Multibody Syst Dyn (2016) 37:15–28
DOI 10.1007/s11044-015-9498-z

Multibody dynamics as a tool for historical research

Study of an 18th century piano action of Johann Andreas Stein

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Received: 18 February 2015 / Accepted: 10 December 2015 / Published online: 5 January 2016

Abstract Ancient musical instruments can tell us much about the way composers of the past centuries wrote their music. Indeed, the sound and playing characteristics of historical instruments are often very different from those of the instruments we are used to. For example, in the case of the Viennese piano actions used by Mozart and his contemporaries, the so-called "escapement height" largely conditions the response of the instrument to the pianist's touch. In this contribution, we aim to define how the Viennese action behaves when the escapement height, usually tuned by piano technicians, is changed.

To do this, a multibody model containing the frame, the key, the hammer, the pawl, and the string has been developed. This paper describes how the model has been carried out; a special focus is put on the detection of the intermittent contacts between bodies, which may look easy in the real action, but is rather complex to model. The model is compared with high-speed imaging data and a parametric study of the escapement height is performed by adjusting the rest position of the pawl. The high sensitivity of this regulation is revealed as a shift of 1 mm of the pawl seems to induce a displacement of the escapement height of 20 mm. It is also shown that a strong but linear decrease of the maximal force between the hammer and the string appears when the escapement height increases.

Keywords Piano action dynamics · Ancient musical instruments · Johann Andreas Stein · Action tuning

This research is funded by the FNRS-FRESH Fonds de la Recherche en Sciences Humaines.

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Fig. 1 The Viennese action of Johann Andreas Stein was used by Mozart and his contemporaries. A virtual 3D representation of the action is used to visualize the simulation results

1 Introduction

While all modern grand pianos roughly contain the same action models, their predecessors display a remarkable variation in shapes and sizes. Nowadays, a professional pianist would probably play one of Mozart's sonatas on a Steinway grand piano, trying to approach the spirit the musical master has put in his piece of music. One thing he might not be aware of is the huge difference which lies in the piano he is used to play, and the one Mozart composed his music with. Such differences can be crucial when interpreting 18th century music with 21st century instruments: some dynamical expressions were impossible, or sometimes easier to produce, touch and weight of the keys were different, tone quality had other standards than today, etc. For these reasons, it is interesting to look back at the capabilities of older, historical instruments. In engineering terms, these "machines" would probably look obsolete, but in the eyes of a historian or musician defending historically informed performances,[1] they are of the highest importance. The action studied in this article has been conceived by Johann Andreas Stein, a famous piano builder, contemporary of Mozart (Fig. 1).

One of the reasons why a multibody model may be interesting in studying historical piano actions is the fact that their dynamic behavior is largely unknown. Even specialists are often limited in their analysis, on the one hand, because of the complex and very fast interaction between parts of the action, and on the other hand, because of small but crucial differences between the different existing models. As we shall see below, some studies have already highlighted differences between two piano actions which were famous for having been appreciated by Mozart. The present paper aims at highlighting the influence of technicians' settings applied to one of these actions on its behavior.

The paper is organized as follows. Section 2 summarizes the previous contributions in modeling piano actions. The modeling procedure and measurements performed on the parts of the model are illustrated in Sect. 3. It sums up the different bodies of the model and the way contact interaction is computed. Then, a comparison between the model and high-speed camera images is developed in Sect. 4. Finally, Sect. 5 explores the simulation results which correspond to one of the important regulations technicians apply on the action.

2 State of the art

Although the piano has been the topic of many acoustical studies in the past, one had to wait for the last decade before the scientific community started to tackle a complete model of its action with a real multibody approach.

[1] The practice of "historically informed performances" consists in playing music while being aware of the historical background of its composition and performance. This usually implies that the piece of music is played on instruments contemporary of the music itself.

