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Solving the kinematics and dynamics of a modular spatial hyper-redundant manipulator by means of screw theory

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Abstract In this contribution, a systematic methodology for solving the kinematic and dynamic analyses of a modular spatial hyper-redundant manipulator built with an optional number of serially connected three-legged in-parallel manipulators are presented.

First, the kinematics and dynamics of the base module are carried out using the theory of screws and the principle of virtual work. Next, the expressions obtained for the base module are extended without significant effort to the spatial hyper-redundant manipulator under study. Finally, the proposed methodology of analysis is applied to a 18 degrees of freedom hyper-redundant manipulator.

Keywords Hyper-redundant manipulator · Modularity · Driving force · Screw theory · Forward dynamics

1 Introduction

Modularity is a key concept to overcome the drawbacks of both parallel and serial manipulators. In fact, the poor dexterity and reduced workspace of parallel manipulators, together with the poor stiffness and accuracy of serial manipulators, can be ameliorated by application of the concept of modular robotic system. Furthermore, if the modules have the possibility of connect to, disconnect from, and relocate to, without external help, then it is said that the robot is self-reconfigurable [1].

On the other hand, the redundancy concept, and its benefits, like avoidance of obstacles and singularities, among many others has been widely studied for serial manipulators, see, for instance [2–8]. In that way, a redundant parallel manipulator can be physically realized

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in three ways: (i) adding at least one more active equivalent limb to the existing nonredundant topology, (ii) increasing the number of active joints in any limb such that the number of active joints is higher than needed, and (iii) modifying one of the existing limbs such that it is totally different than others with more than one active joint; for details see Dasgupta and Mruthyunjaya [9]. It must be noted that while the second way does not increase the motion degree of freedom of the manipulator, the remaining two do, naturally considering that the first option implies that the additional limbs are provided with passive and active joints. With this in mind, the redundancy in parallel manipulators can be divided into two classes [10]: (i) actuation redundancy which refers to activating one or more of the existing passive joints of the manipulator—not increasing the motion degree of freedom, (ii) kinematic redundancy [11, 12]; which refers to either adding a new equivalent limb to the existing limbs of the manipulator or adding one or more active joints to one of the limbs of the mechanism, increasing the motion degree of freedom. According to this classification, the actuation redundancy is a subset of the kinematic redundancy.

In robot kinematics, the concepts of modularity and redundancy can be understood in different ways. In fact, modularity can be considered as the possibility of systematically select and change, the kinematic pairs of serial and parallel manipulators, including hybrid applications, without affecting the performance of the mechanical system under study, especially the freedoms of the mechanism; interchangeability is a key word in this option. In that way, the role of the kinematic pairs can be studied applying a wide variety of strategies, including the finite element method [13]. More popularly, modular robotic systems with redundant freedoms can be obtained when two or more parallel manipulators are assembled in series connection.

Examples of the performance of mechanisms with the above mentioned characteristics, known as modular spatial hyper-redundant manipulators (hereafter MSHRM for brevity) can be found in nature itself, giving birth to mechanical systems with illustrative names like snake [14, 15]; serpentine [16], tentacle [17] or elephant's trunk [18]. However, from a rigorous mechanical point of view, most contributions dealing with the study of modular robotic systems are dedicated or limited to the so-called serial-parallel manipulators, in other words, mechanisms where two manipulators are assembled in series connection, see, for instance [19–25]. Perhaps the handling of several parameters or the lack of a systematic methodology of analysis increases the difficulty of the extension of these works to the full dynamics of MSHRMs built with an optional number of modules.

In this work, the kinematics and dynamics of modular spatial hyper-redundant manipulators formed from Revolute+Prismatic+Spherical and RPS-type limbs are addressed via the theory of screws and the principle of virtual work.

2 Description of the modular spatial hyper-redundant manipulator

The base module of the MSHRM considered here is the well-known 3-RPS parallel manipulator, Fig. 1.

This popular in-parallel manipulator was introduced by Hunt [26] and consists of a moving platform and a fixed platform connected to each other by means of three extendible limbs type RPS. The limbs are connected, respectively, at the moving platform and at the fixed platform by means of three distinct spherical joints and three distinct revolute joints. According to the classical Kutzbach–Grübler criterion, this spatial mechanism has three degrees of freedom, two rotations, and one translation. Usually, the prismatic joints are chosen as the active joints.

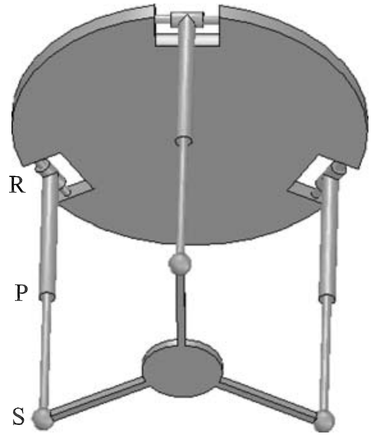


Fig. 1 3-RPS parallel manipulator

The 3-RPS parallel manipulator has been the motive of an exhaustive research field approaching a wide variety of topics like kinematic and dynamic analyses, synthesis, singularities, and so on, see, for instance [27–29]. In particular, screw theory has been proved to be an efficient mathematical resource for determining the kinematic characteristics of 3-RPS parallel manipulators [30–32]; including the instantaneous motion of the mechanism at the level of velocity analysis [33–35]; and the acceleration analysis [36, 37]. In Alizade and Bayram [38], the structural synthesis of two 3-dof parallel manipulators assembled in series connection is carried out firstly identifying the number of different structural groups, and secondly using the principle of interchangeability of kinematic joints. In Lu and Leinonen [24], the position analysis of a mechanism composed of two 3-RPS parallel manipulators assembled in tandem, called the spatial 2(3-RPS) manipulator is investigated. The computation of driving or generalized forces of a similar mechanism, called 2(3-SPR) manipulator, was approached in Lu and Hu [25] applying the principle of virtual work and the so-called CAD variation geometry.

It is well known that lower mobility parallel manipulators, in other words spatial mechanisms with fewer than six freedoms, strangely known as defective parallel manipulators, cannot perform sophisticated motions. Moreover, in order to operate properly a defective parallel manipulator, it is necessary to take into account that when the configuration space is a *constraint singularity*, distinct modes of operation are expected for the parallel manipulator, Zlatanov et al. [39]. Furthermore, with the purpose to improve the mobility and manipulability of a 3-RPS parallel manipulator, a serial-parallel manipulator or double parallel manipulator can be obtained assembling in series two 3-RPS parallel manipulators; a suitable topology is presented in Fig. 2.

