

A unified approach to split scope

Klaus Abels · Luisa Martí

Published online: 27 May 2010
© Springer Science+Business Media B.V. 2010

Abstract The goal of this paper is to propose a unified approach to the split scope readings of negative indefinites, comparative quantifiers, and numerals. There are two main observations that justify this approach. First, split scope shows the same kinds of restrictions across these different quantifiers. Second, split scope always involves low existential force. In our approach, following Sauerland, natural language determiner quantifiers are quantifiers over choice functions, of type $\langle\langle\langle t, e \rangle, t \rangle, t \rangle$. In split readings, the quantifier over choice functions scopes above other operators (such as intensional verbs like *must* or *can*). Determiner quantifiers leave a choice-function trace when they move and this trace combines with the noun restriction, which is interpreted low. That split scope always involves low existential force is derived, without stipulation, from Kratzer's idea that low existential force can be achieved via binding (into the noun restriction).

Keywords Split scope · Negative indefinites · Indefinites · Comparative quantifiers · Numerals · Intensional operators · Choice functions · Pseudo-scope

K. Abels
Department of Linguistics, Chandler House, University College London,
2 Wakefield Street, London WC1N 1PF, UK
e-mail: k.abels@ucl.ac.uk

L. Martí (✉)
Center for Advanced Study in Theoretical Linguistics (CASTL),
Det Humanistiske Fakultet, Universitetet i Tromsø, Tromsø 9037, Norway
e-mail: luisa.marti@uit.no

1 Introduction

Quantificational noun phrases that are not upward monotonic give rise to truth-conditionally distinct, so-called split scope readings across intensional verbs (Heim 2001). Consider (1)–(3):

- (1) The company need fire no employees,
‘It is not the case that the company is obligated to fire employees.’
(Potts 2000)
- (2) At MIT one must publish fewer than three books in order to get tenure.
‘At MIT one must publish at least n books in order to get tenure, and n is less than three.’
(Hackl 2000)
- (3) How many books does Chris want to buy?
‘What is the number n such that Chris wants it to be the case that there are n books that he buys?’
(Rullmann 1995a)

Examples (1) and (2) need not be about actual employees or books, so we can put wide scope, *de re* readings aside. In (1), the split scope reading entails a lack of obligation for the company (it doesn’t have to fire employees), whereas the narrow scope reading does entail an obligation (the company has the obligation to not fire any employees).¹ In (2), the split scope reading says that, in order to get tenure at MIT, the requirement is that one publishes at least one or two books (‘The maximal number of books such that in all possible worlds one publishes that many books is less than three’). The narrow scope reading imposes the implausible requirement that the number of books that you publish be less than three; if you publish more, you don’t get tenure. The split scope reading of (3) is a cardinality question; if the answer is ‘five’, then Chris wants to buy some set of books or other whose cardinality is five, no matter which books they are. The wide scope reading (there is no narrow scope reading here), on the other hand, does care about the identity of those books. If the answer is ‘five’, then Chris wants to buy a particular set of books (e.g., *To Kill a Mockingbird*, *As I Lay Dying*, *Invisible Man*, *A Passage to India*, and *Winesburg, Ohio*), and it so happens that the cardinality of this set is five.

We call the readings of interest here “split scope” readings because they involve noun phrases which seem to scope in two different places at the same time. The overtly expressed quantifier (negation, a maximality operator, a cardinality operator) takes scope above an intensional verb (*need*, *must*, *want*). At the same time, the reference of the nominal restriction (*employees*, *books*) varies with the worlds introduced by the intensional verb, something which is often modeled as a silent low existential operator (the paraphrases of (1) and (3) reflect this better than the paraphrase of (2)).

Split scope readings are available in a number of languages, including English, as shown above. However, English is not the best language to investigate the

¹ See Penka (2007, pp. 89–90) for an argument that split scope readings of negative indefinites are independent of the wide and narrow scope readings these quantifiers give rise to.

properties of split scope. For example, the availability of these readings for negative indefinites seems to be more restricted than in other languages, such as German or Dutch (for German, see Bech 1955/1957; Geurts 1996; Jacobs 1980; Penka 2007; for Dutch, see Rullmann 1995b; de Swart 2000), for reasons that are not clearly understood. Thus, we will limit our attention mostly to German in this paper. The same observations we made above about (1)–(3) obtain for the corresponding German sentences.

There are four generalizations about split scope readings that will be important in the paper:

- (4) *Generalization A*: Split scope readings are possible only across intensional verbs.
Generalization B: Split scope readings are possible only across some intensional verbs, not all.
Generalization C: Split scope readings are not possible in the context of extraposition (even when the intensional verb falls within the class of verbs that allow split scope).
Generalization D: Split scope readings always involve low existential force.

These generalizations are important because they are the main motivation for the unifying approach we take here. The behavior of split scope is independent of the particular quantifier involved: negative indefinites, comparative quantifiers, and numerals all obey the generalizations in (4). Generalization A, when it applies to comparatives, is known as *Kennedy's Generalization* in the literature (after Kennedy 1997; see Heim 2001). We illustrate Generalization A in (5) and (6) for negative indefinites and comparative quantifiers:²

- (5) Genau ein Arzt hat kein Auto.
 exactly one doctor has no car
 'Exactly one doctor has no car.'
- (6) Jeder Arzt hat weniger als drei Autos.
 every doctor has less than three cars
 'Every doctor has less than three cars.'

If the negative indefinite of (5) could split its scope, the sentence would be true if there wasn't exactly one doctor who owned a car (i.e., if either no or more than one doctor owned a car). This reading is clearly unavailable and distinct from the narrow scope reading that the sentence does have: there is exactly one doctor who is carless. If (6) had a split scope reading, then it would be true in a situation in which every doctor has at least two cars—with some doctors having more cars. The paraphrase of the split scope reading is: "The maximal number of cars every doctor has is less than three"—this gives us a minimum threshold. However, (6) is false in this situation. The sentence does have a narrow scope reading in which every doctor is

² We do not deal with *how many*-splits in this paper, and put them aside from now on.

such that s/he owns either one or two cars, but not more. Later on in the paper we illustrate and discuss the four generalizations in much more detail (see Sect. 3).³

To us, the fact that the split scope readings of all of these quantifiers are subject to the same restrictions suggests that the readings have the same source. This unification has never, to our knowledge, been attempted. For example, split scope readings of negative indefinites have received attention in the literature on German and Dutch, for, as suggested above, they are easily available in these languages (Bech 1955/1957; Geurts 1996; Jacobs 1980; Penka 2007; Rullmann 1995b; de Swart 2000, a.o.). However, the treatment of split scope in these accounts does not readily extend to other quantifiers. Hackl (2000) and Heim (2001) are the standard references for split scope readings with comparatives (like *less tall*) and comparative quantifiers (like *fewer than three*), but their treatment of comparative quantifiers does not immediately extend to other quantifiers, like negative indefinites.

We argue that split scope readings result from two basic mechanisms, both of which have been argued for independently in the literature. First, we assume, together with Sauerland (1998, 2004), that quantificational determiners in natural language (i.e., words like *every*, *two*, *most*, *no*, *fewer than five*, etc. and their correspondents in different languages) are quantifiers over *choice functions*, instead of quantifiers over individuals.⁴ This entails an analysis of split scope in which the quantificational determiner by itself scopes high, while the NP restriction of the determiner is interpreted low. Roughly, quantificational determiners leave a trace of type $\langle \text{et}, e \rangle$ (the type of a choice function variable), which takes the NP restriction as an argument and outputs an individual that can then combine with the lexical verb. The second ingredient is *pseudo-scope*, as in Kratzer (1998). Kratzer argues that certain cases where an indefinite is referentially dependent on a higher operator should not be analyzed as involving scope of an existential quantifier but can be modeled in terms of a bound variable. In her case, that variable is a pronoun. In our case, it will be the world index of the common noun. This will allow us to simulate low existential scope via binding. Because these two ingredients are independently justified, we contend that our analysis comes for free.

We discuss the details of the analysis in Sect. 2. First we present our analysis of negative indefinites in terms of choice functions. Then, we show that our account predicts truth-conditionally distinct split scope readings only for quantifiers that are not upward monotonic, a correct prediction (Heim 2001). Thus, we show that a choice-function approach to universals (*every*), indefinites (*a*) and others (*most*,

³ We will also address an exception to Generalization A: negative indefinites, and only them, seem to be able to split their scope across universal quantifiers under special intonation in German (see Penka 2007 for extensive discussion of this point). In Sect. 4.1 we argue that there are good reasons to put this fact aside.

⁴ Interestingly, Sauerland (2000) argues that negative determiners like *no* are not quantifiers, and for split scope readings he advocates a lexical decompositional account (following Jacobs 1980; Rullman 1995b). However, the fact that split scope readings can be generated the way we do below invalidates one of the crucial arguments in Sauerland (2000). As for his proof that negative existential quantifiers are not definable in terms of quantification over choice functions, a crucial assumption in the proof is the availability of scope construals that we argue on independent grounds not to be available (see below on Kennedy's Generalization). Thus, we assume that negative determiners are also quantifiers over choice functions.

more than three) yields split scope readings that are equivalent to narrow scope, *de dicto* readings. For all other quantifiers, we predict genuine split scope readings, as desired. One pleasant surprise here will be that non-monotonic quantifiers, like numerals,⁵ do in fact give rise to genuine split scope readings. This prediction, new as far as we know, turns out to be very useful in dealing with problems that ensue from the interaction between numerals and modals (Carston 1998; Breheny 2008; Geurts 2006, a.o.). In this section we also offer a formal proof that conservativity follows without further assumption in Sauerland's system, which we take to be an argument in favor of his proposal (and therefore ours). (See Winter 2001 for a very similar proof, and Fox 2001.)

Section 3 provides the empirical justification for our approach. We first show that split scope is possible only across intensional verbs. Then we show that only a subset of intensional verbs are possible scope splitters, and only under certain circumstances.

In Sect. 4 we discuss universal quantifiers as scope splitters for negative indefinites and argue that there are good reasons to set apart this kind of split scope from the kind that we have dealt with here. We also compare our approach to other approaches. Section 5 is the conclusion.

While we provide a worked-out analysis of the split scope of negative indefinites, comparative quantifiers like *fewer/less than n*, bare numerals, and *exactly*-numerals, our discussion will be incomplete in the following ways: we don't discuss comparatives like *less tall*, we don't discuss predicative uses (see Winter 2001 for a possible approach compatible with ours), and we don't do justice to the huge literature on the topic of bare and modified numerals (see Geurts 2006; Krifka 1999; Nouwen 2010; and references cited there). Although we have to leave these issues for future research, we think that our approach is, in principle, capable of dealing with them.