Gillespie published in 1996 a model made of the five main bodies of the grand piano action: the key, the wippen, the escapement, the repetition lever, and the hammer [1]. The bodies are rigid and linked with kinematic constraints. In 2004, Hirschkorn published his thesis [2] in which the dynamic behavior of the piano action is deeply analyzed. It is the first study in which every part of the mechanism is considered independently and with its own physical properties. Consequently, there is no need to adjust global parameters with no physical meaning in order to match the model with experimental results. Each body is characterized independently in order to create a design tool for piano makers. On the recommendation of Hirschkorn, Izadbakhsh [3] introduces flexibility to the hammer shank using Rayleigh's method [3]. The articulation of the key, previously in pure rotation, is replaced by a combination of a rotation and translations, more in line with the real movement. The addition of flexibility influences the mechanism only slightly, but seems to increase the amplitude of the vibrations when the hammer strikes the string. In 2009, Vyasarayani [4] integrated a vibrating string in the multibody model developed by Hirschkorn and Izadbakhsh. He observed an increase in the stick and slip between the hammer and the string and also a decrease in the vibrations in the hammer shank. In the same year, Mamou-Mani and Maniguet [5] used the method developed by Hirschkorn in a comparison of historical piano actions which are not available anymore. Their study compares Érard's pianos preceding his invention of the double escapement. The objective of this research is to identify the characteristics of an instrument which is presently unknown, and to establish whether the underlying mechanism was (or not) precursory of the double escapement principle. More recently, Masoudi has worked on the action of an upright piano and proposed a micromechanical model of the felt based on the structural properties of its fibers [6]. Thorin used a different approach by modeling a grand piano action with non-smooth numerical method [7]. A great part of his work is dedicated to validating his model. In this respect, he considers that a model should be driven using displacement instead of force input.

Research on piano actions contemporary to Mozart has been led by Stephen Birkett [8]. His work compares two similar, yet different actions supposedly used during Mozart's time. The analysis is based on high-speed imaging results of existing reconstructions of the action, but does not yet contain a multibody model of the studied actions. This kind of study enables pointing out important characteristics of the action, but does not allow easily modifying some of its properties in order to compare their importance on the overall behavior.

3 Multibody model

The action we have modeled is a reproduction taken from an original 1788 Johann Andreas Stein piano, located at the Germanisches Nationalmuseum (inventory number MIR1097). The reproduction, currently kept at the Musical Instruments Museum (MIM) in Belgium, represents the highest note of the instrument (F6). The original action also contains a string damper, which we have neglected in the model since it does not significantly participate in the overall motion.

The multibody model of the action is composed of five different bodies: the frame, the key, the pawl, the hammer, and the string (Fig. 2). The model is bi-dimensional, and each body possesses one relative degree of freedom in rotation, which is represented by discontinuous lines. In total, the system thus has three degrees of freedom. Some sub-parts of the bodies are of prime importance and deserve special attention; the *Kapsel* is a wooden fork in which the hammer is hinged so that it can rotate, the pawl is kept upright with the help

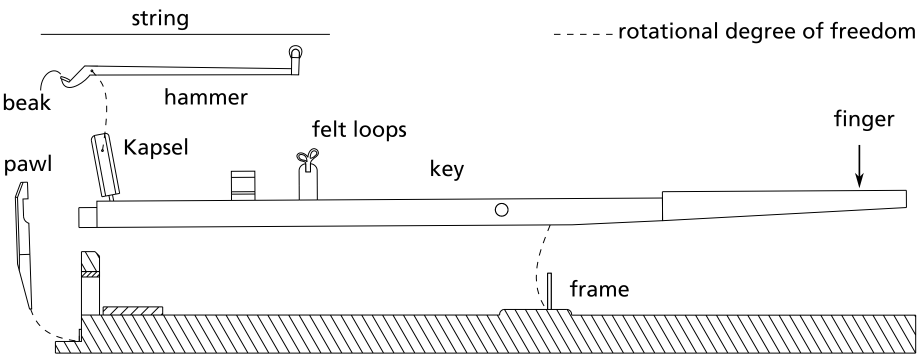


Fig. 2 The model is in 2D and comprises the frame, the key, the hammer, the pawl, and the string

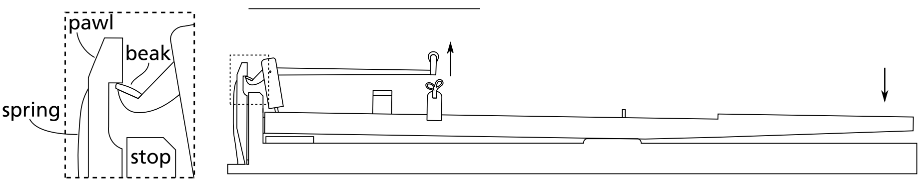


Fig. 3 The rotation of the hammer is caused by the depression of the key and the contact between the hammer beak and the pawl

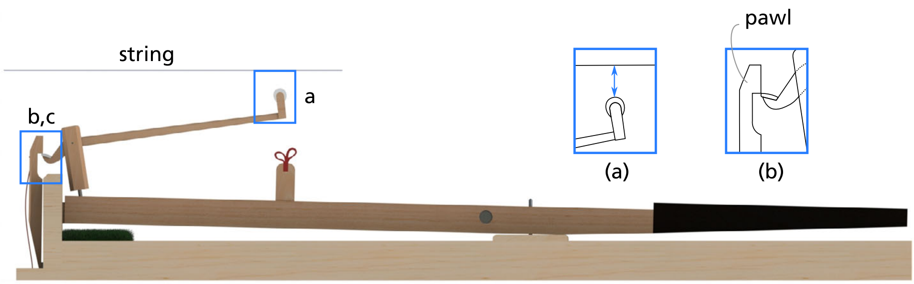


Fig. 4 The hammer escapes when the hammer beak passes the angled corner of the pawl (**b**). The distance between the hammer and the string at that moment is called the "escapement height" (**a**)

of a spring (left side, visible in Fig. 3) and a stop (right side). The hammer beak is a small leather tongue glued on top of the left end of the hammer.