The topology proposed in Fig. 2 has the advantage that unlike the method of position analysis introduced in Lu and Leinonen [24], the kinematics of the first parallel manipulator is applicable, without significative changes to the second parallel manipulator, only it is necessary to take into proper account the corresponding reference frames avoiding the inclusion of a large number of parameters. Furthermore, the topology presented in Fig. 2 is compact and can be extended to any number of modules; this is the idea of the present work. In fact, once a module is chosen as the base of the MSHRM, the next module is added in such a way that the axes of its revolute joints are concurrent at the geometric centers of the corre-

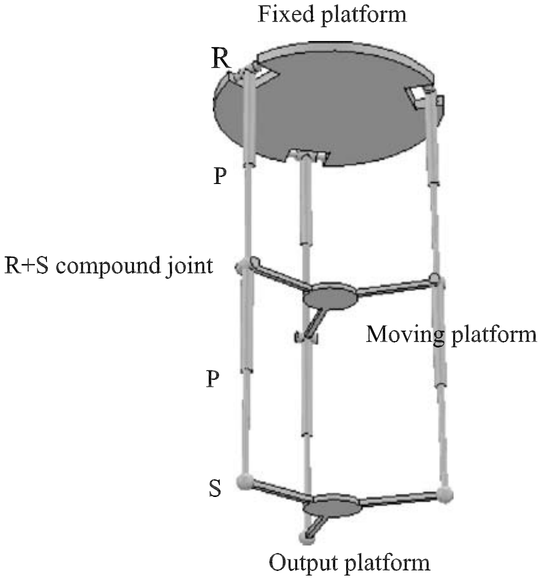


Fig. 2 2(3-RPS) manipulator with a compact topology

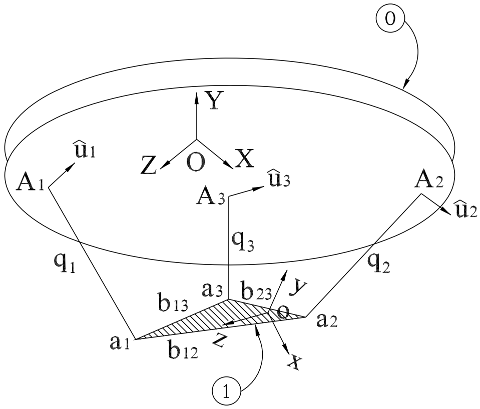


Fig. 3 3-RS structure

sponding spherical joints of the previous module. The process is continued until the specific MSHRM has been reached. In particular, the last platform is called the *output platform*.

3 Kinematic model

3.1 Finite kinematics

In this subsection, the forward position analysis (FPA) of the proposed hyper-redundant manipulator is carried out analytically using simple geometric procedures for a detailed explanation of the FPA of a 3-RPS parallel manipulator, which has a direct connection with this subsection; the reader is referred to [36, 40, 41].

Consider the 3-RPS parallel manipulator shown in Fig. 1. When the three prismatic joints are locked, the parallel manipulator becomes the 3-RS structure shown in Fig. 3.

Under this consideration, the FPA of the parallel manipulator is established as follows. Given the limb lengths q_i, $i \in \{1, 2, 3\}$, compute the feasible locations of the moving platform, body 1, with respect to the fixed platform, body 0, through the computation of the coordinates of the centers of the spherical joints attached at the moving platform, points $\mathbf{a}_i$, $i \in \{1, 2, 3\}$, expressed in the reference frame XYZ.

From the geometry of the mechanism, see Fig. 3, the closure equations of the parallel manipulator can be written as follows

$$\left.\begin{aligned} &(\mathbf{a}_i - \mathbf{A}_i) \cdot \hat{u}_i = 0, \quad i \in \{1, 2, 3\},\\ &(\mathbf{a}_i - \mathbf{A}_i) \cdot (\mathbf{a}_i - \mathbf{A}_i) = q_i^2, \quad i \in \{1, 2, 3\},\\ &(\mathbf{a}_2 - \mathbf{a}_3) \cdot (\mathbf{a}_2 - \mathbf{a}_3) = {b_{23}}^2,\\ &(\mathbf{a}_1 - \mathbf{a}_3) \cdot (\mathbf{a}_1 - \mathbf{a}_3) = {b_{13}}^2,\\ &(\mathbf{a}_1 - \mathbf{a}_2) \cdot (\mathbf{a}_1 - \mathbf{a}_2) = {b_{12}}^2, \end{aligned}\right\} \tag{1}$$

where $\hat{u}_i = (u_{Xi}, 0, u_{Zi})$ is the i-th unit vector along the screw axis of the i-th revolute joint, and $\mathbf{A}_i$, $i \in \{1, 2, 3\}$ are the nominal coordinates of the three distinct revolute joints attached to the fixed platform.

Expressions (1) represent a system of 9 equations in the unknowns X_i, Y_i, Z_i, $i \in \{1, 2, 3\}$. After a few computations, a higher nonlinear system of three equations in the unknowns Z_1, Z_2, and Z_3 are obtained as follows:

$$\left.\begin{aligned} &K_1' Z_2^2 + K_2' Z_3^2 + K_3' Z_2^2 Z_3 + K_4' Z_2 Z_3^2\\ &\quad + K_5' Z_2 Z_3 + K_6' Z_2 + K_7' Z_3 + K_8' = 0,\\ &K_1'' Z_1^2 + K_2'' Z_3^2 + K_3'' Z_1^2 Z_3 + K_4'' Z_1 Z_3^2\\ &\quad + K_5'' Z_2 Z_3 + K_6'' Z_1 + K_7'' Z_3 + K_8'' = 0,\\ &K_1''' Z_1^2 + K_2''' Z_2^2 + K_3''' Z_1^2 Z_2 + K_4''' Z_1 Z_2^2\\ &\quad + K_5''' Z_1 Z_2 + K_6''' Z_1 + K_7''' Z_2 + K_8''' = 0, \end{aligned}\right\} \tag{2}$$

where K_*', K_*'', and K_*''' are coefficients that are calculated according to the parameters and generalized coordinates of the parallel manipulator.

In order to solve (2), the application of the Sylvester dyalitic elimination method leads to

$$\mathrm{M}_1 \begin{bmatrix} Z_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{3}$$

and

$$\mathrm{M}_2 \begin{bmatrix} Z_2^3 \\ Z_2^2 \\ Z_2^1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{4}$$

where

- $\mathrm{M}_1 = \begin{bmatrix} \mathrm{p}_1\mathrm{p}_5 - \mathrm{p}_2\mathrm{p}_4 & \mathrm{p}_1\mathrm{p}_6 - \mathrm{p}_3\mathrm{p}_4 \\ \mathrm{p}_3\mathrm{p}_4 - \mathrm{p}_1\mathrm{p}_6 & \mathrm{p}_3\mathrm{p}_5 - \mathrm{p}_2\mathrm{p}_6 \end{bmatrix}$,

•

$$M_2 = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{13}p_7 - p_{12}p_8 & p_{14}p_7 - p_{12}p_9 & -p_{12}p_{10} & -p_{12}p_{11} \\ p_{12} & p_{13} & p_{14} & 0 \\ p_{12}p_9 - p_7p_{14} & p_{12}p_{10} + p_{13}p_9 - p_8p_{14} & p_{12}p_{11} + p_{13}p_{10} & p_{13}p_{11} \end{bmatrix},$$

- p_1, p_2, p_3 are second-order polynomials in Z_2,
- $p_4, p_5, p_6, \ldots, p_{14}$ are second-order polynomials in Z_1.

