Multidim Syst Sign Process (2017) 28:1481–1497
DOI 10.1007/s11045-016-0430-3

Finite frequency H_∞ control of 2-D continuous systems in Roesser model

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Received: 5 January 2016 / Revised: 25 May 2016 / Accepted: 30 May 2016 /
Published online: 6 June 2016

Abstract This paper investigates the finite frequency (FF) H_∞ control problem of two-dimensional (2-D) continuous systems in Roesser Model. Our attention is focused on designing state feedback controllers guaranteeing the bounded-input-bounded-output stability and FF H_∞ performance of the corresponding closed-loop system. A generalized 2-D Kalman-Yakubovich-Popov (KYP) lemma is presented for 2-D continuous systems. By the generalized 2-D KYP lemma, the existence conditions of H_∞ controllers are obtained in terms of linear matrix inequalities. Two examples are given to validate the proposed methods.

Keywords 2-D continuous system · KYP lemma · Finite frequency · H_∞ control

1 Introduction

The transfer function method and state-space model method are two main approaches to describe, analyze and design dynamic system, from the frequency-domain and time-domain points of view, respectively. It is well known that the celebrated Kalman-Yakubovich-Popov (KYP) lemma (Anderson and Vongpanitlerd 1973; Kalman 1963; Rantzer 1996) effectively builds a bridge between the frequency-domain approach and time-domain approach, which establishes the equivalence relationship for one-dimensional (1-D) systems between frequency-domain inequality representing system properties, such as positive realness and bounded realness (Gahinet and Apkarian 1994; Sun et al. 1994; Xie et al. 1998), and a linear matrix inequality (LMI) for the state space realization (Iwasaki and Hara 2005). Thus, the infinite-dimensional problem can be easily converted to a finite dimensional convex feasibil-

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ity problem with LMI constraints. In particular, the development of the numerical algorithm for LMI further strengthened the position of the KYP lemma in the past two decades.

However, one main drawback of standard KYP lemma is that it is only applicable for the case over the entire frequency (EF) domain, while system properties are often required over a specified finite frequency (FF) range in engineering practice. Thus, the generalized KYP lemma over an FF range proposed in Iwasaki et al. (2000) and Iwasaki and Hara (2005) broke this obstacle. It has proven that generalized KYP lemma is a useful tool when applied in various engineering design problems with FF specifications, such as feedback control synthesis (Iwasaki and Hara 2007), disturbance rejection (Du et al. 2007) and filter design (Gao and Li 2011).

On the other hand, two-dimensional (2-D) systems also have drawn much attention over the past decades due to their significant applications, such as 2-D digital filtering (Lu and Antoniou 1992), image processing (Bracewell 1995) and repetitive processes control (Xu et al. 2003). Two mainly used 2-D models are the Roesser model (Lam et al. 2004; Roesser 1975), and the Fornasini-Marchesini local state-space (FM LSS) model (Fornasini and Marchesini 1978; Wang and Liu 2003). For these two models, bounded realness (Chen and Fong 2006, 2007; Du and Xie 2002; Wu et al. 2008, 2007) and positive realness (Xu et al. 2002, 2003) have been extensively researched. Some general results related to the KYP lemma for 2-D discrete systems have appeared for both EF and FF cases. Bachelier et al. (2008) proposed a KYP lemma for hybrid 2-D Roesser systems, which included the existing bounded real lemma in Du and Xie (2002) and positive real lemma in Xu et al. (2003) as special cases, which is for the EF domain. Bachelier and Mehdi (2006) established the generalized KYP lemma for multi-dimensional hybrid Roesser systems. Yang et al. (2008) developed generalized KYP lemmas for 2-D Roesser models which can directly consider properties of a transfer function over a rectangular FF region. Then, Li and Gao (2012) defined a novel characterization of rectangular finite frequency regions in the context of FM LSS models and proposed a generalized KYP lemma for FM LSS models. Li et al. (2012) employed the results of (Yang et al. 2008) to solve the robust FF H_∞ filtering problem for 2-D Roesser systems. Based on the generalized KYP lemma, Paszke et al. (2013) developed a 2-D systems based finite frequency range iterative learning control law design algorithm and Paszke and Bachelier (2013) solved the robust control problem with finite frequency specification for uncertain discrete linear repetitive process. Lately, H_∞ and H_2 norms of 2-D mixed continuous-discrete-time systems have been studied via rationally-dependent complex Lyapunov function approach (Chesi and Middleton 2015).

Recently, attention has been devoted towards 2-D continuous systems. With the aid of the technique of line integral, several useful results related to 2-D continuous systems are available. The robust state feedback H_∞ control problem for uncertain 2-D continuous state delayed systems in the Roesser model has been solved lately (Ghous and Xiang 2016). The problems of stability and H_∞ control of 2-D continuous switched systems have been studied (Ghous and Xiang 2016). Similarly, attention has also been devoted towards robust H_∞ filtering of uncertain 2-D continuous systems with time-varying delays (El-Kasri et al. 2013). However, all the mentioned above literatures considered the problems for 2-D continuous systems from the point of state-space domain, thus all the obtained results just can be applicable for the case over the EF range. The practical systems face with different performance requirements under different frequency domains, such as noise signal generally from the low frequency range, which motivates us to consider the H_∞ performance over the FF range.

In this paper, we focus on investigating the FF H_∞ control problem for 2-D continuous systems in Roesser model. The main contribution of this paper is summarized as follows:

(1) By using the generalized KYP lemma, FF bounded realness property for 2-D continuous systems is investigated;
(2) A state feedback controller design scheme is proposed to guarantee the bounded-input-bounded-output (BIBO) stability and FF H_∞ performance of the closed-loop system.

The remainder of this paper is organized as follows. Section 2 is devoted to problem formulation and some necessary lemmas. In Sect. 3, the generalized FF KYP lemma for 2-D continuous is presented. One application of the generalized KYP lemma and FF H_∞ controller design are presented in Sect. 4. Two examples are provided in Sect. 5. In Sect. 6, concluding remarks are given.

