V(t_0, x_0)); \end{cases} \tag{11}$$

implies exponential stability of the trivial solution of stochastic impulsive system (10).

In this paper, some standard notations will be used. I_n means the identity matrix of order n . For any random variable ζ , $E(\zeta)$ denoted the expectation value of ζ .

3 Main Results

In this section, we will derive our main results concerning the synchronization of stochastic neural networks by impulsively controlling part of the states. Let λ_M be the largest eigenvalue of the symmetric matrix $-2C + BB^T + M^T M$ and $\alpha = [n + \rho\mu(\mu + 2)]/n$ with ρ being the number of the elements of the index set $\mathcal{J}(t_k)$.

Theorem 1 *Suppose that Assumptions 1 and 2 hold, and that the average impulsive interval of the impulsive sequence $\{t_1, t_2, \dots\}$ is less than T_a . If the following condition*

$$\ln \alpha + (\lambda_M + L^2)T_a < 0 \tag{12}$$

holds, then the globally exponential synchronization in mean square between the drive-response neural networks (1) and (2) is realized by the impulsive controller (5) acted on partial components.

Proof Consider the following Lyapunov function:

$$V(e(t)) = \frac{1}{2}e^T(t)e(t). \tag{13}$$

When $t \in (t_{k-1}, t_k)$, according to Assumptions 1 and 2, we can obtain that

$$\begin{aligned} \mathcal{L}V(e(t)) &= e^T(t)[-Ce(t) + Bf(e(t))] + \frac{1}{2}\text{trace}[g^T(t, e(t))g(t, e(t))] \\ &= -e^T(t)Ce(t) + e^T(t)Bf(e(t)) + \frac{1}{2}\text{trace}[g^T(t, e(t))g(t, e(t))] \\ &\leq -e^T(t)Ce(t) + \frac{1}{2}e^T(t)B \cdot (e^T(t)B)^T + \frac{1}{2}f^T(e(t))f(e(t)) + \frac{1}{2}\|Me(t)\|^2 \\ &\leq -e^T(t)Ce(t) + \frac{1}{2}e^T(t)BB^T e(t) + \frac{1}{2}L^2e^T(t)e(t) + \frac{1}{2}e^T(t)M^T Me(t) \\ &= \frac{1}{2}e^T(t)[-2C + BB^T + M^T M]e(t) + \frac{1}{2}L^2e^T(t)e(t) \\ &\leq \frac{1}{2}(\lambda_M + L^2)e^T(t)e(t) \\ &= (\lambda_M + L^2)V(e(t)). \end{aligned} \tag{14}$$

Since $e(t_k^+) = e(t_k^-) + \mu D(t_k)e(t_k^-)$, we have $e_i(t_k^+) = (1 + \mu)e_i(t_k^-)$ for $i \in \mathcal{J}(t_k)$, and $e_i(t_k^+) = e_i(t_k^-)$ for $i \notin \mathcal{J}(t_k)$.

When $t = t_k$, it follows that

$$\begin{aligned} V(e(t_k^+)) &= \frac{1}{2}e^T(t_k^+)e(t_k^+) \\ &= \frac{1}{2} \sum_{i \in \mathcal{J}(t_k)} e_i^2(t_k^+) + \frac{1}{2} \sum_{i \notin \mathcal{J}(t_k)} e_i^2(t_k^+) \\ &= \frac{1}{2}(1 + \mu)^2 \sum_{i \in \mathcal{J}(t_k)} e_i^2(t_k) + \frac{1}{2} \sum_{i \notin \mathcal{J}(t_k)} e_i^2(t_k). \end{aligned} \tag{15}$$