Thus, from (3), one eliminant is obtained when $\det M_1 = 0$, canceling the variable Z_3. Furthermore, a 16-th order polynomial expression in the unknown Z_1 is derived taking into account that in order to avoid arbitrary solutions necessarily from (4), $\det M_2 = 0$, canceling Z_2. Once the 16 solutions of Z_1 are calculated, the feasible values for the coordinates of the points $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are obtained using (1).

At this point, it is prudent to emphasize that as it is pointed out by one of the reviewers, the solution of a higher nonlinear system of equations using the Sylvester dyalitic elimination method, in general, is not free of spurious solutions. Thus, the user must check in the closure equations each one of the possible 16 solutions.

Once the coordinates of the points $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are calculated,[1] the geometric center of the moving platform expressed in the reference frame XYZ, vector ${}^0\rho^1 = (\rho_X, \rho_Y, \rho_Z)$ results in

$${}^0\rho^1 = (\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3)/3. \tag{5}$$

Furthermore, since the coordinates of the points $\mathbf{a}_i$, $i \in \{1, 2, 3\}$ are easily expressed in the reference frame xyz, the corresponding 4×4 transformation matrix between the reference frames XYZ and xyz, ${}^0T^1$, results in

$${}^0T^1 = \begin{bmatrix} {}^0R^1 & {}^0\rho^1 \\ 0_{1\times 3} & 1 \end{bmatrix}, \tag{6}$$

where ${}^0R^1$ is the rotation matrix.

The computation of the rotation matrix ${}^0R^1$ with the resulting coordinates, expressed in the reference frame XYZ, of the three points $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ can be simplified by choosing appropriate locations for the reference frames XYZ and xyz.

Consider a rigid plate $A_1A_2A_3$; see Fig. 4 with a reference frame XYZ attached to it, and consider that after a Euclidean displacement, the pose of the plate changes according to the coordinates of the points $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, expressed in the reference frame XYZ. Furthermore, by assuming that the Y axis of the reference frame XYZ is perpendicular to the plane $A_1A_2A_3$, while the y axis of the reference frame xyz is perpendicular to the plane $a_1a_2a_3$. The rotation matrix ${}^0R^1$ between these reference frames results in

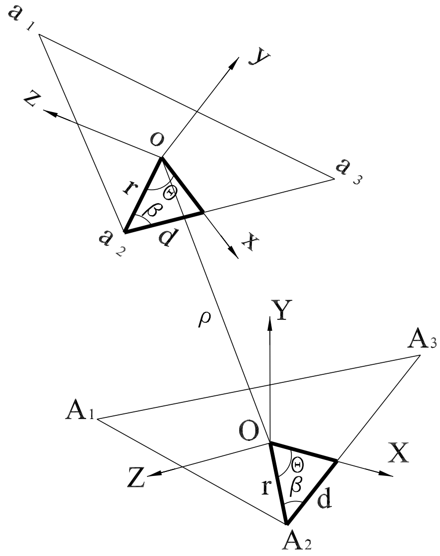
$${}^0R^1 = [\hat{u}_x \quad \hat{u}_y \quad \hat{u}_z], \tag{7}$$

where

- $\hat{u}_x = ((\rho - \mathbf{a}_2) + \lambda(\mathbf{a}_3 - \mathbf{a}_2))/\|((\rho - \mathbf{a}_2) + \lambda(\mathbf{a}_3 - \mathbf{a}_2))\|$ is a unit vector along the x axis,
- $\lambda = d/\|\mathbf{a}_3 - \mathbf{a}_2\|$,

[1] It is straightforward to show that once the feasible values for Z_1 are calculated, the remaining components of the coordinates of the points $\mathbf{a}_i$ are easily obtained from (1).

Fig. 4 Euclidean displacement of a rigid plate



- $d = r\sin\theta/\sin(\pi-\beta-\theta)$,
- $\hat{u}_y = (\mathbf{a}_1-\mathbf{a}_2)\times(\mathbf{a}_3-\mathbf{a}_2)/\|(\mathbf{a}_1-\mathbf{a}_2)\times(\mathbf{a}_3-\mathbf{a}_2)\|$ is a unit vector along the y axis, and
- $\hat{u}_z = \hat{u}_x \times \hat{u}_y$ is a unit vector along the z axis.

The computation of the rotation matrix using the procedure introduced here, as far as the authors are aware, is original and can be considered as an additional novel outcome of this contribution.

Finally, assuming that a MSHRM is composed of n modules, then the transformation matrix of each moving platform, with respect to the fixed platform, body 0 is obtained applying recursively the procedure described in this subsection. Indeed,

$$^0\mathrm{T}^k = \prod_{j=0}^{k-1} {}^j\mathrm{T}^{j+1}, \quad k \in \{1, 2, \dots, n\}. \tag{8}$$

3.2 Infinitesimal kinematics of the modular spatial hyper-redundant manipulator

In this subsection, the velocity and acceleration analyses of the MSHRM are carried out by means of the theory of screws. The kinematics of open serial chains using screw theory is the basis of this subsection; for a detailed explanation of it the reader is referred to [42–44].

Consider the 3-RPS parallel manipulator shown in Fig. 1. In order to satisfy the rank of the Jacobian matrix spanned by the infinitesimal screws of the limbs of the mechanism, the parallel manipulator is modeled as a 3-CPS manipulator (CPS = Cylindrical + Prismatic + Spherical), in which the translational displacements of the cylindrical joints are null. Figure 5 shows the infinitesimal screws of one limb of the 3-CPS parallel manipulator.

