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Multidisciplinary Optimization of Multibody Systems with Application to the Design of Rail Vehicles

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Abstract. A methodology for the design optimization of multibody systems is presented. The methodology has the following features: (1) multibody dynamics is employed to model and simulate complex systems; (2) multidisciplinary optimization (MDO) methods are used to combine multibody systems and additional systems in a synergistic manner; (3) using genetic algorithms (GAs) and other effective search algorithms, the mechanical and other design variables are optimized simultaneously. The methodology is shown to handle the conflicting requirements of rail vehicle design, i.e., lateral stability, curving performance, and ride quality, in an effective manner. By coordinating these conflicting requirements at the system level, three multibody models corresponding to each of these requirements for a rail vehicle are optimized simultaneously.

Keywords: multidisciplinary optimization, multibody systems, genetic algorithms, design of rail vehicles, lateral stability, curving performance, ride quality

1. Introduction

In general, the design optimization of a mechanical system is multidisciplinary [1] and the task is to find effective trade-off solutions for complicated and conflicting design criteria [2]. For example, in rail vehicle suspension design, the task is to search for a compromised design while considering lateral stability, curving performance, and ride quality. To find the compromised design, an effective method is to use a lateral stability model, curving performance model, and ride quality model, each of which can be treated as an analysis discipline that concentrates on a specific aspect of interest, and synthesize the design results based on the three different objective-oriented models [3, 4]. In the case concerned, there are strong interactions among the three models. These interactions make the lateral stability, curving performance, and ride quality a synergistic whole; taking advantage of that synergy is the mark of a good design.

Multidisciplinary Optimization (MDO) is presently of increasing interest in engineering. MDO received recognition in the aeronautical sciences, first for structural optimization and later for aerodynamic design [5]. Currently, we can find the application of MDO to automobile design for safety and NVH (noise, vibration, and harshness) reduction [1, 6, 7]. MDO is also used for ground vehicle suspension design [8].

This paper presents a methodology for the design optimization of multibody systems. The methodology has the following features: (1) multibody dynamics is used for modeling and simulating multibody systems; (2) multidisciplinary optimization (MDO) methods are introduced to combine multibody systems and additional systems; (3) with the scalarization technique, a vector optimization problem is converted into a scalar one; (4) with genetic algorithm (GAs) and other effective search algorithms, the mechanical and other (e.g., control) design variables can be optimized simultaneously. Relevant multidisciplinary optimization formulation methods, multicriteria optimization concepts, and genetic algorithms are also presented.

This methodology is applied to the design of rail vehicles. The proposed multidisciplinary optimization method combines a 17 degrees of freedom (DOF) lateral stability model, a 36 DOF vertical ride quality model, and a 21 DOF nonlinear dynamic curving performance model. For the lateral stability problem, the dynamic equations for the lateral stability model of the rail vehicle are generated and linearized by A'GEM [9]; the corresponding eigenvalue problem is solved. To evaluate curving performance, A'GEM is used to generate and numerically integrate the nonlinear dynamic equations for the curving performance model of the same rail vehicle. For the problem of vertical ride quality, the frequency response of the ride quality model (with car body flexibility) to stochastic inputs is determined by A'GEM. By coordinating the conflicting requirements from lateral stability, curving performance, and vertical ride quality at the system level, the suspension, geometric, and inertial parameters for a rail vehicle are optimized simultaneously by the MDO methodology.

2. A Methodology for Optimizing Multibody Systems

The proposed framework for the design optimization of multibody systems is shown in Figure 1. To generate objective-oriented multibody system models, $M_1, M_2, \dots, M_n$, multibody dynamics is utilized. For example, in the design optimization of rail vehicles, these objective-oriented multibody system models may be lateral stability models, curving performance models, ride quality models, etc. Based on the features of mechanical systems to be optimized, a variety of successful computer programs for multibody systems, e.g., ADAMS, A'GEM, SIMPACK, and VAMPIRE, could be applied for this purpose. For a specified optimization problem, relevant analysis tools, $A_1, A_2, \dots, A_m$, such as stability analysis, modal analysis, power spectral density (PSD) analysis, control algorithms, etc., can be introduced. In addition, systems or analysis disciplines, e.g., control systems, can be included. By means of multidisciplinary optimization formulations, these strongly coupled models, analysis tools, and/or additional systems are integrated as a synergistic whole.

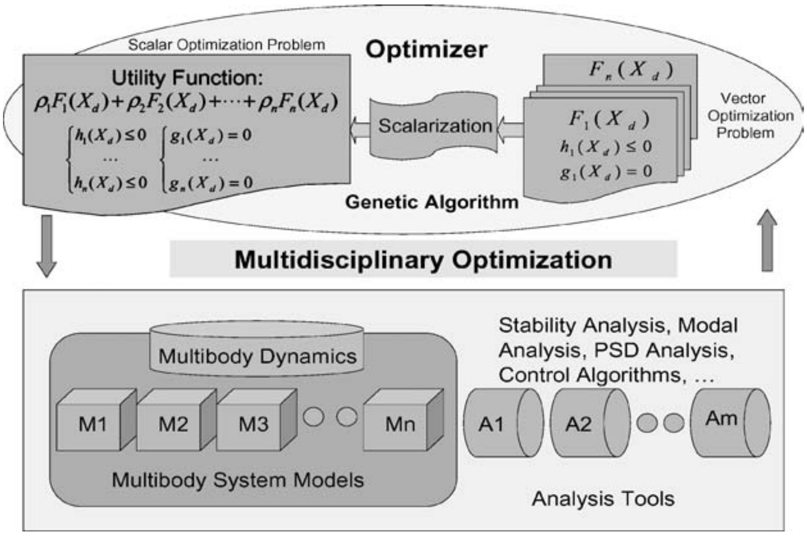

Figure 1. Proposed framework for the design optimization of multibody systems.

With the integration of these multiple systems or disciplines to be optimized, at the system level, one is faced with a vector optimization problem with a set of design criteria, $\mathbf{F}(\mathbf{X}_\mathrm{d})$, where $\mathbf{X}_\mathrm{d}$ is the vector of design variables, and corresponding constraints, $\mathbf{g}(\mathbf{X}_\mathrm{d}) = \mathbf{0}$ and $\mathbf{h}(\mathbf{X}_\mathrm{d}) \leq \mathbf{0}$. The commonly used strategy for dynamic system design is to reduce the vector optimization problem to a scalar one that may be solved by existing optimization algorithms. This strategy has proven to be effective [10]. In the proposed methodology, a scalarization technique is adopted to convert the vector optimization problem into a scalar one with a resulting utility function, $\rho_1 F_1(\mathbf{X}_\mathrm{d}) + \rho_2 F_2(\mathbf{X}_\mathrm{d}) + \cdots + \rho_n F_n(\mathbf{X}_\mathrm{d})$ (where $\rho_i,\ i = 1, 2, \ldots, n$, are weighting factors). The formulation of the utility function and the selection of the weighting factors are discussed in this paper.

Genetic algorithms have the distinguished properties of performing global optimizations, requiring no gradient information, using probability rules to guide their searches, and being suitable for solving complex real-world problems. Therefore, a GA is used as the optimizer to resolve the trade-off relations among the various design criteria at system level.

