# Wavelet Neural Network for Corrections Prediction in Single-Frequency GPS Users

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Abstract Accurate and reliable position determination is a vital component in Global Positioning System (GPS). GPS positioning errors occur from the cumulative effects of receiver, satellite and atmosphere, and also due to the U.S. military intentionally such as Selective Availability (SA). In order to improve the accuracy of positions provided by GPS additional correction information may be used, such as Differential GPS (DGPS) or other sensors to enhance position reliability. The DGPS has the problem of slow updates. To overcome this limitation, DGPS corrections prediction has been proposed. The ability of Neural Networks (NNs) to discover nonlinear relationships in input data makes them ideal for modeling nonlinear dynamic systems. The Wavelet Neural Network (WNN) employing nonlinear wavelet basis function, which are localized in both the time and frequency space, has been developed as an alternative approach to nonlinear fitting problem. Particle Swarm Optimization (PSO), a global optimization method, is used to train the WNN. In this paper, a WNN trained by a PSO algorithm is proposed for DGPS corrections prediction in single-frequency GPS receivers. Experimental results show the feasibility and effectiveness of the proposed method. The results are analyzed and compared with WNN trained by Back Propagation (BP) algorithm. The experimental results show that WNN, trained by the PSO algorithm, is able to reduce RMS errors to less than 1 m with SA on and 0.6 m with SA off.

**Keywords** DGPS corrections prediction · Wavelet neural network · Back propagation · Particle swarm optimization

## 1 Introduction

The Global Positioning System (GPS) is a satellite-based navigation and positioning system developed by the U.S. Department of Defense. It provides at all altitudes, 24-h, continuous,

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Department of Electrical Engineering, Iran University of Science and Technology, Narmak, Tehran, 16846-13114, Iran e-mail: M\_Mosavi@iust.ac.ir and global high accuracy service to military and civilian users. Authorized and civilian users have access to the Precise Positioning Service (PPS) and Standard Positioning Service (SPS), respectively. GPS has wide-spread use in marine, land, and air applications [1].

Each GPS satellite continuously transmits direct-sequence, spread-spectrum signals on which receivers can perform ranging measurements. The PPS uses a 10 MHz chipping rate and transmits at two frequencies: 1575.42 MHz (L1) and 1227.60 MHz (L2). In contrast, the SPS utilizes a chipping rate of 1 MHz and transmits on L1 only. The PPS achieves a higher accuracy because its chipping rate is ten times as great. Additionally, it utilizes measurements at the two frequencies to reduce the effect of ionospheric delays. Finally, SPS accuracy was controlled for security reasons through a program called Selective Availability (SA). The SA was the deliberate degradation by the U.S. DoD of the GPS signal through both the predicted ephemeris and the timing. SA is one of the most important error sources in GPS and when it is activated, it causes error in positioning up to 100 m. The U.S. Government has stopped the intentional degradation of the GPS signals after the end of May 1, 2000. In other words, after this time, the SA was set to zero. In this way, the accuracy of the single point positioning is reduced from 100 m to 15–25 m. Removal of the SA not only enhances the performance of single point positioning, but also expands the application area of single GPS receivers in addition to already in use. These features make it possible to greatly enhance the accuracy of GPS in local areas through Differential GPS (DGPS) operation [2].

A DGPS system employs a local reference station, which has a high-quality GPS receiver and an antenna at a known, surveyed location. The reference station estimates the slowly varying components of the satellite range measurement errors, and transmits them as corrections to users within communications range of the station. The DGPS has the problem of slow updates. It sometimes cannot send correction information for minutes at a time, due to radio interference or loss of signals. Absence of DGPS radio waves means that the accurate position of the unit cannot be identified [3]. To overcome this limitation, DGPS corrections prediction has been proposed.

Neural Network (NN), inspired by the current understanding of biological NN, is a class of adaptive systems consisting of a number of simple processing elements, called neurons, that are interconnected to each other in feed-forward and recurrent ways. Although NN can perform some human brain-like tasks, there is still a huge gap between biological and artificial NN. An important contribution of NN is the ability to learn to perform operations, not only for inputs exactly like the training data, but also for new data that may be incomplete or noisy. NN has also the benefit of easy modification by retraining with an updated data set [4].

