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A new structure and design method for variable fractional-delay 2-D FIR digital filters

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Abstract In this paper, a new structure and design method are proposed for variable fractional-delay (VFD) 2-D FIR digital filters. Basing on the Taylor series expansion of the desired frequency response, a prefilter–subfilter cascaded structure can be derived. For the 1-D differentiating prefilters and the 2-D quadrantally symmetric subfilters, they can be designed simply by the least-squares method. Design examples show that the required number of independent coefficients of the proposed system is much less than that of the existing structure while the performance of the designed VFD 2-D filters is still better under the cost of larger delays.

Keywords Farrow structure · Variable fractional-delay filter · 2-D FIR filter · Least-squares method · 2-D quadrantally symmetric filter · Prefilter

1 Introduction

VFD digital filters belong to the branch of variable digital filters which are applied to where frequency characteristics need to be adjusted online without redesigning the system. For the past decade, several works have been proposed for the design of variable digital filters (Shyu et al. 2009a,b, 2010; Deng 1998a,b, 2001, 2003, 2005, 2007a,b, 2010; Deng et al. 2003; Deng

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and Soma 1995a,b; Zarour and Fahmy 1989; Farrow 1998; Laakso et al. 1996; Lu and Deng 1999; Tseng 2002a,b, 2003; Johansson and Löwenborg 2003; Deng and Lian 2006; Zhao and Yu 2006; Tsui et al. 2007; Kwan and Jiang 2009; Pei and Lin 2009; Tseng and Lee 2010; Deng and Lu 2000) due to their wide applications in signal processing and communication systems. By the function, they are generally classified into two main categories. One is the filters with variable magnitude characteristics such as cutoff frequencies or magnitude responses (Deng 1998a,b, 2001, 2003, 2005; Deng and Soma 1995a,b; Zarour and Fahmy 1989; Deng et al. 2003; Shyu et al. 2009b), and the other is the filters with variable fractional delay (Shyu et al. 2009a, 2010; Farrow 1998; Laakso et al. 1996; Lu and Deng 1999; Tseng 2002a,b, 2003; Johansson and Löwenborg 2003; Deng and Lian 2006; Zhao and Yu 2006; Deng 2007a,b, 2010; Tsui et al. 2007; Kwan and Jiang 2009; Pei and Lin 2009; Tseng and Lee 2010; Deng and Lu 2000).

In this paper, the design of VFD 2-D FIR digital filters will be investigated. Conventionally, the transfer function of a variable fractional-delay (VFD) 2-D FIR digital filter is given by

$$Hc\left(z_1, z_2, p_1, p_2\right) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} h_{n_1 n_2}\left(p_1, p_2\right) z_1^{-n_1} z_2^{-n_2} \tag{1}$$

where

$$h_{n_1 n_2}\left(p_1, p_2\right) = \sum_{m_1=0}^{M} \sum_{m_2=0}^{M} h\left(n_1, n_2, m_1, m_2\right) p_1^{m_1} p_2^{m_2}. \tag{2}$$

Hence, (1) can be represented by

$$Hc\left(z_1, z_2, p_1, p_2\right) = \sum_{m_1=0}^{M} \sum_{m_2=0}^{M} \hat{G}_{m_1 m_2}\left(z_1, z_2\right) p_1^{m_1} p_2^{m_2} \tag{3}$$

where the 2-D subfilters

$$\hat{G}_{m_1 m_2}\left(z_1, z_2\right) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} h\left(n_1, n_2, m_1, m_2\right) z_1^{-n_1} z_2^{-n_2}, \tag{4}$$

and the system can be implemented by a 2-D Farrow structure as in Fig. 1 (Shyu et al. 2009a).

Comparing with the conventional 2-D Farrow structure presented recently in (Shyu et al. 2009a), a prefilter–subfilter cascaded structure is proposed in this paper. The structure is developed based on the Taylor series expansion of the desired frequency response. In (Shyu et al. 2009a), there are four types of 2-D quadrantally symmetric/antisymmetric filters (Pei and Shyu 1995; Zhao and Lai 2011) to be designed. But, only two 1-D differentiating prefilters and one type of 2-D quadrantally symmetric subfilters are needed to be designed in the proposed structure. By the proposed experiments in this paper, it will be shown that the required number of independent coefficients of the designed system is much less than that in (Shyu et al. 2009a) while the performance of the designed filters is still better than that in (Shyu et al. 2009a) under the cost of larger delays.

This paper is organized as follows. In Sect. 2, the proposed prefilter–subfilter cascaded structure is derived from the Taylor series expansion of the desired frequency response. And the design of the mentioned prefilters and subfilters for even M is presented in Sect. 3. For simplicity, the general least-squares method (Shyu et al. 2009a, 2010; Zhao and Lai 2011, 2012) is applied, and design examples will be presented to demonstrate the effectiveness of the presented method. As to the design of VFD 2-D FIR digital filters for odd M, it is shown in Sect. 4 accompanying also a design example. Finally, the conclusions are given in Sect. 5.

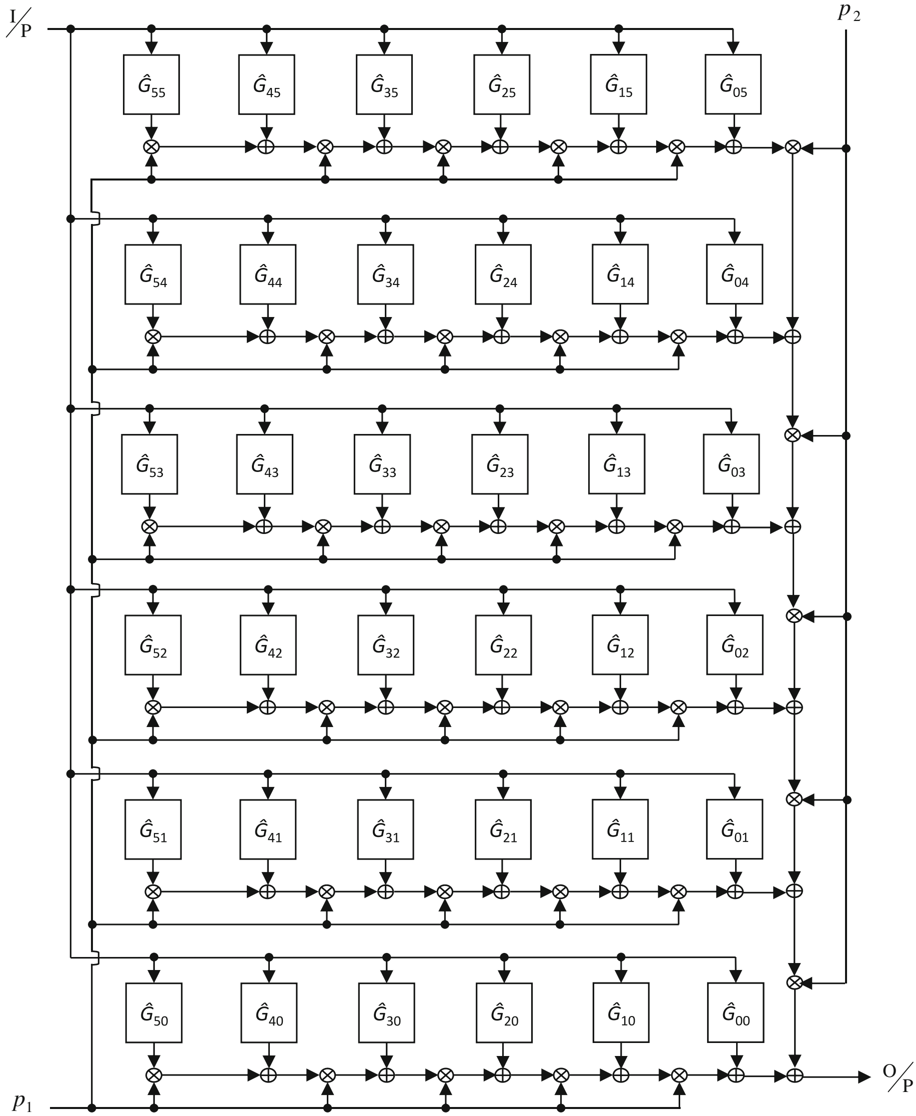

Fig. 1 The conventional structure for a VFD 2-D FIR digital filter. ($M = 5$)