The framework of the methodology consists of the main components, i.e., application of MDO and multicriteria optimization methods, introduction of multibody dynamics, and use of GAs.

3. Multidisciplinary Optimization Formulation Methods

A successful mechanical system design requires harmonization of a number of criteria and constraints. Such a design problem can be modeled as a constrained optimization in the design variable space. However, for such optimization, due to its dimensionality, complexity, and expense for analysis, a decomposition approach is recommended so as to enable concurrent execution of smaller and more manageable tasks [1]. To preserve the couplings that naturally occur among the subsystems of the whole problem, such optimization by various types of decomposition must include a degree of coordination at the system and subsystem levels. MDO offers effective methods for performing the above optimization so as to resolve the trade-off relations among the various design criteria at the system and subsystem levels.

Several MDO formulation methods exist, including All-in-One (A-i-O) [1], Individual Discipline Feasible (IDF) [11], Collaborative Optimization [12], Bi-Level Integrated System Synthesis [13], and Concurrent Subspace Optimization [14], to name a few. Among most of these MDO methods, the shared character is that the system concerned is decomposed into subsystems so that the corresponding optimization subtasks are performed independently in their own modules; then at a system level, the coordination of the different design considerations gives rise to a two-level optimization. One of the most important advantages of this decomposition is the concurrent execution of the subtasks, which is well suited for parallel computations.

For the methodology concerned, the All-in-One (A-i-O) and Individual Discipline Feasible (IDF) methods are adopted.

3.1. ALL-IN-ONE FORMULATION METHOD

The All-in-One method (also known as the Multidisciplinary Feasibility (MDF) method [11]) is commonly used for the solution of MDO problems. When this method is used, the optimization problem can be formulated in the following general format:

$$\begin{cases} \text{minimize} & \mathbf{F}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d})) \\ \text{with respect to} & \mathbf{X}_\mathrm{d} \\ \text{subject to} & \begin{cases} \mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d})) \le \mathbf{0} \\ \mathbf{C}_\mathrm{l} \le \mathbf{X}_\mathrm{d} \le \mathbf{C}_\mathrm{u} \end{cases} \end{cases} \tag{1}$$

where

$$\begin{cases} \mathbf{U}(\mathbf{X}_\mathrm{d}) = \mathbf{A}(\mathbf{X}_\mathrm{d}, \mathbf{Y}) \\ \mathbf{Y} = \mathbf{G}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d})) \end{cases} \tag{2}$$

and $\mathbf{C}_\mathrm{u}$ and $\mathbf{C}_\mathrm{l}$ are the upper and lower bounds on the design variable vector $\mathbf{X}_\mathrm{d}$, $\mathbf{U}(\mathbf{X}_\mathrm{d})$ is the system output variable vector, $\mathbf{A}(\mathbf{X}_\mathrm{d}, \mathbf{Y})$ is the analysis mapping from the input vectors $\mathbf{X}_\mathrm{d}$ and $\mathbf{Y}$ of an analysis discipline to the outputs $\mathbf{U}$, $\mathbf{G}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d}))$

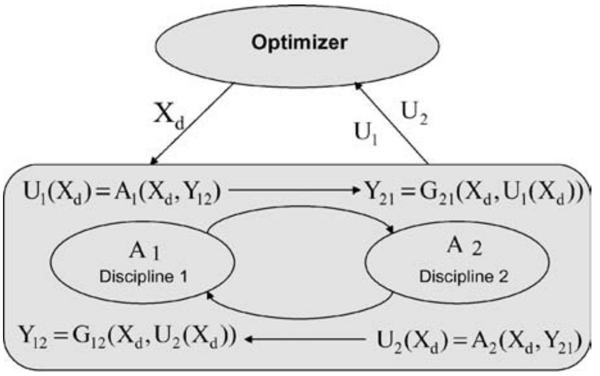

Figure 2. All-in-One (A-i-O) method.

is the mapping to the inputs required for an analysis discipline from the output of another analysis discipline, and $\mathbf{F}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d}))$ and $\mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d}))$ are the objective function vector and constraints, respectively. Note that $\mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}_\mathrm{d}))$ may also contain equality constraints.

Figure 2 shows the data flow in an A-i-O optimization of a problem involving two analysis disciplines. The system consists of an optimizer that controls specified objective $\mathbf{F}$ and constraints $\mathbf{g}$, discipline 1 with analysis solver $\mathbf{A}_1$ and discipline 2 with analysis solver $\mathbf{A}_2$. For a certain iteration in the outer optimization loop, the fixed design variable vector $\mathbf{X}_\mathrm{d}$ is provided by the optimizer to the coupled analysis disciplines. Then in the interior loop between disciplines 1 and 2, a complete discipline 1 analysis and discipline 2 analysis is performed with that fixed vector $\mathbf{X}_\mathrm{d}$ to obtain system output variable vectors $\mathbf{U}_1(\mathbf{X}_\mathrm{d})$ and $\mathbf{U}_2(\mathbf{X}_\mathrm{d})$. Note that this interior loop analysis may also be iterative. The output variable vectors $\mathbf{U}_1(\mathbf{X}_\mathrm{d})$ and $\mathbf{U}_2(\mathbf{X}_\mathrm{d})$ are returned to the optimizer for evaluating the objective $\mathbf{F}(\mathbf{X}_\mathrm{d}, \mathbf{U}_1(\mathbf{X}_\mathrm{d}), \mathbf{U}_2(\mathbf{X}_\mathrm{d}))$ and constraints $\mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}_1(\mathbf{X}_\mathrm{d}), \mathbf{U}_2(\mathbf{X}_\mathrm{d}))$.

In Figure 2, the interdisciplinary mapping $\mathbf{G}_{ij}$ means that given the output variable vector $\mathbf{U}_j$ from discipline j, a suitable variable vector $\mathbf{Y}_{ij}$ can be calculated for use by discipline i. We say that we have single discipline feasibility for discipline i when the solver $\mathbf{A}_i$ has been executed successfully and solved for output variable vector $\mathbf{U}_i$, given the input variable vector $\mathbf{Y}_{ij}$. Here, “feasibility” for a single discipline means that the equations the discipline code is intended to solve are satisfied. For this two analysis discipline case, the optimization problem can be rewritten as

$$\begin{cases} \text{minimize} & \mathbf{F}(\mathbf{X}_\mathrm{d}, \mathbf{U}_1(\mathbf{X}_\mathrm{d}), \mathbf{U}_2(\mathbf{X}_\mathrm{d})) \\ \text{with respect to} & \mathbf{X}_\mathrm{d} \\ \text{subject to} & \begin{cases} \mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}_1(\mathbf{X}_\mathrm{d}), \mathbf{U}_2(\mathbf{X}_\mathrm{d})) \le \mathbf{0} \\ \mathbf{C}_\mathrm{l} \le \mathbf{X}_\mathrm{d} \le \mathbf{C}_\mathrm{u} \end{cases} \end{cases} \tag{3}$$

where

$$\begin{cases} \mathbf{U}_1(\mathbf{X}_\mathrm{d}) = \mathbf{A}_1(\mathbf{X}_\mathrm{d}, \mathbf{Y}_{12}) \\ \mathbf{U}_2(\mathbf{X}_\mathrm{d}) = \mathbf{A}_2(\mathbf{X}_\mathrm{d}, \mathbf{Y}_{21}) \\ \mathbf{Y}_{12} = \mathbf{G}_{12}(\mathbf{X}_\mathrm{d}, \mathbf{U}_2(\mathbf{X}_\mathrm{d})) \\ \mathbf{Y}_{21} = \mathbf{G}_{21}(\mathbf{X}_\mathrm{d}, \mathbf{U}_1(\mathbf{X}_\mathrm{d})) \end{cases} \tag{4}$$

Notice that if a gradient-guided optimization algorithm is used to solve the above problem, then a complete multidisciplinary analysis (MDA) is necessary not just at every iteration of the optimization loop, but at every point where the derivatives are to be evaluated. Thus, it is very expensive to attain multidisciplinary feasibility (i.e., simultaneous feasibility in all disciplines) in realistic applications.