Wavelet transform is a signal analysis method from domain of time-frequency. It has the characteristic of multi-resolution analysis, and it has the ability of representing local features of signal both in the area of time and frequency. It is a localized analysis method that the size of time window and frequency window are stable but their window shape can change. In the part of low-frequency, it has higher frequency-resolution and lower time-resolution, while in the part of high-frequency, it has higher time-resolution and lower frequency-resolution, so it is called as the microscope of signal analysis. In Wavelet Neural Networks (WNNs), the wavelet function replaces the role of sigmoid function in the hidden unit. The wavelet parameters and wavelet shape are adaptively computed to minimize an energy function for finding the optimal representation of the signal [5].

The usually used learning for WNN is gradient descent method. But its disadvantages are slow convergence speed and easy stay at local minimum [6]. The Particle Swarm Optimization (PSO) is a population based optimization method first proposed by Kennedy and Eberhart [7]. Some of the attractive features of the PSO include ease of implementation and

the fact that no gradient information is required. It can be used in NN training and function minimization.

In this paper, a WNN trained by a PSO algorithm is proposed for DGPS corrections prediction in single-frequency GPS receivers. This paper is organized as follows. Section 2 describes proposed predictor architecture. Back Propagation (BP) and PSO learning algorithms for WNN training are presented in Sect. 3. Experimental results are reported in Sect. 4 and finally conclusions are given in Sect. 5.

#### 2 Proposed Predictor Architecture

NN is clearly a nonlinear mapping from the input space to the output space. It has been proved that any continuous nonlinear function can be approximated arbitrarily well over a compact set by a NN consisting of one hidden layer provided that there are infinite neurons in the hidden layer.

In this paper WNN is employed for prediction of DGPS corrections. The DGPS corrections are calculated as  $\vec{C} = \vec{P} - \vec{U}$ , where  $\vec{P} = (x_n, y_n, z_n)$  and  $\vec{U} = (x_b, y_b, z_b)$  at time *n* are the base station position vectors of measured and accurate, respectively. The WNNs are trained and achieve the ability to predict later seconds DGPS corrections. Formally, this can be stated as: find a function  $f : \Re^{p+1} \to \Re$  such as to obtain an estimate of DGPS corrections at time n + N, from the *p* time steps back from time *n*, so that [8]:

$$C(n+N) = f[C(n), C(n-1), \dots, C(n-p)]$$
(1)

Normally N will be one, so that f will be predicting the next value C. The DGPS process removes the position errors and provides accurate position of the rover unit. This accurate position is obtained by subtracting the coordinate DGPS corrections from the moving vehicle data.

The proposed WNN in this research is shown in Fig. 1 The choice of the order for f is also important. In this paper, the order is based on the experimental results.

Let C(n) denotes the  $(p + 1) \times 1$  external input vector applied to the network, Y(n) denotes the output of the network,  $w_{ji}(n)$  presents the weight between the hidden unit *j* and input unit *i*,  $w_j(n)$  denotes the connection weight between the output unit and hidden unit *j*,  $a_j(n)$  and  $b_j(n)$  present dilation and translation coefficients of wavlon in hidden layer at discrete time *n*, respectively. The wavelet function  $\psi(.)$  which we have considered here is the so called "Gauusian-derivative" function as:

$$\psi(x) = -xe^{-\frac{1}{2}x^2} \tag{2}$$

The usual sigmoid function  $\sigma(.)$  of used in this research is as follow:

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{3}$$

We hope to train a WNN to approximate the function f(x). For a WNN with a fixed number of neurons and architecture, the network weights may be arranged into a vector w. Let F(x, w) be the output of mapping implemented by the WNN. The training process is to find an appropriate weight vector w that provides the best possible approximation of f(x). Let  $\{(x_i^*, y_i^*) | i = 1, 2, ..., N\}$  be a set of training input–output data on f(x). We wish to choose a weight vector w so that the output F(x, w) is "close" to the desired output  $y_i^*$  for the input  $x_i^*$ . That is, the training process is to find the weight vector w that minimizes the



Fig. 1 Proposed WNN architecture with (p + 1, q, 1) structure for DGPS corrections prediction

error between the output of the model and actual values. In this study, BP and PSO learning algorithms for WNN training are employed.