2 The proposed structure

For designing a VFD 2-D FIR filter, the desired frequency response is given by

$$H_d(\omega_1, \omega_2, p_1, p_2) = M(\omega_1, \omega_2)\, e^{-j[\omega_1(I_1+p_1)+\omega_2(I_2+p_2)]} \tag{5}$$

where $M(\omega_1, \omega_2)$ is the desired magnitude response, I_1 and I_2 are the prescribed group-delays with respect to ω_1 and ω_2-axis, respectively, and $p_1, p_2 \in [-0.5, 0.5]$ .For simplicity, only quadrantally symmetric magnitude response $M(\omega_1, \omega_2)$ is considered in this paper. By Taylor series expansion,

$$
\begin{aligned}
e^{-j(\omega_1 p_1+\omega_2 p_2)} &= \sum_{m_1=0}^{\infty} \frac{(-j\omega_1 p_1)^{m_1}}{m_1!} \cdot \sum_{m_2=0}^{\infty} \frac{(-j\omega_2 p_2)^{m_2}}{m_2!} \\
&\approx \sum_{m_1=0}^{M} \frac{(-j\omega_1 p_1)^{m_1}}{m_1!} \cdot \sum_{m_2=0}^{M} \frac{(-j\omega_2 p_2)^{m_2}}{m_2!}
\end{aligned} \tag{6}
$$

for sufficiently large M. In this paper, the case for odd M is considered first, and the case for even M will be discussed in Sect. 4. Let $M = 2\hat{M} + 1$, then (6) becomes

$$
\begin{aligned}
e^{-j(\omega_1 p_1+\omega_2 p_2)} \approx& \left[\sum_{m_1=0}^{\hat{M}} (-1)^{m_1} \frac{(\omega_1 p_1)^{2m_1}}{(2m_1)!} + (-j\omega_1)\, p_1 \sum_{m_1=0}^{\hat{M}} \frac{(-1)^{m_1}}{2m_1+1} \frac{(\omega_1 p_1)^{2m_1}}{(2m_1)!}\right] \\
&\times \left[\sum_{m_2=0}^{\hat{M}} (-1)^{m_2} \frac{(\omega_2 p_2)^{2m_2}}{(2m_2)!} + (-j\omega_2)\, p_2 \sum_{m_2=0}^{\hat{M}} \frac{(-1)^{m_2}}{2m_2+1} \frac{(\omega_2 p_2)^{2m_2}}{(2m_2)!}\right] \\
=& \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} (-1)^{m_1+m_2} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!} \\
&+ (-j\omega_1)\, p_1 \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \frac{(-1)^{m_1+m_2}}{2m_1+1} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!} \\
&+ (-j\omega_2)\, p_2 \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \frac{(-1)^{m_1+m_2}}{2m_2+1} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!} \\
&+ (-j\omega_1)\,(-j\omega_2)\, p_1 p_2 \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \frac{(-1)^{m_1+m_2}}{(2m_1+1)\,(2m_2+1)} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!}
\end{aligned} \tag{7}
$$

By (5) and (7), the applied transfer function of the VFD 2-D FIR filter in this section is represented by

$$
\begin{aligned}
H\,(z_1, z_2, p_1, p_2) =& z_1^{-\frac{N_{d1}}{2}} z_2^{-\frac{N_{d2}}{2}} \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} G_{2m_1,2m_2}\,(z_1, z_2) p_1^{2m_1} p_2^{2m_2} \\
&+ z_2^{-\frac{N_{d2}}{2}} D_1\,(z_1) \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \frac{1}{2m_1+1} G_{2m_1,2m_2}\,(z_1, z_2) p_1^{2m_1+1} p_2^{2m_2} \\
&+ z_1^{-\frac{N_{d1}}{2}} D_2\,(z_2) \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \frac{1}{2m_2+1} G_{2m_1,2m_2}\,(z_1, z_2) p_1^{2m_1} p_2^{2m_2+1} \\
&+ D_1\,(z_1)\, D_2\,(z_2) \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \frac{1}{(2m_1+1)\,(2m_2+1)} G_{2m_1,2m_2} \\
&\times (z_1, z_2)\, p_1^{2m_1+1} p_2^{2m_2+1}
\end{aligned} \tag{8}
$$

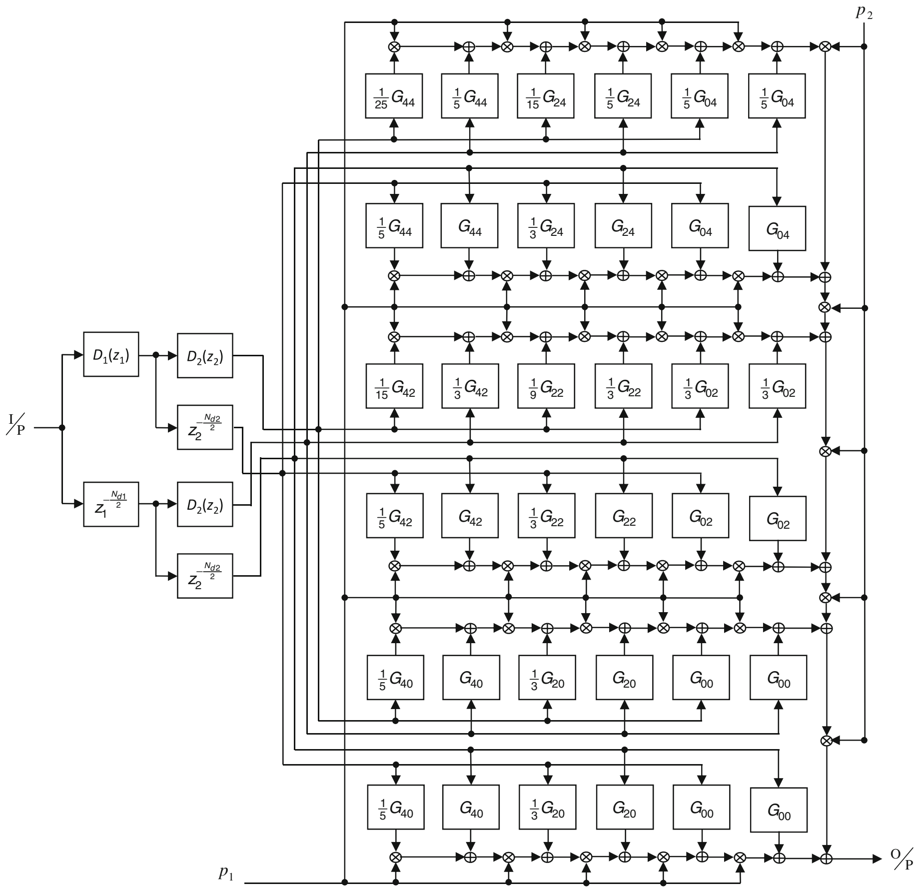

Fig. 2 The proposed structure of a VFD 2-D FIR digital filter. ($M = 5$)

and the proposed structure is shown in Fig. 2. In (8), the quadrantally symmetric subfilters $G_{2m_1,2m_2}(z_1, z_2)$ are characterized by

$$G_{2m_1,2m_2}(z_1, z_2) = \sum_{n_1=0}^{N_g} \sum_{n_2=0}^{N_g} g_{m_1 m_2}(n_1, n_2) z_1^{-n_1} z_2^{-n_2} \tag{9}$$

where N_g is assumed to be even while the Type III linear-phase prefilters $D_i(z_i)$, $i = 1, 2$, are characterized by

$$D_i(z_i) = \sum_{n=0}^{N_{di}} d_i(n) z_i^{-n}, N_{di} : \text{even}, \quad i = 1, 2. \tag{10}$$

