Neural Process Lett (2017) 45:981–993
DOI 10.1007/s11063-016-9549-3

Existence and Global Exponential Stability of Periodic Solution for a Class of Neutral-Type Neural Networks with Time Delays

Yurong Liu[1,3] · **Bo Du**[1,2] · **Ahmed Alsaedi**[4]

Published online: 11 October 2016

Abstract This paper is concerned with the problems of existence and stability of the periodic solution for a class of neutral-type neural networks. The neural network addressed is general where the time delays and difference operator are taken into account. By employing the Mawhin's continuation theorem, the sufficient condition is obtained to guarantee the existence and uniqueness of the periodic solution for the neutral-type neural networks. By constructing a novel Lyapunov functional, a unified framework is established to derive sufficient conditions for the concerned system to be globally exponentially stable. A numerical example is provided to demonstrate the usefulness of the main results obtained.

Keywords Neutral-type neural networks · Periodic solution · Existence · Stability · Mawhin's continuation theorem

1 Introduction

Over the past decades, the neural networks have been widely investigated and found many applications in different areas such as image processing, signal processing, pattern recognition and optimization. The dynamical behaviors of neural networks such as stability, oscillation and convergence issues have been extensively studied. In general, many applications of neural networks are built upon the existence and stability of the equilibrium point. For example,

✉ Yurong Liu
liuyurong@gmail.com

1 Department of Mathematics, Yangzhou University, Yangzhou 225002, China

2 Department of Mathematics, Huaiyin Normal University, Huaian, Jiangsu 223300, People's Republic of China

3 Department of Electrical and Computer Enineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

4 Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

if a neural network is used to solve an optimization problem, it is desirable for the neural network to have a unique globally stable equilibrium. Therefore, the stability analysis and synchronization problem of neural networks has caught many researchers' attention [1–18].

In many biological and artificial neural networks, time delays always exist due to varieties of reasons such as the finite speed of information transmission and processing. As is well known, the time delay is one of main sources for causing instability and bad performances of neural networks [11]. Consequently, the stability analysis problems for delayed neural networks have received considerable research attention. Recently, a great deal of results have been reported in the literature, see e.g. [1,8–10,12,14] and references therein, where the time delays considered can be categorized as constant delays, time-varying delays and distributed delays, the methods used include the M-matrix theory, linear matrix inequality (LMI) approach, Lyapunov functional method and techniques of inequality analysis, and the stability criteria derived contain delay-independent conditions and delay-dependent conditions.

On the other hand, it is common in engineering systems that the time delay occurs not only in system states but also in the derivatives of system states. The systems containing the information of past state derivatives are called neutral-type systems, and such systems can be found in many engineering systems, e.g. chemical reactors, transmission lines, partial element equivalent circuits in very large scale integration (VLSI) systems and Lotka-Volterra systems. Due to the fact that neutral delays may exist in VLSI implementations of neural networks, the stability analysis of neural networks with neutral terms has received increasing attention and a rich body of results has been reported [19–23]. In [20], the delay-dependent exponential stability have been studied for a class of neural networks described by nonlinear delay differential equations of neutral type by means of linear matrix inequalities (LMIs). By utilizing the Lyapunov-Krasovkii functional and the LMI approach, the global exponential stability have been analyzed in [24] for a kind of neutral-type impulsive neural networks. By constructing the new Lyapunov-Krasovskii functional, a unified framework has been established in [25] to derive sufficient conditions for the global exponential mean square stability of a class of Markovian jumping neutral-type neural networks with mode-dependent mixed time-delays.

As has been pointed by Hale [26], the properties of difference operator are crucial for the existence and stability of solutions to neutral functional differential equations (NFDEs). In order to obtain solutions of NFDEs, the definition of stability for difference operator has been introduced in [26]. The properties of difference operator has been studied in [27] when it is not stable. By using the results derived in [27], some results on the existence of periodic solutions to NFDEs have been obtained in [28–30]. However, to the best of the authors' knowledge, the problems of existence and stability of periodic conditions for delayed neural networks with difference operator have not been fully addressed, which constitutes the main motivation of the current research. In this paper, we aim to investigate the existence and stability of periodic solutions for a class of neutral neural networks by using the properties of difference operator. Three fundamental issues emerge as follows: (1) how to prove the existence of the periodic solution of the delayed neural networks with difference operator; (2) how to construct a feasible Lyapunov functional to reflect the influence of the neutral operator in neural networks; (3) how to analyze the stability of the periodic solution for the neutral-type neural networks with difference operator. By using the Mawhin's continuation theorem and Lyapunov functional method, some new sufficient conditions are derived to guarantee the existence, uniqueness, and global exponential stability of the periodic solution for neutral neural networks.

The main contributions of this paper are highlighted as follows. (1) The neural network under consideration shows the neutral features characterized by the operator A_i, which is different from other papers. Hence, when the neutral term is studied as a neutral operator A_i, novel analysis technique is developed since the conventional analysis tool no longer applies; (2) By employing the Mawhin's continuation theorem and Lemma 1 in [27], the sufficient condition is obtained to guarantee the existence of the periodic solution for a class of neutral-type neural networks with delays; and (3) By constructing a novel Lyapunov functional, the sufficient conditions are derived for the concerned systems to be globally exponentially stable.

The following sections are organized as follows: In Sect. 2, the problem under consideration is formulated and some useful lemmas are introduced. In Sect. 3, sufficient conditions are established for the existence of a unique periodic solution of neutral neural networks. The global exponential stability of the periodic solution are investigated in Sect. 4. In Sect. 5, a numerical example is provided to show the feasibility of our results. Finally, we conclude the paper in Sect. 6.