2 Analysis

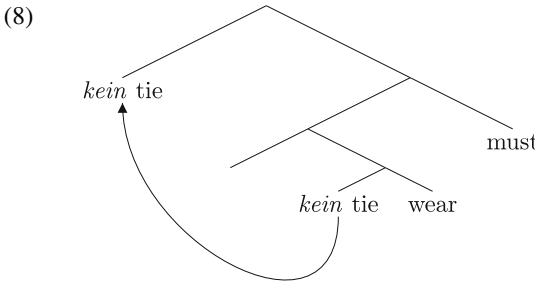
2.1 Split scope readings of negative indefinites

Following Sauerland (1998, 2004), we propose that quantificational noun phrases undergo quantifier raising (QR) and that they leave a copy in the trace position of movement, as in the Copy Theory of movement. Selective deletion will result in one part of the quantifier being interpreted high and the other part low. We exemplify with negative indefinites. The relevant aspects of the derivation of the split scope

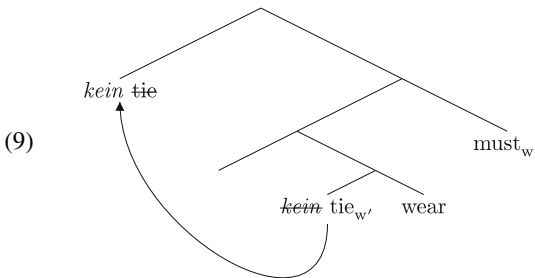
⁵ Whether bare numerals are non-monotonic is subject to considerable debate in the literature (see Geurts 2006 for a recent overview). We follow Geurts (2006) in assuming that unmodified numerals have a non-monotonic, 'exactly' reading. For expository purposes, we also follow Geurts in assuming that they give rise to an additional, 'at least' reading, but note that this latter assumption is not crucial for us.

reading of (7) are depicted in (8) (most words in trees are translated into English for ease of reference):⁶

- (7) Zu dieser Feier musst du keine Krawatte anziehen.
 to this party must you no tie wear
 ‘To this party you don’t have to wear a tie.’



Selective deletion applies and deletes the higher copy of the noun and the lower copy of the quantified determiner, as shown in (9). In this tree we also indicate the binding of the world index of the noun by the intensional verb:



The next issue is how the structure in (9) is interpreted. We need several ingredients—the meaning we assume for *kein* is in (10), and the meaning for the modal verb is in (11):⁷

(10) $\llbracket \mathbf{kein} \rrbracket^w = \lambda R_{\langle \langle et, e \rangle, t \rangle} . \neg \exists f CF(f) \ \& \ R(f) = 1$

(11) $\llbracket \mathbf{must} \rrbracket^w = \lambda p_{\langle st \rangle} . \forall w' R w \rightarrow p(w') = 1$

The expression in (11) is standard: intensional verbs are quantifiers over possible worlds. In this view, necessity modals are universal quantifiers over possible worlds, and possibility modals are existential quantifiers over possible worlds. So, for

⁶ We remain agnostic as to the treatment of subjects here—whether they raise to Spec, IP or equivalent positions, whether there is a null PRO in subject position in these structures, etc.

⁷ We assume Heim and Kratzer’s (1998) rule of Intensional Functional Application and the ensuing intensional system, to keep types maximally simple—we don’t think choosing otherwise has consequences for our main points.

example, if it is true that you don't have to wear a tie, in some worlds you wear no tie though there may be worlds where you do.

It is well known that it is necessary to restrict this quantification, and we assume the usual constraints on the possible worlds that count in (11), but we don't make that explicit here because it is independent of the phenomenon we are interested in ('xRy' in (11) stands for 'x is related to y in the appropriate way'; for some basic discussion, see Kratzer 1991). (10) says that *kein* takes a set of choice functions and gives back a truth value. Here is the definition of choice functions from Sauerland (1998); choice functions take sets of individuals and return a member of the set:⁸

$$(12) \quad f_{\langle\langle e,t \rangle, e \rangle} \text{ is a choice function iff } \forall P_{\langle e,t \rangle} \in \text{domain}(f), P(f(P)) = 1$$

In the trace position, the quantifier leaves a choice function variable that takes a property, denoted by the common noun, as its argument. The interpretation of *tie* in our example, $[[\text{tie}]]^w$, is the set of objects that are ties in w . It is because *kein* is interpreted higher than the intensional verb that we will obtain the result that negation outscopes the intensional verb in the split scope reading. We assume, following standard practice, that common noun denotations are indexed to a world. This index can be bound by intensional operators. It is because of this binding that we obtain the impression that there is existential quantification below the intensional verb (for more on this, see Sect. 2.2.2). This is, in effect, just another way of saying that the split scope reading is a *de dicto* reading.

We obtain the truth conditions in (13), where '@' stands, as usual, for the actual world:

$$(13) \quad [[(7)]]^@ = 1 \text{ iff } \neg \exists f \text{ CF}(f) \ \& \ \forall w'R@, \text{ you wear } f(\text{tie}_{w'}) \text{ in } w'$$

In words: the sentence is predicted to be true if and only if there is no choice function that in all relevant worlds w' picks a tie from w' that you wear in w' . So you don't wear a tie in every world, which is precisely what the split scope reading suggests. To see this, let us go through the tables below, where we also point out a technical problem with our approach (' \ast ' stands for a tie that is picked by a choice function but is not worn, and ' \dots ' indicates that the state of affairs in a particular world with respect to what the choice function picks, or with respect to whether a tie is worn, is irrelevant).⁹ Table 1 specifies things in such a way that we predict (7) to be true:

⁸ This definition entails that there are no choice functions that have the empty set in their domain, for, if $P = \emptyset$, $P(f(P))$ is false. We will use partial choice functions throughout (see Geurts 2000; Reinhart 1997; Winter 1997, 2001 for discussion and possible alternative approaches).

⁹ Sauerland (1998, p. 255) excludes consideration of choice functions that are not point-wise different from each other in order to deal with problems that arise with numeral quantifiers. Two choice functions f and g are point-wise different iff $\forall x \in \text{domain}(f) \cap \text{domain}(g): f(x) \neq g(x)$. This would in effect preclude us from considering some of the choice functions in Tables 1 and 2. However, we show in Sect. 2.3 below that it is not necessary to appeal to point-wise different choice functions in order to address the problems that arise with numeral quantifiers. Thus, we ignore this restriction here.

Table 1 (7) is true and predicted to be true

	W ₁	W ₂	W ₃	W ₄	W ₅	W _n
f ₁	t1	t2	t3	t4	✗	...
f ₂	t1	✗	✗	✗	✗	...
f ₃	✗	t2	✗	✗	✗	...
f ₄	✗	✗	t3	✗	✗	...
f ₅	✗	✗	✗	t4	✗	...
f _m	✗	...

In this table, there is no choice function that in every world¹⁰ picks a tie that you wear. W₅ is a world that prevents the existence of such a choice function. Thus, (13) is true. The split scope reading of (7) is true as well, since for (7) to be true, there must be worlds in which you wear a tie and worlds in which you don't—that is how it comes about that you don't have to wear a tie.

In Table 2, on the other hand, you wear a tie in every world:

Table 2 (7) is false and predicted to be false

	W ₁	W ₂	W ₃	W ₄	W ₅	W _n
f ₁	t1	t2	t3	t4	t5	t _n
f ₂	t1	✗	✗	✗	✗	...
f ₃	✗	t2	✗	✗	✗	...
f ₄	✗	✗	t3	✗	✗	...
f ₅	✗	✗	✗	t4	✗	...
f ₆	✗	✗	✗	✗	t5	...
f _m

This makes (7) false. (13) is also false: you *can* find at least one choice function, namely, f₁, that in every relevant world picks a tie that you wear. Notice that, if (13) is false, (7) cannot be true: as soon as there is a choice function that in every relevant world gives you a tie that you wear, you wear a tie in every relevant world, and (7) is false.

Finally, Table 3 is our problematic case:

Table 3 (7) is false but predicted to be true

	W ₁	W ₂	W _n
f ₁	{t1, t2}→t1	✗ {t1, t2}→t1	...
f ₂	✗ {t1, t2}→t2	{t1, t2}→t2	...
f _m

¹⁰ I.e., every *relevant* world, as before. We drop this restriction from now on.

The problem has been pointed out for choice functions before (see Winter 1997; 2001; Kratzer 1998; Geurts 2000; among others), and it arises as follows: if the set of ties is exactly the same in two worlds w_1 and w_2 , then for all choice functions f , $f(\text{tie}_{w_1}) = f(\text{tie}_{w_2})$. As shown in the Table 3, assume that you are wearing a tie in both worlds and that the tie worn in w_1 is different from that worn in w_2 , a plausible assumption. That is, even though f_1 , for example, picks t_1 in the worlds under consideration, the tie it picks for w_2 is not actually worn in that world—this ensures that neither f_1 nor f_2 counter-exemplify the negative existential claim in (13). There are no other functions different from f_1 and f_2 to consider. (13) is true: you cannot find a choice function that in every relevant world picks a tie that you wear. But (7) may be false, since what we just said is compatible with wearing a tie in every world. The crucial ingredient here is that the set of ties is exactly the same in the two worlds—if that were not the case, then we could entertain at least one additional choice function f_3 that would be different from both f_1 and f_2 and that, in every world, would pick a tie that you wear, making (13) false.

Different solutions to the problem have been entertained in the literature (e.g., adding one more parameter of variation would ensure that the set of ties in any two worlds can never be the same—see Kratzer's (1998) parametrized choice functions), but we don't choose among them here because we believe that our account is unaffected by this choice.¹¹

Thus, split scope readings can be generated by appealing to quantification over choice functions, coupled with the idea that the world index of common nouns can be bound by intensional operators. In the next subsection, we justify the use of these tools, in part by showing what independent motivation there is for them.

2.2 Independent justification of formal tools

2.2.1 ACD, conservativity, and Sauerland (1998, 2004)

First we go through an empirical argument from Sauerland's work for the position that in A' -chains, such as those generated by QR, the restriction of the quantifier must sometimes be represented and interpreted in the trace position. This is a crucial ingredient of our analysis. Sauerland's argument is based on examples concerning Antecedent-Contained Deletion (ACD), exemplified in (14)–(17) and based on well-known examples from Kennedy (1994):

(14) I talked to a bachelor who looked like the one/bachelor you did ~~talk to~~

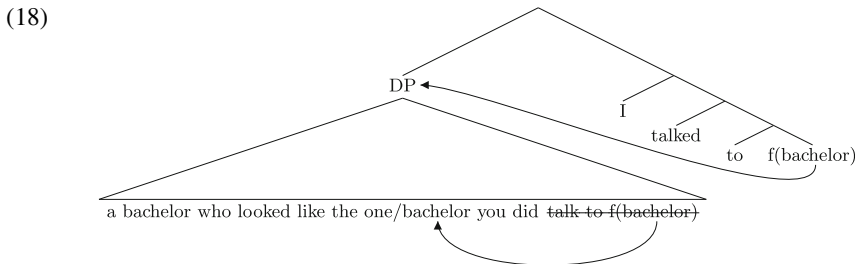
¹¹ Other problems and issues with the use of choice functions have been identified in the literature—see Endriss (2009, Sect. 4.7) for a recent summary and further references. We accept the problem discussed in the text as ours. The other problems have to do with the fact that choice functions are used to account for the exceptional wide scope of indefinites (i.e., for their scope outside of syntactic islands). For example, there is the issue of intermediate scope readings and the contexts in which they are (not) available—certain choice function approaches erroneously predict such readings to be always available (e.g., Reinhart 1997; Winter 1997). Such issues do not affect us. We remain agnostic as to what the best treatment of the exceptional wide scope of indefinites is (see Endriss 2009 for a recent proposal and further references), and note that a non-choice-function approach to it is compatible with our proposal in this paper.