After some algebraic operations, the frequency response of (8) can be represented by

$$H\left(e^{j\omega_1}, e^{j\omega_2}, p_1, p_2\right) = e^{-j\left(\frac{N_{d1}}{2}+\frac{N_g}{2}\right)\omega_1} e^{-j\left(\frac{N_{d2}}{2}+\frac{N_g}{2}\right)\omega_2} \hat{H}(\omega_1, \omega_2, p_1, p_2) \tag{11}$$

where

$$\hat{H}(\omega_1,\omega_2,p_1,p_2)=\sum_{m_1=0}^{\hat{M}}\sum_{m_2=0}^{\hat{M}}\hat{G}_{2m_1,2m_2}(\omega_1,\omega_2)p_1^{2m_1}p_2^{2m_2}$$
$$+j\hat{D}_1(\omega_1)\sum_{m_1=0}^{\hat{M}}\sum_{m_2=0}^{\hat{M}}\frac{1}{2m_1+1}\hat{G}_{2m_1,2m_2}(\omega_1,\omega_2)p_1^{2m_1+1}p_2^{2m_2}$$
$$+j\hat{D}_2(\omega_2)\sum_{m_1=0}^{\hat{M}}\sum_{m_2=0}^{\hat{M}}\frac{1}{2m_2+1}\hat{G}_{2m_1,2m_2}(\omega_1,\omega_2)p_1^{2m_1}p_2^{2m_2+1}$$
$$-\hat{D}_1(\omega_1)\hat{D}_2(\omega_2)\sum_{m_1=0}^{\hat{M}}\sum_{m_2=0}^{\hat{M}}\frac{1}{(2m_1+1)(2m_2+1)}$$
$$\times\hat{G}_{2m_1,2m_2}(\omega_1,\omega_2)\,p_1^{2m_1+1}p_2^{2m_2+1}, \tag{12a}$$

$$\hat{G}_{2m_1,2m_2}(\omega_1,\omega_2)=\sum_{n_1=0}^{\frac{N_g}{2}}\sum_{n_2=0}^{\frac{N_g}{2}}\hat{g}_{m_1m_2}(n_1,n_2)\cos(n_1\omega_1)\cos(n_2\omega_2), \tag{12b}$$

$$\hat{D}_i(\omega_i)=\sum_{n=1}^{\frac{N_{di}}{2}}\hat{d}_i(n)\sin(n\omega_i),\ i=1,2, \tag{12c}$$

$$\hat{g}_{m_1m_2}(n_1,n_2)=\begin{cases}g_{m_1m_2}\left(\frac{N_g}{2},\frac{N_g}{2}\right), & n_1=n_2=0,\\ 2g_{m_1m_2}\left(\frac{N_g}{2}-n_1,\frac{N_g}{2}\right), & 1\le n_1\le\frac{N_g}{2},\quad n_2=0,\\ 2g_{m_1m_2}\left(\frac{N_g}{2},\frac{N_g}{2}-n_2\right), & n_1=0,\quad 1\le n_2\le\frac{N_g}{2},\\ 4g_{m_1m_2}\left(\frac{N_g}{2}-n_1,\frac{N_g}{2}-n_2\right), & 1\le n_1,n_2\le\frac{N_g}{2},\end{cases} \tag{12d}$$

$$\hat{d}_i(n)=2d_i\left(\frac{N_{di}}{2}-n\right),\quad i=1,2. \tag{12e}$$

Obviously, the integers I_1 and I_2 in (5) can be set as $I_i=\frac{N_{di}}{2}+\frac{N_g}{2},\ \ i=1,2.$

3 Design of 2-D VFD FIR digital filters with odd M

In this paper, we first deal with the design of the prefilters $D_1(z_1)$ and $D_2(z_2)$, and then these prefilters will be applied for the design of the subfilters $G_{2m_1,2m_2}(z_1,z_2)$. Design examples will be given to demonstrate the effectiveness of the presented method.

3.1 Design of the prefilters $D_1(z_1)$ and $D_2(z_2)$

By (7) and (8), the prefilters $D_1(z_1)$ and $D_2(z_2)$ are used as differentiators with magnitudes $-\omega_1$ and $-\omega_2$, respectively, and their specifications depend on the magnitude response $M(\omega_1,\omega_2)$ in (5). For example, when the designed filter is an elliptically low-pass VFD filter with

$$M(\omega_1,\omega_2)=\begin{cases}1, & \frac{\omega_1^2}{\omega_{p1}^2}+\frac{\omega_2^2}{\omega_{p2}^2}\le 1,\\ 0, & \frac{\omega_1^2}{\omega_{s1}^2}+\frac{\omega_2^2}{\omega_{s2}^2}\ge 1,\end{cases} \tag{13}$$

the prefilters $D_1(z_1)$ and $D_2(z_2)$ are designed with passband edges ω_{p1} and ω_{p2}, respectively, while their stopband edges are ω_{s1} and ω_{s2}, respectively.

Defining

$$\mathbf{d}_i=\left[\hat{d}_i(1),\ \hat{d}_i(2),\ldots,\hat{d}_i\left(\frac{N_{di}}{2}\right)\right]^T, \tag{14a}$$

$$\mathbf{s}_i(\omega_i)=\left[\sin(\omega_i),\ \sin(2\omega_i),\ldots,\sin\left(\frac{N_{di}}{2}\omega_i\right)\right]^T, \tag{14b}$$

the magnitude responses $\hat{D}_i(\omega_i)$ of the prefilters can be represented by

$$\hat{D}_i(\omega_i)=\mathbf{d}_i^T\mathbf{s}_i(\omega_i),\quad i=1,2 \tag{15}$$

where the superscript T denotes a transpose operator. Hence, the objective error functions for designing the prefilters in least-squares sense can be defined by

$$\begin{aligned}e(\mathbf{d}_i)&=\int_0^{\omega_{pi}}\left[-\omega_i-\hat{D}_i(\omega_i)\right]^2 d\omega_i+\int_{\omega_{si}}^{\pi}\left[\hat{D}_i(\omega_i)\right]^2 d\omega_i\\&=u_i+\mathbf{r}_i^T\mathbf{d}_i+\mathbf{d}_i^T\mathbf{Q}_i\mathbf{d}_i\end{aligned} \tag{16}$$

where

$$u_i=\int_0^{\omega_{pi}}\omega_i^2\,d\omega_i=\frac{\omega_{pi}^2}{3}, \tag{17a}$$

$$\mathbf{r}_i=2\int_0^{\omega_{pi}}\omega_i\mathbf{s}_i(\omega_i)\,d\omega_i, \tag{17b}$$