2 Preliminaries

Consider the following neutral-type neural networks with delays:

$$\begin{cases} (A_i x_i)'(t) = -a_i(t)x_i(t) + \sum_{j=1}^{n}[b_{ij}(t)f_j(t, x_j(t)) + d_{ij}(t)g_j(t, x_j(t-\tau_{ij}(t)))] + I_i(t), \\ x_i(t) = \phi_i(t),\ t \in [-\tau, 0],\ i = 1, 2, \ldots, n, \end{cases} \tag{1}$$

where A_i is a difference operator defined by

$$(A_i x_i)(t) = x_i(t) - c_i(t)x_i(t-\gamma), \tag{2}$$

$x_i(t)$ denotes the state of the ith unit at time t, and $I_i(t)$ is the external bias on the ith at time t, $a_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state when disconnected from the network and external inputs at time t, $\tau_{ij}(t)$ corresponds to the finite speed of the axonal transmission of signal, $b_{ij}(t)$ denotes the strength of the jth unit on the ith unit at time t, $d_{ij}(t)$ denotes the strength of the jth unit on the ith unit at time $t - \tau_{ij}(t)$ and f_j is the signal transmission function. Throughout this paper, it is assumed that $c_i(t),\ a_i(t),\ b_{ij}(t),\ d_{ij}(t),\ \tau_{ij}(t),\ I_i(t)$ are continuously periodic functions defined on $t \in [0, \infty)$ with a common period $\omega > 0$. Moreover, $\gamma,\ a_i(t),\ b_{ij}(t),\ d_{ij}(t)$ are positive everywhere, $f_j(t, x),\ g_j(t, x)$ are continuous and $\omega-$periodic with respect to t.

Let $\tau = \max\{\gamma,\ \tau_{ij}(t),\ 1 \le i, j \le n,\ t \in [0, \infty)\}$. The initial-value functions are as follow:

$$\phi(t) = (\phi_1(t), \phi_2(t), \cdots, \phi_n(t))^T \in C([-\tau, 0], \mathbb{R}^n),$$

where $C([-\tau, 0], \mathbb{R}^n)$ is the Banach space of continuous functions on $[-\tau, 0]$ with norm

$$||\phi|| = \sup_{t\in[-\tau,0]} \max_{1\le i\le n} |\phi_i(t)|.$$

Denote that

$$c_0 = \max_{t\in[0,T]} |c(t)|,\ \ \sigma = \min_{t\in[0,T]} |c(t)|,$$
$$C_T = \{x | x \in C(\mathbb{R}, \mathbb{R}),\ x(t+T) \equiv x(t),\ \forall t \in \mathbb{R}\}$$

with the norm

$$|\varphi|_0 = \max_{t\in[0,T]} |\varphi(t)|, \quad \forall \varphi \in C_T.$$

Clearly, C_T is a Banach space. Define a linear operator as follow:

$$A : C_T \to C_T, \quad [Ax](t) = x(t) - c(t)x(t-\tau), \quad \forall t \in \mathbb{R}.$$

Lemma 2.1 [27] *If $|c(t)| \neq 1$, then the operator A has the continuous inverse A^{-1} on C_T which satisfies*

(i)

$$[A^{-1}f](t) = \begin{cases} f(t) + \sum_{j=1}^{\infty}\prod_{i=1}^{j} c(t-(i-1)\tau)f(t-j\tau), \ c_0 < 1, \ \forall f \in C_T, \\ -\frac{f(t+\tau)}{c(t+\tau)} - \sum_{j=1}^{\infty}\prod_{i=1}^{j+1} \frac{1}{c(t+i\tau)} f(t+j\tau+\tau), \ \sigma > 1, \ \forall f \in C_T. \end{cases}$$

(ii)

$$\int_0^T |[A^{-1}f](t)|dt \leq \begin{cases} \frac{1}{1-c_0}\int_0^T |f(t)|dt, \ c_0 < 1, \ \forall f \in C_T, \\ \frac{1}{\sigma - 1}\int_0^T |f(t)|dt, \ \sigma > 1, \ \forall f \in C_T. \end{cases}$$

(iii)

$$|A^{-1}f|_0 \leq \begin{cases} \frac{1}{1-c_0}|f|_0, \ c_0 < 1, \ \forall f \in C_T, \\ \frac{1}{\sigma-1}|f|_0, \ \sigma > 1, \ \forall f \in C_T. \end{cases}$$

The famous Mawhin's continuation theorem is recalled as follows.

Lemma 2.2 [31] *Suppose that X and Y are two Banach spaces, and $L : D(L) \subset X \to Y$ is a Fredholm operator with index zero. Furthermore, $\Omega \subset X$ is an open bounded set and $N : \bar{\Omega} \to Y$ is L-compact on $\bar{\Omega}$. If all the following conditions hold*

(i) $Lx \neq \lambda Nx, \forall x \in \partial\Omega \cap D(L), \forall \lambda \in (0, 1)$,
(ii) $Nx \notin ImL, \forall x \in \partial\Omega \cap KerL$,
(iii) $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$,

where $J : ImQ \to KerL$ is an isomorphism, then the equation $Lx = Nx$ has a solution on $\bar{\Omega} \cap D(L)$.

3 Existence of Periodic Solution

For convenience, the following notations will be used in this paper:

$$\bar{f} = \frac{1}{\omega}\int_0^{\omega} f(t)dt, \quad f^+ = \max_{t\in[0,\omega]} |f(t)|, \quad f^- = \min_{t\in[0,\omega]} |f(t)|,$$

where f is a continuous $\omega-$periodic function.