(15) *I talked to a bachelor who looked like the woman you did ~~talk to~~

(16) Polly visited every town that's near the one/town Eric did ~~visit~~

(17) *Polly visited every town that's near the lake Eric did ~~visit~~

Whereas (14) and (16) are grammatical, (15) and (17) are not. The contrast suggests that what is left behind in the trace position of QR matters for the resolution of VP-ellipsis, a process involved in ACD. A rough approximation to the LF of (14) is shown in (18):



Recall that, in this system, natural language quantified determiners quantify over choice functions. In the derivation of the LF in (18), there is first QR of the quantificational object of main-clause *talk to*. In Sauerland's system, as we saw, this movement leaves behind a copy of the moved phrase and, after selective deletion, we obtain an interpretation in terms of a choice function variable and the argument of that function, provided by the common noun *bachelor*.¹² Within *who looked like the one/bachelor you did*, there is another relevant instance of A'-movement, which affects the lower relative clause *you did*. This movement proceeds as before and leaves behind a copy that is interpreted in terms of a choice function variable together with its argument *bachelor*. We are now ready for VP-ellipsis: the deleted VP is identical to its antecedent. Now, that *bachelor* in the trace position matters can be shown if we compare (14) to (15): if all that A'-movement left behind were a regular trace, there would be no way of distinguishing these two examples.

That the restriction of the quantified determiner is not only represented but interpreted in the trace position of movement is suggested by (19):

(19) I talked to a bachelor who looked like the guy you did ~~talk to~~

It seems that it is the semantic relation between *guy* (in the ellided VP) and *bachelor* (in the antecedent) that licenses ACD—and thus VP-ellipsis—here. Without representing and interpreting the restriction of quantified determiners in positions other

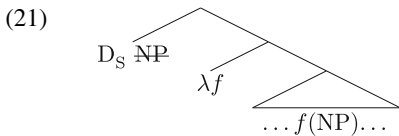
¹² Notice that this means that, for this LF, the relative clause *who looked like the one/bachelor you did* is deleted downstairs and interpreted upstairs. See Sauerland's work for more on this; he shows that there are cases in which even relative clauses are interpreted in the trace position, not just the common noun. In this system, the common noun *bachelor* can either be interpreted just downstairs, the option that we need for split scope, or both upstairs and downstairs.

than the top of a movement chain, it would not be possible to make sense of the ACD data.

In addition to this empirical argument from Sauerland, we would like to offer a theoretical argument for the idea that natural language quantification (at least, nominal quantification) is over choice functions. The argument is that within this view, the conservativity of natural language quantified determiners follows as a theorem. We will offer a formal proof below (for a similar proof, see Winter 2001, as well as Fox 2001).

Consider the semantics and the syntax of quantified phrases in this system (‘D_S’ stands for ‘Sauerland determiner’). Informally:

$$(20) \quad [[D_S]] = \lambda R_{\langle\langle et, e \rangle\rangle}. D\text{-many choice functions } f \text{ are such that } R(f) = 1$$



Recall that conservativity says that, in order to evaluate the truth of a quantificational statement, you only “look” at the set of individuals denoted by the NP-restriction; i.e., the NP-restriction is special. To see intuitively what is behind the formal proof below, notice that the argument of all determiner quantifiers in Sauerland’s system is always a set of choice functions whose domains are NPs. This way of singling out the NP-restriction ensures that this restriction is special.

For the formal proof, we start out by giving the definition of conservativity for both generalized quantifiers, in (22), and quantifiers over choice functions, in (23):

$$(22) \quad D \text{ is conservative iff for any sets } A, B \text{ and } C \text{ such that } A \cap B = C \text{ (i.e., } C \subseteq A \text{) and } C \neq B \text{ (i.e., } B \not\subseteq A \text{), } D(A)(B) = D(A)(A \cap B) = D(A)(C).$$

$$(23) \quad D_S \text{ is conservative iff for any sets } A, B \text{ and } C \text{ such that } A \cap B = C \text{ (i.e., } C \subseteq A \text{) and } C \neq B \text{ (i.e., } B \not\subseteq A \text{), } D_S(\{f: B(f(A))\}) = D_S(\{f: (A \cap B)(f(A))\}) = D_S(\{f: C(f(A))\}).$$

Now we show that $D_S(\{f: C(f(A))\})$ follows from $D_S(\{f: B(f(A))\})$ (with $A, B,$ and C as above):

$$(24) \quad \text{By definition of choice functions, for any choice of } f, f(X) \in X. \text{ Thus, for any choice of } f, B(f(A)) = (B \cap A)(f(A)) = C(f(A))$$

$$\text{By extensionality, } D_S(\{f: B(f(A))\}) = D_S(\{f: (A \cap B)(f(A))\}) = D_S(\{f: C(f(A))\}).$$

QED

So, once you assume that quantified determiners quantify over choice functions, not individuals, conservativity follows as a theorem. Conservativity is one of the few known universal properties of quantified determiners. Sauerland’s system explains this property, which we view as a major advantage.

2.2.2 Binding and Kratzer (1998)

Notice that once one adopts Sauerland's system, it is difficult to prevent higher quantificational operators from binding world indices, pronouns, etc. contained in the low copy of nominal restrictions. That is, once the common noun restriction of a quantified determiner is interpreted below the intensional verb, only an additional ad hoc stipulation can rule out this binding.

To the best of our knowledge, Kratzer (1998) was the first to notice that such binding relations can be used to simulate scope. Kratzer argues that certain cases of intermediate indefinite scope can be modeled using just such a bound variable. In these cases, there is no need to move the indefinite to give it scope; the scope of the indefinite is actually a case of *pseudoscope* that obtains because there is a binding relation. To see this, consider the example in (25) (cf. Abusch 1994 and many others):

- (25) [Every professor]_i rewarded every student who read some book she_i had reviewed for the New York Times.

Sentence (25) has a reading in which the indefinite *some book she had reviewed for the NYT* seems to take scope in between the other two quantificational expressions ('For every professor x there is a book y that x had reviewed for the NYT such that x rewarded every student who read y '). This intermediate scope reading is different from both the widest scope and the narrowest scope readings of the indefinite. In this reading, there is potentially a *different* book that each professor reviewed for the NYT, so it cannot be the widest scope reading. It cannot be the narrowest scope reading either: the reading is not about just any book each professor reviewed, but a particular one for each professor (e.g., the first book ever she reviewed for the NYT).

Kratzer notes that this reading is absent in the minimally different (26):

- (26) Every professor rewarded every student who read some book I had reviewed for the New York Times.

The difference between the two examples is that in (25) the indefinite noun phrase contains a bound variable (bound by *every professor*), whereas (26) contains no bound variable. Kratzer argues that this fact is key in understanding what is going on here. She proposes that it is this binding that is responsible for the intermediate scope reading of (25), in a framework that uses choice functions to interpret indefinites. *Some book she_i had reviewed* is interpreted as in (27), which results in a different output of the choice function for each professor, as desired:

- (27) $f(\text{book she}_i \text{ had reviewed})$

In our account of split scope, we bind world indices instead. To see intuitively how this helps, it is easier to consider an example without negation:

- (28) You must wear a tie.

We understand in (28) that the tie doesn't have to be the same tie in every world; the requirement in this sentence is about tie-wearing, not about the necessity to wear a particular tie. (If the *de re* reading exists at all, it is not very salient.) This is a *de dicto* reading that can be captured by assuming that the intensional verb binds the world index of *tie*. Thus, *tie* is interpreted as in (29):

$$(29) \quad f(\text{tie}_w)$$

The output of the choice function differs from world to world, as desired.

2.3 Split scope readings of other quantifiers

In this subsection we first provide the denotations of *ein* 'a', *jeder* 'every', *die meisten* 'most', and upward-monotonic comparative quantifiers like *mehr als drei* 'more than three', and show that the split scope readings that we derive for these quantifiers are equivalent to narrow scope, *de dicto* readings. This is a good outcome because these quantifiers don't give rise to split-scope readings that are independent and distinct. Then, we provide the choice function analysis of comparative quantifiers like *weniger als drei* 'less than three', bare numerals, and *exactly*-numerals.¹³ We show that split scope readings are predicted here, correctly.

2.3.1 When split scope is equivalent to narrow scope 'de dicto'

Let's start by considering *ein* 'a'.¹⁴ The denotation we envisage for it is in (30):¹⁵

$$(30) \quad \llbracket \text{ein} \rrbracket^w = \lambda R_{\langle \langle \text{et}, \text{e} \rangle, \text{t} \rangle} \exists f \text{ CF}(f) \ \& \ R(f) = 1$$

In combination with a necessity modal, as in (31), the split scope reading is 'There is a choice function *f* such that for all worlds *w'*, you buy *f*(*tie*_{*w*}) in *w'*'.

- (31) Du musst eine Krawatte kaufen.
 you must a tie buy
 'You must buy a tie.'

This entails the narrow scope *de dicto* reading of the indefinite:¹⁶ 'For every world *w'* there is a tie in *w'* that you buy in *w'*'. To see this, consider a choice function *f*₁ that, in every world, outputs a tie that you buy. The existence of this choice function makes the split scope reading true. And, since you are buying a tie in every world,

¹³ We do not provide denotations for quantifiers like *few* or *many* below for reasons of space. It is not difficult to see what a basic account of their contribution would be in our framework once the semantics of quantifiers like *fewer* is in place (Sect. 2.3.3). Split scope readings can be predicted, correctly, for *few*.

¹⁴ What we say here also applies to interrogative words and other (positive) indefinites, since both involve existential quantification. We ignore these here.

¹⁵ We ignore domain restriction unless relevant.

¹⁶ It can't be equivalent to the wide scope reading, for that is a *de re* reading, and our readings are always *de dicto*.

the narrow scope *de dicto* reading is also true. To see the other direction, consider that, if in every world there is a tie that you buy, then (modulo the problem we put aside in Sect. 2.1, which we ignore from now on) there is a way of picking ties (i.e., a choice function) that in every world picks a tie that you buy in that world.

In combination with a possibility modal, as in (32), the split scope reading is ‘There is a choice function f and a world w' such that you buy $f(\text{tie}_{w'})$ in w' ’.

- (32) Du kannst eine Krawatte kaufen.
‘You can buy a tie.’

This is true as long as we can find a choice function that, in at least one world, outputs a tie that you buy. If there is a world in which you buy a tie, then the narrow scope *de dicto* reading is also true (‘There is a world w' in which you buy a tie in w' ’). The entailment between readings also holds in the other direction.

For *jeder* ‘every’, the lexical entry is as in (33):

- (33) $\llbracket \text{jeder} \rrbracket^w = \lambda R_{\langle \text{et}, e \rangle, \langle \rangle}. \forall f \text{ CF}(f) \ \& \ R(f) = 1$

The split scope reading of (34) is ‘All choice functions f and all worlds w' are such that you buy $f(\text{tie}_{w'})$ in w' ’.

- (34) Du musst jede Krawatte kaufen.
‘You must buy every tie.’

This is true when all choice functions output a tie that you buy in every world. If so, then you buy all ties in all worlds. This makes the narrow scope *de dicto* reading true (‘For every world w' and every tie x in w' , you buy x in w' ’). Conversely, if in every world w' you buy all ties in w' , then all ways of choosing ties (i.e., all choice functions) are such that in every world w' you buy in w' a tie that they pick.

For (35), the split scope reading is ‘For all choice functions f there is a world w' such that you buy $f(\text{tie}_{w'})$ in w' ’.