$$\mathbf{Q}_i=\int_0^{\omega_{pi}}\mathbf{s}_i(\omega_i)\mathbf{s}_i^T(\omega_i)\,d\omega_i+\int_{\omega_{si}}^{\pi}\mathbf{s}_i(\omega_i)\mathbf{s}_i^T(\omega_i)\,d\omega_i, \tag{17c}$$

and the solutions are

$$\mathbf{d}_i=-\frac{1}{2}\mathbf{Q}_i^{-1}\mathbf{r}_i,\quad i=1,2. \tag{18}$$

3.2 Design of the subfilters $G_{2m_1,2m_2}(z_1, z_2)$

Similarly, by defining

$$\mathbf{g} = \left[\hat{g}_{00}(0,0), \ldots, \hat{g}_{00}\left(\frac{N_g}{2}, \frac{N_g}{2}\right), \ldots, \hat{g}_{\hat{M}\hat{M}}(0,0), \ldots, \hat{g}_{\hat{M}\hat{M}}\left(\frac{N_g}{2}, \frac{N_g}{2}\right)\right]^T, \tag{19a}$$

$$\mathbf{c}_{ee} = \left[1, \ldots, \cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right), \ldots, p_1^{2\hat{M}} p_2^{2\hat{M}}, \ldots, p_1^{2\hat{M}} p_2^{2\hat{M}} \cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right)\right]^T, \tag{19b}$$

$$\mathbf{c}_{oe} = \left[p_1, \ldots, p_1\cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right), \ldots, \frac{1}{M} p_1^{M} p_2^{2\hat{M}}, \ldots, \frac{1}{M} p_1^{M} p_2^{2\hat{M}} \cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right)\right]^T, \tag{19c}$$

$$\mathbf{c}_{eo} = \left[p_2, \ldots, p_2\cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right), \ldots, \frac{1}{M} p_1^{2\hat{M}} p_2^{M}, \ldots, \frac{1}{M} p_1^{2\hat{M}} p_2^{M} \cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right)\right]^T, \tag{19d}$$

$$\mathbf{c}_{oo} = \left[p_1 p_2, \ldots, p_1 p_2\cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right), \ldots, \frac{1}{M^2} p_1^{M} p_2^{M}, \ldots, \frac{1}{M^2} p_1^{M} p_2^{M} \cos\left(\frac{N_g}{2}\omega_1\right)\cos\left(\frac{N_g}{2}\omega_2\right)\right]^T, \tag{19e}$$

(12a) can be represented by

$$\hat{H}(\omega_1, \omega_2, p_1, p_2) = \mathbf{g}^T\mathbf{c}_{ee} + j\hat{D}_1(\omega_1)\,\mathbf{g}^T\mathbf{c}_{oe} + j\hat{D}_2(\omega_2)\,\mathbf{g}^T\mathbf{c}_{eo} - \hat{D}_1(\omega_1)\,\hat{D}_2(\omega_2)\,\mathbf{g}^T\mathbf{c}_{oo}. \tag{20}$$

Therefore, the objective error function for designing the subfilters $G_{2m_1,2m_2}(z_1, z_2)$ can be defined by

$$\begin{aligned} e(\mathbf{g}) &= \int_R \left|H_d(\omega_1, \omega_2, p_1, p_2) - H\left(e^{j\omega_1}, e^{j\omega_2}, p_1, p_2\right)\right|^2 d\boldsymbol{v} \\ &= \int_R \left|M(\omega_1, \omega_2)\, e^{-j(\omega_1 p_1 + \omega_2 p_2)} - \hat{H}(\omega_1, \omega_2, p_1, p_2)\right|^2 d\boldsymbol{v} \\ &= \int_R \left|M(\omega_1, \omega_2)\cos(\omega_1 p_1 + \omega_2 p_2) - \mathbf{g}^T\mathbf{c}_{ee} + \hat{D}_1(\omega_1)\,\hat{D}_2(\omega_2)\,\mathbf{g}^T\mathbf{c}_{oo}\right|^2 d\boldsymbol{v} \\ &\quad + \int_R \left|-M(\omega_1, \omega_2)\sin(\omega_1 p_1 + \omega_2 p_2) - \hat{D}_1(\omega_1)\,\mathbf{g}^T\mathbf{c}_{oe} - \hat{D}_2(\omega_2)\,\mathbf{g}^T\mathbf{c}_{eo}\right|^2 d\boldsymbol{v} \\ &= u + \mathbf{r}^T\mathbf{g} + \mathbf{g}^T\mathbf{Q}\mathbf{g} \end{aligned} \tag{21}$$

where

$$\int_R \triangleq \iint_{R_p} \iint_{R_\omega}, \tag{22a}$$

$$\boldsymbol{dv} \triangleq d\omega_1 d\omega_2 dp_1 dp_2, \tag{22b}$$

$$R = R_p \cup R_\omega = \{-0.5 \le p_1, p_2 \le 0.5\} \cup \{(\omega_1, \omega_2) \in \text{passbands or } (\omega_1, \omega_2) \in \text{stopbands}\} \tag{22c}$$

and

$$\begin{aligned} u &= \int_R |M(\omega_1, \omega_2) \cos(\omega_1 p_1 + \omega_2 p_2)|^2 \boldsymbol{dv} + \int_R |M(\omega_1, \omega_2) \sin(\omega_1 p_1 + \omega_2 p_2)|^2 \boldsymbol{dv} \\ &= \int_R |M(\omega_1, \omega_2)|^2 \boldsymbol{dv} \\ &= \iint_{R_\omega} |M(\omega_1, \omega_2)|^2 d\omega_1 d\omega_2, \end{aligned} \tag{23a}$$

$$\begin{aligned} \mathbf{r} = &-2 \int_R M(\omega_1, \omega_2) \cos(\omega_1 p_1 + \omega_2 p_2) \left[\mathbf{c}_{ee} - \hat{D}_1(\omega_1) \hat{D}_2(\omega_2) \mathbf{c}_{oo}\right] \boldsymbol{dv} \\ &+ 2 \int_R M(\omega_1, \omega_2) \sin(\omega_1 p_1 + \omega_2 p_2) \left[\hat{D}_1(\omega_1) \mathbf{c}_{oe} + \hat{D}_2(\omega_2) \mathbf{c}_{eo}\right] \boldsymbol{dv}, \end{aligned} \tag{23b}$$

$$\begin{aligned} \mathbf{Q} = &\int_R \left[\mathbf{c}_{ee} - \hat{D}_1(\omega_1) \hat{D}_2(\omega_2) \mathbf{c}_{oo}\right] \left[\mathbf{c}_{ee} - \hat{D}_1(\omega_1) \hat{D}_2(\omega_2) \mathbf{c}_{oo}\right]^T \boldsymbol{dv} \\ &+ \int_R \left[\hat{D}_1(\omega_1) \mathbf{c}_{oe} + \hat{D}_2(\omega_2) \mathbf{c}_{eo}\right] \left[\hat{D}_1(\omega_1) \mathbf{c}_{oe} + \hat{D}_2(\omega_2) \mathbf{c}_{eo}\right]^T \boldsymbol{dv}. \end{aligned} \tag{23c}$$