Denote by C_ω (respectively, C_ω^1) the set of all continuous (respectively, differentiable) ω-periodic functions with respect to $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$ defined on $\mathbb{R}$. Moreover, denote that

$$|x|_0 = \max_{1\le i\le n}\{x_i^+\}, \quad |x|_1 = \max\{|x|_0, |x'|_0\}.$$

Then C_ω and C_ω^1 are Banach spaces with the norms $|\cdot|$ and $|\cdot|_1$, respectively. Define

$$\begin{aligned}
A:\ & C_\omega \to C_\omega, (Ax)(t) = x(t) - C(t)x(t-\gamma), \forall t \in \mathbb{R},\\
L:\ & D(L) \subset C_\omega \to C_\omega^1, (Lx)(t) = (Ax)'(t),\\
N:\ & C_\omega^1 \to C_\omega, (Nx)_i(t) = -a_i(t)x_i(t) + \sum_{j=1}^{n}[b_{ij}(t)f_j(t, x_j(t))\\
& + d_{ij}(t)g_j(t, x_j(t-\tau_{ij}(t)))] + I_i(t),\ i = 1, 2, \cdots, n,
\end{aligned}$$

where $C(t) = \mathrm{diag}(c_1(t), c_2(t), \cdots, c_n(t))$ and $D(L) = \{x : x \in C_\omega^1\}$.
Then system (1) is the operator equation $Lx = Nx$. It is easy to see

$$\mathrm{Im}L = \left\{y : y \in C_\omega, \int_0^\omega y(s)ds = 0\right\}.$$

We have $(x(t) - C(t)x(t-\tau))' = 0\ \forall x \in KerL$. Therefore,

$$x(t) - C(t)x(t-\tau) = \widetilde{c}, \tag{3}$$

where $\widetilde{c} \in \mathbb{R}^n$ is a constant vector. Let $\varphi(t)$ be the unique $\omega-$periodic solution of (3), then $\varphi(t) \ne \mathbf{0}$ and

$$\mathrm{Ker}L = \{a_0\varphi(t) : a_0 \in \mathbb{R}\}.$$

It is now a position to state the result on the existence of the periodic solution.

Theorem 3.1 *Assume that $c_i^+ < 1$ or $c_i^- > 1$ for $t > 0$ and $i = 1, 2, \cdots, n$. Moreover, suppose that $\int_0^T \varphi^2(t)dt \ne 0$ where $\varphi(t)$ is defined in (3), and there exist non-negative constants p_j and q_j such that*

$$|f_j(t, x)| \le p_j, \quad |g_j(t, x)| \le q_j,\ j = 1, 2, \cdots, n.$$

Then system (1) has at least one $\omega-$periodic solution.

Proof Obviously, ImL is a closed in C_ω and $dimKerL = condimImL = n$. So L is a Fredholm operator with index zero. From $\int_0^T \varphi^2(t)dt \ne 0$, define continuous projectors P, Q

$$P : C_\omega \to KerL, (Px)(t) = \frac{\int_0^\omega x(t)\varphi(t)dt}{\int_0^\omega \varphi^2 dt}\varphi(t)$$

and

$$Q : C_\omega \to C_\omega/ImL, Qy = \frac{1}{\omega}\int_0^\omega y(s)ds.$$

Let

$$L_P = L|_{D(L)\cap KerP} : D(L) \cap KerP \to ImL,$$

then

$$L_P^{-1} = K_P : ImL \to D(L) \cap KerP.$$

Since $ImL \subset C_T$ and $D(L) \cap KerP \subset C_T^1$, we know that K_P is an embedding operator. Hence, K_P is a completely operator in ImL. According to the definitions of Q and N, it can be found that $QN(\overline{\Omega})$ is bounded on $\overline{\Omega}$. Then, nonlinear operator N is $L-$compact on $\overline{\Omega}$. Next, we will complete the proof in three steps.

Step 1. Let $\Omega_1 = \{x \in D(L) \subset C_\omega^1 : Lx = \lambda Nx, \lambda \in (0, 1)\}$. We show that Ω_1 is a bounded set. If $\forall x \in \Omega_1$, then $Lx = \lambda Nx$, i.e., for $i = 1, 2, \cdots, n$,

$$(A_i x_i(t))' + \lambda a_i(t)x_i(t) - \lambda \sum_{j=1}^{n}[b_{ij}(t) f_j(t, x_j(t)) + d_{ij}(t)g_j(t, x_j(t - \tau_{ij}(t)))] - \lambda I_i(t) = 0. \tag{4}$$

Notice that there exists $t_i \in [0, \omega]$ such that $Ax_i(t_i) = [Ax_i(t)]^+$. Hence $(A_i x_i)'(t_i) = 0$ which implies that

$$a_i(t_i)x_i(t_i) = \sum_{j=1}^{n}[b_{ij}(t_i) f_j(t_i, x_j(t_i)) + d_{ij}(t_i)g_j(t_i, x_j(t_i - \tau_{ij}(t_i)))] + I_i(t_i). \tag{5}$$

From (5), we have

$$\begin{aligned} |x_i(t_i)| &= \left| \sum_{j=1}^{n} \left[\frac{b_{ij}(t_i)}{a_i(t_i)} f_j(t_i, x_j(t_i)) + \frac{d_{ij}(t_i)}{a_i(t_i)} g_j(t_i, x_j(t_i - \tau_{ij}(t_i))) \right] + \frac{I_i(t_i)}{a_i(t_i)} \right| \\ &\le \sum_{j=1}^{n} p_j \left[\frac{b_{ij}(t_i)}{a_i(t_i)} \right]^+ + \sum_{j=1}^{n} q_j \left[\frac{d_{ij}(t_i)}{a_i(t_i)} \right]^+ + \left[\frac{I_i(t_i)}{a_i(t_i)} \right]^+, \quad i = 1, 2, \cdots, n. \end{aligned} \tag{6}$$