All in all, the functioning of this action is very simple (Fig. 3). When the key is depressed by a pianist's finger, the key turns clockwise, taking with it the hammer which is hinged to the *Kapsel*. The beak of the hammer then comes in contact with the pawl (Fig. 3, dashed rectangle), causing the hammer to turn anticlockwise with respect to the key. This accelerates the hammer's motion and projects it towards the string while the finger keeps pressing the key down. A few millimeters before the hammer hits the string, the leather patch fixed at the beak passes the corner of the pawl (Fig. 3, dashed rectangle). This releases the hammer, allows it to hit the string and freely rebound right after the impact. The hammer then falls down on the felt loops attached to the key which absorb its energy in order to prevent it from bouncing back on the string.

One crucial aspect in the functioning of the action is the so-called "escapement height". Referring to the working process described above, the escapement height is a purely geometrical value characterized by the vertical distance between the hammer and the string (Fig. 4(a)) at the precise moment when the hammer beak escapes from the pawl by passing its angled corner (Fig. 4(b)). In practice, piano technicians adapt the escapement height through successive quasi-static configurations in order to obtain the desired response and touch of a key. The regulation of this key parameter and the precise consequences on the behavior of the action are described in Sect. 5.

3.1 Multibody formulation

The generalized joint accelerations $\ddot{q}$ of the multibody system to which forces and torques are applied are computed via the following dynamic model:

$$M(q,\delta)\ddot{q} + c(q,\dot{q},\delta,f,t,g) = Q(q,\dot{q}) \tag{1}$$

where M is the generalized mass matrix of the system, c the nonlinear dynamic vector (it contains the gyroscopic and centrifugal effects, the external resultant forces f and torques t, and the gravity g), Q the generalized forces and torques applied in the joints, q the relative generalized coordinates, and δ the dynamic parameters of the system (body masses, positions of the center of masses and body inertia tensors). The equations of motion (Eq. (1)) are symbolically generated with ROBOTRAN [9] on the basis of the Newton–Euler recursive scheme and are fully implemented in C language and time integrated with efficient numerical libraries.

3.2 Body characteristics

The components of the model have been scanned in 3D in order to evaluate their dimensions. The action has been disassembled, allowing to measure the mass, the center of mass and the inertia of its components. The inertia has been evaluated via Euler's equation of motion applied to a planar pendulum:

$$I^o\ddot{\theta} + mgd\sin\theta = 0 \tag{2}$$

with I^o being the inertia of the body with respect to rotation axis, θ and $\ddot{\theta}$ its angular position and acceleration, m its mass, and d the distance between the center of rotation and the center of mass. For small movements the equation of motion can be linearized with the approximation $\sin\theta \simeq \theta$. The natural frequency of the system can be rewritten from Eq. (2) as $\nu = \frac{1}{2\pi}\sqrt{\frac{mgd}{I^o}}$. As it is easy to obtain the natural frequency of a pendulum experimentally (Fig. 5), one can retrieve the value of I^o:

$$I^o = \frac{mgd}{(2\pi\nu)^2}, \tag{3}$$

and taking advantage of Steiner's theorem in 2D, one can compute the inertia with respect to the center of mass of the body: $I^G = I^o - m\|d\|^2$.

In the case of the pawl, which was too difficult to disassemble without damaging the action and which is not subjected to a highly dynamic motion, we have determined these physical values with a CAD software.

