Multidim Syst Sign Process (2010) 21:255–275
DOI 10.1007/s11045-010-0104-5

Non-fragile H_2 and H_∞ filter designs for polytopic two-dimensional systems in Roesser model

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Received: 10 July 2009 / Revised: 4 February 2010 / Accepted: 17 March 2010 /
Published online: 1 April 2010

Abstract This paper is concerned with the problem of non-fragile H_2 and H_∞ filter designs for two-dimensional (2-D) discrete systems in Roesser model with polytopic uncertainties. The filters to be designed are assumed to be with additive norm-bounded coefficient variations which reflect the imprecision in filter implementation. The complicated filter design problem is successfully tackled by using the slack variable technique and imposing a structural restriction on the slack matrix. Explicit expressions of the non-fragile H_2 and H_∞ filters are given in terms of solutions to a set of linear matrix inequalities (LMIs). An illustrative example is provided to demonstrate the feasibility and effectiveness of the proposed method.

Keywords Linear matrix inequality (LMI) · Non-fragile · Filter design · Two-dimensional systems

1 Introduction

State estimation for dynamic systems with both process and measurement noise inputs has been one of the fundamental issues in engineering applications (Xie et al. 2004; Xu and Van Dooren 2002). The filter determination is carried out by defining a suitable performance index in terms of the state estimation error variance. It is well known that the H_2 filtering approach is useful to handle the error estimation under a stochastic disturbance input with known statistics (Duan et al. 2006), while the H_∞ filtering approach is more suitable for an energy-bounded disturbance input (Xu and Chen 2003, 2004). The past decade has witnessed

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major developments in the robust filtering problem using various approaches, and many efforts have been made in the direction of reducing the conservativeness of the filter analysis and design methods. The parameter-dependent Lyapunov method has been applied to analysis and design of robust filters (Geromel et al. 2002). However, all the above work is based on an implicit assumption that the filter will be implemented exactly. Many sources, such as imprecision in analogue-digital conversion, fixed word length, finite resolution instrumentation, and numerical roundoff errors, will lead to small perturbations in filter parameters. The designed filters can be sensitive to errors in the filter coefficients. Very recently, the non-fragile H_∞ filter design problem has been considered for one-dimensional (1-D) continuous and discrete-time systems (Che and Yang 2008; Mahmoud 2004; Song et al. 2009; Yang and Che 2008; Yang and Wang 2001). In references Mahmoud (2004), Yang and Che (2008), non-fragile H_∞ filters with norm-bounded uncertainty and interval coefficient uncertainty have been designed.

On the other hand, during the past decades, two-dimensional (2-D) discrete systems have received much attention since 2-D systems have extensive applications in image processing, seismographic data processing, thermal processes, water stream heating, modeling of partial differential equations, and other areas (Kaczorek 1985; Lin and Bruton 1989). A great number of fundamental notions and results of 1-D discrete systems were generalized to 2-D discrete systems. Different state-space models, such as the 2-D Roesser model (Roesser 1975), the 2-D Fornasini-Marchesini local state-space model (Fornasini and Marchesini 1976), and the 2-D general state space model (Kurek 1985), etc. were proposed. Recently, robust H_∞ filtering and mixed H_2/H_∞ filtering for 2-D systems described by Roesser model and Fornasini-Marchesini model have been studied (see, e.g. Du et al. (2000), Gao et al. (2004, 2008a,b), Liu et al. (2009), Tuan et al. (2002), Xu et al. (2005), Yang et al. (2006), and the references therein). However, the problem of non-fragile filtering for 2-D systems has not been investigated to date due to the complexity involved.

In this paper, we study the new problem of non-fragile H_2 and H_∞ filter designs for 2-D discrete systems in Roesser model with polytopic uncertainties. The filters to be designed are assumed to be with additive norm-bounded coefficient variations which reflect the imprecision in filter implementation. We formulate the filter design problem in terms of linear matrix inequalities (LMIs) and solve it using the slack matrix with structural restriction. Furthermore, we also give an alternative method for designing H_2 and H_∞ filters for 2-D systems in Roesser model.

The organization of the paper is as follows. We describe the non-fragile filtering problem in Sect. 2. The analysis on the LMI characterizations for H_2 and H_∞ norm of the estimation error system is presented in Sect. 3. Section 4 gives the design methods of non-fragile H_2 and H_∞ filters. In Sect. 5, an example is provided to demonstrate the applicability and effectiveness of the proposed method. Finally, we give the conclusions in Sect. 6.

Notation. Throughout this paper, for Hermitian matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix of appropriate dimension. For square matrices, $trace(X)$ denotes the trace function of X being equal to the sum of its diagonal elements. The superscript "T" represents the transpose. For the sake of easing the notation of partitioned symmetric matrices, we use an asterisk ($*$) to represent each of its symmetric blocks. $diag\{...\}$ stands for a block-diagonal matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 Problem formulation

Consider the following 2-D discrete-time system described by the Roesser model (Roesser 1975):

$$(\Sigma): \begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = A(\lambda) \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + B(\lambda)\omega(i, j), \tag{1}$$

$$y(i, j) = C(\lambda) \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + D(\lambda)\omega(i, j), \tag{2}$$

$$z(i, j) = H(\lambda) \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix}. \tag{3}$$

The boundary condition is assumed to be

$$x^h(0, j) = 0, \quad \text{for } j = 0, 1, 2, \ldots, \tag{4}$$

$$x^v(i, 0) = 0, \quad \text{for } i = 0, 1, 2, \ldots, \tag{5}$$

where $x^h(i, j) \in \mathbf{R}^{n_h}$, $x^v(i, j) \in \mathbf{R}^{n_v}$ $(n_h + n_v = n)$ are the horizontal state and the vertical state, respectively; $y(i, j) \in \mathbf{R}^l$ is the measured output; $z(i, j) \in \mathbf{R}^q$ is the signal to be estimated; $\omega(i, j) \in \mathbf{R}^p$ is the disturbance input which is assumed to be energy bounded (i.e. belongs to $l_2\{[0, \infty), [0, \infty)\}$) or white noise process. The system matrices $A(\lambda)$, $B(\lambda)$, $C(\lambda)$, $D(\lambda)$ and $H(\lambda)$ are assumed to be unknown (uncertain) but belonging to a known convex compact set of polytopic type, i.e.

$$\Lambda(\lambda) \triangleq \{A(\lambda), B(\lambda), C(\lambda), D(\lambda), H(\lambda)\} \in \Im, \tag{6}$$

where $\Im$ is a given convex bounded polyhedral domain described by s vertices

$$\Im \triangleq \left\{ \Lambda(\lambda) | \Lambda(\lambda) = \sum_{k=1}^{s} \lambda_k \Lambda_k;\ \sum_{k=1}^{s} \lambda_k = 1, \lambda_k \geq 0 \right\}, \tag{7}$$

with $\Lambda_k \triangleq \{A^{(k)}, B^{(k)}, C^{(k)}, D^{(k)}, H^{(k)}\}$ denoting the vertices of the polytope. The objective here is to design a filter with additive coefficient variations of the form

$$(\Sigma_f): \begin{bmatrix} x_f^h(i+1, j) \\ x_f^v(i, j+1) \end{bmatrix} = (A_f + \Delta A_f) \begin{bmatrix} x_f^h(i, j) \\ x_f^v(i, j) \end{bmatrix} + (B_f + \Delta B_f) y(i, j), \tag{8}$$

$$z_f(i, j) = (H_f + \Delta H_f) \begin{bmatrix} x_f^h(i, j) \\ x_f^v(i, j) \end{bmatrix} + (L_f + \Delta L_f) y(i, j), \tag{9}$$

$$x_f^h(0, j) = 0, \quad \text{for } j = 0, 1, 2, \ldots, \tag{10}$$

$$x_f^v(i, 0) = 0, \quad \text{for } i = 0, 1, 2, \ldots, \tag{11}$$

where $x_f^h(i, j) \in \mathbf{R}^{n_h}$, $x_f^v(i, j) \in \mathbf{R}^{n_v}$ are the filter states, and $z_f(i, j)$ is an estimation of $z(i, j)$.A_f, B_f, H_f and L_f are filter matrices of appropriate dimensions to be determined. ΔA_f, ΔB_f, ΔH_f and ΔL_f are real-valued time-varying matrix functions representing norm-bounded type of the additive coefficient variation of the following form

$$\begin{bmatrix} \Delta A_f & \Delta B_f \\ \Delta H_f & \Delta L_f \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \Upsilon(i, j) \begin{bmatrix} F_1 & F_2 \end{bmatrix}, \tag{12}$$

with $\Upsilon(i, j)$ being a real uncertain matrix function satisfying

$$\Upsilon(i,j)^T\Upsilon(i,j) \le I, \tag{13}$$

and E_1, E_2, F_1 and F_2 being known real constant matrices of appropriate dimensions.