## 3 DGPS Corrections Prediction Using WNN

The net internal activity of neuron j at time n, is given by [9]:

$$v_j(n) = \sum_{i=0}^{i=p} w_{ji}(n)C(n-i)$$
(4)

where,  $v_j(n)$  is the weighted sum of inputs to the jth hidden neuron, C(n-i) is the ith input at time *n*. The output of the jth neuron is computed by passing  $v_j(n)$  through the wavelets  $\psi(.)$ , obtaining:

$$\psi[v_j(n)] = \psi\left[\frac{v_j(n) - b_j(n)}{a_j(n)}\right]$$
(5)

The sum of inputs to the output neuron is obtained by:

$$v(n) = \sum_{j=1}^{j=q} w_j(n) \psi[v_j(n)]$$
(6)

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**Fig. 2** Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by BP algorithm (SA on)

The output of the network is computed by passing v(n) through the nonlinear function  $\sigma(.)$ , obtaining:

$$Y(n) = \sigma[v(n)] \tag{7}$$

## 3.1 BP Learning Algorithm for the WNN Training

The standard BP algorithm is based on the steepest descent gradient approach applied to the minimization of an energy function representing the instantaneous error. BP algorithm is adopted to train the WNN and minimize the function which is defined as [10]:

$$E(n) = \frac{1}{2}e^{2}(n) = \frac{1}{2}\left[Y(n) - D(n)\right]^{2}$$
(8)

where, D(n) denote the desired response of output at time *n*. To minimize of above cost function, the method of steepest descent is used. The weight between the hidden unit *j* and input unit *i* can be adjusted according to:

$$\Delta w_{ji}(n) = -\eta \frac{\partial E(n)}{\partial w_{ji}(n)} = -\eta e(n)\sigma'[v(n)]w_j(n)\psi'[v_j(n)]\frac{C(i-n)}{a_j(n)}$$
(9)

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**Fig. 3** Error of Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by BP algorithm (SA on)

where,  $\eta$  is a learning rate parameter, which is used for controlling the convergent speed of the algorithm. The connection weight between the output unit and hidden unit *j* is updated as follow:

$$\Delta w_j(n) = -\eta \frac{\partial E(n)}{\partial w_j(n)} = -\eta e(n)\sigma'[v(n)]\psi[v_j(n)]$$
(10)

The translation coefficient of the jth wavlon in hidden layer can be adjusted according to:

$$\Delta b_j(n) = -\eta \frac{\partial E(n)}{\partial b_j(n)} = \eta e(n) \sigma'[v(n)] w_j(n) \psi'[v_j(n)] \frac{1}{a_j(n)}$$
(11)

The dilation coefficient of the jth wavlon in hidden layer is updated as follow:

$$\Delta a_j(n) = -\eta \frac{\partial E(n)}{\partial a_j(n)} = \eta e(n)\sigma'[v(n)]w_j(n)\psi'[v_j(n)]\frac{v_j(n) - b_j(n)}{a_j(n)^2}$$
(12)

According to Eqs. 4–12, the iterated procedure is repeated until the predefined termination criteria such as the maximum generation and the error goal are reached.

Dx (m)

 $D\mathbf{y}\left(\mathbf{m}\right)$ 

DZ(B)

L د. 0

100



**Fig. 4** Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by BP algorithm (SA off)

500

Time(s)

600

700

800

900

1000

400

## 3.2 PSO Learning Algorithm for the WNN Training

200

300

PSO is an evolutionary computation technique, inspired by social behaviors of bird flocking, which has been successfully applied in many areas. PSO is a population based search process where individuals, referred to as particles, are grouped into a swarm. Each particle in the swarm denotes a candidate solution to the optimization problem. In a PSO system, each particle flies through the multidimensional search space, adjusts its position in search space according to its own experience and that of neighboring particles. A particle therefore makes use of the best position encountered by itself and that of its neighbors to position itself toward an optimal solution. The effect is that particles "fly" toward a minimum, while still searching a wide area around the best solution. The performance of each particle is measured by a predefined fitness function, which encapsulates the characteristics of the optimization problem [11].