The least-squares solution can be obtained by differentiating (21) with respect to the coefficient vector $\mathbf{g}$ and setting the result to zero, which yields

$$\mathbf{g} = -\frac{1}{2} \mathbf{Q}^{-1} \mathbf{r}. \tag{24}$$

3.3 Design examples

In this subsection, design example is presented and the results are compared with those of the conventional method (Shyu et al. 2009a). To evaluate the performance, several measured criterions are defined as below:

$$\varepsilon_{m,rms} = \left[\frac{\int_R \left|H_d(\omega_1, \omega_2, p_1, p_2) - H\left(e^{j\omega_1}, e^{j\omega_2}, p_1, p_2\right)\right|^2 \boldsymbol{dv}}{\int_R |H_d(\omega_1, \omega_2, p_1, p_2)|^2 \boldsymbol{dv}}\right]^{1/2} \times 100\,\%, \tag{25a}$$

$$\begin{aligned} \varepsilon_{mp} = \max \Big\{ &\left|H_d(\omega_1, \omega_2, p_1, p_2) - H\left(e^{j\omega_1}, e^{j\omega_2}, p_1, p_2\right)\right|, \\ &(\omega_1, \omega_2) \in \text{passbands}, -0.5 \le p_1, p_2 \le 0.5 \Big\} \end{aligned} \tag{25b}$$

$$\varepsilon_{ms} = \max\left\{\left|H_d(\omega_1,\omega_2,p_1,p_2) - H\left(e^{j\omega_1},e^{j\omega_2},p_1,p_2\right)\right|,\right.$$
$$\left.(\omega_1,\omega_2)\in \text{stopbands}, -0.5\le p_1,p_2\le 0.5\right\} \tag{25c}$$

$$\varepsilon_{\tau_1,rms} = \left[\frac{\int_R |\tau_{d1}(\omega_1,\omega_2,p_1,p_2)-\tau_1(\omega_1,\omega_2,p_1,p_2)|^2 \boldsymbol{dv}}{\int_R p_1^2 \boldsymbol{dv}}\right]^{1/2}\times 100\,\%, \tag{25d}$$

$$\varepsilon_{\tau_2,rms} = \left[\frac{\int_R |\tau_{d2}(\omega_1,\omega_2,p_1,p_2)-\tau_2(\omega_1,\omega_2,p_1,p_2)|^2 \boldsymbol{dv}}{\int_R p_2^2 \boldsymbol{dv}}\right]^{1/2}\times 100\,\%, \tag{25e}$$

$$\varepsilon_{\tau_1} = \max\{|\tau_{d1}(\omega_1,\omega_2,p_1,p_2)-\tau_1(\omega_1,\omega_2,p_1,p_2)|\ (\omega_1,\omega_2)$$
$$\in \text{passbands}, -0.5\le p_1,p_2\le 0.5\}, \tag{25f}$$

$$\varepsilon_{\tau_2} = \max\{|\tau_{d2}(\omega_1,\omega_2,p_1,p_2)-\tau_2(\omega_1,\omega_2,p_1,p_2)|\,(\omega_1,\omega_2)$$
$$\in \text{passbands}, -0.5\le p_1,p_2\le 0.5\} \tag{25g}$$

where $\tau_{di}(\omega_1,\omega_2,p_1,p_2)$ and $\tau_i(\omega_1,\omega_2,p_1,p_2)$ denote the desired and actual group delays, respectively, with respect to ω_i-direction, $i=1,2$. Meanwhile, the numbers of independent coefficients are also taken into account for comparison, which are computed as below:

Proposed method (including scale factors):

$$N_d + \left(\frac{N_g}{2}+1\right)^2\left(\hat{M}+1\right)^2 + 4\hat{M} + 3\hat{M}^2 \tag{26a}$$

Conventional method (Shyu et al. 2009a):

$$\left(\frac{N}{2}+1\right)^2 (M_c+1)^2 + \left(\frac{N}{2}\right)^2 M_s^2 + 2\left(\frac{N}{2}+1\right)\frac{N}{2}(M_c+1)\,M_s \tag{26b}$$

where

$$\begin{cases} M_c = M_s = \frac{M}{2}, & \text{for even } M,\\ M_c+1 = M_s = \frac{M+1}{2}, & \text{for odd } M. \end{cases} \tag{27}$$

To compute the errors in (25), the frequencies ω_1 and ω_2 are uniformly sampled at step size $\pi/100$, and the variable parameters p_1 and p_2 are uniformly sampled at step size 1/50.

Example 1 In this example, an elliptically symmetric low-pass VFD FIR filter is designed and the desired magnitude response has been given in (13). When $\omega_{p1}=0.45\pi$, $\omega_{p2}=0.6\pi$, $\omega_{s1}=0.7\pi$, $\omega_{s2}=0.85\pi$, $N_{d1}=N_{d2}=30$, $N_g=20$, $M=5$, the obtained magnitude responses for $(p_1,p_2)=(0,0)$, $(0.25,0.25)$, $(0.5,0.5)$, $(0.5,-0.5)$ are shown in Fig. 3a, the group-delay responses at $(p_1,p_2)=(0.25,0.25)$ and $(0.5,-0.5)$ are shown in Fig. 3b, c, while the variable group-delay responses and magnitude responses for both $\omega_2=0$, $p_2=0$ and $\omega_1=0$, $p_1=0$ are shown in Fig. 3d, e, respectively. The errors defined in (25) are tabulated in Table 1, accompanying those of the conventional method with $N=20$.