With this consideration, the velocity state $^0\mathbf{V}_O^1 = [^0\omega^1\ {}^0\mathbf{v}_O^1]^{\mathrm{T}}$ and the reduced acceleration state $^0\mathbf{A}_O^1 = [^0\dot{\omega}^1\ {}^0\mathbf{a}_O^1 - {}^0\omega^1 \times {}^0\mathbf{v}_O^1]^{\mathrm{T}}$ of the moving platform, body 1, with respect to the fixed platform, body 0, can be obtained, respectively, in screw form through any of the i-th

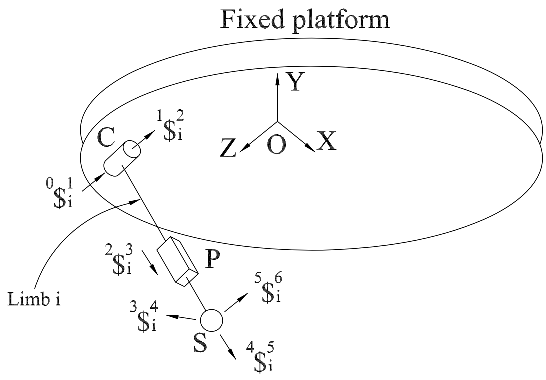


Fig. 5 The limb of a 3-CPS parallel manipulator with its infinitesimal screws

limbs, $i \in \{1, 2, 3\}$, of the parallel manipulator as follows

$$ {}^{0}\mathbf{V}_{O}^{1} = \begin{bmatrix} {}^{0}\omega^{1} \\ {}^{0}\mathbf{v}_{O}^{1} \end{bmatrix} = \sum_{j=0}^{5} {}_{j}\omega_{j+1}^{i} \, {}^{j}\$_{i}^{j+1} \tag{9} $$

and

$$ {}^{0}\mathbf{A}^{1}{}_{O} = \begin{bmatrix} {}^{0}\dot{\omega}^{1} \\ {}^{0}\mathbf{a}_{O}^{1} - {}^{0}\omega^{1} \times {}^{0}\mathbf{v}_{O}^{1} \end{bmatrix} = \sum_{j=0}^{5} {}_{j}\dot{\omega}_{j+1}^{i} \, {}^{j}\$_{i}^{j+1} + \$_{\mathrm{Lie}_i} \tag{10} $$

where

- ${}^{0}\omega^{1}$ and ${}^{0}\dot{\omega}^{1}$ are the angular velocity and acceleration of the moving platform,
- ${}^{0}\mathbf{v}_{O}^{1}$ and ${}^{0}\mathbf{a}_{O}^{1}$ are the velocity and acceleration of a point O fixed to the moving platform which is instantaneously coincident with a point of the fixed platform,
- ${}_{k}\omega_{k+1}^{i}$ and ${}_{k}\dot{\omega}_{k+1}^{i}$ are the joint velocity and acceleration rates of body $k+1$ with respect to the adjacent body k, both in the same limb. Particularly, ${}_{2}\omega_{3}^{i} = \dot{q}_i$ is the joint velocity rate of the i-th actuated prismatic joint associated to the extendible limb, while ${}_{0}\omega_{1}^{i} = 0$ is the joint velocity rate of the i-th prismatic joint associated to the corresponding cylindrical joint,
- $\$_{\mathrm{Lie}_i}$ is the i-th *Lie screw* given by

$$ \$_{\mathrm{Lie}_i} = \sum_{j=0}^{5} \left[{}_{j}\omega_{j+1}^{i} \, {}^{j}\$_{i}^{j+1} \quad \sum_{k=j+1}^{5} {}_{k}\omega_{k+1}^{i} \, {}^{k}\$_{i}^{k+1} \right] $$

  and the brackets $[* \quad *]$ denote the Lie product.

Thus, if the motion of the moving platform is described by the velocity state ${}^{0}\mathbf{V}_{O}^{1}$ and the reduced acceleration state ${}^{0}\mathbf{A}_{O}^{1}$, the required joint rates that satisfy these kinematic states, or in other words, the inverse infinitesimal kinematics of the 3-CPS parallel manipulator are calculated by means of the expressions

$$ {}^{0}\Omega^{1}{}_{i} = \left({}^{0}\mathrm{J}^{1}{}_{i}\right)^{-1} {}^{0}\mathbf{V}_{O}^{1} \tag{11} $$

and

$$ {}^{0}\dot{\Omega}^{1}{}_{i} = \left({}^{0}\mathrm{J}^{1}{}_{i}\right)^{-1} \left({}^{0}\mathbf{A}_{O}^{1} - \$_{\mathrm{Lie}_i}\right), \tag{12} $$

therein

- ${}^0\mathrm{J}^1{}_i = [{}^0\$_i^1\ {}^1\$_i^2\ {}^2\$_i^3\ {}^3\$_i^4\ {}^4\$_i^5\ {}^5\$_i^6]$ is the i-th Jacobian matrix of the i-th limb,
- ${}^0\Omega^1{}_i = [{}_0\omega_1^i\ {}_1\omega_2^i\ {}_2\omega_3^i\ {}_3\omega_4^i\ {}_4\omega_5^i\ {}_5\omega_6^i]^{\mathrm{T}}$ is the i-th matrix of joint velocity rates of the i-th limb,
- $\dot{\Omega}_i = [{}_0\dot{\omega}_1^i\ {}_1\dot{\omega}_2^i\ {}_2\dot{\omega}_3^i\ {}_3\dot{\omega}_4^i\ {}_4\dot{\omega}_5^i\ {}_5\dot{\omega}_6^i]^{\mathrm{T}}$ is the i-th matrix of joint acceleration rates of the i-th limb.

On the other hand, by applying the concept of reciprocal screws, via the Klein form of the Lie algebra $e(3)$, $\{*;*\}$ to expressions (9) and (10), the forward infinitesimal kinematics of the 3-CPS parallel manipulator is solved by means of the expressions

$$\left({}^0\mathrm{J}^1\right)^{\mathrm{T}} \Delta\, {}^0\mathbf{V}_O^1 = {}^0\mathrm{Q}^1{}_{\mathrm{vel}} \tag{13}$$

and

$$\left({}^0\mathrm{J}^1\right)^{\mathrm{T}} \Delta\, {}^0\mathbf{A}^1{}_O = {}^0\mathrm{Q}^1{}_{\mathrm{accel}} \tag{14}$$

where

- ${}^0\mathrm{J}^1 = [{}^4\$_1^5\ {}^4\$_2^5\ {}^4\$_3^5\ {}^3\$_1^4\ {}^3\$_2^4\ {}^3\$_3^4]$ is the active Jacobian matrix of the parallel manipulator,
- ${}^0\mathrm{Q}^1{}_{\mathrm{vel}} = [{}^0\dot{q}^1{}_1\ {}^0\dot{q}^1{}_2\ {}^0\dot{q}^1{}_3\ 0\ 0\ 0]^{\mathrm{T}}$ is a matrix composed of the active joint velocity rates of the parallel manipulator,
- ${}^0\mathrm{Q}^1{}_{\mathrm{accel}} = \begin{bmatrix} {}^0\ddot{q}^1{}_1+\{{}^4\$_1^5;\$_{\mathrm{Lie}_1}\} \\ {}^0\ddot{q}^1{}_2+\{{}^4\$_2^5;\$_{\mathrm{Lie}_2}\} \\ {}^0\ddot{q}^1{}_3+\{{}^4\$_3^5;\$_{\mathrm{Lie}_3}\} \\ \{{}^3\$_1^4;\$_{\mathrm{Lie}_1}\} \\ \{{}^3\$_2^4;\$_{\mathrm{Lie}_2}\} \\ \{{}^3\$_3^4;\$_{\mathrm{Lie}_3}\} \end{bmatrix}$ is a matrix due to the active joint acceleration rates of the parallel manipulator, and
- $\Delta = \begin{bmatrix} \mathbf{0} & \mathrm{I}_3 \\ \mathrm{I}_3 & \mathbf{0} \end{bmatrix}$ is an operator of polarity.