By (6), we obtain

$$[x_i(t)]^+ \le h_i, \quad i = 1, 2, \cdots, n,$$

where h_i is the ith component of vector h, and it is independent of λ. Moreover, it follows from (4) that

$$\begin{aligned} [(A_i x_i(t))']^+ &\le \max_{t\in[0,\omega]} [a_i(t)|x_i(t)| + \sum_{j=1}^{n}[|b_{ij}(t) f_j(t, x_j(t))| + |d_{ij}(t)g_j(t, x_j(t - \tau_{ij}(t)))|] + |I_i(t)|] \\ &\le [a_i(t)]^+ h_i + \sum_{j=1}^{n}[p_j|b_{ij}(t)]^+ + q_i[d_{ij}(t)]^+] + [I_i(t)]^+ := \hbar_i. \end{aligned} \tag{7}$$

From (7) and Lemma 2.1, if $c_i^+ < 1$, we get

$$\begin{aligned} |x_i'(t)| &= |A_i^{-1} A_i x_i'(t)| \\ &\le \frac{1}{1 - c_i^+} \max_{t \in [0,\omega]} \{|A_i x_i'(t)|\} \\ &= \frac{1}{1 - c_0} \max_{t \in [0,\omega]} \{|(A_i x_i)'(t) + c_i'(t) x_i(t - \gamma)|\} \\ &\le \frac{\hbar_i + [c_i'(t)]^+ h_i}{1 - c_i^+}. \end{aligned} \tag{8}$$

Similarly, if $c_i^- > 1$, we have

$$|x_i'(t)| \le \frac{\hbar_i + [c_i'(t)]^+ h_i}{c_i^- - 1}. \tag{9}$$

From (8) and (9), we obtain

$$[x_i'(t)]^+ \le \ell_i.$$

Step 2. From the above proof, it can be found that there exists some $d > 1$ such that

$$dh_i > A \ \text{ for } \ i = 1, 2, \cdots, n,$$

where $A = \max_{1 \le i \le n}\{h_i + 1, \hbar_i + 1\}$. Let $\Omega_2 = \{x \in C_\omega^1 : -dh < x(t) < dh\}$. We shall prove that if $x \in \partial\Omega_2 \subset KerL$, then

$$|(QNx)_i| \ne 0 \ \text{ for } \ i = 1, 2, \cdots, n \tag{10}$$

where

$$\begin{aligned} (QNx)_i = \frac{1}{\omega} \int_0^\omega \Big[&- a_i(t) x_i(t) + \sum_{j=1}^n \big[b_{ij}(t) f_j(t, x_j(t)) \\ &+ d_{ij}(t) g_j(t, x_j(t - \tau_{ij}(t))) \big] + I_i(t) \Big] dt. \end{aligned}$$

Considering $x \in \partial\Omega_2 \subset KerL$, it is obvious that x is a constant vector in $\mathbb{R}^n$ with $|x_i| = dh_i$ for $i = 1, 2, \cdots, n$.

Next, the proof by contradiction will be used. Suppose that there exists some $i \in \{1, 2, \cdots, n\}$ such that $|(QNx)_i| = 0$, i.e.,

$$\int_0^\omega \Big[- a_i(t) x_i(t) + \sum_{j=1}^n \big[b_{ij}(t) f_j(t, x_j(t)) + d_{ij}(t) g_j(t, x_j(t - \tau_{ij}(t))) \big] + I_i(t) \Big] dt = 0.$$

Then there exists some $\xi \in [0, \omega]$ such that

$$-a_i(\xi) x_i + \sum_{j=1}^n \big[b_{ij}(\xi) f_j(\xi, x_j) + d_{ij}(\xi) g_j(\xi, x_j) \big] + I_i(\xi) = 0.$$

Therefore, we have

$$dh_i = |x_i| \le \sum_{j=1}^n \left[\frac{|b_{ij}(\xi)|}{a_i(\xi)} |f_j(\xi, x_j)| + \frac{|d_{ij}(\xi)|}{a_i(\xi)} |g_j(\xi, x_j)| \right] + \frac{|I_i(\xi)|}{a_i(\xi)} \le h_i.$$

In view of $d > 1$, it follows from the above inequality that $dh_i < dh_i$, which is a contradiction. Thus, (10) holds and hence

$$QNx \neq 0, \ \ \forall x \in \partial\Omega_2 \subset KerL.$$

Step 3. We shall prove that the third condition in Lemma 2.2 holds. Take the homotopy

$$H(x, \mu) = \mu \text{diag}(-\bar{a}_1, -\bar{a}_2, \cdots, -\bar{a}_n)x + (1-\mu)QNx, \ x \in \overline{\Omega} \cap KerL, \ \mu \in [0, 1].$$

When $x \in \partial\Omega_2 \subset KerL$, one has $|x_i| = dh_i$ for $i = 1, 2, \cdots, n$. Thus,

$$|H(x, \mu)|_0 = \max_{1 \leq i \leq n} \left\{ -\bar{a}_i x_i + \frac{(1-\mu)}{\omega} \Sigma_{j=1}^n \int_0^\omega [b_{ij}(t) f_j(t, x_j) + d_{ij}(t) f_j(t, x_j)]dt + \bar{I}_i \right\}.$$

We claim that

$$|H(x, \mu)|_0 > 0. \tag{11}$$

Suppose that $|H(x, \mu)|_0 = 0$, then for $i = 1, 2, \cdots, n$ we have

$$-\bar{a}_i x_i + \frac{(1-\mu)}{\omega} \Sigma_{j=1}^n \int_0^\omega [b_{ij}(t) f_j(t, x_j) + d_{ij}(t) f_j(t, x_j)]dt + (1-\mu)\bar{I}_i = 0. \tag{12}$$

According to the integral mean value theorem and (12), there is some $\eta \in [0, \omega]$ such that

$$-\bar{a}_i x_i + (1-\mu)\Sigma_{j=1}^n [b_{ij}(\eta) f_j(\eta, x_j) + d_{ij}(\eta) f_j(\eta, x_j)] + (1-\mu)\bar{I}_i = 0.$$