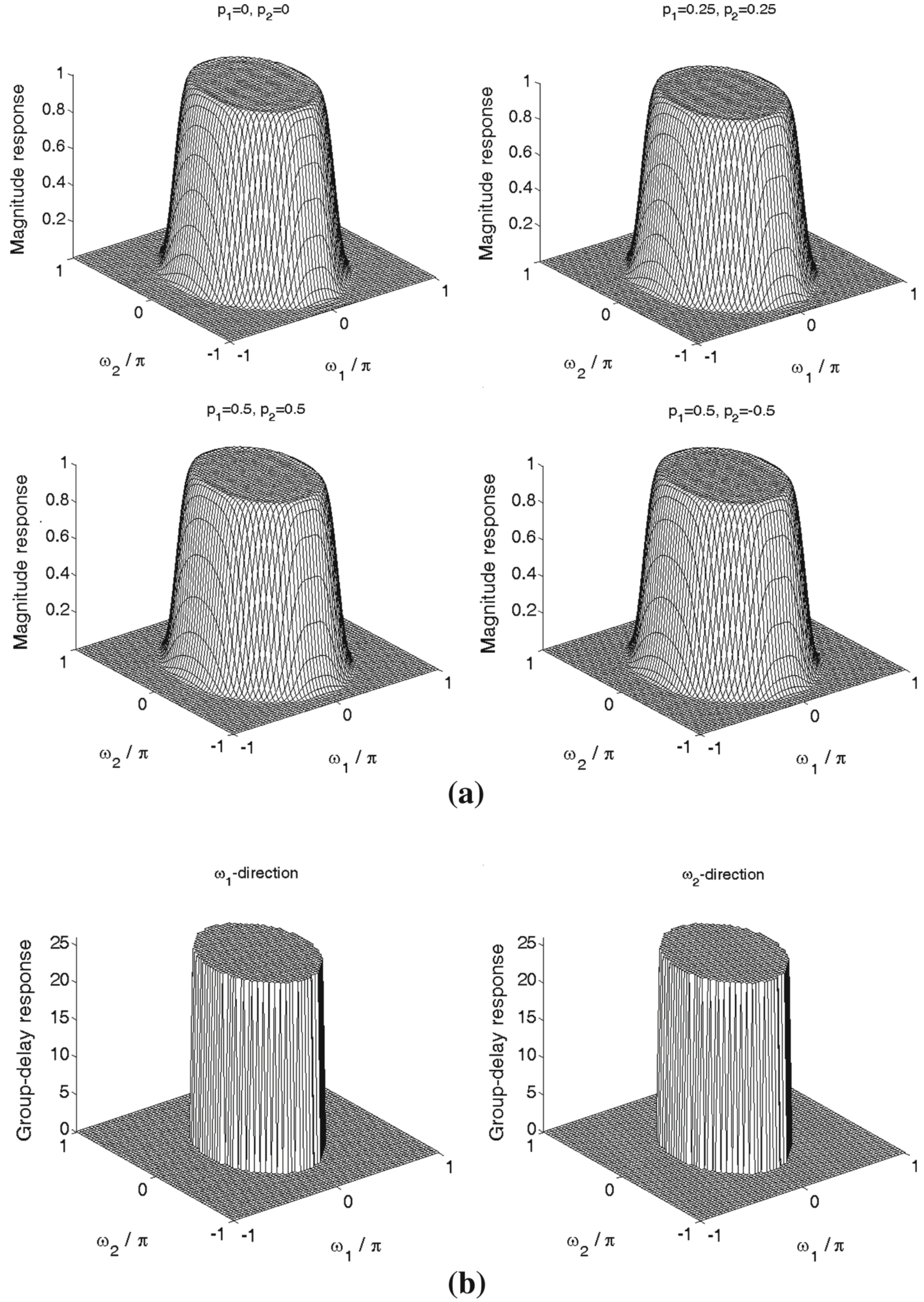

Fig. 3 Design of an elliptically symmetric low-pass VFD FIR filter. **a** Magnitude responses at $(p_1, p_2) = (0, 0), (0.25, 0.25), (0.5, 0.5), (0.5, -0.5)$. **b** ω_1-directional and ω_2-directional group-delay responses in the passband at $(p_1, p_2) = (0.25, 0.25)$. **c** ω_1-directional and ω_2-directional group-delay responses in the passband at $(p_1, p_2) = (0.5, -0.5)$. **d** Variable group-delay response in the passband and magnitude response at $\omega_2 = 0$, $p_2 = 0$. **e** Variable group-delay response in the passband and magnitude response at $\omega_1 = 0$, $p_1 = 0$

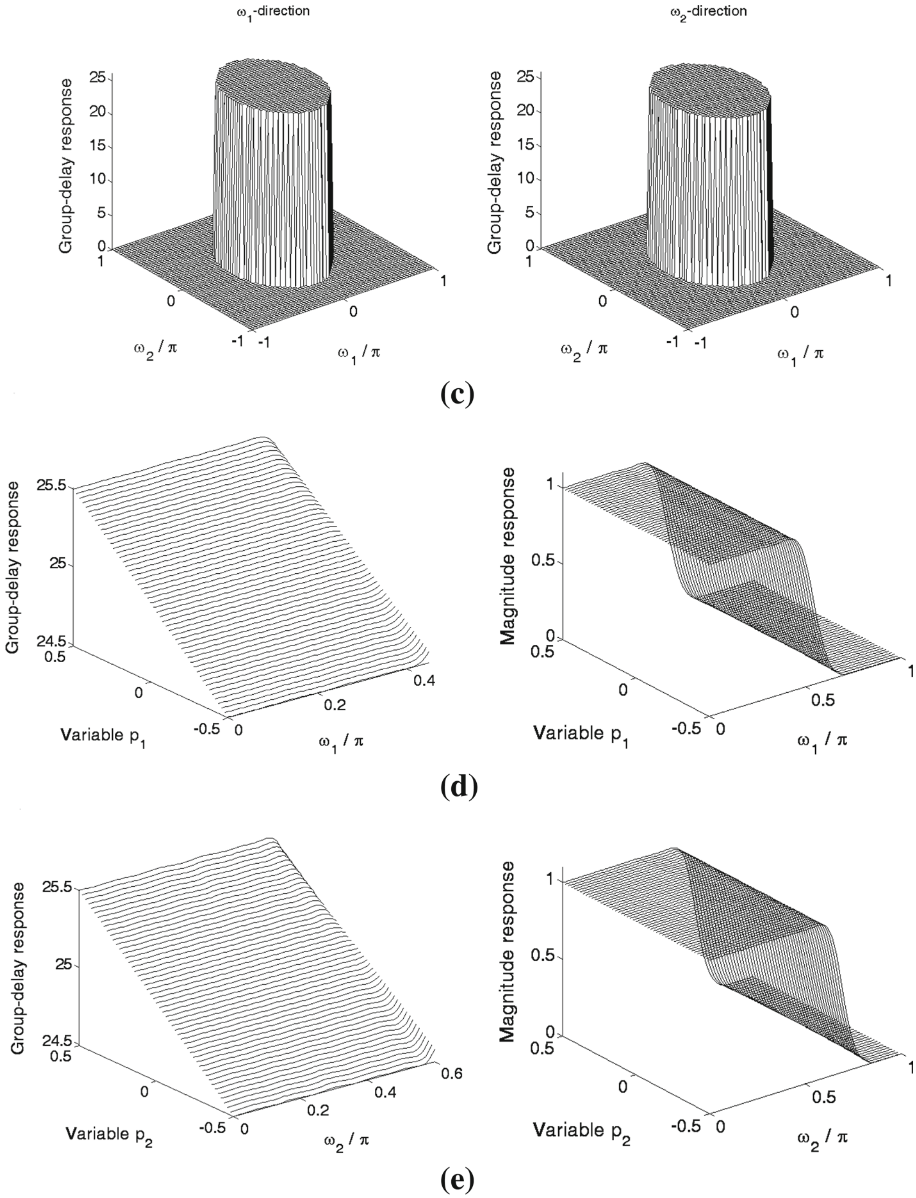

Fig. 3 continued

4 Design of 2-D VFD FIR digital filters with even M

For even M in (6), let $M = 2\hat{M}$ then

$$e^{-j(\omega_1 p_1 + \omega_2 p_2)} \approx \left[\sum_{m_1=0}^{\hat{M}} (-1)^{m_1} \frac{(\omega_1 p_1)^{2m_1}}{(2m_1)!} + (-j\omega_1)\, p_1 \sum_{m_1=0}^{\hat{M}-1} \frac{(-1)^{m_1}}{2m_1+1} \frac{(\omega_1 p_1)^{2m_1}}{(2m_1)!} \right]$$

Table 1 Comparisons for the proposed method and the conventional method (Shyu et al. 2009a)