3.2. INDIVIDUAL DISCIPLINE FEASIBLE FORMULATION METHOD

One way to avoid a complete MDA every time an objective function, constraint, or sensitivity evaluation is needed, is to use the IDF method. The essence of IDF is that this approach maintains individual discipline feasibility, while allowing the optimizer to drive the individual disciplines to multidisciplinary feasibility and optimality by controlling the interdisciplinary coupling variables. In the case of the IDF approach, some specific analysis variables representing communication, or coupling, between analysis disciplines via interdisciplinary mappings are "promoted" to become optimization variables. These optimization variables are indistinguishable from design variables from the point of view of a single analysis discipline solver. The general IDF formulation can be written as follows:

$$\begin{cases} \text{minimize} & \mathbf{F}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X})) \\ \text{with respect to} & \mathbf{X} = (\mathbf{X}_\mathrm{d}, \mathbf{X}_\mathbf{Y}) \\ \text{subject to} & \begin{cases} \mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X})) \le \mathbf{0} \\ \mathbf{C}_\mathrm{aux} \triangleq \mathbf{X}_\mathbf{Y} - \mathbf{G}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X})) = 0 \\ \mathbf{D}_\mathrm{l} \le \mathbf{X} \le \mathbf{D}_\mathrm{u} \end{cases} \end{cases} \tag{5}$$

where

$$\mathbf{U}(\mathbf{X}) = \mathbf{A}(\mathbf{X}) \tag{6}$$

and $\mathbf{D}_\mathrm{u}$ and $\mathbf{D}_\mathrm{l}$ are the upper and lower bounds on the design variable vector $\mathbf{X}$ which consists of the original design variable vector $\mathbf{X}_\mathrm{d}$ and "promoted" design variable vector $\mathbf{X}_\mathbf{Y}$. Since the vector $\mathbf{Y}$ is "promoted" as design variable vector, here $\mathbf{X}_\mathbf{Y}$ is introduced to replace the input variable vector $\mathbf{Y}$ for an analysis discipline. $\mathbf{F}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}))$ and $\mathbf{g}(\mathbf{X}_\mathrm{d}, \mathbf{U}(\mathbf{X}))$ are objective and constraints, respectively. $\mathbf{U}(\mathbf{X})$ is the system output variable vector and $\mathbf{A}(\mathbf{X})$ is the analysis mapping from the inputs

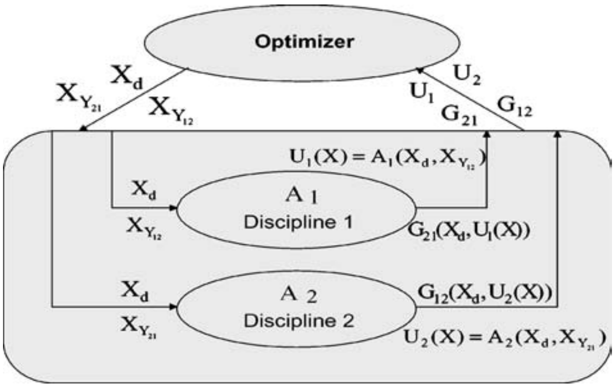

Figure 3. Individual discipline feasible (IDF) method.

$\mathbf{X}_{\mathrm{d}}$ and $\mathbf{X}_{\mathbf{Y}}$. $\mathbf{G}$ represents interdisciplinary mappings and the condition $\mathbf{C}_{\mathrm{aux}} \triangleq \mathbf{X}_{\mathbf{Y}} - \mathbf{G}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}(\mathbf{X})) = \mathbf{0}$ converts the interdisciplinary mappings into auxiliary optimization constraints. Notice that the symbol "$\triangleq$" means "defined as."

It should be noted that an evaluation of $\mathbf{U}(\mathbf{X}) = \mathbf{A}(\mathbf{X})$ involves executing all the single discipline analysis codes simultaneously with available multidisciplinary design variable vector $\mathbf{X}$. Therefore, these very expensive computations can be done independently and concurrently and communication costs are likely to be negligible. It is evident that the IDF method is well-suited for applications with the use of parallel computer system.

Figure 3 shows an application of the IDF method to a system consisting of an optimizer that controls objective $\mathbf{F}$ and constraints $\mathbf{g}$, $\mathbf{C}_{12}$ and $\mathbf{C}_{21}$, discipline 1 with analysis solver $\mathbf{A}_1$, and discipline 2 with analysis solver $\mathbf{A}_2$. For a certain iteration, the fixed design variable vectors $\mathbf{X}_{\mathrm{d}}$, $\mathbf{X}_{Y_{12}}$, and $\mathbf{X}_{Y_{21}}$ are provided by the optimizer to the analysis disciplines 1 and 2. With the offered design variable vectors, each analysis is performed to obtain system output vectors $\mathbf{U}_1(\mathbf{X})$ and $\mathbf{U}_2(\mathbf{X})$ and interdisciplinary mapping vectors $\mathbf{G}_{21}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X}))$ and $\mathbf{G}_{12}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_2(\mathbf{X}))$, respectively. The objective $\mathbf{F}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X}), \mathbf{U}_2(\mathbf{X}))$ and constraints $\mathbf{C}_{12}$, $\mathbf{C}_{21}$, and $\mathbf{g}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X}), \mathbf{U}_2(\mathbf{X}))$ can be evaluated, given the system output vectors $\mathbf{U}_1(\mathbf{X})$ and $\mathbf{U}_2(\mathbf{X})$ and interdisciplinary mapping vectors $\mathbf{G}_{21}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X}))$ and $\mathbf{G}_{12}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_2(\mathbf{X}))$. For this case, the optimization problem is formulated as:

$$
\begin{cases}
\text{minimize} & \mathbf{F}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X}), \mathbf{U}_2(\mathbf{X})) \\
\text{with respect to} & \mathbf{X} = \left(\mathbf{X}_{\mathrm{d}}, \mathbf{X}_{Y_{12}}, \mathbf{X}_{Y_{21}}\right) \\
\text{subject to} & \begin{cases}
\mathbf{g}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X}), \mathbf{U}_2(\mathbf{X})) \le \mathbf{0} \\
\mathbf{C}_{12} \triangleq \mathbf{X}_{Y_{12}} - \mathbf{G}_{12}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_2(\mathbf{X})) = 0 \\
\mathbf{C}_{21} \triangleq \mathbf{X}_{Y_{21}} - \mathbf{G}_{21}(\mathbf{X}_{\mathrm{d}}, \mathbf{U}_1(\mathbf{X})) = 0 \\
\mathbf{C}_{\mathrm{l}} \le \mathbf{X}_{\mathrm{d}} \le \mathbf{C}_{\mathrm{u}}
\end{cases}
\end{cases}
\tag{7}
$$