Taking into proper account the reference frames, the velocity state, and the reduced acceleration state of the k-th platform with $k \in \{1, 2, \ldots, n\}$ are obtained as follows

$$\left.\begin{aligned} {}^0\mathbf{V}_O^k &= \sum_{j=0}^{k-1} {}^j\mathbf{V}_O^{j+1}, \\ {}^0\mathbf{A}_O^k &= \sum_{j=0}^{k-1} {}^j\mathbf{A}_O^{j+1} + \sum_{j=1}^{k-2} \left[{}^j\mathbf{V}_O^{j+1} \quad \sum_{i=j+1}^{k-1} {}^i\mathbf{V}_O^{i+1} \right] \end{aligned}\right\}. \tag{15}$$

Furthermore, the angular velocity of the k-th platform, with respect to the fixed platform, is obtained as the primal part of the velocity state, ${}^0\omega^k = \mathrm{P}({}^0\mathbf{V}_O^k)$, while the translational velocity of the center of the same platform, with respect to the fixed platform, vector $\mathbf{v}_{Ck}$, is calculated according to the condition of helicoidal fields [45, 46]; using the dual part, $\mathrm{D}({}^0\mathbf{V}_O^k)$, of the six-dimensional vector ${}^0\mathbf{V}_O^k$, as follows

$$\mathbf{v}_{Ck} = \mathrm{D}\left({}^0\mathbf{V}_O^k\right) + \mathrm{P}\left({}^0\mathbf{V}_O^k\right) \times {}^0\rho^k, \tag{16}$$

where ${}^0\rho^k$ is a vector from the origin of the reference frame XYZ to the center of the k-th platform.

Similarly, the angular acceleration of the k-th, platform, with respect to the fixed platform, is obtained as the primal part of the accelerator ${}^0\mathbf{A}_O^k$, indeed ${}^0\dot{\omega}^k = \mathrm{P}({}^0\mathbf{A}_O^k)$, while

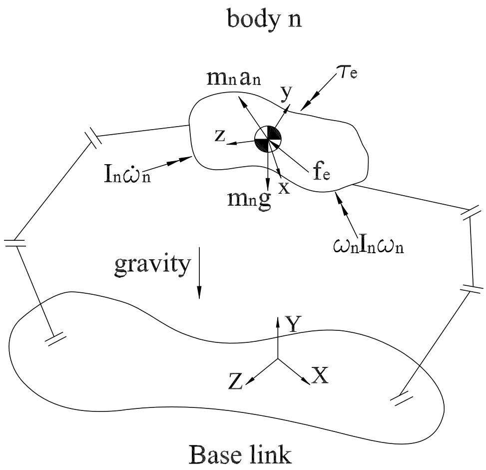


Fig. 6 Closed chain

the translational acceleration of the center of the k-th platform, vector $\mathbf{a}_{Ck}$, expressed in the reference frame XYZ using classical kinematics results in

$$\mathbf{a}_{Ck} = {}^0\mathbf{a}_O^k + {}^0\dot{\omega}^k \times {}^0\rho^k + {}^0\omega^k \times \left({}^0\omega^k \times {}^0\rho^k\right), \tag{17}$$

where the translational acceleration ${}^0\mathbf{a}_O^k$ is calculated from the dual part of the accelerator, $\mathrm{D}({}^0\mathbf{A}_O^k)$, as follows

$${}^0\mathbf{a}_O^k = \mathrm{D}\left({}^0\mathbf{A}_O^k\right) + {}^0\omega^k \times {}^0\mathbf{v}_O^k.$$

4 Computing the driving forces of the modular spatial hyper-redundant manipulator

In this section, the principle of virtual work and the theory of screws are used to compute the generalized forces of the MSHRM; for a detailed explanation of this systematic method, the reader is referred to [47].

The computation of driving forces in parallel manipulators becomes a hazardous task when traditional strategies such as the Newton–Euler method or the Lagrangian formulation are employed. In fact, the Newton–Euler method usually requires large computation time, since it needs the computation of all the internal reactions of constraint of the system, even if they are not employed in the control law of the manipulator. On the other hand, the Lagrangian formulation is based on the computation of the energy of the whole system with the adoption of a generalized coordinate framework. These computations are unnecessary when the theory of screws and the principle of virtual work are used systematically.

Figure 6 shows a closed chain in which a body n of mass m_n and centroidal inertia matrix I_n is under the action of gravitational and inertial forces. In addition, the body n is supporting an external force f_e and an external torque τ_e. The velocity state $\mathbf{V}_n = [\omega_n\ \mathbf{v}_n]^{\mathrm{T}}$ and the reduced acceleration state $\mathbf{A}_n = [\dot{\omega}_n\ \mathbf{a}_n - \omega_n \times \mathbf{v}_n]^{\mathrm{T}}$ describe the motion of rigid body n taking its mass center as representation point, in other words, $\mathbf{v}_n$ and $\mathbf{a}_n$ are, respectively, the translational velocity and acceleration of the mass center. The overall wrench acting on body n, $\mathbf{F}_n = [\mathbf{f}_n\ \tau_n]^{\mathrm{T}}$ is given by

$$\mathbf{F}_n = \mathbf{F}_{inertial} + \mathbf{F}_{grav} + \mathbf{F}_{external}, \tag{18}$$

where

- $\mathbf{F}_{inertial} = \begin{bmatrix} -m_n\mathbf{a}_n \\ -\mathrm{I}_n\dot{\omega}_n - \omega_n \times \mathrm{I}_n\omega_n \end{bmatrix}$ is the inertial wrench according with D'Alembert's principle. In this wrench, the centroidal inertia matrix I_n can be calculated as follows

$$\mathrm{I}_n = \mathrm{R}\mathrm{I}_{\mathrm{xyz}}(\mathrm{R})^{\mathrm{T}},$$

  where $\mathrm{I}_{\mathrm{xyz}}$ is the centroidal inertia matrix of body n expressed in the reference frame xyz and R is the rotation matrix between the frames xyz and XYZ,
- $\mathbf{F}_{grav} = \begin{bmatrix} m_n\mathbf{g} \\ \mathbf{0} \end{bmatrix}$ is the gravity wrench,
- $\mathbf{F}_{external} = \begin{bmatrix} \mathbf{f}_e \\ \tau_e \end{bmatrix}$ is the external wrench.