Then, one has

$$dh_i = |x_i| \leq (1-\mu) \sum_{j=1}^n \left[\frac{|b_{ij}(\eta)|}{\bar{a}_i} |f_j(\eta, x_j)| + \frac{|d_{ij}(\eta)|}{\bar{a}_i} |g_j(\eta, x_j)| \right] + (1-\mu)\frac{|\bar{I}_i|}{\bar{a}_i} \leq h_i$$

which contradicts that $d > 1$. Therefore, Eq. (11) holds. Using the property of topological degree and taking J to be the identity mapping $I : ImQ \to KerL$, we have

$$\begin{aligned} \deg\{JQN, \Omega \cap KerL, 0\} &= \deg\{H(\cdot, 0), \Omega \cap KerL, 0\} \\ &= \deg\{H(\cdot, 1), \Omega \cap KerL, 0\} \\ &= \deg\{\text{diag}(-\bar{a}_1, -\bar{a}_2, \cdots, -\bar{a}_n), \Omega \cap KerL, 0\} \\ &= 1 \neq 0. \end{aligned}$$

Then, by using Lemma 2.2, we obtain that the equation $Lx = Nx$ has at least one $\omega-$periodic solution x in $\bar{\Omega}$. Namely, system (1) has at least one $\omega-$periodic solution. □

Remark 3.2 The neural network system (1) shows the neutral features characterized by the A_i operator, which is different from the corresponding results of other papers, see, e.g., [1,22,24,25]. And also the main results are derived by fully taking advantage of the some properties of operators, such as the characterization of the kernel space of operator L.

If we further assume that $f_j(t, x)$ and $g_j(t, x)$ $(j = 1, 2, \cdots, n)$ are globally Lipschitz with respect to the second variables, then we shall obtain a unique $\omega-$periodic solution for system (1). Then we have the following corollary.

Corollary 3.3 *Assume that all the conditions of Theorem 3.1 hold and there exist nonnegative constants L_j^f and L_j^g such that $|f_j(t, x) - f_j(t, y)| \leq L_j^f |x - y|$ and $|g_j(t, x) - g_j(t, y)| \leq L_j^g |x - y|$, where $t > 0$, $x, y \in \mathbb{R}$, $j = 1, 2, \cdots, n$. Then system (1) has a unique $\omega-$periodic solution $x(t)$ on $[-\tau, \infty]$.*

Since the proof of Corollary 3.3 is similar to Ref. [32], we omit it here.

4 Exponential Stability

In this section, we shall deal with the exponential stability of the periodic solutions.

Definition 4.1 Let $z^*(t) = (z_1^*(t), z_2^*(t), \cdots, z_n^*(t))^\top$ be a periodic solution of system (1). Then the periodic solution $z^*(t)$ is globally exponentially stable if there exist constants $\alpha > 0$ and $\beta \geq 1$ such that, for the any solution of system (1), the following holds

$$|z(t) - z^*(t)| \leq \beta e^{-\alpha t}|z(0) - z^*(0)|, \quad t > 0.$$

Denote that

$$\mathbb{D} = \text{diag}\left(a_1^- - \left(\frac{a_1^+ c_1^+}{1 - c_1^+} + 1\right), a_2^- - \left(\frac{a_2^+ c_2^+}{1 - c_2^+} + 1\right), \cdots, a_n^- - \left(\frac{a_n^+ c_n^+}{1 - c_n^+} + 1\right)\right),$$

$$\mathbb{E} = \left(\frac{b_{ij}^+ L_j^f + d_{ij}^+ L_j^g}{1 - c_j^+}\right)_{n\times n},$$

then we have the following theorem for the stability of system (1).

Theorem 4.2 *Assume that all the conditions of Theorem 3.1 hold,* $\rho(\mathbb{D}^{-1}\mathbb{E}) < 1$, $f_j(t, 0) = g_j(t, 0) = 0$ *and there exist nonnegative constants* L_j^f *and* L_j^g *such that* $|f_j(t, x) - f_j(t, y)| \leq L_j^f|x - y|$ *and* $|g_j(t, x) - g_j(t, y)| \leq L_j^g|x - y|$, *where* $t > 0,\ x, y \in \mathbb{R},\ j = 1, 2, \cdots, n$. *Then system* (1) *has a unique* $\omega-$*periodic solution* $x(t)$ *on* $[-\tau, \infty]$, *which is globally exponentially stable.*

Proof Since $\rho(\mathbb{D}^{-1}\mathbb{E}) < 1$, i.e., the spectral radius of $\mathbb{D}^{-1}\mathbb{E}$ is less than 1, there exists a constant vector $\vartheta = (\vartheta_1, \vartheta_2, \cdots, \vartheta_n)^{\mathrm{T}} > 0$ such that

$$(\mathbb{E}_n - \mathbb{D}^{-1}\mathbb{E})\vartheta > 0, \quad [I_i(t)]^+ \leq \vartheta_i,\ i = 1, 2, \cdots, n,$$

where $\mathbb{E}_n$ is an $n \times n$ unit matrix. Thus

$$\left(-a_i^- + \frac{a_i^+ c_i^+}{1 - c_i^+} + 1\right)\vartheta_i + \sum_{j=1}^{n} \frac{(b_{ij}^+ L_j^f + d_{ij}^+ L_j^g)}{1 - c_j^+}\vartheta_j < 0, \quad i = 1, 2, \cdots, n. \tag{13}$$