Let

$$\bar{x}^h(i,j) = \begin{bmatrix} x^h(i,j) \\ x_f^h(i,j) \end{bmatrix}, \quad \bar{x}^v(i,j) = \begin{bmatrix} x^v(i,j) \\ x_f^v(i,j) \end{bmatrix}, \tag{14}$$

and

$$\bar{z}(i,j) = z(i,j) - z_f(i,j). \tag{15}$$

Then, we can obtain the estimation error system as follows

$$(\bar{\Sigma}): \begin{bmatrix} \bar{x}^h(i+1,j) \\ \bar{x}^v(i,j+1) \end{bmatrix} = \bar{A}(\lambda)\begin{bmatrix} \bar{x}^h(i,j) \\ \bar{x}^v(i,j) \end{bmatrix} + \bar{B}(\lambda)\omega(i,j), \tag{16}$$

$$\bar{z}(i,j) = \bar{H}(\lambda)\begin{bmatrix} \bar{x}^h(i,j) \\ \bar{x}^v(i,j) \end{bmatrix} + \bar{L}(\lambda)\omega(i,j), \tag{17}$$

where

$$\bar{A}(\lambda) = \Omega\tilde{A}(\lambda)\Omega^T, \ \bar{B}(\lambda) = \Omega\tilde{B}(\lambda), \ \bar{H}(\lambda) = \tilde{H}(\lambda)\Omega^T, \ \bar{L}(\lambda) = -(L_f + \Delta L_f)D(\lambda), \tag{18}$$

$$\Omega = \begin{bmatrix} I_{n_h} & 0 & 0 & 0 \\ 0 & 0 & I_{n_h} & 0 \\ 0 & I_{n_v} & 0 & 0 \\ 0 & 0 & 0 & I_{n_v} \end{bmatrix}, \quad \tilde{A}(\lambda) = \begin{bmatrix} A(\lambda) & 0 \\ (B_f + \Delta B_f)C(\lambda) & A_f + \Delta A_f \end{bmatrix}, \tag{19}$$

$$\tilde{B}(\lambda) = \begin{bmatrix} B(\lambda) \\ (B_f + \Delta B_f)D(\lambda) \end{bmatrix}, \quad \tilde{H}(\lambda) = \begin{bmatrix} H(\lambda) - (L_f + \Delta L_f)C(\lambda) & -(H_f + \Delta H_f) \end{bmatrix}. \tag{20}$$

Let $\|G(\lambda)\|_2$ and $\|G(\lambda)\|_\infty$ denote the H_2 norm and H_∞ norm of the estimation error system $(\bar{\Sigma})$ that maps the input $\omega(i,j)$ to the error $\bar{z}(i,j)$, respectively. Then, the non-fragile filtering problem to be addressed in this paper can be expressed as follows.

Non-fragile H_2 filtering: Assume that the noise input $\omega(i,j)$ is a white noise with unit variance. Given a 2-D system (Σ) and $\gamma_2 > 0$, design a filter described by (Σ_f) with the additive coefficient variation of the form (12) and (13), such that the estimation error system $(\bar{\Sigma})$ is robustly asymptotically stable and $\|G(\lambda)\|_2 < \gamma_2$, for all $\lambda \in \Im$.

Non-fragile H_∞ filtering: Assume that the noise input $\omega(i,j)$ belongs to $l_2\{[0,\infty), [0,\infty)\}$. Given a 2-D system (Σ) and $\gamma_\infty > 0$, design a filter described by (Σ_f) with the additive coefficient variation of the form (12) and (13), such that the estimation error system $(\bar{\Sigma})$ is robustly asymptotically stable and $\|G(\lambda)\|_\infty < \gamma_\infty$, for all $\lambda \in \Im$.

3 LMI Characterizations for H_2 and H_∞ norms

To facilitate the derivation of our main results, we characterize the H_2 and H_∞ norms in terms of some new LMIs in this section. For simplicity and convenience, we first consider the standard 2-D system (Σ_0) by replacing the system matrices $A(\lambda)$, $B(\lambda)$, $C(\lambda)$, $D(\lambda)$ and $H(\lambda)$ in 2-D system (Σ) with A, B, C, D and H which are assumed to be exactly known.

The corresponding estimation error system $(\bar{\Sigma})$ can be rewritten as

$$(\bar{\Sigma}_0): \begin{bmatrix} \bar{x}^h(i+1,j) \\ \bar{x}^v(i,j+1) \end{bmatrix} = \bar{A} \begin{bmatrix} \bar{x}^h(i,j) \\ \bar{x}^v(i,j) \end{bmatrix} + \bar{B}\omega(i,j),$$
$$\bar{z}(i,j) = \bar{H} \begin{bmatrix} \bar{x}^h(i,j) \\ \bar{x}^v(i,j) \end{bmatrix} + \bar{L}\omega(i,j),$$

where

$$\bar{A} = \Omega\tilde{A}\Omega^T, \quad \bar{B} = \Omega\tilde{B}, \quad \bar{H} = \tilde{H}\Omega^T, \quad \bar{L} = -(L_f + \Delta L_f)D, \tag{21}$$

$$\tilde{A} = \begin{bmatrix} A & 0 \\ (B_f + \Delta B_f)C & A_f + \Delta A_f \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ (B_f + \Delta B_f)D \end{bmatrix}, \tag{22}$$

$$\tilde{H} = \begin{bmatrix} H - (L_f + \Delta L_f)C & -(H_f + \Delta H_f) \end{bmatrix}. \tag{23}$$

Following section II, let $\|G\|_2$ and $\|G\|_\infty$ denote the H_2 norm and H_∞ norm of the estimation error system $(\bar{\Sigma}_0)$, respectively. We first recall the following results.

Lemma 1 (Yang et al. 2006) *Given a scalar $\gamma_2 > 0$, a 2-D system $(\bar{\Sigma}_0)$ is asymptotically stable and $\|G\|_2 < \gamma_2$ if there exists a matrix $P = diag\{P_h, P_v\} > 0$ with $P_h \in \boldsymbol{R}^{2n_h \times 2n_h}$ and $P_v \in \boldsymbol{R}^{2n_v \times 2n_v}$, such that*

$$\bar{A}^T P\bar{A} + \bar{H}^T\bar{H} - P < 0, \tag{24}$$

$$trace\left(\bar{B}^T P\bar{B} + \bar{L}^T\bar{L}\right) < \gamma_2^2. \tag{25}$$

Lemma 2 (Du and Xie 2002) *Given a scalar $\gamma_\infty > 0$, a 2-D system $(\bar{\Sigma}_0)$ is asymptotically stable and $\|G\|_\infty < \gamma_\infty$ if there exists a matrix $P = diag\{P_h, P_v\} > 0$ with $P_h \in \boldsymbol{R}^{2n_h \times 2n_h}$ and $P_v \in \boldsymbol{R}^{2n_v \times 2n_v}$, such that*

$$\bar{A}^T P\bar{A} - P + (\bar{A}^T P\bar{B} + \bar{H}^T\bar{L})[\gamma_\infty^2 I - (\bar{B}^T P\bar{B} + \bar{L}^T\bar{L})]^{-1}(\bar{B}^T P\bar{A} + \bar{L}^T\bar{H}) + \bar{H}^T\bar{H} < 0, \tag{26}$$

$$\gamma_\infty^2 I - (\bar{B}^T P\bar{B} + \bar{L}^T\bar{L}) > 0. \tag{27}$$

It is well-known that the above two lemmas present basic LMI characterizations of the H_2 and H_∞ norms. To facilitate the design of non-fragile H_2 and H_∞ filters for polytopic 2-D systems, we introduce the following improved versions of Lemmas 1 and 2 by using the slack variable technique (Duan et al. 2006; Xie et al. 2004).

Lemma 3 *Given a scalar $\gamma_2 > 0$, a 2-D system $(\bar{\Sigma}_0)$ is asymptotically stable and $\|G\|_2 < \gamma_2$ if there exist a matrix $P = diag\{P_h, P_v\} > 0$ with $P_h \in \boldsymbol{R}^{2n_h \times 2n_h}$, $P_v \in \boldsymbol{R}^{2n_v \times 2n_v}$, $\Pi > 0$ and a matrix $\Phi = diag\{\Phi_h, \Phi_v\}$ with $\Phi_h \in \boldsymbol{R}^{2n_h \times 2n_h}$ and $\Phi_v \in \boldsymbol{R}^{2n_v \times 2n_v}$ satisfying*

$$\begin{bmatrix} P - \Phi - \Phi^T & 0 & \Phi^T\bar{A} \\ * & -I & \bar{H} \\ * & * & -P \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & P - \Phi - \Phi^T & \Phi^T\bar{B} \\ * & * & -\Pi \end{bmatrix} < 0, \tag{29}$$

$$trace\,(\Pi) < \gamma_2^2. \tag{30}$$

Proof By the Schur complement formula, matrix inequalities (24) and (25) are equivalent to

$$\begin{bmatrix} -P & 0 & P\bar{A} \\ * & -I & \bar{H} \\ * & * & -P \end{bmatrix} < 0, \tag{31}$$

and

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & -P & P\bar{B} \\ * & * & -\Pi \end{bmatrix} < 0, \tag{32}$$

$$trace\ (\Pi) < \gamma_2^2, \tag{33}$$

respectively. It suffices to prove that matrix inequalities (31) and (32) are equivalent to matrix inequalities (28) and (29). Assume that matrix inequalities (31) and (32) hold with matrices $P = diag\{P_h, P_v\} > 0$ and $\Pi > 0$. Then matrix inequalities (28) and (29) hold by choosing $\Phi = \Phi^T = P$.

On the other hand, assume that matrix inequalities (28) and (29) hold with matrices $P = diag\{P_h, P_v\} > 0$ and $\Phi = diag\{\Phi_h, \Phi_v\}$. It is easy to see that $P - \Phi - \Phi^T < 0$, so Φ is nonsingular. Note that $(P-\Phi^T)P^{-1}(P-\Phi) \geq 0$, which implies $-\Phi^T P^{-1}\Phi \leq P-\Phi-\Phi^T$. Hence, from (28) and (29), we have

$$\begin{bmatrix} -\Phi^T P^{-1}\Phi & 0 & \Phi^T\bar{A} \\ * & -I & \bar{H} \\ * & * & -P \end{bmatrix} < 0, \tag{34}$$

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & -\Phi^T P^{-1}\Phi & \Phi^T\bar{B} \\ * & * & -\Pi \end{bmatrix} < 0. \tag{35}$$

Pre- and post-multiplying (34) by $diag\{\Phi^{-1}P,\ I,\ I\}^T$ and $diag\{\Phi^{-1}P,\ I,\ I\}$, respectively; and (35) by $diag\{I,\ \Phi^{-1}P,\ I\}^T$ and $diag\{I,\ \Phi^{-1}P,\ I\}$, respectively, we can obtain matrix inequalities (31) and (32). This completes the proof. □

Similar to the proof of Lemma 3, it is easy to obtain the following result.