where

$$\begin{cases} \mathbf{U}_1(\mathbf{X}) = \mathbf{A}_1\big(\mathbf{X}_\mathrm{d}, \mathbf{X}_{Y_{12}}\big) \\ \mathbf{U}_2(\mathbf{X}) = \mathbf{A}_2\big(\mathbf{X}_\mathrm{d}, \mathbf{X}_{Y_{21}}\big) \end{cases} \tag{8}$$

and $\mathbf{C}_\mathrm{u}$ and $\mathbf{C}_\mathrm{l}$ are the upper and lower bounds on the design variable vector $\mathbf{X}_\mathrm{d}$.

3.3. COMPARISON BETWEEN A-i-O AND IDF METHODS

For the above A-i-O and IDF methods, with moderate or no modification, they all have the advantage of using existing single discipline analysis codes. Compared with the A-i-O method, the IDF method avoids the expensive procedure for achieving multidisciplinary feasibility at each optimization iteration. Moreover, when using the IDF method, one may easily replace one analysis code with another, or add new disciplines, and one can easily implement parallel and distributed computation. On the other hand, the IDF method requires the explicit imposition into the optimization of the nonlinear constraints resulting from the interdisciplinary maps. If gradient-guided optimization algorithms are used, the calculation of additional sensitivities corresponding to the coupling variables between disciplines may be very expensive. Provided the coupling variables and constraints are small, the overall IDF optimization will be significantly more efficient than A-i-O optimization [11].

4. Multicriteria Optimization Concepts

A vector or multicriteria optimization problem can be described as:

$$\begin{cases} \text{minimize} & \mathbf{F}(\mathbf{X}_\mathrm{d}) \\ \text{with respect to} & \mathbf{X}_\mathrm{d} \\ \text{subject to} & \begin{cases} \mathbf{g}(\mathbf{X}_\mathrm{d}) = \mathbf{0} \\ \mathbf{h}(\mathbf{X}_\mathrm{d}) \le \mathbf{0} \end{cases} \end{cases} \tag{9}$$

where $\mathbf{X}_\mathrm{d}$, $\mathbf{F}$, $\mathbf{g}$, and $\mathbf{h}$ are design variable vector, objective function vector, equality constraint vector, and inequality constraint vector, respectively. Since different design criteria are usually conflicting, a design variable vector with which all criteria reach their minimal values simultaneously is not feasible.

Although a unique optimal solution can not be defined generally, nonoptimal designs can be eliminated. For example, for every design variable vector $\mathbf{X}_\mathrm{d}$ that satisfies the constraints shown in (9), if $\mathbf{F}(\mathbf{X}_\mathrm{d}) > \mathbf{F}(\tilde{\mathbf{X}}_\mathrm{d})$ and $\tilde{\mathbf{X}}_\mathrm{d}$ is a feasible design variable vector, the design variable vector $\mathbf{X}_\mathrm{d}$ is not optimal. Design variable vectors $\tilde{\mathbf{X}}_\mathrm{d}$ that satisfy the constraints described in (9) are called Edgeworth-Pareto-optimal (EP-optimal), if there is no feasible design variable vector $\mathbf{X}_\mathrm{d}$ where $F_i(\mathbf{X}_\mathrm{d}) \le F_i(\tilde{\mathbf{X}}_\mathrm{d}),\ \forall i \vee \mathbf{F}(\mathbf{X}_\mathrm{d}) \ne \mathbf{F}(\tilde{\mathbf{X}}_\mathrm{d})$ [15]. Usually, EP-optimal solutions are not unique and design points with different images (e.g., curves or surfaces) are not comparable; all of them have to be considered as optimal. These EP-optimal sets provide trade-off relations between different design criteria, and the engineer must select a design from amongst these solutions using additional information and insight into the design problem.

A whole picture of EP-optimal solutions of multicriteria optimization problems requires many objective function evaluations. For the optimization of dynamic behavior of multibody systems, objective function evaluations involve a time-consuming numerical integration of differential equations of motion. In high-dimensional problems (i.e., with many objective functions), the EP–optimal solution cannot be visualized any more. Therefore, not all multicriteria optimization strategies are appropriate for multibody system design.

A scalarization strategy is often used to convert the vector optimization problem to a scalar one. During the process of scalarization, the objective function vector $\mathbf{F}(\mathbf{X}_\mathrm{d})$ (see Figure 1) are formulated as a scalar utility function $u(\mathbf{F}(\mathbf{X}_\mathrm{d}))$. During the optimization, instead of the objective function vector, the scalar utility function is minimized. In the design space, the utility function should have the property of monotonicity, i.e., for two different scalars F^a and F^b, if $F^a < F^b$, then $u(F^a) < u(F^b)$.

To implement the scalarization, generally, the utility function can be formulated either by the weighted criterion method or by the distance method. For the weighted criterion method, the utility function can be expressed as

$$\begin{cases} u(\mathbf{F}) = \sum_{i=1}^{n} \rho_i F_i \\ \rho_i > 0 \end{cases} \tag{10}$$

where $\rho_i,\ i = 1, 2, \ldots, n$, are weighting factors. In the case of the distance method, the utility function is written as

$$\begin{cases} u(\mathbf{F}) = \left(\sum_{i=1}^{n} |F_i - \bar{F}_i|^{\varrho} \right)^{1/\varrho} \\ 1 \le \varrho < \infty \end{cases} \tag{11}$$

where $\bar{F}_i,\ i = 1, 2, \ldots, n$, are ideal or utopian design goal vectors.

The weighted criterion method is widely applied, but the weighting coefficients (ρ_i) are difficult to choose and the optimization result depends on this choice in a highly nonlinear fashion. To facilitate the implementation of the weighted criterion method, it is recommended that each element of the objective function vector $\mathbf{F}$ be normalized to have a value of one for the initial design variables [10]. This recommendation is based on the requirement that all elements of the objective function vector should be optimized simultaneously. Obviously, the selection of the initial design variables and the corresponding values for each element of the objective function vector are vital for the normalization.

Given the utopian goals $\bar{F}_i$, the distance method is a preferable option to the weighted criterion method, since the tedious job of choosing the weighting coefficients for the latter can be avoided. Usually, the goals have some physical meaning. However, if the goals $\bar{F}_i$ are not utopian solutions, EP-optimality can not be reached.

5. Genetic Algorithms

GAs offer significant advantages over traditional local search methods because of the following characteristics [16]: (a) GAs work on a population of design variables in parallel and not on a unique point, so that GAs have a higher reliability to find the global optima; (b) GAs solve the problem of finding good chromosomes (designs) by manipulating the material in the chromosome without any knowledge of the problem they are solving. The only information they require is an evaluation of each chromosome/design — they do not need the gradients of the objective function and constraints; (c) they are simple yet powerful in their search for improvement and they are not limited by restrictive requirements about the search space, such as continuity or existence of derivatives; (d) GAs guide their searches using probability rules; this enhances their global explorative properties.