- (35) Du kannst jede Krawatte kaufen.
‘You can buy every tie.’

This is true if each choice function, in at least one world, outputs a tie that you buy—that is, if you buy all the ties in at least one world (if there was no world in which you buy all ties, then there would be a choice function that in all worlds picks a tie you *don't* buy, which counter-exemplifies the universal claim). This makes the narrow scope *de dicto* reading true as well (‘There is a world w' such that for all ties x in w' , you buy x in w' ’). In the other direction, if there is a world w' in which you buy all ties, then in that world, every choice function will have to pick a tie that you buy. Therefore, for all choice functions f there is a world in which you buy the tie that f picks.

Concerning numerals we depart from the proposal in Sauerland (1998) and do not count choice functions in order to account for ‘at least’ readings of numerals.

Instead, we existentially quantify over choice functions with the help of a null existential determiner quantifier, and we take the numeral to contribute just a degree. Thus, bare numerals are actually never bare (or, at least, not when they are used quantificationally). We propose the following for *vier* ‘four’:

$$(36) \llbracket \mathbf{vier} \rrbracket^w = 4_{\langle d \rangle}$$

$$(37) \llbracket \exists \rrbracket^w = \lambda d \lambda R_{\langle \langle et, e \rangle, t \rangle}. \exists f \text{ CF}(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x = d\} \ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) \geq d$$

$$(38) \llbracket \exists \mathbf{vier} \rrbracket^w = \lambda R_{\langle \langle et, e \rangle, t \rangle}. \exists f \text{ CF}(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x = 4\} \ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) \geq 4$$

The denotation in (38) is just like the denotation of *ein* in (30) except that it comes with two additional restrictions. The only real difference between *ein* and *vier* in this approach is that, whereas with *ein* the choice function outputs an atomic individual, with *vier* it outputs a plural individual with four or more atomic individuals.

The first restriction (‘ $\text{dom}(f) = \{p \mid \exists x \in p \ \#x = 4\}$ ’) is a restriction on the domain of the choice function. In this domain, all properties are such that you can find at least one member in them that has four atomic individuals (‘ $\#x$ ’ stands for ‘the number of atomic individuals of x ’).¹⁷ This has the effect that *vier*, when combined either with singular predicates or with plural predicates all of whose plural individuals contain less than four atoms, gives rise to a false statement. The second restriction (‘ $\forall p \ p \in \text{dom}(f) \rightarrow \#f(p) \geq 4$ ’) says that all properties that are in the domain of the choice function are such that the output of the choice function applied to the property has four or more atoms. So, *four students left* is true iff there are four or more students and there is a plural individual of students with four or more atoms such that these student atoms left. That is, at least four students left. (37) is the denotation of the silent existential determiner quantifier. It takes a degree argument (provided by the numeral) and then returns a quantifier over choice functions that is just like a (positive) indefinite except that it has the two additional restrictions.

Note that the second restriction involves universal quantification over properties. Why? The reason has to do with the fact that the quantifier over choice functions, for type reasons, is always interpreted in a position from which the nominal restriction, the property-denoting expression, can no longer be directly accessed. In other words, the restriction is too deeply embedded at that point for the choice function quantifier to be able to impose any restrictions on it directly. Existentially quantifying over properties would be very problematic because nothing would guarantee that restrictions are imposed on the right property. Thus, we are left with universal quantification. It remains somewhat unintuitive that reference is made to properties that are not students in examples such as *four students left*. However, the only claim that the resulting truth conditions make about such properties is that, if in the world there are, say, four teachers, then there is a way of picking four teachers.

¹⁷ We follow an approach to plurals in which they denote sets of plural individuals. In the alternative approach, in which they denote sets of sets of individuals, ‘ $\#x$ ’ stands for ‘the cardinality of x ’.

And so on for all other properties different from *students*. This, we take it, is harmless.

The split scope reading of (39) says ‘There is a choice function f whose domain only contains properties that have at least one member with four atomic individuals and which selects individuals with four or more atomic individuals such that in every world w' it picks ties that you buy in w' ’:

- (39) Du musst vier Krawatten kaufen.
‘You must buy four ties.’

In other words, the split reading says that there is a way of choosing quadruplets (or bigger tuples) such that in every world, when you choose from the set of ties, it gives quadruplets (or bigger tuples) of ties that you buy. This is equivalent to the narrow scope, *de dicto* reading, which requires that, in every world, you buy four ties (or more).

For (40), the split scope reading is ‘There is a choice function f whose domain only contains properties that have at least one member with four atomic individuals and which selects individuals with four or more atomic individuals such that in some world w' it picks ties that you buy in w' ’:

- (40) Du kannst vier Krawatten kaufen.
‘You can buy four ties’

This is true as long as there is one choice function and one world such that the choice function outputs a set of four or more ties that you buy in that world. These truth-conditions are equivalent to those of the narrow scope *de dicto* reading.¹⁸

For *die meisten* ‘most’, the lexical entry we assume is as follows:

- (41) $\llbracket \text{die meisten} \rrbracket^w = \lambda R_{\langle \langle \text{et}, e \rangle, t \rangle} . \exists f \text{ CF}(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x = 2\} \ \& \ \forall g \llbracket \text{CF}(g) \ \& \ f(n-o)g \ \& \ \text{dom}(g) = \{p \mid \exists x \in p \ \#x = 2\} \rrbracket \rightarrow \forall p \ \#f(p) > \#g(p)$

The denotation of *die meisten* involves, again, existential quantification over choice functions that is further restricted. In this case, the additional domain restriction is simply that *die meisten* cannot combine with singular nouns. The second restriction makes sure that the choice function in question picks a bigger individual than all alternative choice functions, no matter which property is given as argument. Important in this denotation is the non-overlapping condition ‘ $f(n-o)g$ ’. It says that f and g do not overlap. We will say that two plural individuals do not overlap iff there

¹⁸ If numerals counted choice functions, as in Sauerland (1998), then the truth conditions predicted, for example, for (40) are too weak. The reading Sauerland predicts here is ‘There are four choice functions f and a world w' such that you buy $f(\text{tie}_{w'})$ in w' ’. This is true even if in reality there is only one tie you buy, because the tie four choice functions pick can be the same in some worlds. To avoid this, Sauerland proposes to restrict quantification to choice functions that are sufficiently different from each other (i.e., they are point-wise different): when two choice functions output the same individual, they cannot both be part of the quantification (see footnote 9). The problem doesn’t arise in our version of the story. See Winter (2001) for a different approach, which treats numerals as restrictive modifiers of nouns.

is no atomic individual they share. We will say that two choice functions f and g do not overlap iff for all properties p for which both f and g are defined, $f(p)$ and $g(p)$ do not overlap. Hackl (2009) builds a similar non-overlapping condition into the meaning of *die meisten* because without it, *die meisten* would give rise to universal quantification. Our lexical entry for *die meisten* stays as close as possible to that provided by Hackl (2009).

This entry gives rise to a proportional reading, illustrated in (42):¹⁹

- (42) Hans hat die meisten Bücher gelesen.
 Hans has the most books read
 ‘Hans read most books/The number of books that Hans read is greater than the number of books he didn’t read.’

The reading we obtain is ‘There is a choice function f such that John read the books it picks and f picks more books than any other choice function g that is non-overlapping with f ’. This is true in a situation in which, out of ten books, John read six and didn’t read four. If, on the other hand, John read only four out of ten books, there is no such choice function f , because there is at least one non-overlapping choice function g which picks more books (namely, all six unread ones).

For (43), we predict the following split reading: ‘There is a choice function f such that in every world w' you buy the ties it picks and it picks more ties than any other non-overlapping choice function’.

- (43) Du musst die meisten Krawatten kaufen.
 ‘You must buy most ties.’

In other words, in each world, you buy most of the ties in that world. This is equivalent to the narrow scope, *de dicto* reading, which says ‘In every world w' , the number of ties that you buy is greater than the number of ties you don’t buy’.

For (44), we predict the following split reading: ‘There is a choice function f such that in some world w' you buy the ties it picks and it picks more ties than any other non-overlapping choice function’.

- (44) Du kannst die meisten Krawatten kaufen.
 ‘You can buy most ties.’

In other words, in some world, you buy most of the ties in that world. This is equivalent to the narrow scope, *de dicto* reading.

Finally, for upward-monotonic comparative quantifiers, like *mehr als drei* ‘more than three’, we proceed as follows:

- (45) $\llbracket \text{mehr} \rrbracket^w = \lambda d \lambda R_{\langle \text{set}, e \rangle, t} . \exists f \text{ CF}(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x > d\}$
 $\ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) > d$

¹⁹ The lexical entry in (41) gives rise only to proportional readings, available for both German *die meisten* and English *most*. *Die meisten*, though not *most*, can also give rise to relative readings. We don’t deal with those here.

$$(46) \quad \llbracket \text{als drei} \rrbracket^w = 3_{<d>}$$

$$(47) \quad \llbracket \text{drei} \rrbracket^w = 3_{<d>}$$

$$(48) \quad \llbracket \text{als} \rrbracket^w = \lambda D_{<dt>}. \max(D)$$

$$(49) \quad \llbracket \text{mehr als drei} \rrbracket^w = \lambda R_{<et, e>, t>}. \exists f \text{ CF}(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x > 3\} \ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) > 3$$

Mehr ‘more’ is an existential quantifier over choice functions very similar to those provided before. It also comes with two additional restrictions. The first one ensures that the domain of the choice function in question is not too small (this will depend on the particular argument that *mehr* takes). The second restriction says that the output of applying the choice function to all big-enough properties is greater than the degree provided by the argument of *mehr*.

Mehr takes a *than*-clause as its argument, even in cases like (46), where the argument of *als* does not appear to be clausal. We assume that there is covert clausal structure, *what three is*, meaning ‘ $\lambda d.d = 3$ ’ in such cases (like Hackl 2000). The *than*-clause as a whole denotes a degree. *Than* is a choice function over degrees. In split scope readings, the *than*-clause is interpreted high, with the determiner quantifier. Because (49) is almost identical to the denotation we have assumed for numerals (see (38)), we need not repeat the reasoning about how the split scope reading we predict is equivalent to the narrow scope, *de dicto* reading.

We have shown in this subsection that the split scope readings we predict for upward-entailing quantifiers are truth-conditionally equivalent to narrow scope, *de dicto* readings. Empirically, this is the right result.

A reviewer asks whether this result might be a consequence of us having chosen just the two intensional verbs *müssen* ‘must’ and *können* ‘can’: perhaps with other verbs the two readings are not equivalent and our account overgenerates? Potential problematic verbs might be *verbieten* ‘prohibit’ and *bezweifeln* ‘doubt’, which, of course, have a more involved semantics than *müssen* and *können*. We come back to this issue in Sect. 3.2, where we discuss Generalization B, namely, that not all intensional verbs can split scope. As it turns out, verbs like *verbieten* and *bezweifeln* can never split the scope of quantifiers.