Lemma 4 *Given a scalar $\gamma_\infty > 0$, a 2-D system $(\bar{\Sigma}_0)$ is asymptotically stable and $\|G\|_\infty < \gamma_\infty$ if there exist a matrix $\hat{P} = diag\{\hat{P}_h, \hat{P}_v\} > 0$ with $\hat{P}_h \in \boldsymbol{R}^{2n_h\times 2n_h}$, $\hat{P}_v \in \boldsymbol{R}^{2n_v\times 2n_v}$, and a matrix $\hat{\Phi} = diag\{\hat{\Phi}_h,\ \hat{\Phi}_v\}$ with $\hat{\Phi}_h \in \boldsymbol{R}^{2n_h\times 2n_h}$ and $\hat{\Phi}_v \in \boldsymbol{R}^{2n_v\times 2n_v}$ satisfying*

$$\begin{bmatrix} \hat{P} - \hat{\Phi} - \hat{\Phi}^T & 0 & \hat{\Phi}^T\bar{A} & \hat{\Phi}^T\bar{B} \\ * & -I & \bar{H} & \bar{L} \\ * & * & -\hat{P} & 0 \\ * & * & * & -\gamma_\infty^2 I \end{bmatrix} < 0. \tag{36}$$

It is well-known that by using the slack variable technique, the above two lemmas, in general, will render a less conservative evaluation of the upper-bound of the H_2 and H_∞ norms for a polytopic 2-D system.

Next, we give a new characterization of the H_2 performance by imposing a structural restriction on the slack matrix.

Lemma 5 *Given a scalar $\gamma_2 > 0$, a 2-D system $(\bar{\Sigma}_0)$ is asymptotically stable and $\|G\|_2 < \gamma_2$, if there exist a nonsingular matrix $U = diag\{U_h, U_v\}$ with $U_h \in \boldsymbol{R}^{n_h\times n_h}$ and $U_v \in$*

$\boldsymbol{R}^{n_v \times n_v}$, *and* $Q = diag\{Q_h, Q_v\} > 0$ *with* $Q_h \in \boldsymbol{R}^{2n_h \times 2n_h}$ *and* $Q_v \in \boldsymbol{R}^{2n_v \times 2n_v}$, $\Pi > 0$ *and a matrix* Ψ *with structure*

$$\Psi^T = \Omega \begin{bmatrix} X_1 & Y \\ X_2 & Y \end{bmatrix} \Omega^T, \tag{37}$$

with $X_1 = diag\{X_{1h}, X_{1v}\}$, $X_2 = diag\{X_{2h}, X_{2v}\}$, $Y = diag\{Y_h, Y_v\}$, *and* $X_{1h},\ X_{2h},\ Y_h \in \boldsymbol{R}^{n_h \times n_h}$, $X_{1v},\ X_{2v},\ Y_v \in \boldsymbol{R}^{n_v \times n_v}$, *such that*

$$\begin{bmatrix} Q - \Psi - \Psi^T & 0 & \Psi^T \bar{A}_u \\ * & -I & \bar{H}_u \\ * & * & -Q \end{bmatrix} < 0, \tag{38}$$

$$\begin{bmatrix} -I & 0 & \bar{L}_u \\ * & Q - \Psi - \Psi^T & \Psi^T \bar{B}_u \\ * & * & -\Pi \end{bmatrix} < 0, \tag{39}$$

$$trace\ (\Pi) < \gamma_2^2. \tag{40}$$

where

$$\bar{A}_u = \Omega \tilde{A}_u \Omega^T, \quad \bar{B}_u = \Omega \tilde{B}_u, \quad \bar{H}_u = \tilde{H}_u \Omega^T, \quad \bar{L}_u = -(L_f + \Delta L_f)D, \tag{41}$$

and

$$\tilde{A}_u = \begin{bmatrix} A & 0 \\ (B_{fu} + \Delta B_{fu})C & (A_{fu} + \Delta A_{fu}) \end{bmatrix}, \tag{42}$$

$$\tilde{B}_u = \begin{bmatrix} B \\ (B_{fu} + \Delta B_{fu})D \end{bmatrix}, \tag{43}$$

$$\tilde{H}_u = \begin{bmatrix} H - (L_{fu} + \Delta L_{fu})C & -(H_{fu} + \Delta H_{fu}) \end{bmatrix}, \tag{44}$$

and

$$A_{fu} = U^{-T} A_f U^T, \quad B_{fu} = U^{-T} B_f, \tag{45}$$

$$H_{fu} = H_f U^T, \quad L_{fu} = L_f, \tag{46}$$

$$\Delta A_{fu} = U^{-T} \Delta A_f U^T, \quad \Delta B_{fu} = U^{-T} \Delta B_f, \tag{47}$$

$$\Delta H_{fu} = \Delta H_f U^T, \quad \Delta L_{fu} = \Delta L_f. \tag{48}$$

Proof It suffices to prove that Lemma 5 is equivalent to Lemma 3. Assume that matrix inequalities (28) and (29) hold with $\Phi = diag\{\Phi_h,\ \Phi_v\}$ and $P = diag\{P_h,\ P_v\} > 0$. From (19), $\Omega^T = \Omega^{-1}$. Pre- and post-multiplying (28) by $diag\{\Omega^T, I, \Omega^T\}$ and $diag\{\Omega, I, \Omega\}$, respectively; and (29) by $diag\{I, \Omega^T, I\}$ and $diag\{I, \Omega, I\}$, respectively, we have

$$\begin{bmatrix} \tilde{P} - \tilde{\Phi} - \tilde{\Phi}^T & 0 & \tilde{\Phi}^T \tilde{A} \\ * & -I & \tilde{H} \\ * & * & -\tilde{P} \end{bmatrix} < 0, \tag{49}$$

and

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & \tilde{P} - \tilde{\Phi} - \tilde{\Phi}^T & \tilde{\Phi}^T \tilde{B} \\ * & * & -\Pi \end{bmatrix} < 0, \tag{50}$$

where $\tilde{A} = \Omega^T \bar{A}\Omega$, $\tilde{B} = \Omega^T \bar{B}$, $\tilde{H} = \bar{H}\Omega$, and their block matrix expressions being in (22)–(23). Partition $\tilde{\Phi}^T$ conformably with $\tilde{A}$ as $\tilde{\Phi}^T = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{bmatrix}$. From (49), we have $\tilde{P} - \tilde{\Phi} - \tilde{\Phi}^T < 0$, which implies that both $\tilde{\Phi}^T$ and Φ_4 are nonsingular. If Φ_2 is singular, then there exists a sufficiently small scalar $\alpha > 0$, and $\hat{\Phi}^T = \begin{bmatrix} \Phi_1 & \Phi_2 + \alpha I \\ \Phi_3 & \Phi_4 \end{bmatrix}$ with $\Phi_2 + \alpha I$ being nonsingular, such that matrix inequalities (49) and (50) still hold with $\tilde{\Phi}^T$ replaced by $\hat{\Phi}^T$. Hence, without loss of generality, Φ_2 can be assumed as nonsingular. Since Φ is a block-diagonal matrix, it is easy to verify that $\Phi_m = diag\{\Phi_{mh}, \ \Phi_{mv}\}$ with $\Phi_{mh} \in \mathbf{R}^{n_h \times n_h}$ and $\Phi_{mv} \in \mathbf{R}^{n_v \times n_v}$, $m = 1, 2, 3, 4$. Let

$$U := \Phi_2 \Phi_4^{-1}, \quad \Theta := diag\{I_n, U\}, \tag{51}$$

where $n = n_h + n_v$. Then U is a nonsingular matrix, and $U = diag\{U_h, \ U_v\}$, where $U_h = \Phi_{2h}\Phi_{4h}^{-1} \in \mathbf{R}^{n_h \times n_h}$, $U_v = \Phi_{2v}\Phi_{4v}^{-1} \in \mathbf{R}^{n_v \times n_v}$. Pre- and post-multiplying (49) by $diag\{\Theta, I, \Theta\}$ and $diag\{\Theta^T, I, \Theta^T\}$, respectively; and pre- and post-multiplying (50) by $diag\{I, \Theta, I\}$ and $diag\{I, \Theta^T, I\}$, respectively, we have

$$\begin{bmatrix} \Theta\tilde{P}\Theta^T - \Theta\tilde{\Phi}\Theta^T - \Theta\tilde{\Phi}^T\Theta^T & 0 & \Theta\tilde{\Phi}^T\Theta^T\Theta^{-T}\tilde{A}\Theta^T \\ * & -I & \tilde{H}\Theta^T \\ * & * & -\Theta\tilde{P}\Theta^T \end{bmatrix} < 0, \tag{52}$$

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & \Theta\tilde{P}\Theta^T - \Theta\tilde{\Phi}\Theta^T - \Theta\tilde{\Phi}^T\Theta^T & \Theta\tilde{\Phi}^T\Theta^T\Theta^{-T}\tilde{B} \\ * & * & -\Pi \end{bmatrix} < 0, \tag{53}$$

where $\Theta\tilde{\Phi}^T\Theta^T = \begin{bmatrix} X_1 & Y \\ X_2 & Y \end{bmatrix}$ with $X_1 = \Phi_1$, $X_2 = \Phi_2\Phi_4^{-1}\Phi_3$, $Y = \Phi_2\Phi_4^{-T}\Phi_2^T$, and X_1, X_2, Y all being block-diagonal matrices. From (22), (23), (42)–(48), and (51), it is easy to see

$$\tilde{A}_u = \Theta^{-T}\tilde{A}\Theta^T, \quad \tilde{B}_u = \Theta^{-T}\tilde{B}, \tag{54}$$
$$\tilde{H}_u = \tilde{H}\Theta^T, \quad \bar{L}_u = \bar{L}. \tag{55}$$