A population of designs evolves from generation to generation through the application of genetic operators, the most common being selection, crossover, and mutation.

Selection is a process in which individual strings are copied based on their fitness values. Highly fit strings (good designs) have a higher number of offspring in the succeeding generation. Crossover is a method of combining successful individuals by exchanging equivalent lengths of their chromosomes. The two strings from the reproduced population are mated randomly, and a crossover site is selected at random. Mutation is a technique that introduces new information into the new population at the bit level. A set of bits are selected randomly within the entire population.

After performing selection, crossover and mutation, GAs generate a new population with potentially more individuals of higher fitness value. With enough repetitions of the cycle, the population will converge to the chromosome/design with the highest fitness.

In our research, the GA was implemented using the MechaGen program [17]. The MechaGen program is based on Goldberg's GA [16] and was written in C using pseudo-random number generators linked from the NAG (Numerical Algorithms Group) Fortran library. However, to avoid premature termination of the algorithm, instead of using a weighted roulette wheel based on the fitness sum of the population for the reproduction stage, one based on the ranking of the population according to fitness is used [17]. In addition, to improve the efficiency of the GA, the binary strings and fitness values for each unique design of the current generation are stored in a linear search look-up table. If a design string in the next generation matches one in the table, then the fitness does not have to be re-calculated. This saves significant computing time, especially for expensive fitness evaluations.

6. Design Optimization of Rail Vehicles

To demonstrate the feasibility and efficacy of the MDO methodology for resolving conflicting design requirements of multibody systems, it is applied to the design optimization of rail vehicles. In this application, a hybrid MDO formulation method is proposed: the A-i-O method is used at the subsystem (discipline) level to formulate the subproblems corresponding to lateral stability, curving performance, and ride quality, while the IDF method is utilized at the system level to integrate these three disciplines.

6.1. VEHICLE SYSTEM MODELS

Our models for lateral stability, curving performance, and vertical ride quality all correspond to the same design configuration; however, for the ride quality model, the car body flexibility is also considered. The rail vehicle's configuration is shown in Figure 4, with the leading bogie, car body, and trailing bogie denoted as bodies 2, 4, and 6, respectively. For the ride quality model, the car body is divided into 5 identical rigid bodies denoted as 4(1), 4(2), ..., and 4(5). The adjacent car body sections are connected by a group of bending, torsion and shear springs. The leading bogie, with the leading and trailing wheelsets denoted as 1 and 3, and trailing bogie, with the leading and trailing wheelsets denoted as 5 and 7, are connected to the corresponding car body section by secondary suspensions. Both the leading and trailing bogies, in turn, are connected with their own leading and trailing wheelsets by primary suspensions. Each suspension component consists of a parallel spring and damper, with stiffness and damping coefficients in the three coordinate directions. The

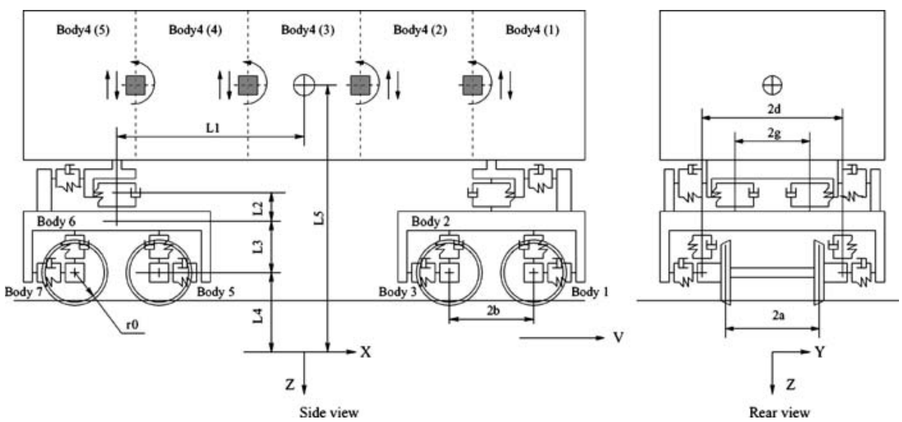

Figure 4. Rail vehicle configuration for dynamic models.

nominal design variables are taken from [18]. Note that the nominal wheel radius of r_0 and conicity λ, as well as the half-distance between contact points a, are fixed in this work.

6.1.1. *Lateral Stability Model*

For the bogies and car body, the motions considered are lateral displacements y_i, yawing ψ_i (about axis z), and rolling ϕ_i (about axis x), where $i = 2, 4, 6$. For the wheelsets, the motions considered are lateral displacement y_i and yawing ψ_i, where $i = 1, 3, 5, 7$. The resulting vehicle model has 17 degrees of freedom (DOF).

Using A'GEM, the following state-space equation is generated automatically:

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q} \tag{12}$$

where $\mathbf{q}$ are the assembled generalized coordinates and $\mathbf{A}$ takes the form:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{17\times 17} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \tag{13}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{I}$ are the inertia, damping, stiffness, and identity matrices, respectively. With the eigenvalues of matrix $\mathbf{A}$, we can analyze the relationship between the critical speed and the suspension parameters.

6.1.2. *Vertical Ride Quality Model*

For ride quality analysis, a 36-DOF model of the rail vehicle is used. The wheelsets are assigned vertical displacements (z_i) and roll motions (ϕ_i), while the bogies and car body sections are assigned lateral displacements (y_i), vertical displacements (z_i), roll motions (ϕ_i), and pitch motions (θ_i). Note that the flexible car body has been represented by a five-element discretization of beam undergoing bending, axial torsion, and lateral and vertical shear. By setting the frequency of the first bending mode to 8.0 Hz, the bending spring stiffnesses were computed to be $3.9586\text{E}+08$ Nm/rad.

The frequency responses of the model, i.e., passenger point accelerations and secondary suspension working spaces, to stochastic rail profile inputs are determined by A'GEM. Once the power spectral density (PSD) is computed for these dynamic responses, the $ISO/(2631-1985)$ ride quality criterion can be evaluated by integrating the PSD over 1/3 octave bands to obtain the root of mean square (RMS) acceleration in the frequency bands.

6.1.3. *Curving Performance Model*

Figure 5 shows the 21-DOF dynamic curving model used by A'GEM to simulate the vehicle as it travels from tangent track, through a spiral of constantly decreasing radius, to a constant radius curve [19].

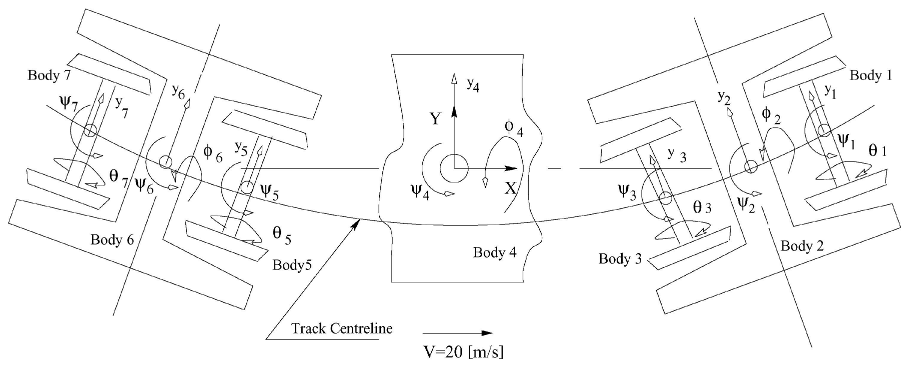

Figure 5. Schematic diagram, showing the degrees of freedom of the curving model.