Consider the output platform, body n, of the SHRM. Then according to (13) and (15), the velocity state of the output platform with respect to the fixed platform, six-dimensional vector ${}^0\mathbf{V}_O^n = [{}^0\omega^n \; {}^0\mathbf{v}_O^n]^{\mathrm{T}}$ results in

$${}^0\mathbf{V}_O^n = \left[\left({}^0\mathrm{J}^1\right)^{\mathrm{T}} \Delta\right]^{-1} {}^0\mathrm{Q}^1{}_{\mathrm{vel}} + \left[\left({}^1\mathrm{J}^2\right)^{\mathrm{T}} \Delta\right]^{-1} {}^1\mathrm{Q}^2{}_{\mathrm{vel}} + \cdots + \left[\left({}^{n-1}\mathrm{J}^n\right)^{\mathrm{T}} \Delta\right]^{-1} {}^{n-1}\mathrm{Q}^n{}_{\mathrm{vel}}. \quad (19)$$

A brief inspection of (19) reveals that the velocity state ${}^0\mathbf{V}_O^n$ depends on the instantaneous geometry and generalized speeds of the SHRM and, therefore, it is possible to rewrite (19) in terms of first order influence coefficients; see [48] as follows:

$${}^0\mathbf{V}_O^n = {}^0\mathrm{M}^1 \, {}^0\mathrm{Q}^1 + {}^1\mathrm{M}^2 \, {}^1\mathrm{Q}^2 + \cdots + {}^{n-1}\mathrm{M}^n \, {}^{n-1}\mathrm{Q}^n \quad (20)$$

where

$${}^{i-1}\mathrm{M}^i = \begin{bmatrix} {}^{i-1}G\omega x_1{}^i & {}^{i-1}G\omega x_2{}^i & {}^{i-1}G\omega x_3{}^i \\ {}^{i-1}G\omega y_1{}^i & {}^{i-1}G\omega y_2{}^i & {}^{i-1}G\omega y_3{}^i \\ {}^{i-1}G\omega z_3{}^i & {}^{i-1}G\omega z_3{}^i & {}^{i-1}G\omega z_3{}^i \\ {}^{i-1}Gvx_1{}^i & {}^{i-1}Gvx_2{}^i & {}^{i-1}Gvx_3{}^i \\ {}^{i-1}Gvy_1{}^i & {}^{i-1}Gvy_2{}^i & {}^{i-1}Gvy_3{}^i \\ {}^{i-1}Gvz_3{}^i & {}^{i-1}Gvz_3{}^i & {}^{i-1}Gvz_3{}^i \end{bmatrix}, \quad i = 1, 2, \ldots, n$$

is the matrix of influence coefficients of the i-th module,[2] and

$${}^{i-1}\mathrm{Q}^i = \begin{bmatrix} {}^{i-1}\dot{q}^i{}_1 \\ {}^{i-1}\dot{q}^i{}_2 \\ {}^{i-1}\dot{q}^i{}_3 \end{bmatrix}, \quad i = 1, 2, \ldots, n$$

is a matrix formed with the generalized speeds of the i-th module.

Assuming that the SHRM undergoes virtual velocities ${}^{i-1}\delta q^i{}_k$, $i = 1, 2, 3$; $k = 1, 2, 3$, the virtual power δw_n produced by the resulting wrench $\mathbf{F}_n$, with $\mathbf{f}_n = f_{nx}\hat{i} + f_{ny}\hat{j} + f_{nz}\hat{k}$ and $\tau_n = \tau_{nx}\hat{i} + \tau_{ny}\hat{j} + \tau_{nz}\hat{k}$, acting on body n on a generic motion ${}^0\mathbf{V}_O^n$, can be calculated using the Klein form as follows

$$\delta w_n = \left\{\mathbf{F}_n; {}^0\mathbf{V}_O^n\right\}. \quad (21)$$

[2] ${}^{i-1}G\omega\mathrm{A}_q{}^i$ and ${}^{i-1}Gv\mathrm{A}_q{}^i$ are the first order influence coefficients, or partial contributions, of the generalized speed q over the angular velocity and the linear velocity of the moving platform of the i-th module along the A axis.

The principle of virtual work states that if a closed chain is in equilibrium under the effect of external actions, then the global work produced by the external forces with any virtual velocity must be null. Under such assumption, (21) leads to

$$\begin{aligned}
&\big({}^{i-1}Gvx_1{}^i\ f_{nx} + {}^{i-1}Gvy_1{}^i\ f_{ny} + Gvz1_i\ f_{nz} \\
&\quad + {}^{i-1}G\omega x_1{}^i\ \tau_{nx} + {}^{i-1}G\omega y_1{}^i\ \tau_{ny} + {}^{i-1}G\omega z_1{}^i\ \tau_{nz} + \xi 1_i\big){}^{i-1}\delta q^i{}_1 \\
&\quad + \big({}^{i-1}Gvx_2{}^i\ f_{nx} + {}^{i-1}Gvy_2{}^i\ f_{ny} + Gvz2_i\ f_{nz} \\
&\quad + {}^{i-1}G\omega x_2{}^i\ \tau_{nx} + {}^{i-1}G\omega y_2{}^i\ \tau_{ny} + {}^{i-1}G\omega z_2{}^i\ \tau_{nz} + \xi 2_i\big){}^{i-1}\delta q^i{}_2 \\
&\quad + \big({}^{i-1}Gvx_3{}^i\ f_{nx} + {}^{i-1}Gvy_3{}^i\ f_{ny} + Gvz3_i\ f_{nz} \\
&\quad + {}^{i-1}G\omega x_3{}^i\ \tau_{nx} + {}^{i-1}G\omega y_3{}^i\ \tau_{ny} + {}^{i-1}G\omega z_3{}^i\ \tau_{nz} + \xi 3_i\big){}^{i-1}\delta q^i{}_3 = 0, \\
&\quad i = 1, 2, \ldots, n
\end{aligned} \tag{22}$$

where $\xi 1_i$, $\xi 2_i$, $\xi 3_i$, $i \in \{1, 2, \ldots, n\}$ are the generalized forces also known as the driving force, acting on the i-th module.