Let $\tilde{Q} := \Theta\tilde{P}\Theta^T$, $\tilde{\Psi} =: \Theta\tilde{\Phi}\Theta^T$. Then $\tilde{Q} > 0$, and matrix inequalities (52) and (53) become

$$\begin{bmatrix} \tilde{Q} - \tilde{\Psi} - \tilde{\Psi}^T & 0 & \tilde{\Psi}^T\tilde{A}_u \\ * & -I & \tilde{H}_u \\ * & * & -\tilde{Q} \end{bmatrix} < 0, \tag{56}$$

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & \tilde{Q} - \tilde{\Psi} - \tilde{\Psi}^T & \tilde{\Psi}^T\tilde{B}_u \\ * & * & -\Pi \end{bmatrix} < 0. \tag{57}$$

Pre- and post-multiplying (56) by $diag\{\Omega, I, \Omega\}$ and $diag\{\Omega^T, I, \Omega^T\}$, respectively; and (57) by $diag\{I, \Omega, I\}$, and $diag\{I, \Omega^T, I\}$, respectively, then matrix inequalities (56) and (57) become (38) and (39) which are given again as follows

$$\begin{bmatrix} Q-\Psi-\Psi^T & 0 & \Psi^T\bar{A}_u \\ * & -I & \bar{H}_u \\ * & * & -Q \end{bmatrix} < 0, \tag{58}$$

$$\begin{bmatrix} -I & 0 & \bar{L}_u \\ * & Q-\Psi-\Psi^T & \Psi^T\bar{B}_u \\ * & * & -\Pi \end{bmatrix} < 0, \tag{59}$$

where $Q = \Omega\tilde{Q}\Omega^T > 0$, with $Q = diag\{Q_h,\ Q_v\}$, $Q_h = diag\{I_{n_h}, U_h\}P_h diag\{I_{n_h}, U_h^T\}$ $\in\mathbf{R}^{2n_h\times 2n_h}$, $Q_v = diag\{I_{n_v}, U_v\}P_v diag\{I_{n_v}, U_v^T\} \in \mathbf{R}^{2n_v\times 2n_v}$, and

$$\Psi^T = \Omega\tilde{\Psi}^T\Omega^T = \Omega\begin{bmatrix} X_1 & Y \\ X_2 & Y \end{bmatrix}\Omega^T.$$

On the other hand, assume that matrix inequalities (38) and (39) hold with Ψ being of structure (37), a nonsingular matrix $U = diag\{U_h, U_v\}$ and $Q = diag\{Q_h, Q_v\} > 0$. Let $\Theta := diag\{I_n, U\}$, $\Gamma := \Omega\Theta^T\Omega^T = diag\{I_h, U_h^T, I_v, U_v^T\}$. Pre- and post-multiplying (38) by $diag\{\Gamma^{-T}, I, \Gamma^{-T}\}$ and $diag\{\Gamma^{-1}, I, \Gamma^{-1}\}$, respectively; and (39) by $diag\{I, \Gamma^{-T}, I\}$ and $diag\{I, \Gamma^{-1}, I\}$, respectively, then (38) and (39) become (28) and (29) given below

$$\begin{bmatrix} P-\Phi-\Phi^T & 0 & \Phi^T\bar{A} \\ * & -I & \bar{H} \\ * & * & -P \end{bmatrix} < 0, \tag{60}$$

$$\begin{bmatrix} -I & 0 & \bar{L} \\ * & P-\Phi-\Phi^T & \Phi^T\bar{B} \\ * & * & -\Pi \end{bmatrix} < 0, \tag{61}$$

where $P = \Gamma^{-T}Q\Gamma^{-1} > 0$, with $P_h = diag\{I_{n_h}, U_h^{-1}\}Q_h diag\{I_{n_h}, U_h^{-T}\} \in \mathbf{R}^{2n_h\times 2n_h}$, $P_v = diag\{I_{n_v}, U_v^{-1}\}Q_v diag\{I_{n_v}, U_v^{-T}\} \in \mathbf{R}^{2n_v\times 2n_v}$, and $\Phi = \Gamma^{-T}\Psi\Gamma^{-1} = diag\{\Phi_h,\ \Phi_v\}$ with

$$\Phi_h = \begin{bmatrix} X_{1h}^T & X_{2h}^T U_h^{-T} \\ U_h^{-1}Y_h^T & U_h^{-1}Y_h^T U_h^{-T} \end{bmatrix} \in \mathbf{R}^{2n_h\times 2n_h},$$
$$\Phi_v = \begin{bmatrix} X_{1v}^T & X_{2v}^T U_v^{-T} \\ U_v^{-1}Y_v^T & U_v^{-1}Y_v^T U_v^{-T} \end{bmatrix} \in \mathbf{R}^{2n_v\times 2n_v},$$

and

$$\bar{A} = \Gamma\bar{A}_u\Gamma^{-1}, \quad \bar{B} = \Gamma\bar{B}_u,$$
$$\bar{H} = \bar{H}_u\Gamma^{-1}, \quad \bar{L} = \bar{L}_u.$$

This completes the proof.

Similar to Lemma 5, we can obtain the following new characterization for the H_∞ norm.

Lemma 6 *Given a scalar $\gamma_\infty > 0$, a 2-D system $(\bar{\Sigma}_0)$ is asymptotically stable and $\|G\|_\infty < \gamma_\infty$ if there exist a nonsingular matrix $U = diag\{U_h, U_v\}$ with $U_h \in \boldsymbol{R}^{n_h\times n_h}$ and $U_v \in \boldsymbol{R}^{n_v\times n_v}$, and $\hat{Q} = diag\{\hat{Q}_h, \hat{Q}_v\} > 0$ with $\hat{Q}_h \in \boldsymbol{R}^{2n_h\times 2n_h}$, $\hat{Q}_v \in \boldsymbol{R}^{2n_v\times 2n_v}$, and a matrix $\hat{\Psi}^T$ with structure*

$$\hat{\Psi}^T = \Omega\begin{bmatrix} \hat{X}_1 & \hat{Y} \\ \hat{X}_2 & \hat{Y} \end{bmatrix}\Omega^T, \tag{62}$$

where $\hat{X}_1 = diag\{\hat{X}_{1h}, \ \hat{X}_{1v}\}$, $\hat{X}_2 = diag\{\hat{X}_{2h}, \ \hat{X}_{2v}\}$, $\hat{Y} = diag\{\hat{Y}_h, \ \hat{Y}_v\}$, *and* $\hat{X}_{1h}$, $\hat{X}_{2h}$, $\hat{Y}_h \in \boldsymbol{R}^{n_h \times n_h}$, $\hat{X}_{1v}$, $\hat{X}_{2v}$, $\hat{Y}_v \in \boldsymbol{R}^{n_v \times n_v}$, *such that*

$$\begin{bmatrix} \hat{Q} - \hat{\Psi} - \hat{\Psi}^T & 0 & \hat{\Psi}^T \bar{A}_u & \hat{\Psi}^T \bar{B}_u \\ * & -I & \bar{H}_u & \bar{L}_u \\ * & * & -\hat{Q} & 0 \\ * & * & * & -\gamma_\infty^2 I \end{bmatrix} < 0. \tag{63}$$

The advantage of the two new characterizations for the H_2 and H_∞ performance given in Lemmas 5 and 6 is that adopting the ideas in these two lemmas, it is more convenient to design 2-D non-fragile filters to be discussed in the next section.

4 2-D Non-fragile filter design

We start this section by recalling a lemma which will be essential in the proof of the main results.

Lemma 7 (Khargonekar et al. 1990) *Let* $\pounds = \pounds^T \in \boldsymbol{R}^{n \times n}$, $E \in \boldsymbol{R}^{n \times r}$ *and* $F \in \boldsymbol{R}^{t \times n}$ *be any given real matrices. Then, the matrix inequality*

$$\pounds + E\Upsilon F + (E\Upsilon F)^T < 0,$$

holds for all Υ *satisfying* $\Upsilon^T \Upsilon \le I$, *if and only if there exists a scalar* $\varepsilon > 0$, *such that*

$$\pounds + \varepsilon E E^T + \varepsilon^{-1} F^T F < 0. \tag{64}$$

In this section, we will present a method for designing 2-D non-fragile H_2 and H_∞ filters with additive norm-bounded coefficient variations. Note that the non-fragile H_2 and H_∞ filtering problem reduces to the standard H_2 and H_∞ filtering problem when a polytopic 2-D system (Σ) reduces to a fixed system (Σ_0) and a designed filter (Σ_f) reduces to a fixed coefficient filter (Σ_f^0) by letting $\Delta A_f = \Delta B_f = \Delta H_f = \Delta L_f = 0$ in (8) and (9). As a result, we can also give an alternative method for designing 2-D H_2 and H_∞ filters.

4.1 2-D Non-fragile H_2 filter design

Similar to the method for a 1-D system in Geromel et al. (2002), it is easy to obtain the following result for a polytopic 2-D system (Σ).