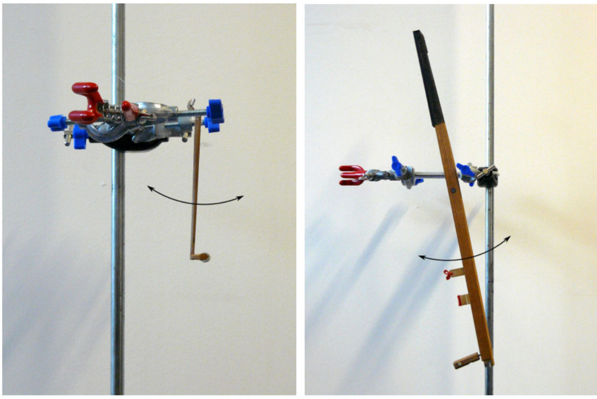


Fig. 5 The inertias of the hammer (*left*) and the key (*right*) have been measured experimentally

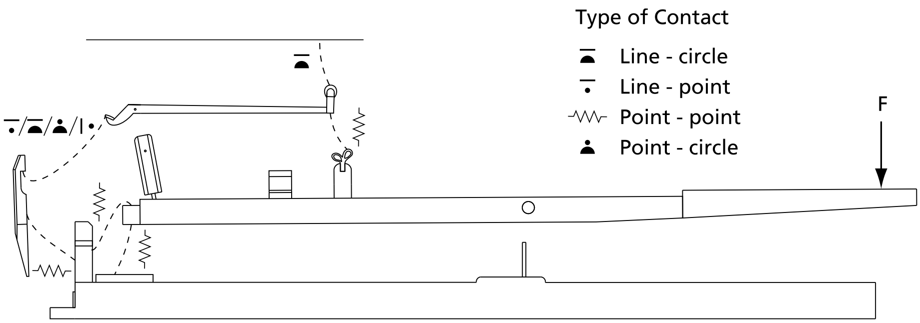


Fig. 6 The contact geometries are approximated with *lines*, *circles*, *points* or a combination of these elements

3.3 Geometrical contact model

Contact modeling requires to carry on two sequential steps. The first one is purely geometrical and consists in finding the contact location between bodies, which can be punctual or linear (rigid case) or areal (flexible case). As regards the latter case, we search for an equivalent contact point to which a contact force resultant will be applied. The second step, which is detailed in Sect. 3.4, is related to the computation of the contact forces issuing from constitutive laws.

The working principle of the piano action is based on multiple percussive contacts which propagate motion through the different parts of the mechanism. Consequently, contact between bodies is of discontinuous nature and thus it should always be ascertained whether pairs of bodies are in contact. In our case, the geometrical contact problem is solved by approximating the body surfaces via simple geometrical primitives: circles, lines, points or a combination of them (Fig. 6). The "point-to-point" case is straightforward since the material contact points involved are known in advance. For all other situations, one needs to find a geometrical contact point on a circle or a line: this can classically be achieved using either an analytical or iterative procedure.

Despite its rudimentary design, the interaction between the hammer beak and the pawl notch (Fig. 3, left) is the trickiest part of the model. The contact being areal, the first step consists in finding the equivalent contact point (Fig. 7). This may sound simple, but is, in

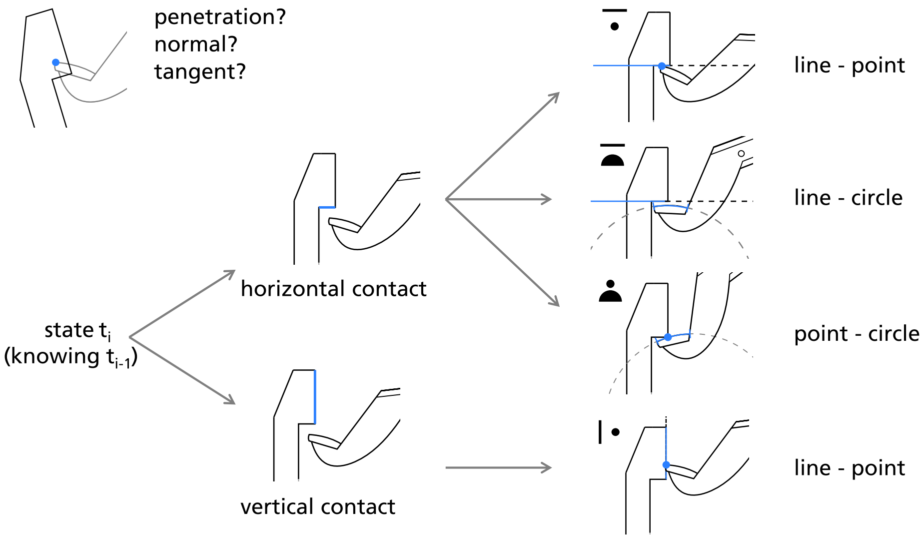


Fig. 7 When the hammer beak and the pawl are in contact, different geometrical primitives can be used to approximate the contact geometry

fact, complex because we have to find a means to choose whether we compute this point relative to the horizontal face of the pawl, its corner or its vertical face.

Once the contact point is found, one needs to define a normal and a tangential direction to the pawl in order to compute the deformation depth and deformation speed of the hammer beak into the pawl.