Notations The symbols $\mathbb{R}$ and $\mathbb{C}$ denote the real number set and complex number set, respectively. $\mathbb{R}^n$ and $\mathbb{C}^n$ denote the sets of real and complex column vectors of dimension n, respectively. $\mathbb{R}^{m\times n}$ and $\mathbb{R}^{m\times n}$ denote, respectively, the sets of real and complex matrices of dimension $m \times n$. The symbols $\mathbb{H}$ and I_n denote the set of Hermitian matrix, and the identity matrix of dimension $n \times n$, respectively. The transpose and complex conjugate transpose of a matrix M are denoted by M^T and M^* , respectively, and $M > 0$ ($M \geq 0$) means that M is positive definite (positive semi-definite). N_M is an arbitrary matrix whose columns form a basis of the null-space of M. $\sigma_{max}(\cdot)$ denotes the maximum singular value of a transfer function. A L_2 norm of a 2-D signal $w(t_1, t_2)$ is given by

$$\|w\|_2 = \sqrt{\int_0^\infty \int_0^\infty w^T\,(t_1, t_2)\, w\,(t_1, t_2) dt_1 dt_2}.$$

2 Problem formulation and preliminaries

Consider the following 2-D continuous Roesser model:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = A \begin{bmatrix} x^h\,(t_1, t_2) \\ x^v\,(t_1, t_2) \end{bmatrix} + Bu\,(t_1, t_2)\,, \tag{1}$$

$$z\,(t_1, t_2) = C \begin{bmatrix} x^h\,(t_1, t_2) \\ x^v\,(t_1, t_2) \end{bmatrix} + Du\,(t_1, t_2)\,, \tag{2}$$

where $x^h \in \mathbb{C}^{n_h},\ x^v \in \mathbb{C}^{n_v},\ u \in \mathbb{C}^{n_u}$ and $z \in \mathbb{C}^{n_z}$ are the horizontal state, vertical state, input and output of the system, respectively; $A,\ B,\ C$ and D are system matrices with appropriate dimensions.

Let $x\,(t_1, t_2) = \begin{bmatrix} x^h\,(t_1, t_2) \\ x^v\,(t_1, t_2) \end{bmatrix}$ with $n = n_h + n_v$, X and U denote Laplace-tranforms of the state variable x and input variable u, respectively. A frequency-domain representation of the Roesser model (1) can be written as

$$\Omega X = AX + BU,\ \Omega = diag\left\{ jw_h I_{n_h}, jw_v I_{n_v} \right\},$$

for $w_h, w_v \in \mathbb{R}$. The transfer function of the system (1) and (2) is

$$G\,(jw_h, jw_v) = C(\Omega - A)^{-1}B + D. \tag{3}$$

To obtain main results of this paper, we present the following lemma.

Lemma 1 (Rantzer 1996) *Given matrices $F, G \in \mathbb{C}^{n\times n}$, vectors $f, g \in \mathbb{C}^n$, then*

(a) $FF^* = GG^*$ if and only if there exists a matrix $\Gamma \in \mathbb{C}^{n\times n}$ such that $\Gamma\Gamma^* = I_n$ and $F = G\Gamma$.

(b) For $g \neq 0$, $fg^* + gf^* = 0$ if and only if there exists a scalar $w \in \mathbb{R}$ such that $f = jw \cdot g$.

Next, we extend the result in Iwasaki and Hara (2007) for 1-D continuous-time system to 2-D continuous system in the following lemma, which establishes equivalence between frequency condition and LMIs.

Lemma 2 *Given two scalars w_{h0}, w_{v0}, and complex vectors $f = \begin{bmatrix} f_h \\ f_v \end{bmatrix}$ and $g = \begin{bmatrix} g_h \\ g_v \end{bmatrix} \in \mathbb{C}^{n_h+n_v}$, the following statements (1) and (2) are equivalent.*

(1) There exist scalars w_h and w_v such that $f = \Omega g$, with $\Omega = diag\left\{jw_h I_{n_h}, jw_v I_{n_v}\right\}$, $w_h \leq w_{h0}$ and $w_v \leq w_{v0}$.
(2) For any complex matrices $P = diag\{P_h, P_v\} \in \mathbb{H}_{n_h+n_v}$ and $Q = diag\{Q_h, Q_v\} \in \mathbb{H}_{n_h+n_v} > 0$, the following inequality holds:

$$\begin{bmatrix} f \\ g \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \begin{bmatrix} f \\ g \end{bmatrix} \leq 0, \tag{4}$$

with $\Omega_0 = diag\left\{jw_{h0} I_{n_h}, jw_{v0} I_{n_v}\right\}$.

Proof If (1) is satisfied, one obtains

$$\begin{aligned}
&\begin{bmatrix} f \\ g \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \begin{bmatrix} f \\ g \end{bmatrix} \\
&= \begin{bmatrix} g \\ g \end{bmatrix}^* \begin{bmatrix} -\Omega^* Q \Omega & \Omega^* P \\ P\Omega & (\Omega_0)^2 Q \end{bmatrix} \begin{bmatrix} g \\ g \end{bmatrix} \\
&= -g^* \Omega^* Q \Omega g + g^* \Omega^* P g + g^* P \Omega g + g^* (\Omega_0)^2 Q g \\
&= -g_h^* (jw_h)^* Q_h (jw_h) g_h + g_h^* (jw_h)^* P_h g_h + g_h^* P_h (jw_h) g_h + g_h^* (jw_{h0})^2 Q_h (jw_h) g_h \\
&\quad - g_v^* (jw_v)^* Q_v (jw_v) g_v + g_v^* (jw_v)^* P_v g_v + g_v^* P_v (jw_v) g_v + g_v^* (jw_{v0})^2 Q_v (jw_v) g_v \\
&= \left[(w_h)^2 g_h^* Q_h g_h - (w_{h0})^2 g_h^* Q_h g_h\right] + \left[(w_v)^2 g_v^* Q_v g_v - (w_{v0})^2 g_v^* Q_v g_v\right] \\
&\leq 0.
\end{aligned}$$

Thus, (2) holds from statement (1).

Conversely, if (2) is satisfied, we have

$$trace\left(-f^* f Q + f^* g P + g^* f P + g^* g (\Omega_0)^2 Q\right) \leq 0.$$

For all Hermitian block diagonal matrices P and positive definite matrices Q, it follows that

$$\begin{aligned}
f_h^* g_h + g_h^* f_h &= 0, \\
f_v^* g_v + g_v^* f_v &= 0, \\
-f_h^* f_h + {g_h}^* g_h (jw_{h0})^2 &\leq 0, \\
-f_v^* f_v + {g_v}^* g_v (jw_{v0})^2 &\leq 0.
\end{aligned}$$

According to Lemma 1, it can be verified that 1) holds. This completes the proof. □

3 Generalized 2-D KYP lemma over FF domain

In this section, we present a generalized KYP lemma for 2-D continuous systems in Roesser model over a rectangular FF domain.