Lemma 8 *Given a polytopic* 2-*D system* (Σ) *and a scalar* $\gamma_2 > 0$, *the estimation error system* $(\bar{\Sigma})$ *is robustly asymptotically stable and* $\|G(\lambda)\|_2 < \gamma_2$ *if there exist a nonsingular matrix* $U = diag\{U_h, U_v\}$ *with* $U_h \in \boldsymbol{R}^{n_h \times n_h}$ *and* $U_v \in \boldsymbol{R}^{n_v \times n_v}$, *and* $Q^{(k)} = diag\{Q_h^{(k)}, Q_v^{(k)}\} > 0$ *with* $Q_h^{(k)} \in \boldsymbol{R}^{2n_h \times 2n_h}$, $Q_v^{(k)} \in \boldsymbol{R}^{2n_v \times 2n_v}$, $\Pi^{(k)} > 0$ *and a matrix* Ψ *with structure*

$$\Psi^T = \Omega \begin{bmatrix} X_1 & Y \\ X_2 & Y \end{bmatrix} \Omega^T, \tag{65}$$

where $X_1 = diag\{X_{1h},\ X_{1v}\}$, $X_2 = diag\{X_{2h},\ X_{2v}\}$, $Y = diag\{Y_h,\ Y_v\}$, *and* X_{1h}, $X_{2h},\ Y_h \in \boldsymbol{R}^{n_h \times n_h}$, $X_{1v},\ X_{2v},\ Y_v \in \boldsymbol{R}^{n_v \times n_v}$, *such that*

$$\begin{bmatrix} Q^{(k)} - \Psi - \Psi^T & 0 & \Psi^T \bar{A}_u^{(k)} \\ * & -I & \bar{H}_u^{(k)} \\ * & * & -Q^{(k)} \end{bmatrix} < 0, \tag{66}$$

$$\begin{bmatrix} -I & 0 & \bar{L}_u^{(k)} \\ * & Q^{(k)} - \Psi - \Psi^T & \Psi^T \bar{B}_u^{(k)} \\ * & * & -\Pi^{(k)} \end{bmatrix} < 0, \tag{67}$$

$$trace\ (\Pi^{(k)}) < \gamma_2^2, \tag{68}$$

where

$$\begin{aligned} \bar{A}_u^{(k)} &= \Omega \tilde{A}_u^{(k)} \Omega^T, \quad \bar{B}_u^{(k)} = \Omega \tilde{B}_u^{(k)}, \\ \bar{H}_u^{(k)} &= \tilde{H}_u^{(k)} \Omega^T, \quad \bar{L}_u^{(k)} = -(L_{fu} + \Delta L_{fu}) D^{(k)}, \\ \tilde{A}_u^{(k)} &= \begin{bmatrix} A^{(k)} & 0 \\ (B_{fu} + \Delta B_{fu}) C^{(k)} & A_{fu} + \Delta A_{fu} \end{bmatrix}, \qquad (69) \\ \tilde{B}_u^{(k)} &= \begin{bmatrix} B^{(k)} \\ (B_{fu} + \Delta B_{fu}) D^{(k)} \end{bmatrix}, \qquad (70) \\ \tilde{H}_u^{(k)} &= \begin{bmatrix} H^{(k)} - (L_{fu} + \Delta L_{fu}) C^{(k)} & -(H_{fu} + \Delta H_{fu}) \end{bmatrix}, \qquad (71) \end{aligned}$$

for $k = 1, 2 \ldots s$, *and* $A_{fu}, \Delta A_{fu}, B_{fu}, \Delta B_{fu}, H_{fu}, \Delta H_{fu}, L_{fu}, \Delta L_{fu}$ *are given as in* (45)–(48). □

Proof Suppose that matrix inequalities (66) and (67) hold for $k = 1, 2 \ldots s$. Then, for each $k = 1, 2 \ldots s$, we multiply inequality (66) by the uncertain parameter $\lambda_k > 0$, and then sum the s inequalities together. Similarly, we do the same for inequality (67). This leads to

$$\begin{bmatrix} Q(\lambda) - \Psi - \Psi^T & 0 & \Psi^T \bar{A}_u(\lambda) \\ * & -I & \bar{H}_u(\lambda) \\ * & * & -Q(\lambda) \end{bmatrix} < 0, \tag{72}$$

$$\begin{bmatrix} -I & 0 & \bar{L}_u(\lambda) \\ * & Q(\lambda) - \Psi - \Psi^T & \Psi^T \bar{B}_u(\lambda) \\ * & * & -\Pi(\lambda) \end{bmatrix} < 0, \tag{73}$$

where

$$Q(\lambda) = diag\{Q_h(\lambda),\ Q_v(\lambda)\} > 0, \quad Q_h(\lambda) = \sum_{k=1}^{s} \lambda_k Q_h^{(k)}, \quad Q_v(\lambda) = \sum_{k=1}^{s} \lambda_k Q_v^{(k)},$$

and

$$\begin{aligned} \bar{A}_u(\lambda) &= \sum_{k=1}^{s} \lambda_k \bar{A}_u^{(k)}, \quad \bar{B}_u(\lambda) = \sum_{k=1}^{s} \lambda_k \bar{B}_u^{(k)}, \\ \bar{H}_u(\lambda) &= \sum_{k=1}^{s} \lambda_k \bar{H}_u^{(k)}, \quad \bar{L}_u(\lambda) = \sum_{k=1}^{s} \lambda_k \bar{L}_u^{(k)}, \\ \Pi(\lambda) &= \sum_{k=1}^{s} \lambda_k \Pi^{(k)}. \end{aligned}$$

From the proof of Lemma 5, it is easy to show that inequalities (72) and (73) are equivalent to

$$\begin{bmatrix} P(\lambda)-\Phi-\Phi^T & 0 & \Phi^T\bar{A}(\lambda) \\ * & -I & \bar{H}(\lambda) \\ * & * & -P(\lambda) \end{bmatrix} < 0, \tag{74}$$

$$\begin{bmatrix} -I & 0 & \bar{L}(\lambda) \\ * & P(\lambda)-\Phi-\Phi^T & \Phi^T\bar{B}(\lambda) \\ * & * & -\Pi(\lambda) \end{bmatrix} < 0, \tag{75}$$

where $\Phi = diag\{\Phi_h, \Phi_v\}$ and

$$P(\lambda) = diag\{P_h(\lambda), P_v(\lambda)\} > 0,$$

with

$$P_h(\lambda) = \sum_{k=1}^{s} \lambda_k P_h^{(k)}, \quad P_v(\lambda) = \sum_{k=1}^{s} \lambda_k P_v^{(k)}.$$

Furthermore, from (74) and (75), and the Schur complement formula, we can obtain

$$\bar{A}^T(\lambda)P(\lambda)\bar{A}(\lambda) + \bar{H}^T(\lambda)\bar{H}(\lambda) - P(\lambda) < 0, \tag{76}$$

and

$$\begin{aligned} \|G(\lambda)\|_2^2 &\le trace\,[\bar{B}^T(\lambda)P(\lambda)\bar{B}(\lambda) + \bar{L}^T(\lambda)\bar{L}(\lambda)] \\ &\le trace\,(\Pi(\lambda)). \end{aligned}$$

Inequality (76) shows that $\bar{A}^T(\lambda)P(\lambda)\bar{A}(\lambda) - P(\lambda) < 0$, which implies that the 2-D system $(\bar{\Sigma})$ is robustly asymptotically stable.

Noting $\Pi(\lambda) = \sum_{k=1}^{s} \lambda_k \Pi^{(k)}$, it ia easy to see that

$$\begin{aligned} trace\,(\Pi(\lambda)) &= trace \sum_{k=1}^{s} \lambda_k \Pi^{(k)} \\ &\le \max_{k=1,\ldots,s} trace\,(\Pi^{(k)}) < \gamma_2^2. \end{aligned}$$

That is, $\|G(\lambda)\|_2 < \gamma_2$. This completes the proof. □

Now, we present a LMI solution to the design of 2-D non-fragile H_2 filters.

Theorem 1 *Given a polytopic 2-D system* (Σ) *and a scalar* $\gamma_2 > 0$. *A 2-D filter* (Σ_f) *that solves the non-fragile* H_2 *filtering problem exists if for some given* $\delta_1 \neq 0, \delta_2 \neq 0$, $U = diag\{\delta_1 I_{n_h}, \ \delta_2 I_{n_v}\}$, *there exist matrices* $Q_{11}^{(k)} = diag\{Q_{11h}^{(k)}, Q_{11v}^{(k)}\} > 0$, $Q_{22}^{(k)} = diag\{Q_{22h}^{(k)}, Q_{22v}^{(k)}\} > 0$, $Q_{12}^{(k)} = diag\{Q_{12h}^{(k)}, Q_{12v}^{(k)}\}$, $X_1 = diag\{X_{1h}, \ X_{1v}\}$, $X_2 = diag\{X_{2h}, \ X_{2v}\}$, $Y = diag\{Y_h, \ Y_v\}$, $N, M, Z, K, \Pi^{(k)} > 0$, *and scalars* $\varepsilon_1^{(k)} > 0$, $\varepsilon_2^{(k)} > 0$ *such that the following LMIs hold*

$$\begin{bmatrix} W_1^{(k)} & J_1 & \varepsilon_1^{(k)} J_2^{(k)T} \\ * & -\varepsilon_1^{(k)} I & 0 \\ * & * & -\varepsilon_1^{(k)} I \end{bmatrix} < 0, \tag{77}$$

$$\begin{bmatrix} W_2^{(k)} & J_3 & \varepsilon_2^{(k)} J_4^{(k)T} \\ * & -\varepsilon_2^{(k)} I & 0 \\ * & * & -\varepsilon_2^{(k)} I \end{bmatrix} < 0, \tag{78}$$

$$trace\ (\Pi^{(k)}) < \gamma_2^2, \tag{79}$$

where

$$W_1^{(k)} = \begin{bmatrix} W_{11}^{(k)} & 0 & W_{12}^{(k)} & \bar{K} \\ * & -I & H^{(k)} - MC^{(k)} & -N \\ * & * & -Q_{11}^{(k)} & -Q_{12}^{(k)} \\ * & * & * & -Q_{22}^{(k)} \end{bmatrix},$$