Based on high-speed camera observations of the interaction between the hammer beak and the pawl, we have defined a first division; contact occurs either on the horizontal face of the pawl notch or on its vertical face (Fig. 7). Next, we have defined four distinct contact configurations which follow a temporal sequence of functioning when the key is depressed. Reading the last column of Fig. 7 from top to down, these phases consist of a contact between (i) the left end of the beak and the horizontal part of the pawl, (ii) the upper surface of the beak and the horizontal part of the pawl, (iii) the upper surface of the beak and the corner of the pawl, and (iv) the left end of the beak and the vertical part of the pawl.

In some cases, it would be impossible to determine which configuration has to be used at $t = t_i$ without knowing the previous state $t = t_{i-1}$ of the two bodies. Even if this representation is exaggerated, it is, for example, impossible to know how to compute the penetration of the hammer beak in Fig. 7, upper left corner. The most correct solution will arise from the sequence of motion, which means the algorithm will always first assess whether the configuration in t_{i-1} is still valid and if it is not the case, test the other configurations.

3.4 Force and torque laws

The interacting parts of the action are mainly composed of wood, leather and felt. Although Masoudi [6] has made substantial efforts to find a suitable law for these scarcely studied materials, we have chosen, for the sake of effectiveness and simplicity, to use the same laws as in Hirschkorn [2]. Force and torque are computed using a derivative of Hunt and Crosley's law [10], changing the power function into a third order polynomial fit. The hysteresis is taken into account thanks to the damping factor D, which has been calibrated to fit the

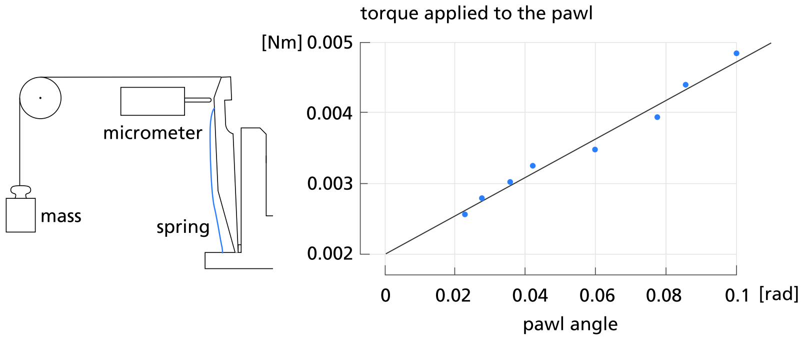


Fig. 8 Considering the importance of the pawl in the action, the characteristics of its spring have been measured using a weight and a displacement sensor

curves at a typical speed for a compression and decompression cycle when the action is used under normal playing conditions. The expression of the normal force is given by

$$f_n = f_{\text{fit}}(x)(1 + D\dot{x}_n) \tag{4}$$

with $f_{\text{fit}} = ax^3 + bx^2 + cx$, x being the deformation of the bodies, D the damping factor, and $\dot{x}_n$ the relative normal velocity of the bodies. The tangential component of the force is modeled with the Coulomb friction, using Cull and Tucker's [11] formula which takes both static and dynamic coefficients of friction into account:

$$\mu = A\left(\tanh\left(\frac{\dot{x}_t}{v_t}\right) + \frac{B_1\dot{x}_t/v_t}{1 + B_2(\dot{x}_t/v_t)^4}\right) \tag{5}$$

with μ being the friction coefficient, $\dot{x}_t$ the relative tangential velocity, v_t the relative threshold velocity, A the dynamic coefficient of friction, B_1 and B_2 coefficients adjusted in order to obtain the right force peak corresponding to the static coefficient of friction. The parameters of the contact model have first been chosen on the basis of their similarity with the materials used in a grand piano action. We have thus taken the parameters from [2], and adapted some of them with experimental measurements in order to make them fit with our particular action.

Torque due to friction in the joints is computed with a law of the same form as in Eq. (5), but applied to a rotational movement. Torque due to the rotational spring at the back of the pawl is of the form

$$\tau = k_\theta(\theta - \theta_0) \tag{6}$$

with k_θ being the equivalent angular stiffness of the pawl return spring, θ the instantaneous, and θ_0 the neutral angle of the pawl. As the resistance of the rotation of the pawl is of high importance for the response of the action, we have evaluated k_θ and θ_0 experimentally by submitting the pawl to various loads and by measuring its resulting inclination with a micrometer (Fig. 8).