2.3.2 Non-upward-monotonic comparative quantifiers

Non-upward-monotonic comparative quantifiers give rise to distinct split scope readings. We already saw example (2) in the introduction, repeated here in its German variant:

- (50) Am MIT muss man weniger als drei Bücher veröffentlichen,
 at.the MIT must one less than three books publish
 um fest angestellt zu werden.
 in.order permanently employed to be
 ‘At MIT one must publish at least n books in order to get tenure, and n is
 less than three.’ (Hackl 2000)

Weniger ‘fewer’, according to us, is essentially the negation of *mehr* ‘more’. Its denotation is as follows:

$$(51) \quad \llbracket \text{weniger} \rrbracket^w = \lambda d \lambda R_{\langle \langle \text{et}, e \rangle, t \rangle} \cdot \neg \exists f \text{ CF}(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x \geq d\} \ \& \ \forall p \in \text{dom}(f) \rightarrow \#f(p) \geq d$$

Weniger is a negative existential, like negative indefinites. However, it comes with additional restrictions. Because it is a negative existential it will give rise to split scope readings that are genuine readings. The predicted split scope reading in an example like (52) is ‘There is no choice function that picks triplets or bigger tuples such that in all worlds w' you buy the triplets or bigger tuples of ties that it picks’.

- (52) Du musst weniger als drei Krawatten kaufen.
 ‘You must buy fewer than three ties.’

In other words, there can be a choice function that picks such triplets or bigger tuples in some worlds, but there can’t be one that does so in all worlds. Therefore, at least in some worlds, you buy less than three ties. We thus generate the split scope reading discussed in Hackl (2000): ‘The maximal degree d such that in all worlds w' you buy d -many ties is less than three’. This reading is a lower-bound reading that requires it to be the case that the minimum amount of ties bought in all worlds is less than three. It is truth-conditionally distinct from the narrow scope, *de dicto* reading, which says: ‘In all worlds w' you buy less than three ties’. The split scope reading allows for the case in which you buy three or more ties in some worlds, whereas the narrow scope, *de dicto* reading does not. Both readings exist (though the narrow scope reading may be pragmatically disfavored in examples such as (50)).

For (53), the split scope reading we predict is ‘There is no choice function that picks triplets or bigger tuples such that in some world w' you buy the triplets or bigger tuples of ties that it picks’:

- (53) Du kannst weniger als drei Krawatten kaufen.
 ‘You can buy fewer than three ties.’

This is an upper-bound reading: there is no world in which you buy three or more ties. Again, this is truth-conditionally distinct from the narrow scope, *de dicto* reading, ‘There is a world w' such that you buy less than three ties in w' ’, which is very weak. Both readings exist, though the narrow scope reading might be available only in certain contexts.

Hackl (2000) accounts for the split scope that these comparative quantifiers give rise to by lexically decomposing *fewer* into a degree quantifier (a maximality operator) and a *many*-quantifier with existential force. While the degree quantifier scopes above the intensional verb, the existential, *many*-quantifier stays low and gives rise to low existential scope. In his framework, the paraphrase of example (50) is ‘At MIT the maximum number n , such that in all relevant worlds there is a set of books one publishes and the cardinality of that set is n , is less than three’. Suppose that $n = 2$. Then what the truth conditions require is that two be the maximum

number of books one publishes in all worlds. This is a minimality requirement: in every world, one must publish two books, and in some worlds, one publishes more.²⁰ In our terms: ‘There is no choice function that picks triplets or bigger tuples such that in all worlds w' where you are hired permanently you publish the triplets or bigger tuples of books that it picks’ (as noted before, the sentence also has a pragmatically implausible narrow scope reading).

It is possible to decompose our *fewer* further (and our *more* from Sect. 2.3.1), of course, but we note that one of Hackl’s main motivations for decomposing *fewer* are split scope readings. If our approach is right, split scope readings do not furnish the motivation to further decompose *fewer*—though they do provide motivation for interpreting parts of quantificational noun phrases in different places in the tree, which is the kind of account we are advocating here.

The results of Sect. 2 so far form the basis for the claim that a unified treatment of split scope is possible. We have demonstrated that a choice function analysis can generate split scope readings for negative indefinites and for comparative quantifiers like *fewer than n* .

2.3.3 Exactly-numerals

Sentences like the following have sometimes been discussed in the literature on numerals (Breheny 2008; Carston 1998; Geurts 2006, a.o.):

- (54) She can have 2000 calories without putting on weight.
- (55) You may attend six courses.
- (56) You need to have exactly one good idea to become famous in America nowadays.

These sentences (and their German counterparts) are problematic for standard accounts because neither a wide nor a narrow scope analysis does justice to their intuitive truth conditions. In particular, the narrow scope readings are either very weak or pragmatically odd, and the wide scope readings are sometimes implausible, *de re* readings. The narrow scope reading of (54) says that there is a world in which she has 2000 calories or more without putting on weight—but this is compatible with there being other worlds in which she also has 2000 calories or more but does put on weight. The most prominent interpretation of the sentence, on the other hand, is that she can have *up to* 2000 calories without putting on weight. This is a stronger interpretation, according to which there is no world in which she has more than 2000 calories without putting on weight. Similarly for (55), whose most prominent reading is that you may attend up to six courses. The narrow scope reading of (56) says that in all worlds you have exactly one good idea and become famous, so there

²⁰ This example presupposes that at least one book must be published in order to get tenure. So, even though zero is less than three, publishing no books will not do. This is also the case in other examples with comparative quantifiers. We assume this presupposition exists but don’t make it explicit here. Thanks to a reviewer for reminding us of this point.

can't be a world in which you have more than one good idea and in which you become famous—having more good ideas would actually *prevent* you from becoming famous. The more plausible interpretation says that you have to have at least one good idea to become famous—in all worlds in which you become famous, you have one or more good ideas.

Breheny (2008) and Geurts (2006) have recently discussed the important observation that numerals get an ‘at most’ interpretation only in the context of possibility modals. We would like to add here that *exactly*-modified numerals allow the ‘at least’ interpretation only in the context of necessity modals, and that they, like bare numerals, allow the ‘at most’ interpretation only in the context of possibility modals. Consider the following example:

- (57) [A rule of a research funding body:]
One can be the PI in exactly three projects.

The most prominent reading of (57) is one in which there is no world in which one is the principal investigator in more than three projects. This is a different reading from the very weak narrow scope reading, which merely requires there to be at least one world in which one is the principal investigator in exactly three projects, without imposing any constraints on any other worlds. So it could well be that in other worlds one is the principal investigator in more projects. Intuitively, it is very surprising that *exactly*-modified numerals could give rise to ‘at least’ and ‘at most’ interpretations.

We will now show that the ‘at least’ and ‘at most’ interpretations of *exactly*-modified numerals are predicted split scope readings in our approach. We assumed earlier that bare numerals give rise to an ‘at least’ reading. For expository simplicity, we adopt the view, expressed recently by Geurts (2006), that bare numerals are ambiguous: they also have an ‘exactly’ reading. On this view, an important function of the modifier *exactly* is to disambiguate the numeral.

Translating into our terms, this means that the silent existential operator is ambiguous between two readings. We analyze *exactly n* as ‘at least n and no more than n’:

$$(58) \quad \llbracket \exists_1 \rrbracket^w = \lambda d \lambda R_{\langle \langle et, e \rangle, t \rangle}. \exists f CF(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x = d\} \ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) \geq d \quad (= (37))$$

$$(59) \quad \llbracket \exists_2 \rrbracket^w = \lambda d \lambda R_{\langle \langle et, e \rangle, t \rangle}. \exists f CF(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x = d\} \ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) \geq d \ \& \ \neg \exists g CF(g) \ \& \ R(g) = 1 \ \& \ \text{dom}(g) = \{p \mid \exists x \in p \ \& \ \#x > d\} \ \& \ \forall p \ p \in \text{dom}(g) \rightarrow \#g(p) > d$$

$$(60) \quad \llbracket \exists_2 \text{ 2000} \rrbracket^w = \lambda R_{\langle \langle et, e \rangle, t \rangle}. \exists f CF(f) \ \& \ R(f) = 1 \ \& \ \text{dom}(f) = \{p \mid \exists x \in p \ \#x = 2000\} \ \& \ \forall p \ p \in \text{dom}(f) \rightarrow \#f(p) \geq 2000 \ \& \ \neg \exists g CF(g) \ \& \ R(g) = 1 \ \& \ \text{dom}(g) = \{p \mid \exists x \in p \ \& \ \#x > 2000\} \ \& \ \forall p \ p \in \text{dom}(g) \rightarrow \#g(p) > 2000$$

Our previous lexical entry for \exists , in (37), is now the lexical entry for \exists_1 ((58)). \exists_1 gives rise to ‘at least’ readings. \exists_2 is just like \exists_1 except that it comes with yet

another restriction ($\neg \exists g \text{ CF}(g) \ \& \ R(g) = 1 \ \& \ \text{dom}(g) = \{p \mid \exists x \in p \ \& \ \#x > d\} \ \& \ \forall p \ p \in \text{dom}(g) \rightarrow \#g(p) > d$). This restriction adds the upper bound missing in ‘at least’ readings. Here is what this gives rise to in a simple example:

(61) She had exactly 2000 calories.

We have shown before that up to the restriction we’ve just introduced, we obtain the truth conditions that she had at least 2000 calories. With the new restriction, we add that there is no choice function g that, independently of the property it applies to, picks tuples greater than 2000 such that she had the tuple of calories picked by g . This sets the necessary upper bound.

In (54), the split scope reading says: ‘There is a choice function f such that it picks 2000-or-greater-tuples and there is a world in which she has the calories picked by f , and there is no choice function g such that it picks greater-than-2000-tuples and there is a world in which she has the calories picked by g ’. This is the upper-bound reading we were after: there is a world in which she has at least 2000 calories and doesn’t put on weight, and there is no world in which she has more than 2000 calories and doesn’t put on weight. If there was such a world, there would be a way of picking those calories; that is, there would be a choice function that counter-exemplifies the negative existential statement. To see how the truth-conditions we generate are equivalent to the upper-bound reading, consider that a different way of paraphrasing the reading is ‘She can have 2000 calories or more and not put on weight and she must not have more than 2000 calories’—this paraphrase puts the quantificational operators in the same relations as they appear in ours and it is clearer that it is the upper-bound reading.

In (56), the split scope reading says: ‘There is a choice function f such that it picks individuals of one or more atoms and in all worlds where you become famous you have the idea that f picks, and there is no choice function g such that it picks proper plural individuals and in all worlds where you become famous you have the ideas that g picks’. This is the lower-bound reading we were after: in all worlds where you become famous, you have at least one good idea, and there is no way of choosing proper pluralities of good ideas of yours in all worlds in which you become famous—this means that in some worlds you may have more than one good idea, but you don’t have more than one good idea in all worlds. Here, the alternative paraphrase we offer is: ‘You must have at least one good idea in order to become famous and you don’t have to have more than one, though you can’.

We provide an argument for our semantic treatment of these readings, as opposed to a pragmatic one (see Breheny 2008), in Sect. 3.3.

3 Empirical motivation

We now turn to the empirical motivation behind our approach. The main argument we would like to develop here is based on a comparison of the behavior of the split scope of negative indefinites, comparative quantifiers, and numerals. As noted

earlier, these different quantifiers behave strikingly alike with respect to split scope—hence, a unified approach to split scope.