For $t > 0$, define

$$\Xi_i(t) = \left[t - a_i^- + \left(\frac{a_i^+ c_i^+}{1 - c_i^+} + 1\right)e^{t\gamma}\right]\vartheta_i + \sum_{j=1}^{n} \frac{(b_{ij}^+ L_j^f + d_{ij}^+ L_j^g e^{t\tau_{ij}^+})}{1 - c_i^+}\vartheta_j, \quad i = 1, 2, \cdots, n.$$

It is easy to verify that $\Xi_i(t),\ i = 1, 2, \cdots, n$ are all continuous on interval $[0, \lambda_0]$. Then we have

$$\Xi_i(0) = \left(-a_i^- + \frac{a_i^+ c_i^+}{1 - c_i^+} + 1\right)\vartheta_i + \sum_{j=1}^{n} \frac{(b_{ij}^+ L_j^f + d_{ij}^+ L_j^g)}{1 - c_j^+}\vartheta_j < 0, \quad i = 1, 2, \cdots, n$$

and there is a constant $\lambda \in [0, \lambda_0]$ such that

$$\Xi_i(\lambda) = \left[\lambda - a_i^- + \left(\frac{a_i^+ c_i^+}{1 - c_i^+} + 1\right)e^{\lambda\gamma}\right]\vartheta_i + \sum_{j=1}^{n} \frac{(b_{ij}^+ L_j^f + d_{ij}^+ L_j^g e^{\lambda\tau_{ij}^+})}{1 - c_j^+}\vartheta_j$$
$$< 0, \quad i = 1, 2, \cdots, n.$$

For the above λ, we choose the following Lyapunov functional:

$$V_i(t) = |(A_i x_i)(t)|e^{\lambda t}, \quad t > 0, \ i = 1, 2, \cdots, n.$$

We claim that

$$V_i(t) = |(A_i x_i)(t)|e^{\lambda t} < \xi_i, \quad t > 0, \ i = 1, 2, \cdots, n. \tag{14}$$

Otherwise, there must exist an $i \in \{1, 2, \cdots, n\}$ and $t_i > 0$ with $t_i \leq \gamma$ such that

$$V_i(t_i) = \xi_i \text{ and } V_j(t) < \xi_j, \ i = 1, 2, \cdots, n, \ t < t_i.$$

Calculating the time derivative of $V_i(t)$ along the trajectories of system (1), we have

$$\begin{aligned}
0 \leq D^+ V_i(t_i) &= \mathrm{sgn}\{(A_i x_i)(t_i)\}(A_i x_i)'(t_i)e^{\lambda t_i} + \lambda |(A_i x_i)(t_i)|e^{\lambda t_i} \\
&= \mathrm{sgn}\{(A_i x_i)(t_i)\}\Big[- a_i(t_i)x_i(t_i) \\
&\quad + \sum_{j=1}^{n} [b_{ij}(t_i) f_j(t, x_j(t_i)) + d_{ij}(t_i) g_j(t_i, x_j(t_i - \tau_{ij}(t_i)))] + I_i(t_i)\Big] e^{\lambda t_i} \\
&\quad + \lambda |(A_i x_i)(t_i)|e^{\lambda t_i} \\
&\leq (\lambda - a_i^-)|(A_i x_i)(t_i)|e^{\lambda t_i} + a_i^+ c_i^+ |x_i(t_i - \gamma)|e^{\lambda(t_i - \gamma)} e^{\lambda \gamma} + [I_i(t)]^+ e^{\lambda \gamma} \\
&\quad + \sum_{j=1}^{n} b_{ij}^+ L_j^f |x_j(t_i)|e^{\lambda t_i} + \sum_{j=1}^{n} d_{ij}^+ L_j^g |x_j(t_i - \tau_{ij}(t_i))|e^{\lambda(t_i - \tau_{ij}(t_i))} e^{\lambda \tau_{ij}(t_i)} \\
&\leq \left[\lambda - a_i^- + (a_i^+ c_i^+ \frac{1}{1 - c_i^+} + 1)e^{\lambda \gamma} \right] \xi_i \\
&\quad + \sum_{j=1}^{n} (b_{ij}^+ L_j^f + d_{ij}^+ L_j^g e^{\lambda \tau_{ij}^+}) \frac{1}{1 - c_j^+} \xi_j,
\end{aligned} \tag{15}$$

where the following inequalities are used

$$\begin{aligned}
|x_i(t_i - \gamma)| &= |A_i^{-1} A_i x_i(t_i - \gamma)| \leq \left| \frac{A_i x_i(t_i - \gamma)}{1 - c_i^+} \right|, \\
|x_j(t_i)| &= |A_j^{-1} A_j x_j(t_i)| \leq \left| \frac{A_j x_j(t_i)}{1 - c_j^+} \right|, \quad |x_j(t_i)| = |A_j^{-1} A_j x_j(t_i - \tau_{ij}(t_i))| \\
&\leq \left| \frac{A_j x_j(t_i - \tau_{ij}(t_i))}{1 - c_j^+} \right|.
\end{aligned}$$

It is obvious that (15) contradicts (13), and then (14) holds. It follows from (14) that, for $i = 1, 2, \cdots, n$ and $t > 0$,

$$|x_i(t)| = |A_i^{-1} A_i x_i(t)| \leq \left| \frac{A_i x_i(t)}{1 - c_i^+} \right| \leq \frac{\vartheta_i}{1 - c_i^+} e^{-\lambda t} \leq M_\phi ||\phi|| e^{-\lambda t},$$

where M_ϕ is a constant such that $M_\phi ||\phi|| \geq \frac{\vartheta_i}{1 - c_i^+}$. The proof is now completed. □

If we denote that

$$\tilde{\mathbb{D}} = \text{diag}\left(a_1^- - \left(\frac{a_1^+ c_1^+}{c_1^- - 1} + 1\right), a_2^- - \left(\frac{a_2^+ c_2^+}{c_2^- - 1} + 1\right), \cdots, a_n^- - \left(\frac{a_n^+ c_n^+}{c_n^- - 1} + 1\right)\right),$$

$$\tilde{\mathbb{E}} = \left(\frac{b_{ij}^+ L_j^f + d_{ij}^+ L_j^g}{c_j^- - 1}\right)_{n\times n},$$

then we get the following corollary.