4 Comparison with experimental data

The multibody model presented in the previous section has been compared with the real action in motion. The experimental set-up allows reproducing its behavior easily in the model;

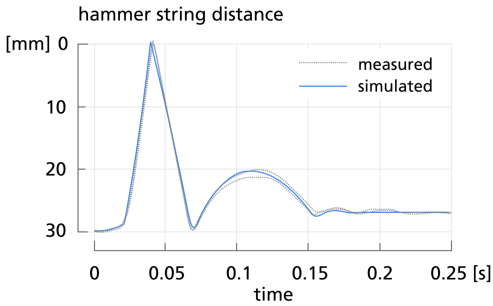


Fig. 9 The evolution of the hammer–string distance of the model is compared with high-speed imaging data of the action submitted to a falling mass of 100 g

for instance, via a suspended mass of 100 g which is flush with the key surface and let off by cutting the wire which supports it. The vertical position of the hammer is then recorded with the help of high speed camera. The free fall of the mass is easily reproduced in the model, and the latter is virtually calibrated in order to match the initial and final positions of the action (Fig. 9).

For the few remaining unknown parameters, such as the stiffness and the damping of the loops, coefficients for Eq. (4) and the friction coefficient between the hammer beak and the pawl, they have been chosen, within a suitable range, by comparing the simulated and the real action, according to the two following criteria. The first criterion is the difference between the measured and the simulated distance between the hammer and the string. The second criterion is the dynamic escapement height of the hammer, which precisely occurs at 2.2 mm of the string when the real action is depressed with the 100 g mass. The result of this identification process gives us a mean discrepancy of 0.014 mm between experimental data and the model (Fig. 9).

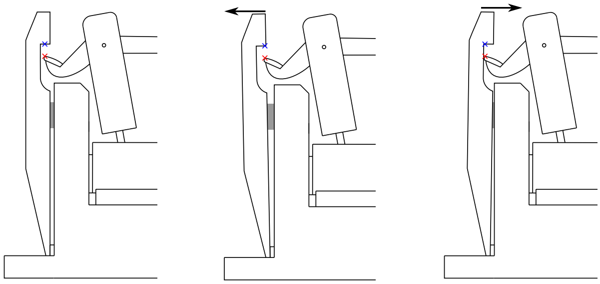
5 Simulation results

In this section, we perform simulations similar to the regulations piano technicians apply to these actions. The aim is, on the one hand, to quantify the precision required to regulate the action, and on the other hand, to evaluate its influence on the playing characteristics of the instrument. For all the simulation results shown in this section, the input is a free falling mass of 100 g which is flush with the key surface. This means the key is kept depressed after the string has been hit.

5.1 Regulation sensitivity of the escapement height

As mentioned previously, the escapement height strongly influences the way the action behaves, and consequently, the sense of touch felt by the pianist. When the escapement height is small, it means the hammer escapes near the string. In this case, the hammer is in free flight during a limited portion of its travel towards the string. According to our technician colleagues of the Musical Instruments Museum Brussels, actions set up with a small escapement height are felt by pianists as being "stiff", but seem to ensure a good control of the action response. A drawback of such a regulation is that the hammer can get jammed against the string, particularly if the key is hit with much force. Increasing the escapement height gives the feeling of a smoother action, but goes together with a loss of precision; it will, for example, be much more complex to produce a controlled *piano* sound because the

Fig. 10 The escapement height is regulated by changing the width of the pawl stop (*grey rectangle*). Compared to the reference case (**a**), the rest position of the pawl moves to the left when plies of paper are added (**b**), and to the right when they are removed (**c**)



portion of stroke during which the hammer is in free flight is much larger. Another drawback is the fact that the sound loses power.

The multibody model allows us to illustrate these behaviors and to confirm the observations made by the technicians when regulating them. In practice, technicians adjust the escapement height in two different ways. The most common one is the horizontal displacement of the stop which defines the rest position of the pawl (grey rectangle in Fig. 10). The second one consists in slightly changing the stiffness of the spring which keeps the pawl in position by bending it.[2] In this work, we will only consider the first type of regulation.

In our particular action, the rest position of the pawl is changed by adding or removing thin pieces of paper behind the stop.[3] A geometrical escapement height of 2–3 mm is ideal because it is supposed to offer a compromise between hammer speed at the impact of the string, sensation of free flight of the hammer and precision of the touch of the action. Referring to a normal setting (Fig. 10a), adding some paper to the stop will move the rest position of the pawl to the left (Fig. 10b). On the other hand, suppressing some paper will shift the pawl to the right (Fig. 10c).