3.1 Generalization A: Kennedy's generalization

Kennedy (1997) observes that degree expressions (e.g., *less tall*) can scope above (at least some) intensional verbs, but not above other scope-bearing items, like DP quantifiers or quantificational adverbs. Applied to comparative quantifiers, this means that, while a split scope reading is available in (62), repeated from before, it is unavailable in (63) (with a DP quantifier) and (64) (with a quantificational adverb):

- (62) Am MIT muss man weniger als drei Bücher veröffentlichen,
 at.the MIT must one less than three books publish
 um fest angestellt zu werden.
 in.order permanently employed to be
 'At MIT one must publish at least n books in order to get tenure, and n
 is less than three.' (Hackl 2000)
- (63) Jeder Professor hat weniger als drei Bücher geschrieben.
 every professor has less than three books written
 'Every professor wrote less than three books.'
 *'Every professor wrote at least n books, and n is less than three.'
- (64) Hans hat immer weniger als €300 auf seinem Bankkonto.
 Hans has always less than €300 on his bank.account
 'Hans always has less than €300 in his bank account.'
 *'Hans always has at least an amount of money n in his bank account, and
 n is less than €300.'

Example (63) doesn't have a split scope reading: the sentence cannot be understood to say that there is no choice function f that picks triplets or bigger tuples such that every professor wrote the books f picks.²¹ In this reading, the minimum amount of books that every professor wrote is less than three—a lower-bound reading, which says nothing about the maximum number of books written. However, the sentence is not about a minimum but about a maximum: it says that no professor wrote more than two books—that's the narrow scope reading. (64) doesn't have a split scope reading either. That reading would say 'There is no choice function f that picks 300-or-bigger tuples such that Hans always has the euros that f picks', which would be true if Hans always had a minimum in his bank account (the minimum being a number that is less than €300). This allows for the possibility of him having amounts greater than €300 sometimes. In actuality, the sentence only has a narrow scope reading, which is true if Hans never has €300 or more in his bank account.

²¹ To get the referential dependency between professors and books, we would have to postulate a silent pronoun (e.g., *books of his*).

Exactly the same restriction is observed in the case of negative indefinites. We have already seen parallel examples with *müssen* ‘must’ in which a split scope reading is available for a negative indefinite across the intensional verb. We have also seen that a split scope reading is not available in (65), and we note now that such a reading is not available in (66) either:

- (65) Genau ein Arzt hat kein Auto.
 Exactly one doctor has no car
 ‘Exactly one doctor has no car.’
 *‘There isn’t exactly one doctor who has a car.’

- (66) Hans hat immer kein Geld.
 Hans always has no money.
 *‘Hans doesn’t always have money.’

Example (66) cannot be understood to mean that Hans doesn’t always have money—that’s the split reading. The sentence is understood to mean that always, he has no money—that’s the narrow scope reading.²²

Exactly-numerals display the same pattern:

- (67) Am MIT muss man genau drei Bücher schreiben,
 at.the MIT must one exactly three books write
 um fest angestellt zu werden.
 in.order permanently employed to be
 ‘At MIT one must publish at least three books in order to be hired permanently.’
- (68) Jeder Professor hat genau drei Bücher geschrieben.
 ‘Every professor wrote exactly three books.’
 *‘Every professor wrote at least three books.’
- (69) Hans hat immer genau €300 auf seinem Bankkonto.
 ‘Hans always has exactly €300 in his bank account.’
 *‘Hans always has at least €300 in his bank account.’

Sentence (67) has a prominent split scope reading, which is true in a situation in which there is a minimum requirement on the amount of books published: three books (rather than a requirement to publish no more and no less than three books). (68), on the other hand, does not have a split scope reading in which the minimum amount of books written by every professor is three—this allows some professors to have written more than three books. Instead, the sentence has a narrow scope reading in which every professor wrote no more and no less than three books. (69) does not have a split scope reading²³ in which the minimum amount of money Hans

²² As noted earlier, we postpone discussion of split scope readings under the hat contour until Sect. 4.

²³ Or, at the very least, it is very marked.

always has in his account is €300, which allows him to have more than that sometimes. Instead, the sentence has the somewhat strange narrow scope reading that always, Hans has exactly €300 in his bank account.

In our account, Generalization A is stated as a constraint on deletion. Recall that, following Sauerland, we implemented scope splitting as a combination of movement and complementary deletion (and binding). Given the existence of QR, Kennedy's Generalization has to be stated as a locality constraint on complementary, partial deletions. A different approach, which would be syntactically closer to Hackl (2000), would assume movement of the determiner quantifier alone—the nominal restriction would never move. Under this alternative implementation, Kennedy's Generalization can be stated in terms of constraints on movement. We currently don't have strong reasons for choosing one over the other. We decided to stick to Sauerland's implementation.²⁴

3.2 Generalization B

The verbs that can split the scope of negative indefinites are the same verbs that can do so with comparative quantifiers and *exactly*-numerals.

Many verbs allow split scope with these quantifiers: *müssen* 'must', *können* 'can', *brauchen* 'need', *anfangen* 'begin', *erlauben* 'allow', *wagen* 'dare', and others.²⁵ We have already seen examples with *müssen* and *können*, which we don't repeat here. Consider now some examples with *erlauben* 'allow':

(70) Ich habe ihm weniger als drei Bücher zu schreiben
 I have him less than three books to write
 erlaubt.
 allowed
 'This is what I allowed him to do: write a maximum of n books, n being less than three.'

(71) Ich habe ihm keine Bücher zu schreiben erlaubt.
 'I didn't allow him to write books.'

(72) Ich habe ihm genau drei Bücher zu schreiben erlaubt.
 'This is what I allowed him to do: write a maximum of three books.'

The split reading of (70) is: 'There is no choice function that picks triples or bigger tuples such that there is a world w' compatible with what I allow and he writes the books picked by the choice function in w' '. In other words, I granted him permission to write a number of books which was less than three—according to what I said, he shouldn't write more than that. This is very different from the narrow scope reading,

²⁴ While scope splitting is constrained by Kennedy's Generalization, QR is not. This contrast is reminiscent of the contrast between *combien* splits vs. full argument movement. This suggests that split interpretations of noun phrases are, in general, much more sensitive to locality than unsplit ones.

²⁵ With some verbs their lexical properties are such that it may be difficult to tease apart split scope readings from other readings.

where, according to what I said, he had the option of writing less than three books, though it was also possible for him to write more. The split scope reading in (71) ('There is no choice function such that there is a world w' and he writes the books that it picks in w' ') is different from the narrow scope reading in that, in the latter, I allowed him to do something, namely, write no books. But he could have ended up writing books, which is incompatible with what I say in the split scope reading. The split scope reading of (72) is 'There is a choice function f that picks triplets or bigger tuples and there is a world w' compatible with what I allow such that he writes the books that f picks in w' , but there is no choice function g that picks quadruplets or bigger tuples such that there is a world w'' compatible with what I allow and he writes the books g picks in w'' '. This says that there are worlds in which he writes three or less books but there is no world in which he writes more than three.

Here are examples with *brauchen* 'need':

- (73) Während der Untersuchung brauchen weniger als drei
 during the examination need less than three
 Chirurgen im Raum zu sein.
 surgeons in.the room to be
 'During the examination, there have to be at least n surgeons in the room,
 and n is less than three.'
- (74) Während der Untersuchung brauchen keine Chirurgen im Raum zu sein.
 'During the examination, it's not necessary for surgeons to be in the room.'

Brauchen 'need' is a negative polarity item (NPI) and thus it must appear in a downward-entailing environment. This is going to have the following consequences for us: split scope readings will not be available for *exactly*-numerals, since they are not downward entailing, and sentences with negative indefinites or comparative quantifiers will lack a narrow scope reading. (73) has a prominent, split scope reading. (It is true in case, in all worlds, you find at least one or two surgeons present. In some worlds, there may be more; if you look at all the worlds, what they have in common is that at least one or two surgeons are in the room.) (74), too, has a split scope reading, as expected.

Beschließen 'decide', *aufgeben* 'give up', *sich weigern* 'refuse', and others prevent split scope with negative indefinites, comparative quantifiers, and *exactly*-numerals. Consider first *sich weigern* 'refuse':

- (75) Hans hat sich weniger als €3000 zu bezahlen
 Hans has self less than €3000 to pay
 Geweigert.
 refused
 'Hans refused to pay less than €3000.'
 *'Hans refused to pay an amount of money n , and n is less than €3000.'

- (76) Hans hat sich kein Geld zu bezahlen geweigert.
 ‘Hans refused to pay no money.’
 *‘Hans didn’t refuse to pay money.’
- (77) Hans hat sich genau drei Filme zu machen geweigert.
 Hans has self exactly three movies to make refused
 ‘Hans refused to make exactly three movies—he will make movies as long
 as they don’t amount to three movies.’²⁶

Split scope readings in (75)–(77) are either unavailable or extremely marginal. In (75), the reading in question says that Hans refused to pay an amount that is less than €3000 and there is no amount above €3000 that he refused to pay. This reading is, crucially, non-committal about the threshold—it may be that Hans did not refuse to pay some *other* amount below €3000. This is very different from the available, narrow scope reading, which says that there are no worlds in which there is no way of picking an amount of €3000 or above such that Hans pays that amount—in other words, Hans refused to pay *any* amount below €3000. In (76), the unavailable split scope reading says that Hans didn’t refuse to pay money—he didn’t express a refusal. A reading the sentence does have is the narrow scope reading, which says that Hans did make a refusal about something: he refused not to pay money.

The (unavailable) split and (available) narrow scope readings of (77) are quite difficult to grasp, and so we provide a little story here that makes them as plausible pragmatically as possible. Imagine that Hans is a moviemaker with a superstition about the number 3. This superstition could make Hans act in a number of different ways. Consider two of these ways. First, Hans could decide that, no matter how many movies he is offered to make, he is always going to leave a residue of three movies that he doesn’t make. Second, Hans could decide that, no matter how many movies he is offered to make, he is always going to make sure that the number of movies he makes is *different* from three—so, in this case, his decision is to make sure he always makes one, two, four, five, etc., movies, and never three. The first kind of scenario makes the split scope reading of (77) true (and, if the total number of movies offered to him is exactly six, then the narrow scope reading is false). The second kind of scenario makes the narrow scope reading of the sentence true (and, if the number of movies he is offered is, for example, two, then the split scope reading is false). Our paraphrase for the split scope reading is: ‘There is a choice function *f* such that it picks triplets or more and in no world compatible with Hans’ wishes does Hans make the movies that *f* picks, and there is no choice function *g* that picks quadruples or more such that in no world compatible with Hans’ wishes does Hans make the movies that *g* picks’. Simplifying: ‘In every world, there are three or more movies that Hans doesn’t make and in every world there are no more than three movies that Hans doesn’t make’. This is the unavailable split scope reading. Admittedly, the scenario that makes this reading true is a bit contrived (though not

²⁶ We can’t find a reasonably-sounding English translation for the unavailable split scope reading here.

implausible completely), but it is very hard to come up with pragmatically plausible scenarios for the split scope reading in this case.

Then, consider *beschließen* ‘decide’:

- (78) Hans hat weniger als drei Bücher zu schreiben
 Hans has less than three books to write
 beschlossen.
 decided
 ‘Hans decided to write less than three books.’
 *‘Hans decided to write at least n books, and n is less than three.’
- (79) Hans hat kein Buch über seine neue Idee zu schreiben beschlossen.
 ‘Hans decided to write no book about his new idea.’
 *‘Hans didn’t decide to write a book about his new idea.’
- (80) Hans hat genau drei Bücher zu schreiben beschlossen.
 ‘Hans decided to write exactly three books.’
 *‘Hans decided to write at least three books.’