Lemma 3 *Consider system (1) with* $\det(\Omega - A) \neq 0$, $\Omega = diag\left\{jw_h I_{n_h}, jw_v I_{n_v}\right\}$, *for a Hermitian matrix* $\Theta \in \mathbb{H}_{n+n_u}$, *with* $n = n_h + n_v$, *if there exist Hermitian matrices* $P = diag\{P_h, P_v\} \in \mathbb{H}_n$ *and* $Q = diag\{Q_h, Q_v\} \in \mathbb{H}_n > 0$ *such that*

$$\Theta < \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \tag{5}$$

holds with $\Omega_0 = diag\left\{jw_{h0} I_{n_h}, jw_{v0} I_{n_v}\right\}$ *and* $w_{h0},\ w_{v0} \geq 0$ *being given scalars, then the following condition is satisfied for all* $|w_h| \leq w_{h0}$ *and* $|w_v| \leq w_{v0}$.

$$\begin{bmatrix} (\Omega - A)^{-1} B \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (\Omega - A)^{-1} B \\ I \end{bmatrix} \leq 0. \tag{6}$$

Proof From (5), we have

$$\Theta - \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} < 0.$$

Thus, the following inequality

$$\varphi^* \left[\Theta - \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \right] \varphi < 0 \tag{7}$$

holds for any $\varphi \in \left\{ \begin{bmatrix} X \\ U \end{bmatrix} \in {}^{n+m} : \Omega X = AX + BU, \forall |w_h| \leq w_{h0},\ |w_v| \leq w_{v0} \right\}$. Letting $f = AX + BU$ and $g = X$, it is clear that $f = \Omega g$. Thus, (7) can be rewritten as:

$$\varphi^* \Theta \varphi - \phi^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \phi < 0, \tag{8}$$

with $\phi = \begin{bmatrix} f \\ g \end{bmatrix}$. According to Lemma 2, we have

$$\phi^* \begin{bmatrix} -Q & P \\ P & (\Omega_0)^2 Q \end{bmatrix} \phi < 0. \tag{9}$$

Then, combining (8) and (9) leads to

$$\varphi^* \Theta \varphi < 0, \tag{10}$$

which implies that the following inequality holds for any $U \in \mathbb{C}^{n_u}$.

$$U^* \left\{ \begin{bmatrix} (\Omega - A)^{-1} B \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (\Omega - A)^{-1} B \\ I \end{bmatrix} \right\} U < 0. \tag{11}$$

Thus, (6) is satisfied. This completes the proof. □

Remark 1 It should be noted that Lemma 3 gives only a sufficient but not a necessary condition for existence of FF property, since P and Q are required to be diagonal matrices. This is in contrast with 1-D generalized KYP lemma (Iwasaki and Hara 2007), which provides a necessary and sufficient condition for the existence of FF property.

Remark 2 In Lemma 3, we present a generalized 2-D KYP lemma for 2-D Roesser model in the continuous-time domain, which is a special case of the result in Bachelier and Mehdi (2006).

Since A, B, M and Θ are general complex matrices, the positive definiteness of a complex matrix is detected by the following lemma.

Lemma 4 (Iwasaki and Hara 2007) *Let $X = X^R + iX^I \in \mathbb{H}_n$, with $X^R, X^I \in \mathbb{R}^{n\times n}$. Then, $X > 0$ if and only if*

$$\begin{bmatrix} X^R & -X^I \\ X^I & X^R \end{bmatrix} > 0. \tag{12}$$

Also, let $Y = Y^R + iY^I \in \mathbb{C}^{n\times n}$ with $Y^R, Y^I \in {}^{n\times n}$. Then, $Y^*XY > 0$ if and only if

$$\begin{bmatrix} Y^R & -Y^I \\ Y^I & Y^R \end{bmatrix}^T \begin{bmatrix} X^R & -X^I \\ X^I & X^R \end{bmatrix} \begin{bmatrix} Y^R & -Y^I \\ Y^I & Y^R \end{bmatrix} > 0. \tag{13}$$

Lemma 3 gives a sufficient condition for the existence of a performance characterization specified over a rectangular low frequency domain. The next lemma presents a sufficient condition over any given rectangular frequency domain.

Lemma 5 *Consider system (1) with $\det(\Omega - A) \neq 0$, $\Omega = diag\left\{jw_h I_{n_h}, jw_v I_{n_v}\right\}$, for scalars w_{h1}, w_{h2}, w_{v1} and w_{v2} satisfying $w_{h1} \leq w_{v2}$, $w_{v1} \leq w_{v2}$, if there exist Hermitian matrices $P = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix} \in \mathbb{H}_n$ and $Q = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} \in \mathbb{H}_n > 0$ such that*

$$\Theta + \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \left\{ \begin{bmatrix} -Q & P + \Omega_c Q \\ P - \Omega_c Q & WQ \end{bmatrix} \right\} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} < 0, \tag{14}$$

with

$$W = \begin{bmatrix} -w_{h1}w_{h2}I_{n_h} & 0 \\ 0 & -w_{v1}w_{v2}I_{n_v} \end{bmatrix}, \quad \Omega_c = \begin{bmatrix} jw_{hc}I_{n_h} & 0 \\ 0 & jw_{vc}I_{n_v} \end{bmatrix},$$
$$w_{hc} = \frac{w_{h1} + w_{h2}}{2}, \quad w_{vc} = \frac{w_{v1} + w_{v2}}{2},$$

then, the following inequality

$$\begin{bmatrix} (\Omega - A)^{-1}B \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (\Omega - A)^{-1}B \\ I \end{bmatrix} < 0 \tag{15}$$

holds for all $w_{h1} \leq w_h \leq w_{h2}$ and $w_{v1} \leq w_v \leq w_{v2}$.