$$W_2^{(k)} = \begin{bmatrix} -I & 0 & -MD^{(k)} \\ * & W_{11}^{(k)} & W_{13}^{(k)} \\ * & * & -\Pi^{(k)} \end{bmatrix}, \quad \bar{K} = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix},$$

$$W_{11}^{(k)} = \begin{bmatrix} Q_{11}^{(k)} - X_1 - X_1^T & Q_{12}^{(k)} - Y - X_2^T \\ * & Q_{22}^{(k)} - Y - Y^T \end{bmatrix},$$

$$W_{12}^{(k)} = \begin{bmatrix} X_1 A^{(k)} + ZC^{(k)} \\ X_2 A^{(k)} + ZC^{(k)} \end{bmatrix}, \quad W_{13}^{(k)} = \begin{bmatrix} X_1 B^{(k)} + ZD^{(k)} \\ X_2 B^{(k)} + ZD^{(k)} \end{bmatrix},$$

$$J_1 = \begin{bmatrix} E_1^T U^{-1} Y^T & E_1^T U^{-1} Y^T & -E_2^T & 0 & 0 \\ E_1^T U^{-1} Y^T & E_1^T U^{-1} Y^T & -E_2^T & 0 & 0 \end{bmatrix}^T, \ J_2^{(k)} = \begin{bmatrix} 0 & 0 & 0 & F_2 C^{(k)} & 0 \\ 0 & 0 & 0 & 0 & F_1 U^T \end{bmatrix},$$

$$J_3 = \begin{bmatrix} -E_2^T & E_1^T U^{-1} Y^T & E_1^T U^{-1} Y^T & 0 \end{bmatrix}^T, \quad J_4^{(k)} = \begin{bmatrix} 0 & 0 & 0 & F_2 D^{(k)} \end{bmatrix},$$

for $k = 1, 2 \ldots s$. Then, the non-fragile H_2 filter with the additive Coefficient variation of the form (12) *and* (13) *can be designed by*

$$A_f = U^T Y^{-1} K U^{-T}, \ B_f = U^T Y^{-1} Z, \ H_f = N U^{-T}, \ L_f = M. \tag{80}$$

Proof By Lemma 8, the estimation error system $(\bar{\Sigma})$ is robustly asymptotically stable and $\|G(\lambda)\|_2 < \gamma_2$ if for some given $\delta_1 \neq 0, \delta_2 \neq 0, U = diag\{\delta_1 I_{n_h}, \ \delta_2 I_{n_v}\}$, there exist a matrix $Q^{(k)} = diag\{Q_h^{(k)}, Q_v^{(k)}\} > 0$ with $Q_h^{(k)} \in \mathbf{R}^{2n_h \times 2n_h}$ and $Q_v^{(k)} \in \mathbf{R}^{2n_v \times 2n_v}$, and Ψ with structure

$$\Psi^T = \Omega \begin{bmatrix} X_1 & Y \\ X_2 & Y \end{bmatrix} \Omega^T, \tag{81}$$

where $X_1 = diag\{X_{1h}, \ X_{1v}\}, \ X_2 = diag\{X_{2h}, \ X_{2v}\}, \ Y = diag\{Y_h, \ Y_v\}$, such that

$$\begin{bmatrix} Q^{(k)} - \Psi - \Psi^T & 0 & \Psi^T \bar{A}_u^{(k)} \\ * & -I & \bar{H}_u^{(k)} \\ * & * & -Q^{(k)} \end{bmatrix} < 0, \tag{82}$$

$$\begin{bmatrix} -I & 0 & \bar{L}_u^{(k)} \\ * & Q^{(k)} - \Psi - \Psi^T & \Psi^T \bar{B}_u^{(k)} \\ * & * & -\Pi^{(k)} \end{bmatrix} < 0, \tag{83}$$

$$trace\ (\Pi^{(k)}) < \gamma_2^2. \tag{84}$$

It is easy to see that $Q^{(k)} - \Psi - \Psi^T < 0$, which implies that Ψ is nonsingular. Then, Y is nonsingular. Pre- and post-multiplying (82) by $diag\{\Omega^T, I, \Omega^T\}$ and $diag\{\Omega, I, \Omega\}$, respectively; and (83) by $diag\{I, \Omega^T, I\}$ and $diag\{I, \Omega, I\}$, respectively, matrix inequalities (82) and (83) are equivalent to

$$\begin{bmatrix} \tilde{Q}^{(k)} - \tilde{\Psi} - \tilde{\Psi}^T & 0 & \tilde{\Psi}^T \tilde{A}_u^{(k)} \\ * & -I & \tilde{H}_u^{(k)} \\ * & * & -\tilde{Q}^{(k)} \end{bmatrix} < 0, \tag{85}$$

$$\begin{bmatrix} -I & 0 & \bar{L}_u^{(k)} \\ * & \tilde{Q}^{(k)} - \tilde{\Psi} - \tilde{\Psi}^T & \tilde{\Psi}^T \tilde{B}_u^{(k)} \\ * & * & -\Pi^{(k)} \end{bmatrix} < 0, \tag{86}$$

where

$$\tilde{\Psi}^T = \begin{bmatrix} X_1 & Y \\ X_2 & Y \end{bmatrix},$$

$$\tilde{Q}^{(k)} = \Omega^T Q^{(k)} \Omega = \begin{bmatrix} Q_{11}^{(k)} & Q_{12}^{(k)} \\ * & Q_{22}^{(k)} \end{bmatrix}.$$

Substituting $\tilde{A}_u^{(k)}$, $\tilde{B}_u^{(k)}$, $\tilde{H}_u^{(k)}$ and $\bar{L}_u^{(k)}$ given in (69)–(71) into (85) and (86), and making use of (12), (47) and (48), letting

$$K := YA_{fu}, \quad N := H_{fu}, \tag{87}$$

$$Z := YB_{fu}, \quad M := L_{fu}, \tag{88}$$

and by the Schur complement formula, matrix inequalities (85) and (86) are equivalent to

$$W_1^{(k)} + J_1 \tilde{\Upsilon}(i,j) J_2^{(k)} + [J_1 \tilde{\Upsilon}(i,j) J_2^{(k)}]^T < 0, \tag{89}$$

$$W_2^{(k)} + J_3 \Upsilon(i,j) J_4^{(k)} + [J_3 \Upsilon(i,j) J_4^{(k)}]^T < 0, \tag{90}$$

where $\tilde{\Upsilon}(i,j) = diag\{\Upsilon(i,j), \Upsilon(i,j)\}$, and $\Upsilon(i,j)$ is given in (13). By Lemma 7 and the Schur complement formula, we have that matrix inequalities (89) and (90) are equivalent to (77) and (78). Furthermore, from (45), (46), (87) and (88), and with some manipulations, a non-fragile H_2 filter can be designed and is given by (80). This completes the proof. □

Remark 1 Observe that once δ_1, δ_2 are given, (77) and (78) are linear in $Q_{11}^{(k)}$, $Q_{22}^{(k)}$, $Q_{12}^{(k)}$, X_1, X_2, Y, N, M, Z, K, $\Pi^{(k)}$, $\varepsilon_1^{(k)}$, $\varepsilon_2^{(k)}$. This provides convex optimization so that the solutions can be solved by the LMI toolbox (Gahinet et al. 1995).

Remark 2 From the proof of Theorem 1, it is easy to see that the nonsingular matrix U only appears on the designed filter for the standard H_2 filtering problem. In such a case, the matrix U is "absorbed" into part of the unknown filter coefficient matrices. Thus the H_2 filter can be designed by A_{fu}, B_{fu}, H_{fu}, L_{fu} without loss of generality.

As a result of Theorem 1 and Remark 2, we present an alternative method for the standard H_2 filter design problem for a 2-D system (Σ_0).

Corollary 1 *Given a 2-D system* (Σ_0) *and a scalar* $\gamma_2 > 0$. *A 2-D filter* (Σ_f^0) *can be designed to solve the* H_2 *filtering problem if there exist matrices* $Q_{11} = diag\{Q_{11h}, Q_{11v}\} > 0$, $Q_{22} = diag\{Q_{22h}, Q_{22v}\} > 0$, $Q_{12} = diag\{Q_{12h}, Q_{12v}\}$, $\Pi > 0$, *and matrices* $X_1 =$

$diag\{X_{1h},\ X_{1v}\}$, $X_2 = diag\{X_{2h},\ X_{2v}\}$, $Y = diag\{Y_h,\ Y_v\}$, N, M, Z and K, such that $W_1 < 0$, $W_2 < 0$, $trace(\Pi) < \gamma_2^2$. Then the H_2 filter is given

$$A_f = Y^{-1}K, \quad B_f = Y^{-1}Z, \quad H_f = N, \quad L_f = M. \tag{91}$$

4.2 2-D Non-fragile H_∞ filter design

In this sub-section, we give the following new result for non-fragile H_∞ filter design. The proofs are similar to their H_2 counterparts and hence omitted here for brevity.