Finally, in order to avoid arbitrary and trivial solutions, (22) requires necessarily that

$$\left.\begin{aligned}
&{}^{i-1}Gvx_1{}^i\ f_{nx} + {}^{i-1}Gvy_1{}^i\ f_{ny} + Gvz1_i\ f_{nz} \\
&\quad + {}^{i-1}G\omega x_1{}^i\ \tau_{nx} + {}^{i-1}G\omega y_1{}^i\ \tau_{ny} + {}^{i-1}G\omega z_1{}^i\ \tau_{nz} + \xi 1_i = 0, \\
&{}^{i-1}Gvx_2{}^i\ f_{nx} + {}^{i-1}Gvy_2{}^i\ f_{ny} + Gvz2_i\ f_{nz} \\
&\quad + {}^{i-1}G\omega x_2{}^i\ \tau_{nx} + {}^{i-1}G\omega y_2{}^i\ \tau_{ny} + {}^{i-1}G\omega z_2{}^i\ \tau_{nz} + \xi 2_i = 0, \\
&{}^{i-1}Gvx_3{}^i\ f_{nx} + {}^{i-1}Gvy_3{}^i\ f_{ny} + Gvz3_i\ f_{nz} \\
&\quad + {}^{i-1}G\omega x_3{}^i\ \tau_{nx} + {}^{i-1}G\omega y_3{}^i\ \tau_{ny} + {}^{i-1}G\omega z_3{}^i\ \tau_{nz} + \xi 3_i = 0, \\
&\quad i = 1, 2, \ldots, n
\end{aligned}\right\}, \tag{23}$$

which allows to compute directly the driving forces $\xi 1_i$, $\xi 2_i$, and $\xi 3_i$ in each module of the SHRM required for controlling the motion of the output platform n.

Finally, it is straightforward to demonstrate that the procedure described here is available to compute the driving forces required to control the motion of any body of the SHRM.

5 Case study

In order to illustrate the proposed methodology of analysis, in this section, a numerical example is provided which consists of solving the kinematics and dynamics of a MSHRM built with six modules.

The parameters of the base module of the MSHRM mechanism, using SI units, are listed in Table 1, only the masses and the centroidal inertia matrices of the moving platforms are considered in this section. While the generalized coordinates are governed by the periodical functions provided in Table 2.

On the other hand, the acceleration of the gravity is given by $g = -9.80665\hat{j}$, and the external wrench expressed in the reference frame attached at the output platform over the

Table 1 Parameters of the base module

$\mathbf{A}_1 = (.353, 0., .353), \mathbf{A}_2 = (.129, 0., -.482), \mathbf{A}_3 = (-.482, 0., .129)$
$\hat{u}_1 = (.7071, 0., -.7071), \hat{u}_2 = (-.965, 0., -.258), \hat{u}_3 = (.258, 0., .965)$
$\mathrm{b}_{12} = \mathrm{b}_{13} = \mathrm{b}_{23} = \sqrt{3}/2$
Mass platform $= 18.914$
Centroidal inertia matrix $= \begin{bmatrix} 0.809 & 0 & 0 \\ 0 & 1.615 & 0 \\ 0 & 0 & 0.809 \end{bmatrix}$

Table 2 Periodical functions selected for the active joints

Module 1:
$q_1 = 0.5 + 0.2\sin(t)\cos(t), q_2 = 0.5 + 0.2\sin^2(t)$,
$q_3 = 0.5 + 0.125\sin(t)$
Module 2:
$q_1 = 0.5 + 0.175\sin(t), q_2 = 0.5 + 0.25\sin(t)\cos(t)$,
$q_3 = 0.5 + 0.175\sin(t)$
Module 3:
$q_1 = 0.5 + 0.125\sin(t)(1.0 + \sin(t)\cos^2(t)), q_2 = 0.5 + 0.25\sin^2(t)$,
$q_3 = 0.5 + 0.275\sin(t)\cos^2(t)$
Module 4:
$q_1 = 0.5 + 0.15\sin(t), q_2 = 0.5 + 0.15\sin(t)$,
$q_3 = 0.5 + 0.15\sin(t)(1.0 + \sin(t)\cos(t))$
Module 5:
$q_1 = 0.5 + 0.15\sin(t)\sin(t), q_2 = 0.5 + 0.135\sin^2(t)$,
$q_3 = 0.5 + 0.125\sin^2(t)$
Module 6:
$q_1 = 0.5 + 0.125\sin^2(t), q_2 = 0.5 + 0.1\sin(t)\cos(t)(\sin(t) + \cos(t))$,
$q_3 = 0.5 + 0.125\sin(t)\cos(t)$
$0 \le t \le 2\pi$

output platform is chosen as follows

$$\mathbf{F}_{external} = \begin{bmatrix} 0 \\ -2500 \\ 0 \\ 0 \\ 150 \\ 0 \end{bmatrix}.$$

Thus, the 16 solutions of the variable Z_1 of the base module are listed in Table 3.

Table 3 The sixteen values of Z_1

−0.08861590520	2.692797540	2.694972365	−0.3513357122
0.1395527109	0.02668240816	0.3536130188	0.3538438343
0.5303476302	0.1392160780	0.1392419181	0.3541170683
2.701360478	0.8026916179	−0.3534021282	−0.3526948579

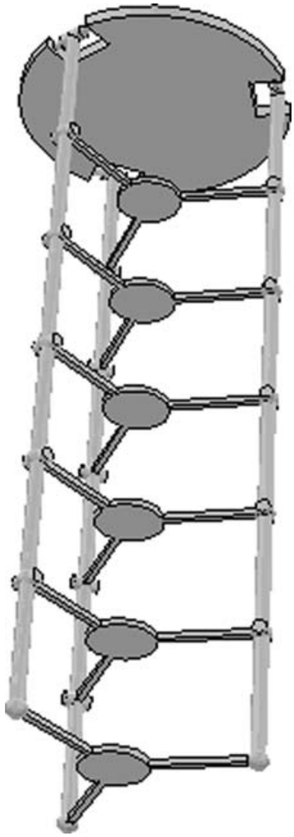


Fig. 7 Home position of the MSHRM

Ignoring spurious solutions, there is only one real solution available for the forward position analysis of the first module:

$$\left.\begin{aligned} \mathbf{a}_1 &= (.3536130188, -.4999992483, .3536130188) \\ \mathbf{a}_2 &= (.1292239221, -.4999992482, -.4828375377) \\ \mathbf{a}_3 &= (-.4811619757, -.4999992475, .1287759479) \end{aligned}\right\}$$

Naturally, according to the periodical functions assigned to the generalized coordinates, this solution is the same taking properly the local reference frames attached to the corresponding moving platforms, for the remaining modules and, therefore, this solution is chosen as the home position of the SHRM, Fig. 7.