Let $m(t_k) = \min\{|e_i(t_k)| : i \in \mathcal{J}(t_k)\}$ and $M(t_k) = \max\{|e_i(t_k)| : i \notin \mathcal{J}(t_k)\}$. According to (7), we can conclude that $0 \leq M(t_k) \leq m(t_k)$. Considering $\alpha = [n + \rho\mu(\mu + 2)]/(1 - \alpha) \in (0, 1)$, we have $n - \rho = \rho[\alpha - (1 + \mu)^2]/n \geq 0$, and hence one obtains that

$$\begin{aligned} \sum_{i \notin \mathcal{J}(t_k)} e_i^2(t_k) &\leq (n - \rho)M^2(t_k) \\ &\leq (n - \rho)m^2(t_k) \\ &= \frac{\alpha - (1 + \mu)^2}{1 - \alpha} \rho \cdot m^2(t_k) \\ &\leq \frac{\alpha - (1 + \mu)^2}{1 - \alpha} \sum_{i \in \mathcal{J}(t_k)} e_i^2(t_k). \end{aligned} \tag{16}$$

Further, we have

$$(1 - \alpha) \sum_{i \notin \mathcal{J}(t_k)} e_i^2(t_k) \leq [\alpha - (1 + \mu)^2] \sum_{i \in \mathcal{J}(t_k)} e_i^2(t_k). \tag{17}$$

Then, one can conclude that

$$\begin{aligned} (1 + \mu)^2 \sum_{i \in \mathcal{J}(t_k)} e_i^2(t_k) + \sum_{i \notin \mathcal{J}(t_k)} e_i^2(t_k) &\leq \left[\alpha \sum_{i \in \mathcal{J}(t_k)} e_i^2(t_k) + \alpha \sum_{i \notin \mathcal{J}(t_k)} e_i^2(t_k) \right] \\ &= \alpha \sum_{i=1}^n e_i^2(t_k) \\ &= 2\alpha V(e(t_k)), \end{aligned} \tag{18}$$

which follows from (15) that

$$V(e(t_k^+)) \leq \alpha V(e(t_k)). \tag{19}$$

Combining (14) and (19) gives the following comparison system

$$\begin{cases} \dot{\omega}(t) = (\lambda_M + L^2)\omega(t), & t \geq t_0, t \neq t_k, k = 1, 2, \dots; \\ \omega(t_k^+) = \alpha\omega(t_k^-), & \alpha \in (0, 1); \\ \omega(t_0) = E(V(e(t_0))). \end{cases} \tag{20}$$

Let $N_\zeta(t, t_0)$ denote the number of impulses in the time interval (t_0, t) . Solving the differential equation (20) gives that, for any $t \in [t_0, +\infty)$,

$$\omega(t) = E(V(e(t_0))) \cdot e^{(\lambda_M + L^2)(t - t_0)} \cdot \alpha^{N_\zeta(t, t_0)}. \tag{21}$$

According to the definition of average impulsive interval, we have that

$$\begin{aligned} \omega(t) &\leq E(V(e(t_0))) \cdot e^{(\lambda_M + L^2)(t - t_0)} \cdot \alpha^{\frac{t - t_0}{T_a} - N_0} \\ &= \alpha^{-N_0} E(V(e(t_0))) \cdot e^{(\lambda_M + L^2)(t - t_0)} \cdot e^{\frac{ln\alpha}{T_a}(t - t_0)} \\ &= \alpha^{-N_0} E(V(e(t_0))) \cdot e^{[(\lambda_M + L^2) + \frac{ln\alpha}{T_a}](t - t_0)}. \end{aligned} \tag{22}$$

The exponential stability of the trivial solution of (20) can be concluded from the condition of $ln\alpha + (\lambda_M + L^2)T_a < 0$. Hence, by the comparison theorem, we can conclude that the globally exponential synchronization in mean square between the drive-response neural networks (1) and (2) can be realized by the impulsive controller (5) acted on partial states. The proof is completed.

Remark 1 In [12, 14], the synchronization problem of neural networks was studied by designing impulsive controllers, and in both papers, all the states of the response neural networks should be controlled. From the aspect of practical application, our method is required to only control part of the states, and hence would be more easy and less cost to implement in practical systems.

Remark 2 Moreover, in [6, 12, 14], the lower or upper bound of the impulsive intervals was used to describe the impulsive sequence and further derived the main results. According to the analysis presented in [22], our result is related to the average impulsive interval, which can allow the upper bound of the impulsive intervals very big and hence would make our result less conservative.