Proof Note that the condition $w_{h1} \leq w_h \leq w_{h2}$ is equivalent to $|w_h - w_{hc}| \leq w_{h\max}$, and $w_{v1} \leq w_v \leq w_{v2}$ is equivalent to $|w_v - w_{vc}| \leq w_{v\max}$, with

$$w_{h\max} = \frac{w_{h2} - w_{h1}}{2}, \quad w_{v\max} = \frac{w_{v2} - w_{v1}}{2}.$$

Introducing the transformation $\tilde{A} = A - \Omega_c$, it can be obtained that

$$\Omega - A = \tilde{\Omega} - \tilde{A}, \tag{16}$$

with

$$\tilde{\Omega} = \begin{bmatrix} j(w_h - w_{hc})I_{n_h} & 0 \\ 0 & j(w_v - w_{vc})I_{n_v} \end{bmatrix}.$$

According to Lemma 3, (21) holds if there exist Hermitian matrices $P = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix} \in \mathbb{H}_n$ and $Q = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} \in \mathbb{H}_n > 0$ such that

$$\Theta + \begin{bmatrix} \tilde{A} & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_{\max})^2 Q \end{bmatrix} \begin{bmatrix} \tilde{A} & B \\ I & 0 \end{bmatrix} < 0, \tag{17}$$

with $\Omega_{\max} = \begin{bmatrix} j w_{h\max} I_{n_h} & 0 \\ 0 & j w_{v\max} I_{n_v} \end{bmatrix}$. Note that

$$\begin{aligned}
&\begin{bmatrix} \tilde{A} & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_{\max})^2 Q \end{bmatrix} \begin{bmatrix} \tilde{A} & B \\ I & 0 \end{bmatrix} \\
&= \begin{bmatrix} A - \Omega_c & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q & P \\ P & (\Omega_{\max})^2 Q \end{bmatrix} \begin{bmatrix} A - \Omega c & B \\ I & 0 \end{bmatrix} \\
&= \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \left\{ \begin{bmatrix} I & 0 \\ \Omega_c & I \end{bmatrix} \begin{bmatrix} -Q & P \\ P & (\Omega_{\max})^2 Q \end{bmatrix} \begin{bmatrix} I & -\Omega_c \\ 0 & I \end{bmatrix} \right\} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \\
&= \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \left\{ \begin{bmatrix} -Q & P + \Omega_c Q \\ P - \Omega_c Q & WQ \end{bmatrix} \right\} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}.
\end{aligned} \tag{18}$$

Substituting (18) into (17) gets (14). This completes the proof. □

4 H_∞ control

4.1 FF bounded real lemma

Lemma 5 gives sufficient condition for the existence of a performance characterization specified over a rectangular FF domain. Thus, inequality (15) can be checked by solving the finite-dimensional convex feasibility problem of (14). Furthermore, appropriate choices of Θ in (15) allow us to represent various system properties including bounded-realness. Therefore, one application on bounded-realness of generalized 2-D KYP lemma is presented in the following.

Define a rectangular FF domain Σ as follows:

$$\Sigma = \{(w_h, w_v) : w_{h1} \leq w_h \leq w_{h2},\ w_{v1} \leq w_v \leq w_{v2}\}. \tag{19}$$

Motivated by the theory of bounded realness for 1-D systems (Anderson and Vongpanitlerd 1973), the corresponding definition for 2-D systems can be defined as follows.

Definition 1 Given a scalar $\gamma > 0$ and a rectangular FF domain Σ defined in (19), 2-D continuous system (1) and (2) is said to be bounded real if its transfer function $G(w_h, w_v)$ satisfies

$$G(w_h, w_v)^* G(w_h, w_v) < \gamma^2 I,\ \forall (w_h, w_v) \in \Sigma, \tag{20}$$

or equivalently

$$\|G\|_\infty^\Sigma = \sup_{(w_h, w_v) \in \Sigma} \sigma_{\max}[G(w_h, w_v)] < \gamma,\ \forall (w_h, w_v) \in \Sigma. \tag{21}$$

Remark 3 In Xu et al. (2005), the H_∞ norm of 2-D continuous system (1) and (2) is defined as

$$\|G\|_\infty = \sup_{w_h, w_v \in \mathbb{R}} \sigma_{\max}\left[G\left(jw_h, jw_v\right)\right] \leq \gamma. \tag{22}$$

By using 2-D Parseval's theorem, Lu and Antoniou (1992) proved that H_∞ norm in (22) is equavalent to L_2-gain $J_0 = \sup\limits_{u:\|u\|_2 \neq 0} \frac{\|z\|_2}{\|u\|_2}$. Inspired by the definition of H_∞ norm in (22), $\|G\|_\infty^\Sigma$ in (21) is called FF H_∞ norm in this paper.

The FF bounded realness property of 2-D continuous system (1) and (2) is presented in the following lemma.

Lemma 6 *Consider system (1) with* $\det(\Omega - A) \neq 0$, $\Omega = diag\left\{jw_h I_{n_h}, jw_v I_{n_v}\right\}$, *for scalars* w_{h1}, w_{h2}, w_{v1} *and* w_{v2} *satisfying* $w_{h1} \leq w_{h2}$, $w_{v1} \leq w_{v2}$, *and a positive constant* $\gamma > 0$, *if there exist Hermitian matrices* $P = diag\{P_h, P_v\} \in \mathbb{H}_n$ *and* $Q = diag\{Q_h, Q_v\} \in \mathbb{H}_n > 0$ *such that*

$$\begin{bmatrix} C^*C & C^*D \\ D^*C & -\gamma^2 I + D^*D \end{bmatrix} + \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \left\{\begin{bmatrix} -Q & P + \Omega_c Q \\ P - \Omega_c Q & WQ \end{bmatrix}\right\} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} < 0, \tag{23}$$

with W, Ω_c, w_{hc} *and* w_{vc} *defined in Lemma 5, then 2-D continuous system (1) and (2) is bounded real within a rectangular FF domain* Σ *defined in (19).*

Proof Taking $\Theta = \begin{bmatrix} C^*C & C^*D \\ D^*C & -\gamma^2 I + D^*D \end{bmatrix}$ and applying Lemma 5, we have

$$\begin{bmatrix} (\Omega - A)^{-1} B \\ I \end{bmatrix}^* \begin{bmatrix} C^*C & C^*D \\ D^*C & -\gamma^2 I + D^*D \end{bmatrix} \begin{bmatrix} (\Omega - A)^{-1} B \\ I \end{bmatrix} < 0. \tag{24}$$

It follows that

$$\left(C(\Omega - A)^{-1}B - D\right)^* \left(C(\Omega - A)^{-1}B - D\right) < \gamma^2 I. \tag{25}$$

Substituting (3) into (25) gives $G(w_h, w_v)^* G(w_h, w_v) < \gamma^2 I$. According to Definition 1, 2-D continuous system (1) and (2) is bounded real within a rectangular FF domain Σ. This completes the proof. □

4.2 FF H_∞ controller design

In this subsection, with the aid of the bounded real lemma, we are concerned with the FF H_∞ control of the following system

$$\begin{aligned} \begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} &= A \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + Bu(t_1,t_2) + B_1 w(t_1,t_2), \\ z(t_1,t_2) &= C \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + Du(t_1,t_2) + D_1 w(t_1,t_2), \end{aligned} \tag{26}$$

where the frequency of the exogenous noise $w(t_1, t_2) \in \mathbb{C}^{n_w}$ are assumed to belong to a known rectangular region, $B_1 \in \mathbb{C}^{n \times n_w}$, $D_1 \in \mathbb{C}^{n_z \times n_w}$ and other notations are the same as those in (1) and (2).