This model takes linear wheel-rail geometry with two points of contact into account. The effect of wheel load changes on creep coefficients, creep force saturation due to combined actions of longitudinal, lateral and spin creepages, and nonlinear creep-force relationships are considered. Nonlinear suspension elements, i.e., lateral bump stops at each secondary suspension, are included to restrict the relative lateral motion between the car body and the bogie frame. Specified vehicle and track parameters are offered in [20].

6.2. FORMULATION OF THE OPTIMIZATION PROBLEM

For simplicity, if the analysis disciplines corresponding to vertical ride quality and curving performance are temporarily excluded, the multidisciplinary optimization problem formulation can be illustrated in Figure 6. Three strongly coupled analysis disciplines, i.e., Multibody Dynamics with A'GEM software, Dynamic Mode Tracking with DMT algorithm, and Critical Speed Identification with sequential quadratic programming (SQP) algorithm, will cooperate to find the critical speed V_c above which rail vehicles will lose stability [18]. Compared with the computer time used by A'GEM for generating the system matrix $\mathbf{A}$ (see equation (13)) or the time used by DMT for modal analysis, the time consumed by SQP for critical speed identification is much longer. In addition, if the IDF method is used for finding the critical speed, the large number of "promoted" design variables will result in expensive communication costs. With the above considerations, the A-i-O method is used for optimizing the lateral stability of the corresponding vehicle model.

As shown in Figure 6, during the kth iterative search for the critical speed, with a design variable vector $\mathbf{X}_d$, SQP sends the potential critical speed V^k_{sqp} to DMT and A'GEM. With the design variable vector $\mathbf{X}_d$, A'GEM first generates the system mass, stiffness, and damping matrices, i.e., $\mathbf{M}$, $\mathbf{K}_0$, and $\mathbf{C}_0$ and these matrices are stored for later use. Once the potential critical speed V^k_{sqp} is offered, the speed dependent nonconservative forces or the creep forces between the wheels and rails

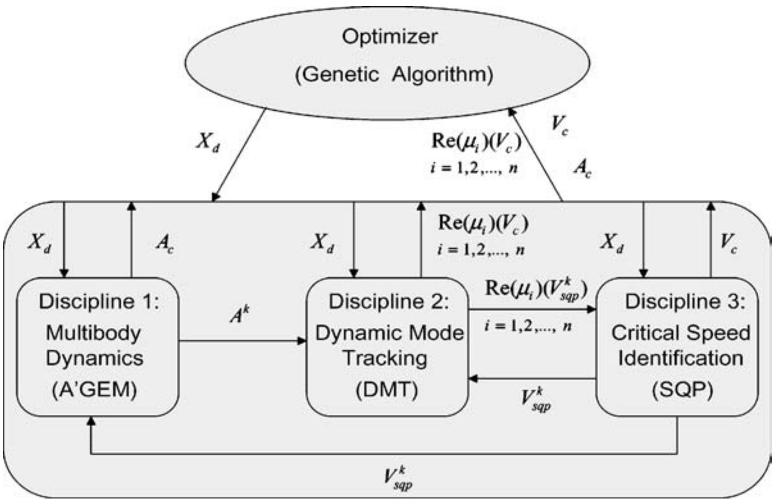

Figure 6. All-in-One (A-i-O) formulation for optimizing the lateral stability.

are added to the stiffness and damping matrices to form the resulting stiffness matrix $\mathbf{K}^k$ and damping matrix $\mathbf{C}^k$ accordingly. A'GEM assembles matrices $\mathbf{M}$, $\mathbf{K}^k$, and $\mathbf{C}^k$ to form the system matrix $\mathbf{A}^k$ (defined in (13)) and offer the system matrix to DMT. With the given speed V^k_{sqp} from SQP and the system matrix $\mathbf{A}^k$ from A'GEM, DMT will perform mode tracking from speed V^{k-1}_{sqp} to speed V^k_{sqp} and return to SQP the required real parts of corresponding eigenvalues $\mathrm{Re}(\mu_i)(V^k_{\mathrm{sqp}})$, for all $i = 1, 2, \ldots, 34$. This process will continue until the corresponding critical speed V_{c} is determined within some error tolerance. At the end of the process, the resulting V_{c} from SQP, $\mathbf{A}_{\mathrm{c}}$ from A'GEM, and $\mathrm{Re}(\mu_i)(V_{\mathrm{c}})$ from DMT are returned to the system optimizer, the GA, for further use.

Similarly, if the analysis disciplines corresponding to lateral stability and vertical ride quality are temporarily excluded, the optimization of curving performance alone can also be regarded as an application of the A-i-O method. In this case, the optimizer, i.e., the genetic algorithm, and three analysis disciplines, i.e., wheel/rail geometry model, wheel/rail creep force model, and multibody dynamics with R'GEM program (from A'GEM) for automatic generation of the equations of motion make the system a synergistic whole. In this MDO method, once the required design variables are provided to the above coupled analysis disciplines, a complete multidisciplinary analysis (MDA) is carried out via an iterative process to obtain the system (MDA) output variables that are later utilized for evaluating the objective function value and the required constraints.

To simultaneously optimize the lateral stability, curving performance, and vertical ride quality of the rail vehicle model, the IDF method is used to incorporate the above lateral stability optimization, curving performance optimization, as well as vertical ride quality optimization. In the IDF formulation, the problems of lateral

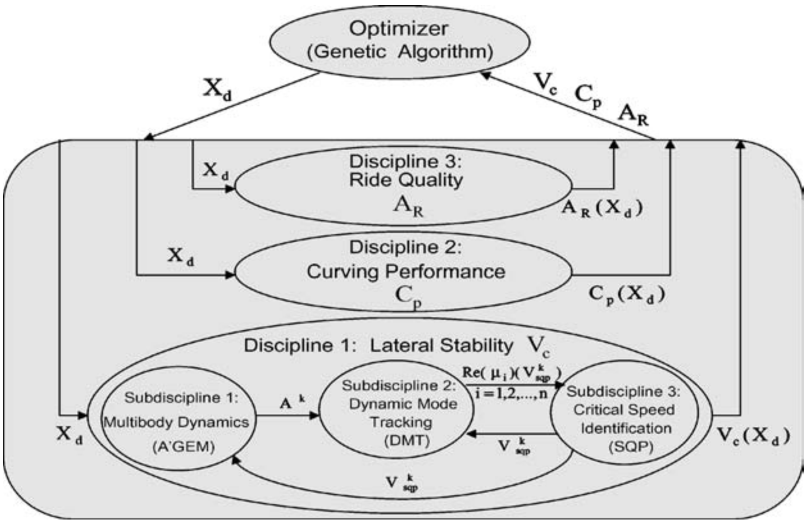

Figure 7. Hybrid MDO method combining IDF and A-i-O for optimizing the lateral stability, curving performance, and vertical ride quality simultaneously.