The basic PSO model consists of a swarm of particles moving in a *D*-dimensional search space where a certain quality measure, the fitness, can be calculated. Each particle has a position represented by a vector  $x_{id}$  and a velocity represented by a vector  $v_{id}$ . Each particle



**Fig. 5** Error of Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by BP algorithm (SA off)

remembers its own previous best position so far in a vector  $pbest_{id}$  (the position where it achieves its best fitness). The best previous vector among all the neighbors of a particle is then stored in the particle as a vector  $gbest_{id}$ . The velocity at every iteration step is updated and the particle is moved to a new position. The update of the velocity and position from the previous particle are determined by Eq. 13 and 14 [12]:

$$v_{id}(k+1) = \omega v_{id}(k) + c_1 rand_1(pbest_{id}(k) - x_{id}(k))$$

$$+c_2 rand_2(gbest_{id}(k) - x_{id}(k)) \tag{13}$$

$$x_{id}(k+1) = x_{id}(k) + v_{id}(k+1)$$
(14)

where  $\omega$  is inertia coefficient, and its value usually is confirmed decreasing linearly from 0.9 to 0.4 along with evolution generation.  $c_1$  and  $c_2$  are the positive constants;  $rand_1$  and  $rand_2$  are two random numbers in the range [0,1]. These random numbers are selected with a uniform distribution. The PSO is employed for optimal obtaining of WNN weights as follows: It is assumed that a third-layered WNN is chosen for all application cases. Without loss of generality, it is denoted that  $W_1$  is the connection weight matrix between the input and the hidden layer,  $W_2$  is the weight matrix between the hidden and the output layer, and A



**Fig. 6** Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by PSO algorithm (SA on)

and *B* are dilation and translation coefficients matrix of wavlon in hidden layer. It is further denoted as  $W_i = (W_{1,i}, W_{2,i}, A_i, B_i)$ , where *i* is represented as the index of the particle. Thus in the optimal process of WNN, *W* becomes the optimization variables according to Eq. 13 and 14.

## 4 Experimental Results

In this research, a low cost GPS engine manufactured by Rockwell Company was used. The Rockwell "Microtraker Low Power (MLP)" receiver is a single board, five parallel-channels, L1-only Coarse Acquisition (C/A) code capability. Performance of the proposed WNNs was evaluated by data sets that were collected on the building of Computer Control and Fuzzy Logic Research Lab in the Iran University of Science and Technology. The optimal selection of proposed methods parameters was based on the experimental results. Increasing the order of WNNs and also training patterns improves the proposed WNNs performance. Increasing the order of WNNs and training patterns increases the memory for software implementation and also the structure complexity for hardware implementation. Therefore, a trade-off



**Fig. 7** Error of Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by PSO algorithm (SA on)

in selecting the order of WNNs and training patterns between CPU time and accuracy of methods is required.

Dx, Dy and Dz are errors of position components. They are obtained difference between the measurement and accurate values, i.e.: dx, dy,  $dz_{\text{Error}} = dx$ , dy,  $dz_{\text{Measurement}} - dx$ , dy,  $dz_{\text{Accurate}}$  [13].

Figures 2, 3, 4, 5, 6, 7, 8 and 9 show Dx, Dy and Dz real, predicted and prediction error values for 1000 test data using proposed WNNs, before and after SA was turned off.

Tables 1, 2, 3 and 4 present prediction errors (the difference between the predicted and real values) statistical characteristics for 1000 test data using proposed WNNs, with and without SA was turned off.

Root Mean Square (RMS) was used to evaluate approximations results [14]. The RMS value is computed using:

$$RMS = \sqrt{\frac{1}{M} \sum_{i=1}^{i=M} (DGPS \text{ Corrections}_{\text{Predicted}} - DGPS \text{Corrections}_{\text{Real}})^2}$$
(15)

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**Fig. 8** Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by PSO algorithm (SA off)

where M is number of tests. Table 5 shows the comparing DGPS corrections prediction accuracy using proposed WNN trained by BP and PSO algorithms.

As shown in Table 5, the WNN trained by learning algorithm based on PSO has better accuracy for DGPS corrections prediction, since RMS error of prediction in this method is lower than another method. It can reduce RMS errors to less than 1 m with SA on and 0.6 m with SA off.