Let us define Δx as the displacement of the stop, with $\Delta x > 0$ meaning a displacement of the pawl to the right. With respect to a reference regulation for which the hammer escapes dynamically[4] at 2.2 mm (Fig. 11), simulations confirm that a shift of the stop to the right ($\Delta x > 0$) causes a decrease of the escapement height, whereas a shift to the left ($\Delta x < 0$) causes an increase of this value. More interestingly, one notes the steep slope of this quasi-linear relationship between escapement height and the initial position of the pawl. Indeed, the relationship of $1/20$ means that adding a sheet of paper of 0.1 mm width changes the escapement position with 2 mm. Knowing that a homogeneous touch of the piano throughout its entire range can only be assured if the escapement height is regulated with a precision in the order of 1 mm, one can understand how meticulous the technician proceeds in his work. It is also remarkable to note how limited the range of action the technician disposes of; outside of the pictured bounds, the model shows that the action does not work anymore, that is, if the pawl stop is shifted more than 1.25 mm to left or 0.2 mm to the right. When $\Delta x < 1.25$ mm, the pawl notch is simply too far from the hammer beak for both bodies to be in contact. Letting the pawl lean too much to the right prevents the hammer from escaping.

[2] Note that changing the spring preload not only changes its behavior during the motion of the action, but also changes the rest position of pawl, just like the first type of regulation.

[3] In later Viennese actions, the stop is an adjustable screw, which makes its regulation easier. Nonetheless, fine regulation can still be performed with pieces of paper.

[4] Let us recall that the key is depressed with a 100 g mass, causing a movement that cannot be considered as quasi-static anymore.

Fig. 11 The relationship between the pawl stop and the dynamic escapement height is quasi-linear. Negative displacement values indicate a shift of the stop to the left. The *grey zones* indicate when the action is not working anymore

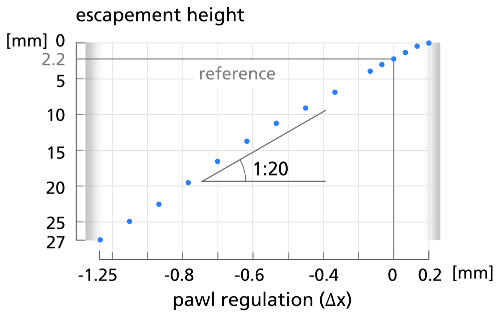
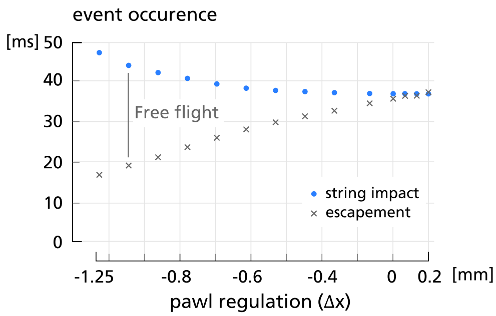


Fig. 12 The moment at which the hammer escapes and at which it hits the string is conditioned by the regulation of the pawl. The free flight of the hammer is the period between those two events, when the hammer flies independently of the action of the pianos



5.2 Event timing of the action

In order to have a more precise idea of what happens when the escapement height varies, it is interesting to observe the timing of two events during a key stroke for each regulation. Figure 12 has to be read bottom-up, starting at $t = 0$ s where the key is hit. The first marker shows the moment at which the hammer escapes; the more the pawl is shifted to the left, the earlier the hammer escapes. This is also in agreement with the increase in escapement height (Fig. 11); escaping earlier means the action escapes further from the string. Conversely, it is fairly logical that the escapement is delayed as the pawl is moved more to the right.

The upper markers indicate the moment at which the hammer hits the string. Going from left to right, this event appears asymptotically sooner when the pawl is moved to the right. This is a first hint indicating the sense of control the action confers to the pianist; the earlier the hammer escapes, the later the sound will be produced. Another important factor is the so-called “free flight” during which the hammer moves towards the strings, without the pianist being able to control it anymore. The free flight of the hammer happens between the escapement and the impact of the string. Intuitively, it seems logical that being unable to control the hammer over a long period of time gives rise to a less controllable action.