stability, curving performance, and vertical ride quality are treated as three analysis disciplines. As shown in Figure 7, three ellipses represent the three disciplines, i.e., Ride Quality, Curving Performance, and Lateral Stability. For the discipline of the lateral stability, the three sub-disciplines or subsystems (multibody dynamics, dynamic mode tracking, and critical speed identification) and their coupling relations are also illustrated. With the systems shown in Figure 7, the individual discipline feasible (IDF) method is used to synthesize the three disciplines at the system level and the A-i-O method is applied at the subsystem level, as previously explained. At the system level, a GA is used as the optimizer. Due to the fact that the formulation method used here is a combination of the IDF and the A-i-O methods, we call it a hybrid MDO method. With the hybrid MDO method and the selected optimization algorithms, the optimal design variables are searched in the design space so that the rail vehicle's lateral stability, vertical ride quality, and curving performance can be optimized simultaneously.

As shown in Figure 7, for the three analysis disciplines of the lateral stability, curving performance, and vertical ride quality, the corresponding analysis solvers are denoted as V_c, C_p, and A_R respectively. A comparison of Figure 7 with Figure 3 reveals that in the case of Figure 7, the "promoted" variables (vector $\mathbf{X}_Y$) representing communication, or coupling, between analysis disciplines vanish. Thus, the communication costs at the system level are cheap. Furthermore, there are no explicit interdisciplinary mappings (vector $\mathbf{G}$) among the three disciplines. However, the three disciplines are coupled by means of the original design variable vector $\mathbf{X}_\mathrm{d}$ and their implicit interdisciplinary mappings are coordinated and manipulated by the optimizer at the system or discipline level.

Note that in our application of the IDF method, the basic principal of the MDO formulation is followed: a complex optimization problem is decomposed into smaller and more manageable tasks that should be concurrently executed [1, 5, 6, 11]. In our case, the complicated rail vehicle design problem is decomposed into the lateral stability, curving performance, and vertical ride quality sub-problems. With respect to the optimizer, i.e., the genetic algorithm, these three sub-problems are formulated in a parallel pattern so that they can be analyzed simultaneously. Due to the nature of the design optimization problem, the "promoted" variable vector $\mathbf{X}_Y$ and explicit interdisciplinary mapping vector $\mathbf{G}$ vanish. The vanishing of the vectors $\mathbf{X}_Y$ and $\mathbf{G}$ does not change the structure of the IDF formulation. Our application can be treated as a special IDF case where $\mathbf{X}_Y$ and $\mathbf{G}$ become vacant vectors. Moreover, as mentioned previously, provided the coupling variables and constraints become small or vacant, the overall optimization based on the IDF method will be significantly more efficient than those based on other MDO formulation methods [11]. Thus, from the view-point of computational efficiency, with vacant vectors $\mathbf{X}_Y$ and $\mathbf{G}$, the IDF method is preferable to other MDO formulation methods.

6.3. OPTIMIZATION PROBLEM AND IMPLEMENTATION

6.3.1. *Objective Function, Constraints, and Design Variables*

For the combined rail vehicle model including the lateral stability model, dynamic curving model, and vertical ride quality model, the design variable vector $\mathbf{X}_\mathrm{d}$ consists of suspension stiffness and damping coefficients ($\bar{\mathbf{S}}$), inertial property parameters ($\bar{\mathbf{I}}$), and geometric parameters ($\bar{\mathbf{G}}$). The total number of design variables reaches 29. The vehicle system parameters are offered in [18].

For the lateral stability problem, if the real parts of all eigenvalues $(Re(\mu_i)(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}}, V), i = 1, \ldots, 34)$ of matrix $\mathbf{A}$ are negative or zero, the time response of the system is stable. If one or more eigenvalues has zero real part and all others have negative real parts, the vehicle is traveling at the so-called critical speed. Thus, the lateral stability objective function and constraints may be expressed as

$$\begin{cases} \text{maximize} & V_\mathrm{c}(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}}, V) \\ \text{subject to} & \mathrm{Re}(\mu_i)(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}}, V) \leq 0, \quad i = 1, 2, \ldots, 34 \end{cases} \tag{14}$$

where V_c and V are the critical speed and vehicle forward speed, respectively.

For the ride quality discipline, the function (A_R) to be minimized is a combination of root of mean square (RMS) acceleration values at different points of the car body and secondary suspension working spaces:

$$A_\mathrm{R} = \left[\int_{\omega_\mathrm{l}}^{\omega_\mathrm{u}} S_{\ddot{z}_4(i)}(\omega)\,\mathrm{d}\omega\right]^{1/2} + \upsilon\ \max\left[0, \left(\left[\int_{\omega_\mathrm{l}}^{\omega_\mathrm{u}} S_{h_k}(\omega)\,\mathrm{d}\omega\right]^{1/2} - h_k\right)\right] \tag{15}$$

where $i = 1, 2, \ldots, 5$, υ is a weighting fact, $h_k(k = 1 \ldots 4)$ are limits on the secondary suspension working spaces, ω_{l} and ω_{u} define the frequency interval of interest, and $S_{\ddot{z}_4(i)}(\omega)$ and $S_{h_k}(\omega)$ are the PSDs of the car body vertical acceleration and the working space of the kth secondary suspension, respectively. Note that A_{R} is a function of design variables $\bar{\mathbf{S}}$, $\bar{\mathbf{I}}$, and $\bar{\mathbf{G}}$.

For the curve performance discipline, the function (C_{p}) to be minimized is a combination of angles of attack $\mathbf{A}_{an}$, and lateral to vertical (L/V) force ratios $\mathbf{L}_v$:

$$C_{\mathrm{p}} = \xi \ \max\left(\left|\frac{A_{\mathrm{ani}}(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}})}{\tilde{A}_{\mathrm{ani}}}\right|\right) + \eta \ \max\left(\left|\frac{L_{vk}(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}})}{\tilde{L}_{vk}}\right|\right) \tag{16}$$

where $i = 1, 2, 3, 4$, $k = 1, 2, \ldots, 8$, $\tilde{A}_{\mathrm{ani}}$ and $\tilde{L}_{vk}$ are the angle of attack and L/V ratio when $A_{\mathrm{ani}}(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}})$ and $L_{vk}(\bar{\mathbf{S}}, \bar{\mathbf{I}}, \bar{\mathbf{G}})$ take nominal values, respectively, and ξ and η are weighting factors.

For optimizing the three criteria, a utility function to be minimized was introduced:

$$\nu_1\left\{\xi \ \max\left(\left|\frac{A_{\mathrm{ani}}}{\tilde{A}_{\mathrm{ani}}}\right|\right) + \eta \ \max\left(\left|\frac{L_{vk}}{\tilde{L}_{vk}}\right|\right)\right\} + \nu_2\left(\frac{A_{\mathrm{R}}}{\tilde{A}_{\mathrm{R}}}\right) + \nu_3\left(\frac{\tilde{V}_{\mathrm{c}}}{V_{\mathrm{c}}}\right) \tag{17}$$

where ν_1, ν_2, ν_3 are the weighting factors, $i = 1, 2, 3, 4$, $k = 1, 2, \ldots, 8$, and $\tilde{A}_{\mathrm{R}}$ and $\tilde{V}_{\mathrm{c}}$ are the nominal values of A_{R} and V_{c} respectively.