Remark 3 In this paper, the parameters of drive and response systems are exactly the same, and the complete synchronization problem is studied. In fact, the results can be extended to the models of coupled systems with non-identical parameters, and the synchronization with errors can be well studied using our method.

Recalling the symbols in (12), we can observe that λ_M and L come from the dynamics of neural networks, $\alpha = 1 + \frac{\rho}{n}\mu(\mu + 2)$ is determined by the proportion of the controlled states $\frac{\rho}{n}$ and impulsive strength μ , and T_a determines the impulsive frequency of the impulsive sequence. From this observation, Theorem 1 presents an explicit relationship among these key factors affecting the synchronization of neural networks.

If only one state is impulsively controlled, that is $\rho = 1$, then we can figure out T_a by using (12) to guarantee the synchronization. Following please find a corollary to present this idea.

Corollary 1 *Suppose that Assumptions 1 and 2 hold, and the average impulsive interval of the impulsive sequence $\{t_1, t_2, \dots\}$ is less than T_a . The drive-response neural networks (1) and (2) can be synchronized in mean square by single impulsive controller (5) acted on one component, if the following condition is satisfied:*

$$T_a < -\ln \left(1 + \frac{\mu(\mu + 2)}{n} \right) / (\lambda_M + L^2). \tag{23}$$

Remark 4 The synchronization problem of chaotic systems with uncertain bounded parameters and/or delays has been widely studied. The main idea in this paper can be used to handle that kind of synchronization problem to further reduce the conservativeness and also reduce the cost. In this paper, the impulsive strength is assumed to be the same at each time instant just for simplicity. In fact, we can similarly derive some interesting results that the impulsive strength could be different at different instants.

4 Numerical Example

In this section, numerical examples are presented to illustrate our theoretical results. A chaotic neural network [42] is used for illustration, and its dynamic is described by the following stochastic differential equation:

$$dx(t) = [-Cx(t) + B\bar{f}(x(t)) + I]dt + \bar{g}(t, x(t))dw(t), \tag{24}$$

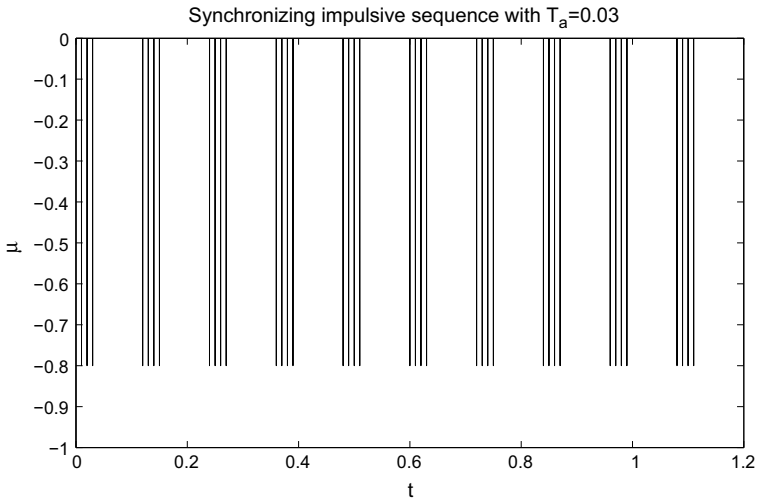


Fig. 1 Impulsive sequence acted on the response neural network

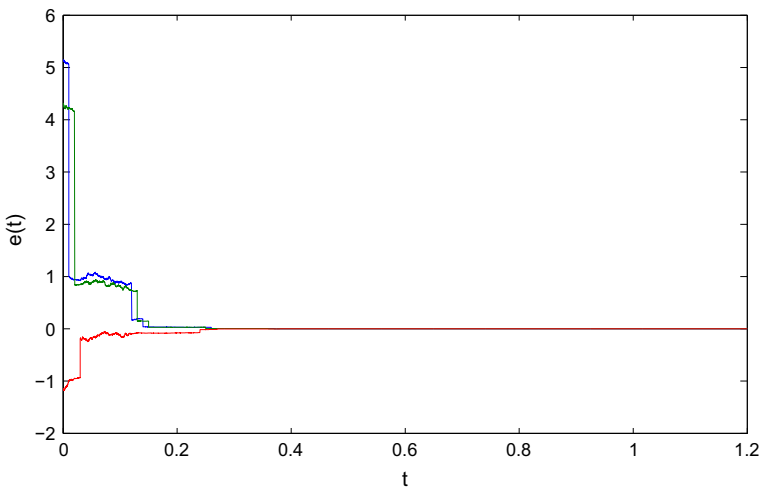


Fig. 2 The error state $e(t)$ of the drive and response neural networks under impulsive controller