The following state feedback controller is used in this paper.

$$u(t_1,t_2) = K \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix}, \tag{27}$$

where K is an appropriately dimensioned controller gain matrix to be determined. Thus, the corresponding closed-loop system can be formulated by

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \bar{A} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \bar{B} w(t_1,t_2),$$
$$z(t_1,t_2) = \bar{C} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \bar{D} w(t_1,t_2), \tag{28}$$

where $\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} = \begin{bmatrix} A + BK & B_1 \\ C + DK & D_1 \end{bmatrix}$. The transfer function of system (28) from $w(t_1,t_2)$ to $z(t_1,t_2)$ is given by

$$\bar{G}(w_h, w_v) = \bar{C}\left(\Omega - \bar{A}\right)^{-1}\bar{B} + \bar{D},$$

with $\Omega = diag\left\{jw_h I_{n_h}, jw_v I_{n_v}\right\}$.

Throughout the paper, we adopt the following stability definition.

Definition 2 (Xu et al. 2008) The 2-D continuous system

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = A \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} \tag{29}$$

is BIBO stable if, under the zero initial condition, the output of the system is bounded for any bounded input.

Definition 3 Given a scalar $\gamma > 0$ and a rectangular FF domain Σ defined in (19), 2-D continuous system (28) is said to have an FF H_∞ performance level γ, if it is BIBO stable when $w(t_1,t_2) = 0$, and its transfer function $\bar{G}(w_h, w_v)$ satisfies

$$\bar{G}(w_h, w_v)^* \bar{G}(w_h, w_v) < \gamma^2 I, \forall (w_h, w_v) \in \Sigma, \tag{30}$$

when $w(t_1,t_2) \neq 0$.

Lemma 7 (Gahinet and Apkarian 1994) *Let R, Z and Γ be given. There exists a matrix Y satisfying $R^*YZ + Z^*Y^*R + \Gamma < 0$ if and only if the following projection inequalities hold:*

$$N_R^* \Gamma N_R < 0, \; N_Z^* \Gamma N_Z < 0.$$

Lemma 8 (Xu et al. 2008) *The 2-D continuous system (29) is BIBO stable if there exists a Hermitian matrix $P = diag\{P_h, P_v\} \in \mathbb{H}_n > 0$ satisfying the LMI: $A^*P + PA < 0$.*

To ensure the BIBO stability and specification (30) for the closed-loop system (28), we need to resort to Lemma 6 and Lemma 8, repectively. By combining these two results, the following lemma can be obtained.

Lemma 9 *Consider system (26), for a scalar $\gamma > 0$, if there exist Hermitian matrices $\bar{P} = \begin{bmatrix} \bar{P}_h & 0 \\ 0 & \bar{P}_v \end{bmatrix} \in \mathbb{H}_n$, $\hat{P} = \begin{bmatrix} \hat{P}_h & 0 \\ 0 & \hat{P}_v \end{bmatrix} \in \mathbb{H}_n$ and $\bar{Q} = \begin{bmatrix} \bar{Q}_h & 0 \\ 0 & \bar{Q}_v \end{bmatrix} \in \mathbb{H}_n$ such that $\bar{Q} > 0$, $\hat{P} > 0$ and*

$$\begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix}^* \left\{ \begin{bmatrix} -\bar{Q} & \bar{P} + \Omega_c \bar{Q} \\ \bar{P} - \Omega_c \bar{Q} & W\bar{Q} \end{bmatrix} \right\} \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} \bar{C}^*\bar{C} & \bar{C}^*\bar{D} \\ \bar{D}^*\bar{C} & -\gamma^2 I + \bar{D}^*\bar{D} \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} \bar{A} \\ I \end{bmatrix}^* \begin{bmatrix} 0 & \hat{P} \\ \hat{P} & 0 \end{bmatrix} \begin{bmatrix} \bar{A} \\ I \end{bmatrix} < 0, \tag{32}$$

with W and Ω_c defined in Lemma 5, then under the state feedback controller (27), the corresponding closed-loop system (28) is BIBO stable with specification (30),

To design an FF H_∞ controller, it is necessary to decouple the product terms of $\bar{P}$, $\hat{P}$, $\bar{Q}$ and system matrices. In the sequel, an FF H_∞ controller design scheme will be given.

Theorem 1 *Consider system (26), for a scalar $\gamma > 0$, if there exist matrices $\bar{F} = diag\{\bar{F}_h, \bar{F}_v\} \in \mathbb{C}^{n\times n}$, $V \in \mathbb{C}^{n\times n_u}$ and Hermitian matrices $\tilde{P} = \begin{bmatrix} \tilde{P}_h & 0 \\ 0 & \tilde{P}_v \end{bmatrix} \in \mathbb{H}_n$, $\widehat{P} = \begin{bmatrix} \widehat{P}_h & 0 \\ 0 & \widehat{P}_v \end{bmatrix} \in \mathbb{H}_n$ and $\tilde{Q} = \begin{bmatrix} \tilde{Q}_h & 0 \\ 0 & \tilde{Q}_v \end{bmatrix} \in \mathbb{H}_n$ such that $\tilde{Q} > 0$, $\widehat{P} > 0$ and*