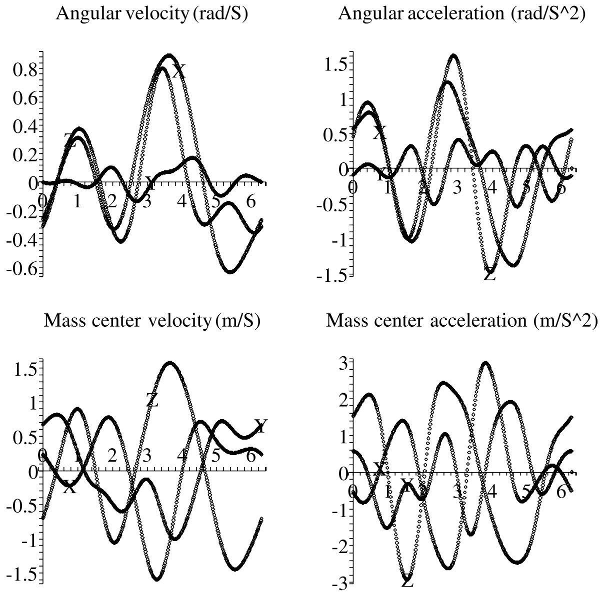


Fig. 8 Time history of the forward kinematics of the output platform of the MSHRM

Finally, the most relevant numerical results generated for the forward kinematics of the output platform are summarized in Fig. 8, whereas the resulting driving forces to control each one of the moving platforms of the MSHRM are provided in Fig. 9.

6 Conclusions

In this work, the kinematic and dynamic analyses, including the computation of the driving forces of a modular spatial hyper-redundant manipulator formed from identical mechanical modules are approached using an harmonious combination of the theory of screws and the principle of virtual work.

Firstly, the kinematics up to the acceleration analysis of the base module, which is a three-legged in-parallel manipulator with linear active joints is carried out by means of the theory of screws. Secondly, the expressions thus obtained for the base module are applied recursively to accomplish the kinematics of the modular spatial hyper-redundant manipulator under study. To this end, the application of the concept of reciprocal screws allows to simplify considerably the forward acceleration analysis. Finally, the expressions to solve the driving forces of the MSHRM are systematically obtained using the Klein form of the Lie algebra. Conveniently, unlike the classical Newton–Euler method, such expressions do not require the instantaneous values of the internal reactions of constraint nor the computation of the energy of the whole system, which is an unavoidable step of the Lagrangian formulation. In order to illustrate the efficacy of the chosen method of analysis, a case study which consists of solving the kinematics, dynamics, and computation of driving forces of a MSHRM built with 6 modules (18 degrees of freedom) is provided.

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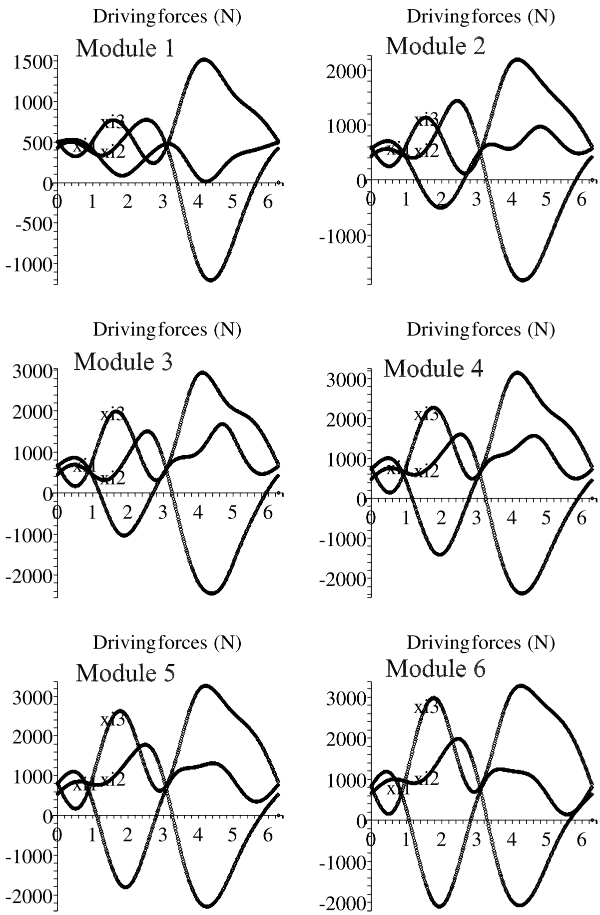

Fig. 9 Time history of the driving forces in each module of the MSHRM

Appendix A: The Lie algebra $e(3)$ of the Euclidean group E(3)

In this Appendix, a brief summary of well-known concepts dealing with the Lie algebra $e(3)$ of the Euclidean group E(3), which is isomorphic to the theory of screws is recalled.

A screw is a straight line with which a definite linear magnitude termed the pitch is associated, Ball [49]. Furthermore, a screw, $\$ = (\hat{s}, \mathbf{s}_O)$, can be considered as a six-dimensional vector composed of a primal part and a dual part, where the primal part, $\hat{s}$, is a unit vector along the screw axis, while the dual part, $\mathbf{s}_O$, is the moment produced by $\hat{s}$ about a selected point O which is instantaneously coincident with a point of a reference frame. The moment $\mathbf{s}_O$ is calculated as follows

$$\mathbf{s}_O = h\hat{s} + \hat{s} \times \mathbf{r}_{O/P},$$

where $\mathbf{r}_{O/P}$ is a vector pointed from a point P on the screw axis to O. Any lower kinematic pair can be represented either by a screw or a group of screws.

The screw algebra is the set of elements of the form $\$ = (\hat{s}, \mathbf{s}_O)$ with the following operations.

$\forall \$_1 = (\hat{s}_1, \mathbf{s}_{O1}), \$_2 = (\hat{s}_2, \mathbf{s}_{O2}) \in e(3)$ and $\lambda \in \Re$

- Addition

$$\$_1 + \$_2 = (\hat{s}_1 + \hat{s}_2, \mathbf{s}_{O1} + \mathbf{s}_{O2}),$$

- Multiplication by a scalar

$$\lambda \$_1 = (\lambda \hat{s}_1, \lambda \mathbf{s}_{O1}),$$

- The Lie product, also known as dual motor product

$$[\$_1 \quad \$_2] = (\hat{s}_1 \times \hat{s}_2, \hat{s}_1 \times \mathbf{s}_{O2} - \hat{s}_2 \times \mathbf{s}_{O1}),$$

- The Killing form

$$Ki : e(3) \times e(3) \rightarrow \Re, \quad Ki(\$_1, \$_2) = \hat{s}_1 \bullet \hat{s}_2,$$

- The Klein form

$$\{\$_1; \$_2\} : e(3) \times e(3) \rightarrow \Re, \quad \{\$_1; \$_2\} = \hat{s}_1 \bullet \mathbf{s}_{O2} + \hat{s}_2 \bullet \mathbf{s}_{O1},$$

where $\times$ and $\bullet$ denote, respectively, the cross product and the inner product of the usual three-dimensional vectorial algebra.

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