Lemma 9 *Given a polytopic 2-D system (Σ) and a scalar $\gamma_\infty > 0$, the estimation error system $(\bar{\Sigma})$ is robustly asymptotically stable and $\|G(\lambda)\|_\infty < \gamma_\infty$ if there exist a nonsingular matrix $U = diag\{U_h, U_v\}$ and $\hat{Q}^{(k)} = diag\{\hat{Q}_h^{(k)}, \hat{Q}_v^{(k)}\} > 0$ with $\hat{Q}_h^{(k)} \in \boldsymbol{R}^{2n_h \times 2n_h}$, $\hat{Q}_v^{(k)} \in \boldsymbol{R}^{2n_v \times 2n_v}$, and a matrix $\hat{\Psi}$ with structure*

$$\hat{\Psi}^T = \Omega \begin{bmatrix} \hat{X}_1 & \hat{Y} \\ \hat{X}_2 & \hat{Y} \end{bmatrix} \Omega^T, \tag{92}$$

where $\hat{X}_1 = diag\{\hat{X}_{1h},\ \hat{X}_{1v}\}$, $\hat{X}_2 = diag\{\hat{X}_{2h},\ \hat{X}_{2v}\}$, $\hat{Y} = diag\{\hat{Y}_h,\ \hat{Y}_v\}$, and $\hat{X}_{1h},\ \hat{X}_{2h},\ \hat{Y}_h \in \boldsymbol{R}^{n_h \times n_h}$, $\hat{X}_{1v},\ \hat{X}_{2v},\ \hat{Y}_v \in \boldsymbol{R}^{n_v \times n_v}$, such that

$$\begin{bmatrix} \hat{Q}^{(k)} - \hat{\Psi} - \hat{\Psi}^T & 0 & \hat{\Psi}^T \bar{A}_u^{(k)} & \hat{\Psi}^T \bar{B}_u^{(k)} \\ * & -I & \bar{H}_u^{(k)} & \bar{L}_u^{(k)} \\ * & * & -\hat{Q}^{(k)} & 0 \\ * & * & * & -\gamma_\infty^2 I \end{bmatrix} < 0. \tag{93}$$

Using this lemma, the 2-D non-fragile H_∞ filter design is given as follows.

Theorem 2 *Given a polytopic 2-D system (Σ) and a scalar $\gamma_\infty > 0$. A 2-D filter (Σ_f) that solves the non-fragile H_∞ filtering problem exists if for some given $\hat{\delta}_1 \neq 0, \hat{\delta}_2 \neq 0$, $\hat{U} = diag\{\hat{\delta}_1 I_{n_h}, \hat{\delta}_2 I_{n_v}\}$, there exist matrices $\hat{Q}_{11}^{(k)} = diag\{\hat{Q}_{11h}^{(k)}, \hat{Q}_{11v}^{(k)}\} > 0$, $\hat{Q}_{22}^{(k)} = diag\{\hat{Q}_{22h}^{(k)}, \hat{Q}_{22v}^{(k)}\} > 0$, $\hat{Q}_{12}^{(k)} = diag\{\hat{Q}_{12h}^{(k)}, \hat{Q}_{12v}^{(k)}\}$, $\hat{X}_1 = diag\{\hat{X}_{1h},\ \hat{X}_{1v}\}$, $\hat{X}_2 = diag\{\hat{X}_{2h},\ \hat{X}_{2v}\}$, $\hat{Y} = diag\{\hat{Y}_h,\ \hat{Y}_v\}$, $\hat{N}, \hat{M}, \hat{Z}, \hat{K}$, and scalars $\mu^{(k)} > 0$, such that the following LMI holds*

$$\begin{bmatrix} W^{(k)} & S_1 & \mu^{(k)} S_2^{(k)T} \\ * & -\mu^{(k)} I & 0 \\ * & * & -\mu^{(k)} I \end{bmatrix} < 0, \tag{94}$$

where

$$W^{(k)} = \begin{bmatrix} \Pi_1^{(k)} & 0 & L_1^{(k)} & L_2^{(k)} \\ * & -I & G^{(k)} & -MD^{(k)} \\ * & * & \Pi_2^{(k)} & 0 \\ * & * & * & -\gamma_\infty^2 I \end{bmatrix},$$

$$\Pi_1^{(k)} = \begin{bmatrix} \hat{Q}_{11}^{(k)} - \hat{X}_1 - \hat{X}_1^T & \hat{Q}_{12}^{(k)} - \hat{Y} - \hat{X}_2^T \\ * & \hat{Q}_{22}^{(k)} - \hat{Y} - \hat{Y}^T \end{bmatrix},$$

$$L_1^{(k)} = \begin{bmatrix} \hat{X}_1 A^{(k)} + \hat{Z} C^{(k)} & \hat{K} \\ \hat{X}_2 A^{(k)} + \hat{Z} C^{(k)} & \hat{K} \end{bmatrix}, \quad \Pi_2^{(k)} = \begin{bmatrix} -\hat{Q}_{11}^{(k)} & -\hat{Q}_{12}^{(k)} \\ * & -\hat{Q}_{22}^{(k)} \end{bmatrix},$$

$$L_2^{(k)} = \begin{bmatrix} \hat{X}_1 B^{(k)} + \hat{Z} D^{(k)} \\ \hat{X}_2 B^{(k)} + \hat{Z} D^{(k)} \end{bmatrix}, \quad G^{(k)} = \begin{bmatrix} H^{(k)} - \hat{M} C^{(k)} & -\hat{N} \end{bmatrix},$$

$$S_1 = \begin{bmatrix} E_1^T \hat{U}^{-1} \hat{Y}^T & E_1^T \hat{U}^{-1} \hat{Y}^T & -E_2^T & 0 & 0 & 0 \\ E_1^T \hat{U}^{-1} \hat{Y}^T & E_1^T \hat{U}^{-1} \hat{Y}^T & -E_2^T & 0 & 0 & 0 \\ E_1^T \hat{U}^{-1} \hat{Y}^T & E_1^T \hat{U}^{-1} \hat{Y}^T & -E_2^T & 0 & 0 & 0 \end{bmatrix}^T,$$

$$S_2^{(k)} = \begin{bmatrix} 0 & 0 & 0 & F_2 C^{(k)} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_1 \hat{U}^T & 0 \\ 0 & 0 & 0 & 0 & 0 & F_2 D^{(k)} \end{bmatrix},$$

for $k = 1, 2 \ldots s$. Then, the non-fragile H_∞ filter with the additive coefficient variation of the form (12) *and* (13) *can be designed by*

$$A_f = \hat{U}^T \hat{Y}^{-1} \hat{K} \hat{U}^{-T}, \quad B_f = \hat{U}^T \hat{Y}^{-1} \hat{Z}, \quad H_f = \hat{N} \hat{U}^{-T}, \quad L_f = \hat{M}. \tag{95}$$

Remark 3 Observe that once $\hat{\delta}_1$, $\hat{\delta}_2$ are given, (94) is linear in $\hat{Q}_{11}^{(k)}$, $\hat{Q}_{22}^{(k)}$, $\hat{Q}_{12}^{(k)}$, $\hat{X}_1$, $\hat{X}_2$, $\hat{Y}$, $\hat{N}$, $\hat{M}$, $\hat{Z}$, $\hat{K}$, $\mu^{(k)}$. This provides convex optimization such that the solutions can be solved by the LMI toolbox (Gahinet et al. 1995).

Remark 4 Similar to Remark 2, the standard H_∞ filter can be designed by A_{fu}, B_{fu}, H_{fu}, L_{fu} without loss of generality.

From Theorem 2 and Remark 4, the following result gives a solution to the standard H_∞ filtering problem for a 2-D system (Σ_0).

Corollary 2 *Given a 2-D system (Σ_0) and a scalar $\gamma_\infty > 0$. A 2-D filter (Σ_f^0) can be designed to solve the H_∞ filtering problem if there exist matrices $\hat{Q}_{11} = diag\{\hat{Q}_{11h}, \hat{Q}_{11v}\} > 0$, $\hat{Q}_{22} = diag\{\hat{Q}_{22h}, \hat{Q}_{22v}\} > 0$, $\hat{Q}_{12} = diag\{\hat{Q}_{12h}, \hat{Q}_{12v}\}$ and matrices $\hat{X}_1 = diag\{\hat{X}_{1h}, \hat{X}_{1v}\}$, $\hat{X}_2 = diag\{\hat{X}_{2h}, \hat{X}_{2v}\}$, $\hat{Y} = diag\{\hat{Y}_h, \hat{Y}_v\}$, $\hat{N}$, $\hat{M}$, $\hat{Z}$, $\hat{K}$ such that $W < 0$. Then the H_∞ filter is given*

$$A_f = \hat{Y}^{-1} \hat{K}, \quad B_f = \hat{Y}^{-1} \hat{Z}, \quad H_f = \hat{N}, \quad L_f = \hat{M}. \tag{96}$$

Remark 5 In Theorems 1 and 2, a natural question is how to find the optimal values of $\delta_1, \delta_2, \hat{\delta}_1$ and $\hat{\delta}_2$ to minimize the upper bound of the H_2 and H_∞ norms. The simple linear search method can be used to obtain the optimal values of $\delta_1, \delta_2, \hat{\delta}_1$ and $\hat{\delta}_2$ with finite intervals. Note that as U and $\hat{U}$ are restricted to the form of $U = diag\{\delta_1 I_{n_h}, \ \delta_2 I_{n_v}\}$ and $\hat{U} = diag\{\hat{\delta}_1 I_{n_h}, \hat{\delta}_2 I_{n_v}\}$, Theorems 1 and 2 have some conservativeness. How to overcome this conservativeness requires further investigation.