Example Method	Example 1 Proposed	Conventional	Example 2 Proposed	Conventional
Filter order	$N_{d1} = N_{d2} = 30$ $N_g = 20$	$N = 20$	$N_{d1} = N_{d2} = 30$ $N_g = 16$	$N = 16$
Number of independent coefficients	1,139	3,969	768	1,849
Average delays	ω_1-direction:25 ω_1-direction:25	ω_1-direction:10 ω_1-direction:10	ω_1-direction:23 ω_1-direction:23	ω_1-direction:8 ω_1-direction:8
$\varepsilon_{m,rms}(\%)$	0.21486344	0.24878285	0.67991394	0.7187766
ε_{mp}	0.01013381	0.01140162	0.02642647	0.02685282
ε_{ms}	0.00844768	0.00992308	0.02541064	0.02507116
$\varepsilon_{\tau_1,rms}(\%)$	0.00141645	0.02759631	0.00529623	0.08887992
$\varepsilon_{\tau_2,rms}(\%)$	0.00241521	0.05946121	0.00529623	0.08887992
ε_{τ_1}	0.03280007	0.09599047	0.03507453	0.09010251
ε_{τ_2}	0.03488614	0.12677654	0.03507453	0.09010251

$$
\begin{aligned}
&\times \left[\sum_{m_2=0}^{\hat{M}} (-1)^{m_2} \frac{(\omega_2 p_2)^{2m_2}}{(2m_2)!} + (-j\omega_2)\, p_2 \sum_{m_2=0}^{\hat{M}-1} \frac{(-1)^{m_2}}{2m_2+1} \frac{(\omega_2 p_2)^{2m_2}}{(2m_2)!} \right] \\
&= \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} (-1)^{m_1+m_2} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!} \\
&\quad + (-j\omega_1)\, p_1 \sum_{m_1=0}^{\hat{M}-1} \sum_{m_2=0}^{\hat{M}} \frac{(-1)^{m_1+m_2}}{2m_1+1} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!} \\
&\quad + (-j\omega_2)\, p_2 \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}-1} \frac{(-1)^{m_1+m_2}}{2m_2+1} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!} \\
&\quad + (-j\omega_1)(-j\omega_2)\, p_1 p_2 \sum_{m_1=0}^{\hat{M}-1} \sum_{m_2=0}^{\hat{M}-1} \frac{(-1)^{m_1+m_2}}{(2m_1+1)(2m_2+1)} \frac{(\omega_1 p_1)^{2m_1} (\omega_2 p_2)^{2m_2}}{(2m_1)!\,(2m_2)!}
\end{aligned}
\tag{28}
$$

Hence, the applied transfer function in this section is represented by

$$
\begin{aligned}
H(z_1, z_2, p_1, p_2) &= z_1^{-\frac{N_{d1}}{2}} z_2^{-\frac{N_{d2}}{2}} \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} G_{2m_1,2m_2}(z_1, z_2)\, p_1^{2m_1} p_2^{2m_2} \\
&\quad + z_2^{-\frac{N_{d2}}{2}} D_1(z_1) \sum_{m_1=0}^{\hat{M}-1} \sum_{m_2=0}^{\hat{M}} \frac{1}{2m_1+1} G_{2m_1,2m_2}(z_1, z_2)\, p_1^{2m_1+1} p_2^{2m_2}
\end{aligned}
$$

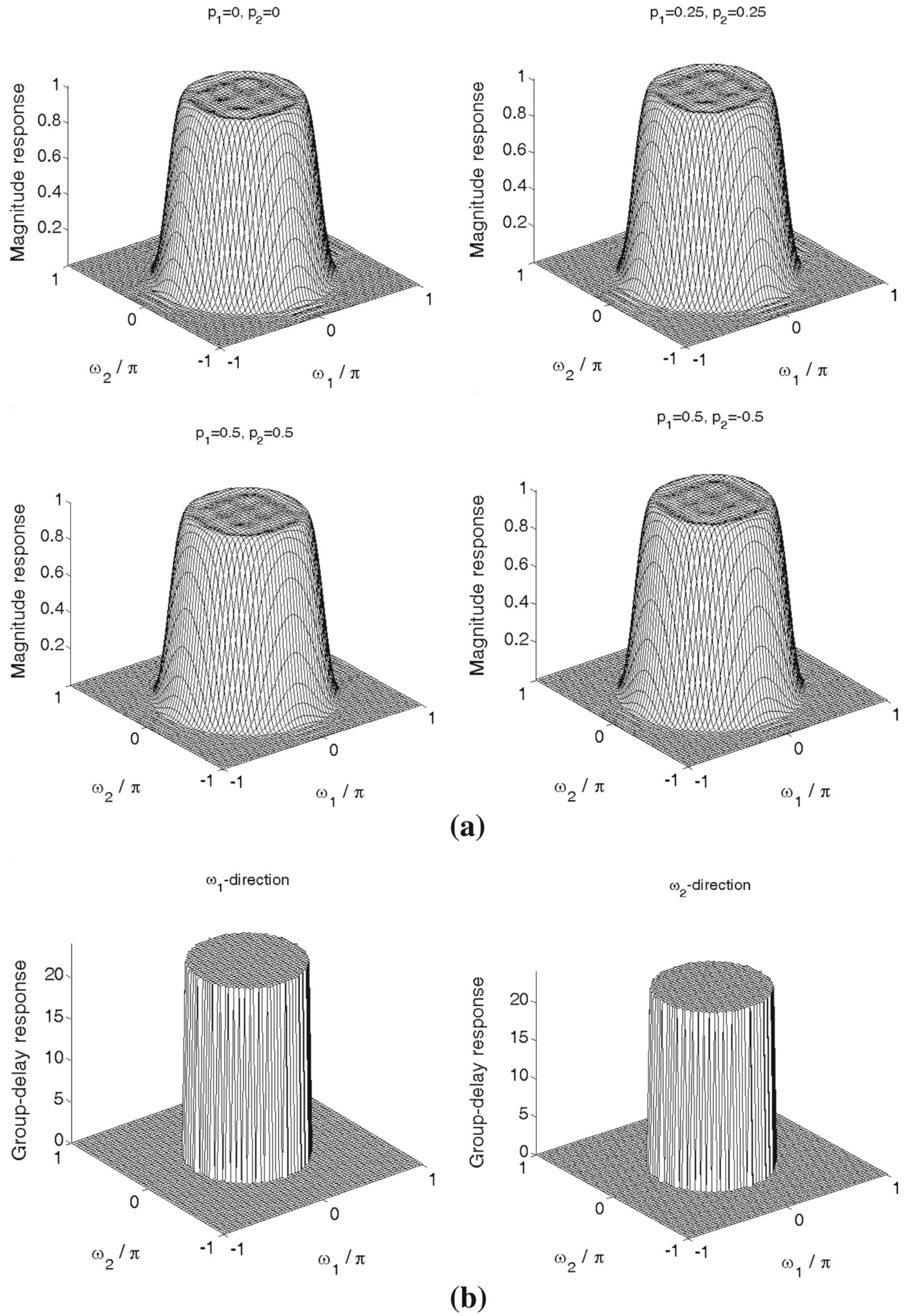

Fig. 4 Design of a circularly symmetric low-pass VFD FIR filter. **a** Magnitude responses at $(p_1, p_2) = (0, 0), (0.25, 0.25), (0.5, 0.5), (0.5, -0.5)$. **b** ω_1-directional and ω_2-directional group-delay responses in the passband at $(p_1, p_2) = (0.25, 0.25)$. **c** ω_1-directional and ω_2-directional group-delay responses in the passband at $(p_1, p_2) = (0.5, -0.5)$. **d** Variable group-delay response in the passband and magnitude response at $\omega_2 = 0$, $p_2 = 0$. **e** Variable group-delay response in the passband and magnitude response at $\omega_1 = 0$, $p_1 = 0$