In the unavailable split scope reading of (78), ‘There is no choice function that picks triplets or bigger tuples such that in all worlds compatible with Hans’ resolutions, he writes the books that it picks’, the minimum number of books that Hans has decided to write is less than three—i.e., he writes n -many books or more in all worlds and n is less than three. In the available narrow scope reading, Hans has made the decision to write less than three books, so he is going to write no more than two books in every world. In the unavailable split scope reading of (79), Hans hasn’t made a decision concerning the writing of a book about his new idea—in the end, he may or may not write books about it. In the available narrow scope reading, Hans has, on the contrary, made a decision: to write no books about his new idea. The unavailable split scope reading of (80) is ‘There is a choice function f such that it picks triplets or more and in all worlds compatible with Hans’ resolutions he writes the books that f picks, and there is no choice function g that picks quadruples or more such that in every world compatible with Hans’ resolutions he writes the books that g picks’. Simplifying: ‘In every world, there are three or more books that Hans writes, but it is false that he writes more than three books in all worlds’. This means that, in every world, he writes at least three books. In the available narrow scope reading, Hans has made the following decision: he will write no more and no less than three books.

The availability of split scope appears to be roughly aligned with Wurmbrand’s (2003) classification of restructuring predicates. Those predicates that are more strongly restructuring (i.e., her lexical restructuring verbs, those that allow non-focus scrambling, long passive, etc.) allow split scope more easily than those predicates that are less restructuring. In future research we hope to be able to demonstrate this correlation experimentally, but we would like to note now that it might be understood in terms of movement, as a number of the restructuring properties themselves involve movement and movement is a precondition for split scope.

Finally, we come back to a reviewer's point about our strategy in Sect. 2.3. In that section, we checked the equivalence between split scope readings and narrow scope, *de dicto* readings with examples containing *müssen* 'must' and *können* 'can', and only with those verbs. The objection was that the equivalence might hold with these verbs but not with others, so our approach might generate inexistent readings in some cases. This is true in principle. However, in this section we have shown that some intensional verbs do, and some intensional verbs don't, split scope. The ones that do are upward monotonic in their nuclear scope (*müssen* 'must', *können* 'can', *erlauben* 'allow'), and for these the equivalences discussed in Sect. 2.3 hold. Downward-monotonic verbs, like *sich weigern* 'refuse', *verbieten* 'prohibit', and others, would give rise, in the company of upward-monotonic DP quantifiers like *ein* 'a', *mehr als n* 'more than n', etc., to truth-conditionally distinct split scope readings, if these verbs were scope splitters. Independently of the nature of the DP quantifiers, these verbs are not scope splitters. Thus, our account doesn't overgenerate here either.²⁷

3.3 Generalization C

There is a rich literature on different classes of infinitival embedding and their restructuring properties in German (see Wurmbrand 2003 and references therein). Among scope splitting verbs, some force their infinitival complement to be intraposed while others allow optional extraposition. This latter class is of particular interest to us, for two reasons. First, it is a well-known observation that scope splitting with negative indefinites requires intraposition of the infinitival complement containing the negative indefinite. Second, extraposed clauses exhibit fewer restructuring properties than intraposed ones; in particular, they are islands for non-contrastive scrambling and scope.

Generalization C is that the split scope readings of negative indefinites, comparative quantifiers, and *exactly*-numerals disappear if the infinitival complement containing the quantified DP is extraposed. Compare (81), (82), and (83) with (70), (71), and (72), respectively:

- (81) Ich habe ihm erlaubt [weniger als drei Bücher zu schreiben]
 I have him allowed less than three books to write
 'I allowed him to write n books, and n is less than three.'
 *'This is what I allowed him to do: write a maximum of n books, n being less than three.'
- (82) Ich habe ihm erlaubt [keine Bücher zu schreiben]
 'I allowed him to write no books (at all).'
 *'I didn't allow him to write books.'

²⁷ The fact that no downward-monotonic verb is a scope splitter recalls Ross's (1984) inner-island effect. This is a very intriguing connection that, unfortunately, we cannot pursue here. See also footnote 24.

- (83) Ich habe ihm erlaubt [genau drei Bücher zu schreiben]
 ‘I allowed him to write n books, and n is exactly three.’
 *‘This is what I allowed him to do: write a maximum of three books.’

The split scope readings of these examples are clearly unavailable.

Extrapolated infinitivals are islands for QR. Since scope splitting is more restricted than QR, the expectation is that it will not be possible to split the scope of quantifiers inside of extrapolated infinitivals.

In Sect. 2.3.3, we provided a structural account of upper- and lower-bounded readings of *exactly*-numerals—they are split scope readings in our account. The only other accounts known to us are pragmatic (Carston 1998; Geurts 2006; Breheny 2008). The fact that the crucial readings disappear under extraposition, i.e., are sensitive to syntactic structure, constitutes one *prima facie* argument for a structural account like ours. Furthermore, we have seen that this behavior is part of a broader pattern, and it is hard to see what a uniform pragmatic approach would be able to say about it.

3.4 Generalization D

Generalization D says that all instances of split scope involve low existential force. This fact provides another reason for pursuing a unified account. We have proposed that there is no low existential *quantifier*, but only referential dependency on a higher operator. Such a dependency can only create the illusion of low existential scope—not of any other scope relation. This means that not only does our account unify split scope, it also derives Generalization D. Other accounts might achieve one of these without doing the other.

4 Other approaches to split scope

The evidence provided in the previous section strongly argues in favor of a uniform treatment of negative indefinites, comparative quantifiers, and *exactly*-numerals: their split scope readings are allowed and blocked in the same circumstances. Our unification breaks with a tradition that treats split scope with comparatives as a separate phenomenon from split scope with negative indefinites (see in particular Penka 2007). In this tradition, one set of facts is usually considered to be part of the empirical domain of negative indefinite split scope which we have mentioned only in passing so far: negative indefinites, unlike the other quantifiers discussed here, can split their scope across universal DPs under the hat contour (Jacobs 1980; Penka 2007, among others; for more on the hat contour, see Büring 1995). In this section, we argue that this set of facts is a separate phenomenon from the negative indefinite split scope we have discussed in this paper. We have shown above that there are good reasons to treat split scope with comparative quantifiers and *exactly*-numerals together with split scope of negative indefinites, and we show now that split scope conditioned by the hat contour cannot be treated in the same way. The considerations that give rise to

this conclusion are strictly empirical, not theoretical. Therefore, approaches within the tradition we are breaking with will also be forced to reflect the empirical distinctions in the theory. Such theories will thus end up with a three-way distinction: split scope with negative indefinites under the hat contour, other split scope with negative indefinites, and split scope with other quantifiers. Crucially, the similarities we have noted here between split scope with negative indefinites and split scope with other quantifiers are purely accidental in these accounts. In our approach, by contrast, we only need to make a two-way distinction: split scope with negative indefinites under the hat contour, and other split scope. At the end of this section we briefly address the prospects of extending existing accounts of split scope to the full paradigm.

4.1 Split scope of negative indefinites under the hat contour

Let us look at negative indefinite split scope across universal DPs in more detail. Consider (84) (‘/’ indicates a rise, ‘\’ indicates a fall, and ‘%’ indicates that the example is grammatical under the indicated reading for a subset of speakers only):

- (84) %/JEDER Arzt hat KEIN\ Auto.
 every doctor has no car
 ‘Not every doctor has a car.’

For many speakers, (84) has a very prominent split scope reading, as reflected in the translation. Other quantifiers don’t give rise to split scope in these circumstances, even for those speakers that accept a split scope in (84) (Penka 2007, p. 138, Magdalena Schwager, p.c.):

- (85) ??/JEDER Student hat WENIGER\ als drei Bücher.
 every student has less than three books
 ‘Every student has at least n books, and n is less than three.’
- (86) ??/JEDER Arzt hat WENIGER\ als drei Autos.
 every doctor has less than three cars
 ‘Every doctor has at least n cars, and n is less than three.’ (cf. (6))
- (87) ??/JEDER Arzt hat weniger als DREI\ Autos.
 every doctor has less than three cars
 ‘Every doctor has at least n cars, and n is less than three.’ (cf. (6))
- (88) */JEDER Arzt hat GENAU\ drei Autos.
 every doctor has less three cars
 ‘Every doctor has at least three cars.’
- (89) */JEDER Arzt hat genau DREI\ Autos.
 every doctor has less three cars
 ‘Every doctor has at least three cars.’

Example (85) is Penka's. In (86)–(89) we provide two different intonation patterns to make sure that that doesn't affect the availability of the split reading. An important reason to think that (84) is really a different phenomenon from the negative indefinite split scope we have seen elsewhere (see Sects. 1–3) is that, across German dialects and across languages, the two do not pattern together.

Thus, while all German speakers allow split scope of negative indefinites across intensional verbs (i.e., a subset thereof, as described in Sect. 3.2), only a subset of German speakers allow it across universal quantifiers under the hat contour. It is not yet clear what the delimitations of the phenomenon are, but there are suggestions in the literature (see Jacobs 1980, p. 126) that it is available more easily for Southern speakers of standard German (e.g., speakers of standard Austrian, as well as speakers in Bavaria, Frankonia, etc.). The crosslinguistic generalization is that, if a language allows split scope, then it allows it across (some) intensional verbs, but not necessarily across universal quantifiers (with or without special intonation). For example, English and Norwegian allow split scope, but only across (some) intensional verbs. In this respect, they are like those speakers of standard German who do not get a split scope reading in (84). Some examples from English are in (90) (cf. Norwegian in Svenonius 2002, p. 125):

- (90) English (Potts 2000; Doris Penka, p.c.)
- a. The company need fire no employees.
'It is not the case that the company is obligated to fire employees.'
 - b. There can be no doubt.
'There can't be any doubt.'
 - c. The company must fire no employees .
* 'It is not the case that the company is obligated to fire employees.'
 - d. All doctors have no car.
* 'Not every doctor has a car.' (independently of intonation)

Dutch (see de Swart 2000) and the dialects of German mentioned above, on the other hand, do allow split scope in the two kinds of contexts. The point is that split scope across intensional verbs can exist on its own.

Another reason for making a distinction is that, in the dialects of German that allow it, split scope across universal quantifiers always requires the hat contour. Split scope across intensional verbs does not require it, not even in standard Austrian or the other dialects of German that allow split scope in (84).

4.2 Comparison with other approaches

Given that the properties of negative indefinite split scope under the hat contour are not well understood,²⁸ all approaches to split scope, including ours, have to make an exception for it. In our approach, that is the only division we make in the empirical domain: we have negative indefinite split scope under the hat contour on the one

²⁸ For example, it is not yet known whether it is only universal quantifiers that are scope-splitters, or if all universal quantifiers can do it.

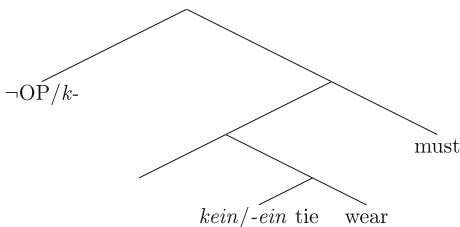
hand, and all other split scope on the other. Because we treat all other split scope alike (that includes other negative indefinite split scope, and the split scope of comparative quantifiers and numerals), in our account the similarities we observed in Sect. 3 are expected: since these split scopes all have the same source, we expect them to be available in the same set of circumstances. In any approach that posits a different source for the split scope of negative indefinites and the split scope of comparative quantifiers and numerals, however, not one but two divisions are made in the empirical domain. But then the common properties we discussed in Sect. 3 are purely accidental—an undesirable outcome.