Corollary 4.3 *Assume that all the conditions of Theorem 3.1 hold,* $\rho(\tilde{\mathbb{D}}^{-1}\tilde{\mathbb{E}}) < 1$, $f_j(t,0) = g_j(t,0) = 0$ *and there exist nonnegative constants* L_j^f *and* L_j^g *such that* $|f_j(t,x) - f_j(t,y)| \le L_j^f |x-y|$ *and* $|g_j(t,x) - g_j(t,y)| \le L_j^g |x-y|$, *where* $t > 0$, $x, y \in \mathbb{R}$, $j = 1, 2, \cdots, n$. *Then system* (1) *has a unique* $\omega-$*periodic solution* $x(t)$ *on* $[-\tau, \infty]$, *which is globally exponentially stable.*

5 Numerical Example

In order to verify the feasibility of our results, consider the following neutral-type neural network:

$$\begin{cases} (A_1x_1)'(t) = -a_1(t)x_1(t) + \sum_{j=1}^{2}[b_{1j}(t)f_j(t,x_j(t)) + d_{1j}(t)g_j(t,x_j(t-\tau_{1j}(t)))] + I_1(t), \\ (A_2x_2)'(t) = -a_2(t)x_2(t) + \sum_{j=1}^{2}[b_{2j}(t)f_j(t,x_j(t)) + d_{2j}(t)g_j(t,x_j(t-\tau_{2j}(t)))] + I_2(t), \end{cases}$$

where

$(A_1x_1)(t) = x_1(t) - c_1(t)x_1(t-\gamma)$, $(A_2x_2)(t) = x_2(t) - c_2(t)x_2(t-\gamma)$, $\omega = 2\pi$, $\gamma = 100$, $I_1(t) = I_2(t) = \sin t$, $a_1(t) = a_2(t) = 2$, $c_1(t) = c_2(t) = 0.01\cos t$, $b_{ij}(t) = d_{ij}(t) = 0.1$, $\tau_{ij}(t) = \frac{1}{2\pi}\sin t$, $f_j(t,u) = g_j(t,u) = 0.2\sin u$.

For $i, j = 1, 2$, we have

$$c_i^+ = 0.01 < 1,\ L_j^f = L_j^g = 0.2,\ \mathbb{D} = \frac{97}{99}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \mathbb{E} = \frac{4}{99}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$\mathbb{D}^{-1}\mathbb{E} = \frac{4}{97}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

then $\rho(\mathbb{D}^{-1}\mathbb{E}) = \frac{8}{97} < 1$. It follows from Theorem 4.2 that the periodic solution of the above system is globally exponentially stable.

6 Conclusions

In this paper, the problem of stability analysis has been discussed for a class of neutral-type neural networks with time delays and difference operator. By using Mawhin's continuation theorem and Lyapunov functional method, some results have been derived for the existence, uniqueness and global exponential stability of periodic solution for the concerned systems. A numerical example has been provided to illustrate the effectiveness of the obtained results. The results about existence and global exponential stability of periodic solution for neutral-type neural networks proposed in this paper could be further utilized for other related problems, such as control and filtering [35–37] the non-fragile state estimation in [38,39], the distributed state estimation for sensor networks as considered in [40,41]. The results derived in this paper can be also extended into Sensor networks or social networks, which is now a hot research topic [42–44].

Acknowledgements This work was supported in part by the National Natural Science Foundation of China under Grants 61374010, 61074129, 11671008, and 61175111, the Natural Science Foundation of Jiangsu Province of China under Grant BK2012682, and the Six Talents Peak Project of Jiangsu Province (2012).

References

1. Arik S (2014) An analysis of stability of neutral-type neural systems with constant time delays. J Franklin Inst 351:4949–4959
2. Liu Y, Liu W, Obaid MA, Abbas IA (2016) Exponential stability of Markovian jumping Cohen–Grossberg neural networks with mixed mode-dependent time-delays. Neurocomputing 177:409–415
3. Li L, Ho DWC, Lu J (2013) A unified approach to practical consensus with quantized data and time delay. IEEE Trans Circuits Syst I 60(10):2668–2678
4. Li L, Ho DWC, Cao J, Lu J (2016) Pinning cluster synchronization in an array of coupled neural networks under event-based mechanism. Neural Netw 76:1–12
5. Lu J, Ho DWC (2011) Stabilization of complex dynamical networks with noise disturbance under performance constraint. Nonlinear Anal Ser B 12:1974–1984
6. Lu J, Wang Z, Cao J, Ho DWC, Kurths J (2012) Pinning impulsive stabilization of nonlinear dynamical networks with time-varying delay. Int J Bifurc Chaos 22(7):1250176
7. Kao Y, Shi L, Xie J, Karimi HR (2015) Global exponential stability of delayed Markovian jump fuzzy cellular neural networks with generally incomplete transition probability. Neural Netw 63:18–30
8. Zhao H, Wang K (2006) Dynamical behaviors of Cohen-Grossberg neural networks with delays and reaction-diffusion terms. Neurocomputing 70:536–543
9. Cao J, Wang J (2005) Global asymptotic and robust stability of recurrent neural networks with time delays. IEEE Trans Circuits Syst I 52:417–426
10. Arik S (2014) An improved robust stability result for uncertain neural networks with multiple time delays. Neural Netw 54:1–10
11. Baldi P, Atiya AF (1994) How delays affect neural dynamics and learning. IEEE Trans Neural Netw 5:612–621
12. Zhu Q, Cao J (2010) Robust exponential stability of Markovian jump impulsive stochastic Cohen–Grossberg neural networks with mixed time delays. IEEE Trans Neural Netw 21:1314–1325
13. Song Q, Cao J (2010) On pinning synchronization of directed and undirected complex dynamical networks. IEEE Trans Circuits Syst I 57:672–680
14. Song Q, Cao J (2007) Impulsive effects on stability of fuzzy Cohen–Grossberg neural networks with time-varying delays. IEEE Trans Syst Man Cybern B 37:733–741
15. Yang F, Wang Z, Hung YS (2002) Robust Kalman filtering for discrete time-varying uncertain systems with multiplicative noise. IEEE Trans Autom Control 47:1179–1183
16. Zhai G, Xu X, Ho DWC (2012) Stability of switched linear discrete-time descriptor systems: a new commutation condition. Int J Control 85:1779–1788
17. Dong H, Wang Z, Ho DWC, Gao H (2010) Robust H-infinity fuzzy output-feedback control with multiple probabilistic delays and multiple missing measurements. IEEE Trans Fuzzy Syst 18:712–725
18. Lu J, Wang Z, Cao J, Ho DWC, Kurths J (2012) Pinning impulsive stabilization of nonlinear dynamical networks with time-varying delay. Int J Bifurc Chaos 22:1250176
19. Gui Z, Ge W, Yang X (2007) Periodic oscillation for a Hopfield neural networks with neutral delays. Phys Lett A 364:267–273
20. Xu S, Lam J, Ho D, Zou Y (2005) Delay-dependent exponential stability for a class of neural networks with time delays. J Comput Appl Math 183:16–28