where $x(t) \in R^3$, and $C = \text{diag}\{1.2, 1.2, 1.2\}$, $B = \begin{pmatrix} 1.16 & -1.5 & -1.5 \\ -1.5 & 1.16 & -2.0 \\ -1.2 & 2.0 & 1.16 \end{pmatrix}$, the activation function is $\bar{f}(x(t)) = (\tanh(x_1), \tanh(x_2), \tanh(x_3))^T$, $I = (0, 0, 0)^T$, and the noise intensity function matrix is $\bar{g}(t, x(t)) = 0.5 \cdot \|x(t)\| \cdot I_3$. Then Assumptions 1 and 2 are satisfied with parameters' values being $L = 1$ and $M = 0.5I_3$. Simple computation gives that $\lambda_M = 10.8253$.

The response neural network with impulsive controllers is described by the following equation:

$$dy(t) = [-Cy(t) + B\bar{f}(y(t)) + I] dt + \bar{g}(t, y(t))dw(t) + u(t)dt, \tag{25}$$

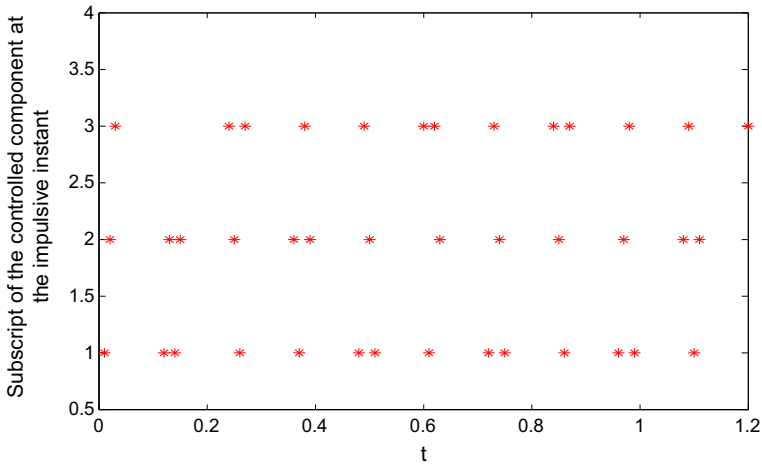


Fig. 3 The subscript of the controlled component at the impulsive instant

where $u(t) \in R^3$ is the impulsive controller determined by (5). In this example, we assume that only one component is controlled at each impulsive instant and the impulsive control gain is $\mu = -0.8$. Following Corollary 1, one can conclude that the drive and response neural networks can be synchronized in mean square if $T_a < 0.0326$. In the numerical example, we choose an impulsive sequence with $T_a = 0.03$, which is presented in Fig. 1. By randomly uniformly choosing the initial values from $[-5, 5]$, Fig. 2 displays the synchronization behavior of impulsively coupled neural networks. The subscript of the controlled component at the impulsive instant is shown Fig. 3. It can be observed from the numerical simulations that our proposed impulsive controllers acting only on part of the components are very effective.

5 Conclusion

In this paper, the synchronization problem between drive and response neural networks is studied. New impulsive controllers are designed to synchronize the drive and response neural networks by only controlling part components of the state at discrete instants. Sufficient and efficient criterion is derived to guarantee synchronization, and the criterion is closely related to these factors including system dynamics itself, the proportion of the controlled components, impulsive frequency, and impulsive gain. The relationship among these factors is explicitly and clearly presented. We finally present a numerical example to illustrate our main theoretical results. Further, effective impulsive control strategy will be considered for the synchronization and control of chaotic systems with time delay and parameters mismatch.

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