When $\Delta x > 0$, the markers are almost indistinguishable. With a closer look, one even realizes that the hammer escapes after having hit the string for a regulation near $\Delta x =$ 0.2 mm. When this happens, the finger of the pianist controls the hammer until it reaches the string, but has to force the movement of the key in order to let the hammer beak escape from the pawl notch. This phenomenon goes together with an audible “click” caused by the forced escapement of the hammer. The musician masters the behavior of the hammer throughout its entire movement, but the produced sound will certainly be poor and muffled because the prolonged contact of the hammer on the string mutes the first vibrations of

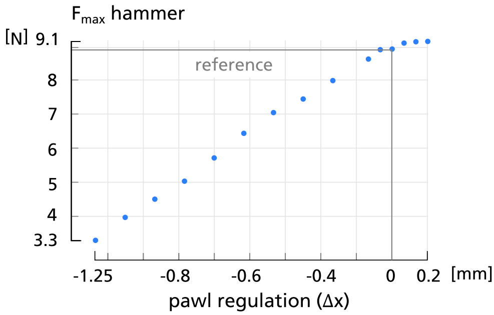


Fig. 13 The maximal force applied by the hammer on the string diminishes linearly when the pawl is moved to the left. Shifting the pawl to the right does not significantly increase the force which saturates

the latter. From the touch point of view, forcing the escapement is not acceptable since it requires a more important force and leads to a non-smooth action.

5.3 String impact force

Another important and logical characteristic of the free flight of the hammer is the fact that the hammer stops being propelled by the pawl notch, causing it to decelerate. This does not only affect the response time of the action, but also the force with which the string is struck (Fig. 13). The relation between the peak force on the string and the displacement of the pawl is linear to a large extent. This is specially the case for $\Delta x < 0$; on this side of the reference, the action loses approximately 1 N for every 0.2 mm the pawl stop is displaced. However, when $\Delta x > 0$, the force keeps approximately constant. When this happens, the pianist controls the hammer up to the point where it hits the string, meaning the hammer never decelerates, but is also unable to accelerate even more. Having reached a threshold limit in terms of impact speed, the force is also limited.

5.4 Discussion

Let us summarize these different observations and confront them with the experience of pianists and technicians. It seems the terms "rigid" and "smooth" used to define the feedback of the key is influenced by the instant of escapement and string impact, and probably even more by the duration of the free flight of the hammer. The sense of control is linked to the same values, as a large escapement height not only induces an increase of free flight duration (that is, of uncontrolled hammer travel), but also a longer time between the key depression and the produced sound. Considering the auditory feedback, it appears that pushing the pawl to the left potentially has a strong decreasing influence on the amplitude of the sound. Conversely, diminishing the escapement height even more than in the reference cases induces a very limited effect.

Having observed this via the model, we can appreciate the advantages of a well regulated action with the escapement height set between 2 mm and 3 mm. Set at this distance, the free flight is limited to a very small portion of the hammer travel; it enables the pianist to control the motion of the hammer up to the very end and to benefit from an ideal maximal force.

6 Conclusions and prospects

In this contribution, we have presented a multibody model of a Viennese action made in 1788 by Johann Andreas Stein. Besides the common elements found in such a model, like the constitutive laws for materials and the experimental comparison between a real action and the model, this paper has given the principles to address the modeling issues due to the very specific interaction between the hammer beak and the pawl which is of central importance regarding the action functioning.

The model in itself has allowed us to evaluate the order of precision required by technicians when they regulate the action; shifting the stop of the pawl by 1 mm moves the escapement height by 20 mm. Knowing that the escapement height has to be regulated with a precision of approximately 1 mm, it follows that technicians have to adjust the stop of the pawl with a precision of 0.05 mm. With a well-regulated action, the pianist feels he has a good control over the whole motion of the action. A loss of control could be due to the response time of the hammer but also to the escapement height that we have quantified for different regulations of the pawl. Finally, we have pointed out that the potential of the action to hit the string with force diminishes as the escapement height increases. Conversely, decreasing the escapement height more than its prescribed value does not lead to a significant increase in hammer–string force and can even cause the hammer to get temporarily jammed against the strings.

Future research concerning the Viennese actions relates to both the modeling part and the historical aspects of the project. As regards the engineering part, we are working on a real-time simulator of the action so that the regulation process could be performed dynamically, without interrupting the simulation. Another enhancement concerns the key input we use for the real action. For this part, we are developing an actuator which can reproduce a more realistic and repeatable input, equivalent to the finger force exerted on the key.

As regards the historical question, musicologists have observed that Viennese actions varied only slightly in design during the time they were used. Only one of its components, the pawl, has shown a noticeable evolution in shape. Specifically, the inclination of the pawl notch was of 90° in the earlier instruments similar to the one studied in this paper. Later instruments almost systematically had a notch angle which was less than 90°. Musicologists are not sure about the reason of this evolution and a dynamic model in which the pawl inclination can be changed might enhance our understanding of this morphological transformation. This part of the project is being carried out with Paul Poletti (Museu de la Música, Barcelona), a piano maker specialized in ancient pianofortes.

Acknowledgements The authors would like to thank the Musical Instruments Museum Brussels for its help, and more particularly Pierre Gevaert, the piano technician of the museum.

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