21. Lien C, Yu K, Lin Y, Chung Y, Chung L (2009) Exponential convergence rate estimation for uncertain delayed neural networks of neutral type. Chao Solit Fract 40:2491–2499
22. Zhang Y, Xu S, Chu Y, Lu J (2010) Robust global synchronization of complex networks with neutral-type delayed nodes. Appl Math Comput 216:768–778
23. Li X (2009) Global exponential stability for a class of neural networks. Appl Math Lett 22:1235–1239
24. Rakkiyappan R, Balasubramaniam P, Cao J (2010) Global exponential stability results for neutral-type impulsive neural networks. Nonlinear Anal Real 11:122–130
25. Liu Y, Wang Z, Liu X (2012) Stability analysis for a class of neutral-type neural networks with Markovian jumping parameters and mode-dependent mixed delays. Neurocomputing 94:46–53
26. Hale J (1977) Theory of functional differential equations. Applied mathematical science, vol 3. Springer, NewYork
27. Du B, Guo L, Ge W, Lu S (2009) Periodic solutions for generalized Liénard neutral equation with variable parameter. Nonlinear Anal 70:2387–2394
28. Du B (2013) Periodic solutions to p-laplacian neutral Lienard type equation with variable parameter. Math Slovaca 2:1–15
29. Du B, Sun B (2011) Periodic solutions to a p-Laplacian neutral Duffing equation with variable parameter. Electron J Qual Theo 55:1–18
30. Bai C, Du B (2013) Periodic Solutions for a kind of neutral Rayleigh equations with variable parameter. Results Math 63:567–580
31. Gaines R, Mawhin J (1977) Coincidence degree and nonlinear differential equations. Springer, Berlin
32. Zhang A, Qiu J, She J (2014) Existence and global exponential stability of periodic solution for high-order discrete-time BAM neural networks. Neural Netw 50:98–109
33. Li T, Zheng W, Lin C (2011) Delay-slope dependent stability results of recurrent neural networks. IEEE Trans Neural Netw 22:2138–2143
34. Wang Z, Wang Y, Liu Y (2010) Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time delays. IEEE Trans Neural Netw 21:11–25
35. Han Q-L, Liu Y, Yang F (2016) Optimal communication network-based H-infinity quantized control with packet dropouts for a class of discrete-time neural networks with distributed time delay. IEEE Trans Neural Netw Learn Syst 27(2):426–434
36. Liu Y, Wang Y, Zhu X, Liu X (2014) Optimal guaranteed cost control of a class of hybrid systems with mode-dependent mixed time delays. Int J Syst Sci 45(7):1528–1538
37. Liu Y, Alsaadi F, Yin X, Wang Y (2015) Robust H-infinity filtering for discrete nonlinear delayed stochastic systems with missing measurements and randomly occurring nonlinearities. Int J Gen Syst 44(2):169–181
38. Hou N, Dong H, Wang Z, Ren W, Alsaadi FE (2016) Non-fragile state estimation for discrete Markovian jumping neural networks. Neurocomputing 179(29):238–245
39. Yu Y, Dong H, Wang Z, Ren W, Alsaadi FE (2016) Design of non-fragile state estimators for discrete time-delayed neural networks with parameter uncertainties. Neurocomputing 182(19):18–24
40. Dong H, Wang Z, Alsaadi FE, Ahmad B (2015) Event-triggered robust distributed state estimation for sensor networks with state-dependent noises. Int J Gen Syst 44(2):254–266
41. Dong H, Wang Z, Ding S, Gao H (2015) Finite-horizon reliable control with randomly occurring uncertainties and nonlinearities subject to output quantization. Automatica 52:355–362
42. Shen J, Tan H, Wang J, Wang J, Lee S (2015) A novel routing protocol providing good transmission reliability in underwater sensor networks. J Internet Technol 16(1):171–178
43. Xie S, Wang Y (2014) Construction of tree network with limited delivery latency in homogeneous wireless sensor networks. Wirel Pers Commun 78(1):231–246
44. Ma T et al (2015) Social network and tag sources based augmenting collaborative recommender system. IEICE Trans Inf Syst E98–D(4):902–910