In principle, though, the unified approach developed here is not the only conceivable unified approach. Could one extend existing accounts of split scope to cover the empirical domain the same way that ours covers it? We think that the answer is negative for one type of account, and positive for the other.

We don't think that accounts of split scope like Jacobs (1980), Penka (2007), or Rullmann (1995b) can be extended to cover the whole empirical domain of split scope—that is, we don't think that the mechanisms these proposals invoke for negative indefinites can be the source of split scope in the case of comparative quantifiers and numerals.

Jacobs (1980), Penka (2007), and Rullmann (1995b) are all non-movement approaches. For both Jacobs and Rullmann, negative indefinites are semantically negative. They are decomposed into two parts, one being clausal negation and the other an indefinite. Clausal negation is sometimes visible morphologically (e.g., the *k-* of *kein*). These two parts are put together via “amalgamation” or some such process under linear adjacency. Split scope arises because material can interfere between the two parts hierarchically, as shown in (91). Penka, on the other hand, argues that negative indefinites are semantically positive generalized quantifiers. Negative indefinites differ from regular indefinites in that the former, but not the latter, need to establish an agreement relation with a null negative operator (under linear adjacency), a need that is sometimes visible morphologically (e.g., the *k-* of *kein*). Split scope arises, as before, because material can hierarchically interfere between the abstract negative operator and the negative indefinite.²⁹

(91)



Our main worry with these approaches is that it does not seem plausible to provide an amalgamation or agreement analysis for comparative quantifiers or numerals. For Jacobs and Rullmann, this would involve positing an abstract degree quantifier that

²⁹ The empty branch in (91) represents the VP-internal thematic position of the subject of *wear*. It is phonologically empty and needs to be disregarded under both approaches for adjacency purposes.

gets together with the rest of the comparative quantifier or the numeral, a proposal that is *ad hoc* at best. In the case of Penka, an additional problem is that agreement with a negative operator is responsible also for negative concord. So if the same process was involved in the analysis of comparative quantifiers and numerals, we would expect these quantifiers to participate in some sort of “comparative/numeral” concord, but this does not seem to be empirically correct.³⁰

We do think that it is possible to generalize Hackl’s (2000) approach so that it covers negative indefinites and numerals as well. This would involve conceiving of both negative indefinites and numerals as quantifiers over degrees, which we think is possible (though for negative indefinites, one would have to appeal to zero degrees, which we don’t think is a trivial matter). Even though we don’t investigate such an approach here, it seems to us that one problem it will have is that it won’t be capable of deriving Generalization D, since Hackl must himself stipulate that there is low existential scope in comparative quantifier split scope readings.

5 Conclusion

In this paper we have developed a unified analysis of split scope. We have proposed that the split scope of negative indefinites (putting aside that which arises under the hat contour; see Sect. 4.1), comparative quantifiers, and *exactly*-numerals has a single source. The analysis had two key features. First, we used quantification over choice functions as our approach to natural language determiner quantification in general, following Sauerland (1998, 2004). Then, we used binding of world indices as a way of simulating existential low scope, inspired by Kratzer’s (1998) binding of pronouns for the same purpose. Once binding of the world index of common nouns is allowed in principle, Sauerland’s system automatically generates the LFs and readings we are interested in—that is, our proposal derives the fact that split scope readings always involve low existential force (Generalization D). Further justification for our position came from the fact that conservativity follows as a theorem in Sauerland’s approach.

On the empirical side, we developed an extended argument that the split scope of negative indefinites, comparative quantifiers, and *exactly*-numerals is available in the same set of circumstances. We proposed the following generalizations: split scope readings are only possible across intensional verbs (Kennedy’s Generalization or Generalization A); split scope readings are possible only across some intensional verbs, not all (Generalization B); and split scope readings are never possible in the context of extraposition, even when the intensional verb falls within the class of

³⁰ There are other movement approaches to negative indefinite split scope, such as Geurts (1996) and de Swart (2000). We take it that these could be extended to cover comparative quantifiers and numerals, and that they may be able to capture the similarities between negative indefinites and other quantifiers that we pointed out in Sect. 3. Geurts and de Swart, however, account for negative indefinite split scope at the expense of postulating lexical ambiguity. Geurts postulates ambiguity in the noun, and de Swart proposes that negative indefinite determiners can themselves be ambiguous. Without going into the details of these proposals, we take it that it is better if one can do without lexical ambiguity, as in our approach.

verbs that allow split scope (Generalization C). These generalizations suggest a unified approach.

We argued that apparent problems for our analysis turn out not to be problems. First, our analysis predicts split scope to arise for all determiner quantifiers. We showed that this prediction is unproblematic for upward-monotonic quantifiers either because the readings we generate are equivalent to readings that can be generated independently on all approaches, or because the verbs involved are not scope splitters. Second, only negative indefinites seem to allow split scope across universal DPs under the hat contour. We argued that this case should be considered a separate phenomenon from all other split scope, since it arises under different circumstances, with a subset of quantifiers, and in a subset of languages/dialects.

One surprising but seemingly correct consequence of our proposal is that *exactly*-numerals can, in some contexts, give rise to split scope. The readings we predict have not been treated as split scope readings before, but we think that doing so has advantages over existing accounts, which we hope to be able to explore further in future work.

Acknowledgments We would like to thank Paul Elbourne, Anastasia Giannakidou, Øystein Nilsen, Rick Nouwen, Doris Penka, Maribel Romero, Philippe Schlenker, Magdalena Schwager, Yael Sharvit, Arnim von Stechow, two very helpful anonymous reviewers and audiences at the University of Tromsø, University College London, the University of York, the *23rd Comparative Germanic Syntax Workshop*, and *Sinn und Bedeutung 13* for their many comments, questions and suggestions. The second author gratefully acknowledges the support of the Research Council of Norway (*Norges Forskningsrådet*). All responsibilities are our own, of course.

References

- Abusch, D. 1994. The scope of indefinites. *Natural Language Semantics* 2: 83–136.
- Bech, G. 1955/1957. *Studien über das deutsche Verbum infinitum*. Danske Vidensk-absernes Selskab: Historisk-filologiske meddelelser. Copenhagen: Munksgaard.
- Breheny, R. 2008. A new look at the semantics and pragmatics of numerically quantified noun phrases. *Journal of Semantics* 25: 93–140.
- Büring, D. 1995. The 59th Street Bridge Accent. On the meaning of topic and focus. PhD dissertation, Universität Tübingen.
- Carston, R. 1998. Informativeness, relevance and scalar implicature. In *Relevance theory: Applications and implications*, ed. R. Carston and S. Uchida, 179–236. Amsterdam: Benjamins.
- de Swart, H. 2000. Scope ambiguities with negative quantifiers. In *Reference and anaphoric relations*, ed. K. von Stechow and U. Egli, 109–132. Dordrecht: Kluwer.
- Endriss, C. 2009. *Quantificational topics—A scopal treatment of exceptional wide scope phenomena* (Studies in Linguistics & Philosophy 86). Berlin: Springer.
- Fox, D. 2001. The Syntax and semantics of traces, talk given at the University of Connecticut, Storrs, November 2001.
- Geurts, B. 1996. On *No*. *Journal of Semantics* 13: 67–86.
- Geurts, B. 2000. Indefinites and choice functions. *Linguistic Inquiry* 31: 731–738.
- Geurts, B. 2006. Take “five”: the meaning and use of a number word. In *Non-definiteness and plurality*, ed. S. Vogeeler and L. Tasmowski, 311–329. Amsterdam: Benjamins.
- Hackl, M. 2000. Comparative quantifiers. PhD dissertation, MIT.
- Hackl, M. 2009. On the grammar and processing of proportional quantifiers: *Most* versus *more than half*. *Natural Language Semantics* 17: 63–98.
- Heim, I. 2001. Degree operators and scope. In *Audiatur Vox Sapientiae: A Festschrift for Arnim von Stechow* (Studia Grammatica 52), ed. C. Féry and W. Sternefeld, 214–239. Berlin: Akademie Verlag.
- Heim, I., and A. Kratzer 1998. *Semantics in generative grammar*. Malden: Blackwell.

- Jacobs, J. 1980. Lexical decomposition in Montague Grammar. *Theoretical Linguistics* 7: 121–136.
- Kennedy, C. 1994. Argument contained ellipsis. Linguistics Research Center Report LRC-94-03. San Diego: UCSC.
- Kennedy, C. 1997. Projecting the adjective. PhD dissertation, UCSC.
- Kratzer, A. 1991. Modality. In *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, ed. A. von Stechow and D. Wunderlich, 639–650. Berlin: de Gruyter.
- Kratzer, A. 1998. Scope or pseudoscope? Are there wide scope indefinites? In *Events in grammar*, ed. S. Rothstein, 163–197. Dordrecht: Kluwer.
- Krifka, M. 1999. At least some determiners aren't determiners. In *The semantics/pragmatics interface from different points of view*, ed. K. Turner, 257–291. Oxford: Elsevier.
- Nouwen, R. 2010. Two kinds of modified numerals. *Semantics and Pragmatics* 3: 1–41.
- Penka, D. 2007. Negative indefinites. PhD dissertation, Universität Tübingen.
- Potts, C. 2000. When even *no*'s NEG is Splitsville, contribution posted to *Jorge Hankamer WebFest*. <http://www.stanford.edu/~cgpotts/papers/potts00-hankamer-webfest.pdf>.
- Reinhart, T. 1997. Quantifier scope. How labor is divided between QR and choice functions. *Linguistics and Philosophy* 20: 335–397.
- Ross, H. 1984. Inner islands. In *Proceedings of the tenth annual meeting of the Berkeley Linguistics Society*, ed. C. Brugman, M. Macauley et al., 258–265. Berkeley: Berkeley Linguistics Society.
- Rullmann, H. 1995a. Maximality in the semantics of wh-constructions. PhD dissertation, University of Massachusetts at Amherst.
- Rullmann, H. 1995b. *Geen einheid*. *Tabu* 25: 194–197.
- Sauerland, U. 1998. The meaning of chains. PhD dissertation, MIT.
- Sauerland, U. 2000. No *no*: On the cross-linguist absence of a determiner *no*. In *Proceedings of the Tsukuba workshop on determiners and quantification*, 415–444. Tsukuba University, Tsukuba.
- Sauerland, U. 2004. The interpretation of traces. *Natural Language Semantics* 12: 63–127.
- Svenonius, P. 2002. Strains of negation in Norwegian. *Working Papers in Scandinavian Syntax* 69: 121–146.
- Winter, Y. 1997. Choice functions and the scopal semantics of indefinites. *Linguistics and Philosophy* 20: 399–467.
- Winter, Y. 2001. *Flexibility principles in Boolean semantics: Coordination, plurality and scope in natural language*. Cambridge, MA: MIT Press.
- Wurmbrand, S. 2003. *Infinitives, restructuring and clause structure*. Berlin: de Gruyter.