5 Example

In this section, we shall consider the stationary random field model described by the following differential equation (Du et al. 2000; Gao et al. 2008b; Katayama and Kosaka 1979):

$$\eta(i+1, j+1) = a_1 \eta(i, j+1) + a_2 \eta(i+1, j) - a_1 a_2 \eta(i, j) + \omega(i, j), \tag{97}$$

where $\eta(i,j)$ is the state of the random field at spacial coordinate (i,j), and $i = 0, 1, \ldots$ and $j = 0, 1, \ldots$ are the vertical and horizontal position variables. a_1 and a_2 are the vertical and horizontal correlative coefficients of the random field respectively, satisfying $a_1^2 < 1$ and $a_2^2 < 1$. Assume that the measured equation and the signal to be estimated are

$$y(i,j) = c(\eta(i,j+1) - a_2\eta(i,j)) + \eta(i,j) + \omega(i,j), \tag{98}$$

$$z(i,j) = \eta(i,j), \tag{99}$$

where c is a coefficient given arbitrarily. Denote

$$x^h(i,j) = \eta(i,j+1) - a_2\eta(i,j),$$
$$x^v(i,j) = \eta(i,j).$$

It is easy to see that the (97)–(99) can be described by the following 2-D Roesser model

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A(\lambda)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + B(\lambda)\omega(i,j), \tag{100}$$

$$y(i,j) = C(\lambda)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + D(\lambda)\omega(i,j), \tag{101}$$

$$z(i,j) = H(\lambda)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, \tag{102}$$

where

$$A(\lambda) = \begin{bmatrix} a_1 & 0 \\ 1 & a_2 \end{bmatrix}, \quad B(\lambda) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C(\lambda) = \begin{bmatrix} c & 1 \end{bmatrix}, \quad H(\lambda) = \begin{bmatrix} 0 & 1 \end{bmatrix}, \; D(\lambda) = 1\,.$$

5.1 2-D H_∞ filter design

Let $c = 0$, $a_1 = 0.3$, $a_2 = 0.2$. Using Corollary 2, we can obtain the optimal level of H_∞ noise attenuation level $\gamma_\infty^* = 1.0001$, and the associated matrices for the H_∞ filter given by

$$A_f = \begin{bmatrix} 0.1302 & 0.0684 \\ 0.6376 & -0.0501 \end{bmatrix},$$
$$B_f = \begin{bmatrix} 0.0501 \\ -0.2487 \end{bmatrix},$$
$$H_f = \begin{bmatrix} 0.0503 & -0.1392 \end{bmatrix} \times 10^{-3},$$
$$L_f = 0.9999.$$

The transfer function of the estimation error system is obtained as follows

$$G(z_1,z_2) = \bar{H}\left[I(z_1,z_2) - \bar{A}\right]^{-1}\bar{B} + \bar{L},$$

where $I(z_1,z_2) = diag\{z_1 I_{2n_h}, z_2 I_{2n_v}\}$, and $\bar{A}$, $\bar{B}$, $\bar{H}$, $\bar{L}$ are given in (21)–(23) with $\Delta A_f = \Delta B_f = \Delta H_f = \Delta L_f = 0$. Figure 1 shows the frequency response of the estimation error system over all frequencies, i.e., $\left|G(e^{jw_1}, e^{jw_2})\right|$, $0 \le w_1 \le 2\pi$, $0 \le w_2 \le 2\pi$. It is easy to see that the amplitude response of the estimation error transfer function is below the H_∞ noise attenuation level $\gamma_\infty^* = 1.0001$.

For comparison, by Theorem 4.1 in the reference Du et al. (2000), the obtained minimum H_∞ performance is $\gamma_\infty^* = 1.01$, which is higher than that obtained by our method. It shows

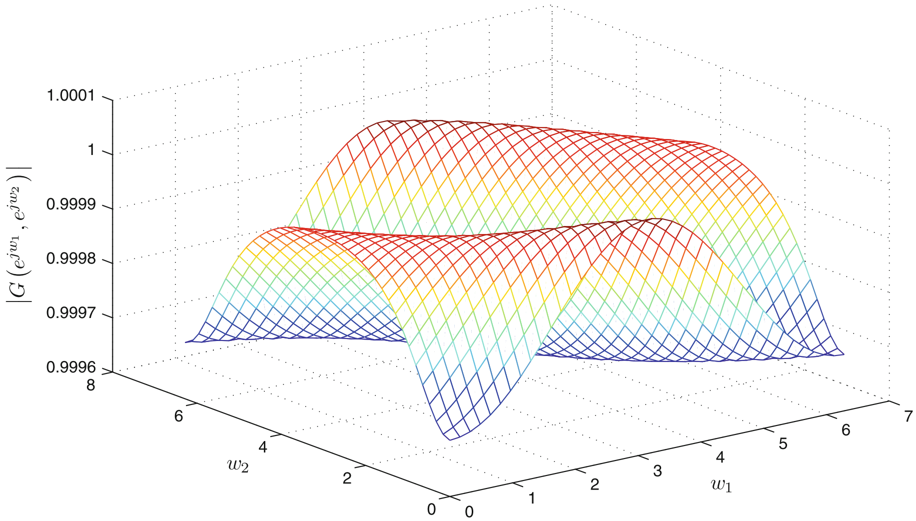

Fig. 1 Frequency response of the estimation error system

Table 1 Non-fragile filter design (with $\delta_1 = \delta_2 = \hat{\delta}_1 = \hat{\delta}_2 = 0.1$.)

Filters	Non-fragile H_2 filter	Non-fragile γ_∞ filter
γ	$\gamma_2^\star = 1.5504$	$\gamma_\infty^\star = 2.7013$
A_f	$\begin{bmatrix} -0.1066 & 6.5624 \\ -0.0012 & 0.5284 \end{bmatrix}$	$\begin{bmatrix} -0.0275 & 5.6011 \\ -0.0011 & -0.0377 \end{bmatrix}$
B_f	$\begin{bmatrix} 0.6650 \\ -0.0444 \end{bmatrix}$	$\begin{bmatrix} 0.5542 \\ -0.0919 \end{bmatrix}$
H_f	$\begin{bmatrix} -0.0018 & -2.8362 \end{bmatrix}$	$\begin{bmatrix} -0.0053 & -6.5852 \end{bmatrix}$
L_f	[0.6739]	[0.2968]

that the structural restriction on the slack matrix does not result in any conservativeness for the standard H_∞ filter design.

5.2 2-D Non-fragile H_2 and H_∞ filter designs

Let $0.15 \leq c = a_1 \leq 0.45$, and $0.35 \leq a_2 \leq 0.85$, respectively. Then the system (100)–(102) represents a four-vertex polytopic system. We design the 2-D non-fragile H_2 and H_∞ filters with the additive coefficient variation of the form (12) and (13), where

$$E_1 = \begin{bmatrix} 0.1 & 0 \end{bmatrix}^T, \ E_2 = 0.1, \tag{103}$$

$$F_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ F_2 = -1. \tag{104}$$

Consider now $\delta_1, \delta_2, \hat{\delta}_1, \hat{\delta}_2 \in [0.1, 1]$. Applying Theorems 1 and 2, the optimal scaling parameters are obtained as $\delta_1 = \delta_2 = \hat{\delta}_1 = \hat{\delta}_2 = 0.1$ using the simple linear search method. The minimum H_2 and H_∞ performances and the non-fragile H_2 and H_∞ filters are given in Table 1.

5.3 Comparison with the 2-D robust H_∞ filter design

Note that the non-fragile H_2 and H_∞ filtering problem reduces to the robust H_2 and H_∞ filtering problem when the designed filter is described by (Σ_f^0) for a polytopic 2-D system (Σ). We now compare the non-fragile H_∞ filter given in Table 1 with a robust H_∞ filter Σ_f^r designed for the same system (100)–(102). Consider the above four-vertex polytopic system. Applying Theorem 2 with $E_1 = E_2 = F_1 = F_2 = 0$, the robust H_∞ filter Σ_f^r is given by

$$A_f = \begin{bmatrix} 0.3253 & 0.2769 \\ 0.0501 & -0.2351 \end{bmatrix}, \quad L_f = 0.5605, \tag{105}$$

$$B_f = \begin{bmatrix} -0.1102 \\ -1.1114 \end{bmatrix}, \quad H_f = \begin{bmatrix} 0.0757 & -0.3865 \end{bmatrix}. \tag{106}$$

which renders the estimation error system robust asymptotically stable with the optimal H_∞ noise attenuation $\gamma^* = 1.4624$, provided that there is no filter coefficient variation. However, with the same filter coefficient variation (103) and (104) for the robust H_∞ filter Σ_f^r, i.e., A_f, B_f, H_f, L_f in (105) and (106) now become $A_f + \Delta A_f$, $B_f + \Delta B_f$, $H_f + \Delta H_f$, $L_f + \Delta L_f$, where

$$\Delta A_f = \begin{bmatrix} \sigma & \sigma \\ 0 & 0 \end{bmatrix}, \quad \Delta B_f = \begin{bmatrix} -\sigma \\ 0 \end{bmatrix},$$
$$\Delta H_f = \begin{bmatrix} \sigma & \sigma \end{bmatrix}, \quad \Delta L_f = -\sigma,$$

with $0 \leq \sigma \leq 0.1$. When $\sigma = 0.1$, the actual achieved H_∞ noise attenuation at a vertex system with $a_1 = c = 0.45$, $a_2 = 0.85$ is $\gamma = 4.3549 > 2.7013$. Thus, it can be seen that the 2-D non-fragile filter is more robust against filter coefficient variations since the variation has been incorporated into the design procedure.

6 Conclusions

In this paper we have considered the new problem of non-fragile H_2 and H_∞ filter designs for 2-D discrete systems in Roesser model with polytopic uncertainties. The filters designed are assumed to be with the additive norm-bounded coefficient variation. As both 2-D systems and filters are allowed to have uncertainties, the filter design problem is quite involved and difficult to solve with conventional methods. By using the technique of imposing a structural restriction on the slack matrix, we have greatly simplified the problem at hand and successfully solved it. An illustrative example has been provided to demonstrate the feasibility and effectiveness of the proposed method.

Acknowledgments This work is supported by A*STAR, Singapore under grants SERC 072 134 0056. The authors would like to thank the anonymous reviewers and the Associate Editor whose insightful and helpful comments greatly improved this paper.

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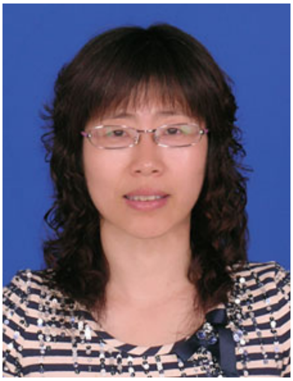
Author Biographies



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