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & 0 \\ \Lambda_{12}^* & \Lambda_{22} & \Lambda_{23} & \bar{F}C^* + \mathrm{V}D^* \\ \Lambda_{13}^* & \Lambda_{23}^* & \Lambda_{33} & {D_1}^* \\ 0 & \left(\bar{F}C^* + \mathrm{V}D^*\right)^* & D_1 & -I_{n_z} \end{bmatrix} < 0, \tag{33}$$

$$\begin{bmatrix} -\bar{F}^* - \bar{F} & \widehat{P} + A\bar{F}^* + BV^* - \bar{F} \\ \widehat{P} + \bar{F}A^* + VB^* - \bar{F}^* & A\bar{F}^* + BV^* + \bar{F}A^* + VB^* \end{bmatrix} < 0, \tag{34}$$

where

$$\begin{aligned} &\Lambda_{11} = -\tilde{Q} - \bar{F}^* - \bar{F},\ \Lambda_{12} = \tilde{P} + \Omega_c\tilde{Q} + A\bar{F}^* + BV^* - \bar{F}, \\ &\Lambda_{13} = B_1,\ \Lambda_{22} = W\tilde{Q} + A\bar{F}^* + \bar{F}A^* + BV^* + VB^*, \\ &\Lambda_{23} = B_1,\ \Lambda_{33} = -\gamma^2 I_{n_w}, \end{aligned}$$

with W and Ω_c defined in Lemma 5, then under the state feedback controller (27), the closed-loop system (28) is BIBO stable with specification (30). Moreover, the controller gain matrix in (27) is given by $K = \left(\bar{F}^{-1}V\right)^*$.

Proof Let

$$\Gamma = \begin{bmatrix} -\bar{Q} & \bar{P} + \Omega_c\bar{Q} & 0 \\ \bar{P} - \Omega_c\bar{Q} & W\bar{Q} + \bar{C}^*\bar{C} & \bar{C}^*\bar{D} \\ 0 & \bar{D}^*\bar{C} & -\gamma^2 I_{n_w} + \bar{D}^*\bar{D} \end{bmatrix},$$
$$Z = \begin{bmatrix} -I & \bar{A} & \bar{B} \end{bmatrix},\ R = I_{2n+n_w},\ Y = \begin{bmatrix} F^* & F^* & 0 \end{bmatrix}^*.$$

By Schur complement, the following inequality

$$R^*YZ + Z^*Y^*R + \Gamma < 0, \tag{35}$$

is equivalent to

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 \\ \Phi_{12}^* & \Phi_{22} & \Phi_{23} & \bar{C}^* \\ \Phi_{13}^* & \Phi_{23}^* & \Phi_{33} & \bar{D}^* \\ 0 & \bar{C} & \bar{D} & -I_{n_z} \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned} &\Phi_{11} = -\bar{Q} - F - F^*,\ \Phi_{12} = \bar{P} + \Omega_c\bar{Q} + F\bar{A} - F^*, \\ &\Phi_{13} = F\bar{B},\ \Phi_{22} = W\bar{Q} + F\bar{A} + \bar{A}^*F^*, \\ &\Phi_{23} = F\bar{B},\ \Phi_{33} = -\gamma^2 I_{n_w}. \end{aligned}$$

Choosing $N_Z = \begin{bmatrix} \hat{A} & \hat{B} \\ I & 0 \\ 0 & I \end{bmatrix}$ and applying Lemma 7, we obtain from (35) that (31) holds.

Set $\Gamma = \begin{bmatrix} 0 & \hat{P} \\ \hat{P} & 0 \end{bmatrix}$, $Y = \begin{bmatrix} F \\ F \end{bmatrix}$, $Z = \begin{bmatrix} -I & \bar{A} \end{bmatrix}$, $R = I_{2n}$ and $N_Z = \begin{bmatrix} \bar{A} \\ I \end{bmatrix}$. By Lemma 7, (32) holds if the following inequality is satisfied.

$$R^T YZ + Z^T Y^T R + \Gamma = \begin{bmatrix} -F - F^* & \hat{P} + F\bar{A} - F^* \\ \hat{P} + \bar{A}^* F^* - F & F\bar{A} + \bar{A}^* F^* \end{bmatrix} < 0. \tag{37}$$

Let $\bar{F} = F^{-1}$, $\tilde{Q} = \bar{F}\bar{Q}\bar{F}^*$, $\tilde{P} = \bar{F}\bar{P}\bar{F}^*$. Pre- and post- multiplying (36) by nonsingular matrices $J = diag\left\{F^{-1}, F^{-1}, I, I\right\}$ and J^* respectively, we obtain that (36) is equivalent to

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 \\ \Xi_{12}^* & \Xi_{22} & \Xi_{23} & \bar{F}\bar{C}^* \\ \Xi_{13}^* & \Xi_{23}^* & \Xi_{33} & \bar{D}^* \\ 0 & \bar{C}\bar{F}^* & \bar{D} & -I_{n_z} \end{bmatrix} < 0, \tag{38}$$

where

$$\begin{aligned} &\Xi_{11} = -\tilde{Q} - \bar{F}^* - \bar{F}, \quad \Xi_{12} = \tilde{P} + \Omega_c\tilde{Q} + \bar{A}\bar{F}^* - \bar{F}, \\ &\Xi_{13} = \bar{B}, \quad \Xi_{22} = W\tilde{Q} + \bar{A}\bar{F}^* + \bar{F}\bar{A}^*, \\ &\Xi_{23} = \bar{B}, \quad \Xi_{33} = -\gamma^2 I_{n_w}. \end{aligned}$$

Set $\widehat{P} = F^{-1}\hat{P}F^{-*}$. Pre- and post- multiplying (37) by $\hat{J} = diag\left\{F^{-1}, F^{-1}\right\}$ and $\hat{J}^*$, respectively, we obtain that (37) is equivalent to

$$\begin{bmatrix} -\bar{F}^* - \bar{F} & \widehat{P} + \bar{A}\bar{F}^* - \bar{F} \\ \widehat{P} + \bar{F}\bar{A}^* - \bar{F}^* & \bar{A}\bar{F}^* + \bar{F}\bar{A}^* \end{bmatrix} < 0. \tag{39}$$

Let $\mathrm{V} = \bar{F}K^*$. Due to (28), (38) and (39) are equivalent to (33) and (34), respectively. The proof is completed. □

Remark 4 It should be noted that (34) guarantees $-\bar{F} - \bar{F}^* < 0$, which implies that $\bar{F}$ is nonsingular. By the slack matrix F, the product terms of $\bar{P}$, $\bar{Q}$, $\hat{P}$ in Lemma 9 and system matrices have been decoupled, which is helpful to H_∞ controller design. However, the fact that F is the same in (33) and (34) unavoidably is a part of conservatism.