There are few papers (for examples [15, 16]) that estimate differential GPS corrections using NNs. The proposed WNNs (trained by BP and PSO algorithms) in this paper have more accuracy than them.

## 5 Conclusions

The accuracy of GPS position is primarily dependent on the satellite position, signal delay, and various environment noises such as ionospheric delay effects, ephemeris errors, satellite clock errors, multi-path distortion, tropospheric delay effects, and numerical errors. The DGPS is the use of differential correction data to remove the errors in the position measured



**Fig. 9** Error of Dx, Dy and Dz predictions for 1000 test data using proposed WNN with (3,3,1) structure trained by PSO algorithm (SA off)

| Parameters | X component   | Y component  | Z component  |
|------------|---|--|--|
| Max        | 4.4234  | 1.6632   | 1.6296   |
| Min        | -6.0472   | -2.1593  | -1.4908  |
| Average    | -0.0456   | 0.0740   | 0.0057   |
| RMS        | 0.8991  | 0.5599   | 0.4858   |
| Total RMS  | 1.1654  |  |  |
|            | Parameters<br>Max<br>Min<br>Average<br>RMS<br>Total RMS | ParametersX componentMax4.4234Min-6.0472Average-0.0456RMS0.8991Total RMS1.1654 | Parameters         X component         Y component           Max         4.4234         1.6632           Min         -6.0472         -2.1593           Average         -0.0456         0.0740           RMS         0.8991         0.5599           Total RMS         1.1654 |

by a standalone GPS receiver. The DGPS has the problem of slow updates. Any interruption of the DGPS service causes a loss of navigation guidance. In this paper, the WNN was used for errors prediction of single-frequency GPS receivers. The BP and PSO learning algorithms were employed for WNN training. Experimental results for DGPS corrections prediction in single-frequency GPS users showed which the WNN trained by PSO has better accuracy; so that RMS error reduced to less than 1 and 0.6 m, before and after SA was turned off, respectively.

| Table 2       Prediction errors         statistical characteristics using       proposed WNN trained by BP         algorithm (SA off)    | Parameters         | X compo  | nent Y compo     | nent Z component  |
|--|--------------------|----------|------------------|-------------------|
|  | Max                | 1.4934   | 1.5420           | 1.4976            |
|  | Min                | -1.9665  | -1.6186          | -1.4473           |
|  | Average            | -0.0001  | 0.0104           | 0.0046            |
|  | RMS                | 0.3546   | 0.4055           | 0.3547            |
|  | Total RMS          | 0.6449   |                  |                   |
| Table 3         Prediction errors  |                    |          |                  |                   |
| statistical characteristics using<br>proposed WNN trained by PSO<br>algorithm (SA on)  | Parameters         | X compoi | nent Y compo     | nent Z component  |
|  | Max                | 2.8414   | 3.2295           | 1.5808            |
|  | Min                | -4.4234  | -2.2396          | -2.8039           |
|  | Average            | 0.0510   | -0.0630          | -0.0108           |
|  | RMS                | 0.5713   | 0.5343           | 0.5076            |
|  | Total RMS          | 0.9324   |                  |                   |
| Table 4 Dradiation arrors  |                    |          |                  |                   |
| Table 4       Prediction errors         statistical characteristics using         proposed WNN trained by PSO         algorithm (SA off) | Parameters         | X compos | nent Y compo     | nent Z component  |
|  | Max                | 2.0434   | 1.6334           | 2.3861            |
|  | Min                | -1.4895  | -1.6552          | -1.3857           |
|  | Average            | 0.0020   | -0.0007          | -0.0072           |
|  | RMS                | 0.3289   | 0.3654           | 0.3273            |
|  | Total RMS          | 0.5906   |                  |                   |
|  |                    |          |                  |                   |
| Table 5Comparing DGPScorrections prediction accuracyusing proposed WNN trained byBP and PSO algorithms                                   | Prediction me      | thod     | Accuracy (SA on) | Accuracy (SA off) |
|  | WNN trained by BP  |          | 1.1654           | 0.6449            |
|  | WNN trained by PSO |          | 0.9324           | 0.5906            |

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