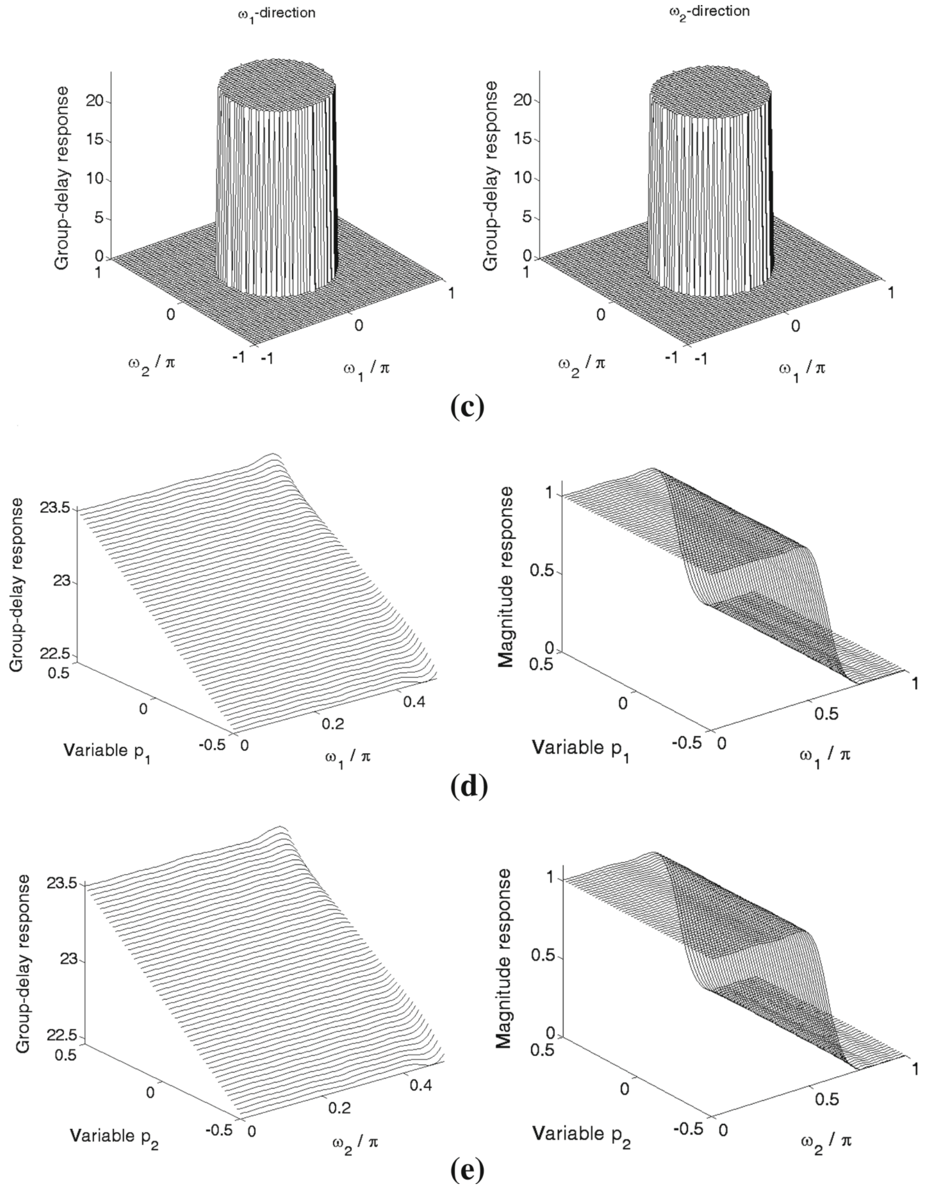

Fig. 4 continued

$$+z_1^{-\frac{N_{d1}}{2}} D_2(z_2) \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}-1} \frac{1}{2m_2+1} G_{2m_1,2m_2}(z_1,z_2)\ p_1^{2m_1} p_2^{2m_1+1}$$
$$+D_1(z_1) D_2(z_2) \times \sum_{m_1=0}^{\hat{M}-1} \sum_{m_2=0}^{\hat{M}-1} \frac{1}{(2m_1+1)(2m_2+1)}$$
$$\times G_{2m_1,2m_2}(z_1,z_2)\ p_1^{2m_1+1} p_2^{2m_1+1} \tag{29}$$

where $G_{2m_1,2m_2}(z_1, z_2)$ and $D_i(z_i)$, $i = 1, 2$ have been characterized in (9) and (10), respectively, and the frequency response of (29) is

$$H\left(e^{j\omega_1}, e^{j\omega_2}, p_1, p_2\right) = e^{-j\left(\frac{N_{d1}}{2}+\frac{N_g}{2}\right)\omega_1} e^{-j\left(\frac{N_{d2}}{2}+\frac{N_g}{2}\right)\omega_2} \hat{H}\left(\omega_1, \omega_2, p_1, p_2\right) \quad (30)$$

where

$$\begin{aligned}\hat{H}\left(\omega_1, \omega_2, p_1, p_2\right) = &\sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}} \hat{G}_{2m_1,2m_2}\left(\omega_1, \omega_2\right) p_1^{2m_1} p_2^{2m_2} \\ &+ j\hat{D}_1\left(\omega_1\right) \sum_{m_1=0}^{\hat{M}-1} \sum_{m_2=0}^{\hat{M}} \frac{1}{2m_1+1} \hat{G}_{2m_1,2m_2}\left(\omega_1, \omega_2\right) p_1^{2m_1+1} p_2^{2m_2} \\ &+ j\hat{D}_2\left(\omega_2\right) \sum_{m_1=0}^{\hat{M}} \sum_{m_2=0}^{\hat{M}-1} \frac{1}{2m_2+1} \hat{G}_{2m_1,2m_2}\left(\omega_1, \omega_2\right) p_1^{2m_1} p_2^{2m_2+1} \\ &- \hat{D}_1\left(\omega_1\right)\hat{D}_2\left(\omega_2\right) \sum_{m_1=0}^{\hat{M}-1} \sum_{m_2=0}^{\hat{M}-1} \frac{1}{\left(2m_1+1\right)\left(2m_2+1\right)} \\ &\times \hat{G}_{2m_1,2m_2}\left(\omega_1, \omega_2\right) p_1^{2m_1+1} p_2^{2m_2+1} \end{aligned} \quad (31)$$

in which $\hat{G}_{2m_1,2m_2}\left(\omega_1, \omega_2\right)$ and $\hat{D}_i\left(\omega_i\right)$, $i = 1, 2$ are the same as (12b) and (12c), respectively. So, the technique in Sect. 3 can also be applied to the design of 2-D VFD FIR filters with even M.

Example 2 This example will deal with the design of a circularly symmetric low-pass VFD FIR filter whose magnitude response is shown in (13) with $\omega_{p1} = \omega_{p2} = \omega_p$ and $\omega_{s1} = \omega_{s2} = \omega_s$. Figure 4a presents the obtained magnitude responses for $(p_1, p_2) = (0, 0)$, $(0.25, 0.25)$, $(0.5, 0.5)$, $(0.5, -0.5)$ if $N_{d1} = N_{d2} = 30$, $N_g = 16$, $M = 4$, $\omega_p = 0.5\pi$, $\omega_s = 0.75\pi$ are used, Fig. 4b, c present the group-delay responses at $(p_1, p_2) = (0.25, 0.25)$ and $(0.5, -0.5)$, and Fig. 4d, e present the variable group-delay responses and magnitude responses for $\omega_2 = 0$, $p_2 = 0$ and $\omega_1 = 0$, $p_1 = 0$, respectively. The error defined in (25) are also tabulated in Table 1, accompanying those of the conventional method with $N = 16$, $M = 4$.

5 Conclusions

In this paper, a prefilter–subfilter cascaded structure for the design of VFD 2-D FIR digital filters has been proposed, which is derived basing on the Taylor series expansion of the desired frequency response. By the specified relationships among the presented structure, it has been shown that the required number of independent coefficients is much less than that of the existing structure, while the performance of the designed filters is still better. Design examples have been presented to demonstrate the effectiveness of the presented method.

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