Remark 5 Like most of the LMI results dedicated to the stability or H_∞ analysis of 2-D models, the conservatism of our results partially come from the fact that P and Q are assumed to be block-diagonal. Very recently, Bachelier et al. (2016) proposed a less conservative LMI stability criteria for 2-D systems by reducing the polynomial-based tests of stability to that of LMIs. A promising perspective is the extension to H_∞ control which would be our future work.

5 Illustrative examples

In this section, two examples will be provided to demonstrate the effectiveness of the proposed results.

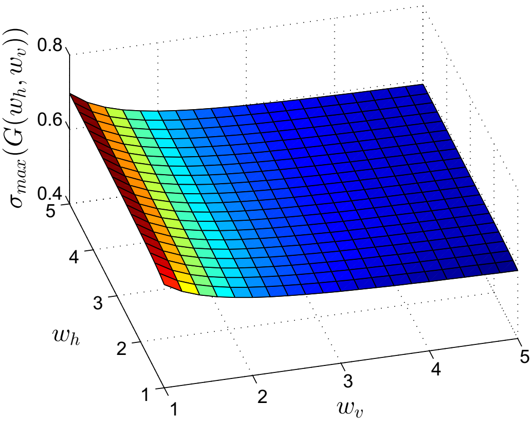


Fig. 1 $\sigma_{max}(G(w_h, w_v))$ of the open-loop system in Example 1

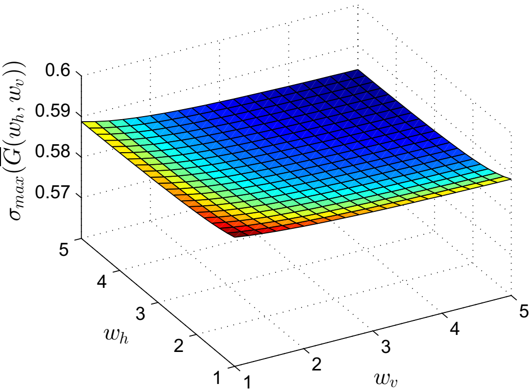


Fig. 2 $\sigma_{max}(\overline{G}(w_h, w_v))$ of the closed-loop system in Example 1

Example 1 Consider system (26) with the following parameters:

$$A = \begin{bmatrix} -0.5 & 0.2 \\ 0.6 & -0.2 \end{bmatrix}, B = \begin{bmatrix} 2.0 \\ -0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix},$$
$$C = \begin{bmatrix} -0.8 & -0.9 \\ 0.5 & -0.3 \end{bmatrix}, D = \begin{bmatrix} 0.6 \\ -0.1 \end{bmatrix}, D_1 = \begin{bmatrix} -0.5 \\ 0.3 \end{bmatrix}. \tag{40}$$

By simulation, the open-loop system (26) with (40) is not bounded real for given $\gamma = 0.6$ within the FF domain $\bar{\Sigma} = \{(w_h, w_v) : 1 \leq w_h \leq 5, 1 \leq w_v \leq 5\}$, which can be seen from Fig. 1.

According to Theorem 1, a desired FF H_∞ state feedback controller gain matrix can be computed:

$$K = \begin{bmatrix} -0.8992 & 1.3341 \end{bmatrix}. \tag{41}$$

To illustrate the effectiveness of the proposed controller, the $\sigma_{max}(\overline{G}(w_h, w_v))$ of the corresponding closed-loop system is depicted in Fig. 2. It is clearly shown that within the considered frequency region, all the singular values of the closed-loop transfer function are smaller than $\gamma = 0.6$. State trajectories of the closed-loop system are given in Figs. 3, 4, which show that the closed-loop system is BIBO stable. The effectiveness of the designed controller is demonstrated.

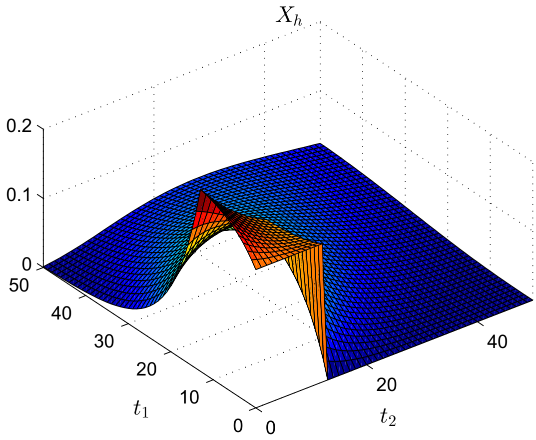


Fig. 3 State trajectory of $x_h(t_1, t_2)$ in Example 1

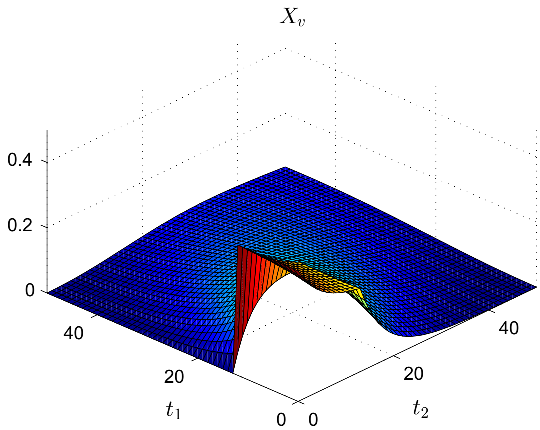


Fig. 4 State trajectory of $x_v(t_1, t_2)$ in Example 1

Example 2 In this example, we consider the Darboux equation which can be used to describe some process of gas absorption,water stream heating and air drying (Kaczorek 1985):

$$\frac{\partial^2 s(x,t)}{\partial x \partial t} = a_1 \frac{\partial s(x,t)}{\partial t} + a_2 \frac{\partial s(x,t)}{\partial x} + a_0 s(x,t) + a_3 w(x,t) + bu(x,t), \tag{42}$$

where $s(x,t)$ is an unknown function at $x \in [0, x_f]$(space) and $t \in [0, \infty)$(time), a_0, a_1, a_2, a_3 and b are real coefficients, $u(x,t)$ is a given input function and $w(x,t)$ is a disturbance.

Let us define

$$r(x,t) = \frac{\partial s(x,t)}{\partial t} - a_2 s(x,t). \tag{43}$$

Using (43), we can transform (42) into an equivalent system of first order differential equations of the form.

$$\begin{bmatrix} \frac{\partial r(x,t)}{\partial x} \\ \frac{\partial s(x,t)}{\partial t} \end{bmatrix} = \begin{bmatrix} a_1 & a_1 a_2 + a_0 \\ 1 & a_2 \end{bmatrix} \begin{bmatrix} r(x,t) \\ s(x,t) \end{bmatrix} + \begin{bmatrix} a_3 \\ 0 \end{bmatrix} w(x,t) + \begin{bmatrix} b \\ 0 \end{bmatrix} u(x,t). \tag{44}$$

Fig. 5 $\sigma_{max}(G(w_h, w_v))$ of the open-loop system in Example 2

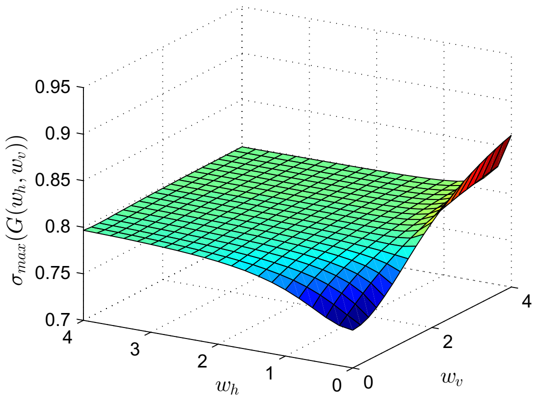
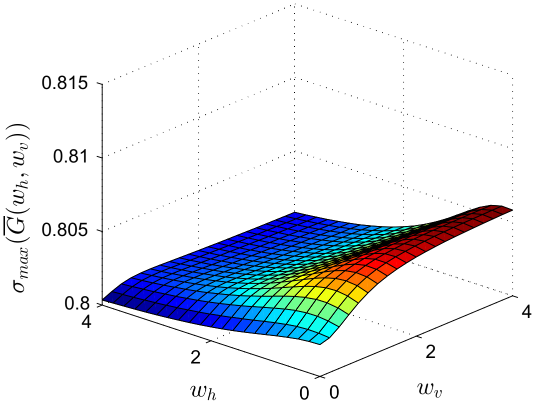


Fig. 6 $\sigma_{max}(\overline{G}(w_h, w_v))$ of the closed-loop system in Example 2



From (43), we know that

$$r(0,t) = \left.\frac{\partial s(x,t)}{\partial t}\right|_{x=0} - a_2 s(0,t) = \frac{ds(0,t)}{dt} - a_2 s(0,t) = R(t). \tag{45}$$

Let $x^h(t_1, t_2) = r(x,t)$, $x^v(t_1, t_2) = s(x,t)$ and take $a_0 = 0.69$, $a_1 = -0.3$, $a_2 = -0.7$, $a_3 = 0.5$, $b = 0.1$. Then we obtain a 2-D continuous system (26) with the following parameters

$$\begin{aligned} A &= \begin{bmatrix} -0.3 & 0.9 \\ 1 & -0.7 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0.3 & 0.2 \end{bmatrix}, D = 0.1, D_1 = 0.8. \end{aligned} \tag{46}$$

Using Theorem 1, we can obtain a state feedback controller satisfying the FF H_∞ performance level $\gamma = 0.82$ as follows:

$$u(t_1, t_2) = \begin{bmatrix} -2.1687 & -2.3162 \end{bmatrix} \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix}. \tag{47}$$

Within the FF domain $\bar{\Sigma} = \{(w_h, w_v) : 0 \le w_h \le 4, 0 \le w_v \le 4\}$, some of the singular values $\sigma_{\max}(G(w_h, w_v))$ of the open-loop system are larger than $\gamma = 0.82$ as shown in Fig. 5, while all the singular values $\sigma_{\max}\left(\overline{G}(w_h, w_v)\right)$ of the closed-loop system are smaller

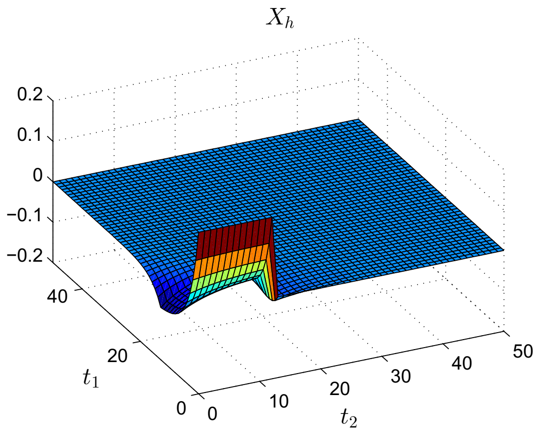


Fig. 7 State trajectory of $x_h(t_1, t_2)$ in Example 2

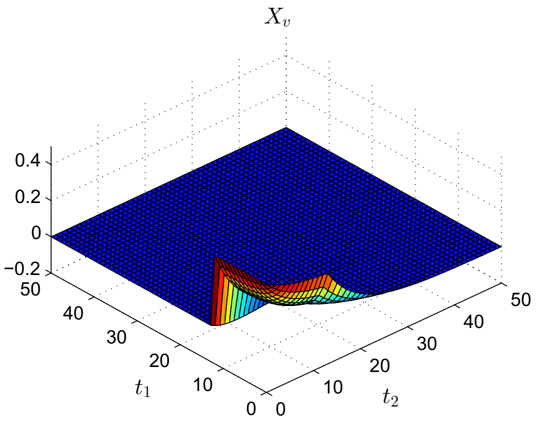


Fig. 8 State trajectory of $x_v(t_1, t_2)$ in Example 2

than $\gamma = 0.82$, which can be seen from Fig. 6. State trajectories of the closed-loop system are shown in Figs. 7, 8. Simulation results demonstrate the effectiveness of the proposed method.

6 Conclusions

This paper studied the FF H_∞ control problem for 2-D continuous systems in Roesser model. The frequencies of the exogenous noises are assumed to reside in a known rectangular region. The generalized KYP lemma for 2-D continuous systems provided sufficient conditions in terms of LMI for general quadratic properties of the transfer function over a rectangular FF region. Then, one application of generalized KYP lemma to FF bounded realness was given. Furthermore, by using the FF bounded real lemma, a systematic method was proposed for the design of H_∞ controllers which guarantee the BIBO stability and FF H_∞ performance level of the corresponding closed-loop system. These results are expected to be useful for analysis and synthesis of systems. Two examples were given to validate the proposed method.

Acknowledgements This work was supported by the National Natural Science Foundation of China under Grant No. 61273120.

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