6.3.2. *Implementation of the Optimization Problem*

As shown in Figure 8, the hybrid MDO method combining IDF and A-i-O discussed previously is implemented using: MechaGen program (a GA), E04UCF routine (an SQP) from the NAG library, Dynamic Mode Tracking (DMT) technique, A'GEM 'Stability' module (STABLE program for lateral stability analysis), A'GEM 'Ride' module (RLRIDE program for vertical ride quality analysis), and A'GEM 'Curve' module (RACES routine for curving performance analysis).

As shown in Figure 8, each set of design parameters of a population generated by the GA is forwarded to the corresponding A'GEM module for calculating the required performance indices for curve performance, vertical ride quality, and lateral stability. With the given set of design variables, the corresponding programs of A'GEM generate the required equations of motion or system matrices automatically. For the cases of curve performance and vertical ride quality, after numerical integration in the time domain and necessary transformation in the frequency domain, respectively, the performance indices can be obtained directly. For lateral stability, however, with the system matrix generated in the form of equation (13),

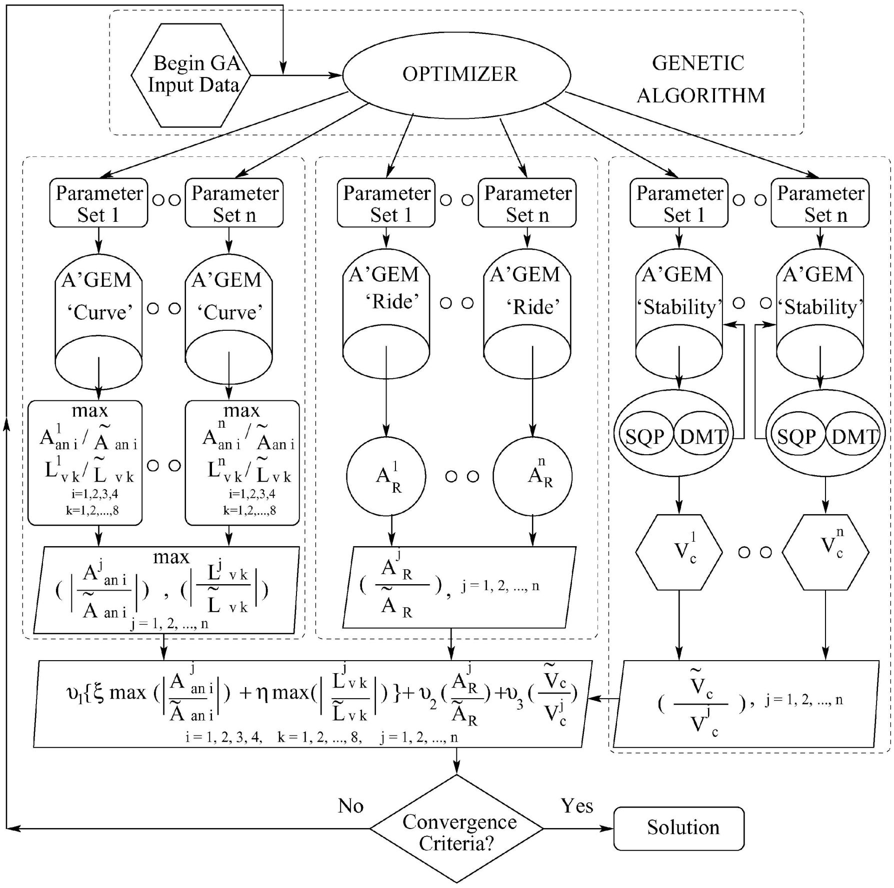

Figure 8. Schematic representation of the implementation of the optimization algorithm.

the SQP and DMT are used to determine the critical speed. Then the corresponding fitness value is obtained by converting the vector optimization problem into a scalar optimization problem using the concept of scalarization by introducing an utility function in the format of (17). Note that the scalarization is performed at the system level so that the performance indices of the three disciplines (i.e., the lateral stability, curving performance, and vertical ride quality) are coordinated and manipulated at the system or discipline level by the genetic algorithm. It should be noted that the total number of fitness values is the same as that of the individual design parameter sets in the population. At this point, if the convergence criteria are satisfied, the calculation terminates; otherwise these fitness values are returned to the GA. Based on the returned fitness values corresponding to the given sets of design variables, the GA produces the next generation of design variable sets using reproduction, crossover and mutation. This procedure repeats until the optimized design variable set is found.

6.4. RESULTS AND DISCUSSION

6.4.1. *Conflicting Requirements on Design Variables*

Since the design criteria for the optimization of the lateral stability, curving performance, and vertical ride quality are different, they impose different or even conflicting requirements on the specific design variable or variables. Table I offers selected numerical results based on the optimization of lateral stability (V_c) and the optimization of curving performance (C_p). For both optimization problems, 10 design variables (i.e., the relevant stiffness and damping coefficients for the secondary and primary suspensions) are permitted to vary by ±20% from their nominal values.

Table I shows the optimized design variables from both optimization problems and the corresponding nominal and bound values for the design variables. Since primary suspension parameters have much more significant effect on curving performance and lateral stability of rail vehicles than secondary suspension parameters [21], the following discussion places emphasis on the primary suspension parameters. As shown in Table I, for the curving performance optimization problem, among the optimized primary suspension parameters, the longitudinal, lateral, and vertical damping coefficients, c_{1x}, c_{1y}, and c_{1z}, take lower values than the corresponding nominal values, and the longitudinal and lateral spring stiffness coefficients, k_{1x} and k_{1y}, take the corresponding lower bound values. However, for the lateral stability optimization problem, among the optimized primary suspension parameters,

Table I. Optimized suspension variables (permitted to vary by ±20% from their nominal values)

	k_{1x} (N/m)	k_{1y} (N/m)	k_{1z} (N/m)	c_{1x} (N/m/s)	c_{1y} (N/m/s)
Nominal Values	3.1500×10^7	3.9600×10^6	2.1000×10^6	666.00	5220.00
Upper Bounds	3.7800×10^7	4.7520×10^6	2.5200×10^6	799.20	6264.00
Lower Bounds	2.5200×10^7	3.1680×10^6	1.6800×10^6	532.80	4176.00
Cp Optimized	2.5200×10^7	3.1680×10^6	2.2964×10^6	654.80	4212.80
Vc Optimized	3.7800×10^7	3.1802×10^6	2.5148×10^6	736.20	4218.90
	c_{1z} (N/m/s)	k_{2y} (N/m)	k_{2z} (N/m)	c_{2y} (N/m/s)	c_{2z} (N/m/s)
Nominal Values	9910.00	1.9700×10^5	6.8700×10^5	4.270×10^4	4.270×10^4
Upper Bounds	11892.0	2.3640×10^5	8.2440×10^5	5.124×10^4	5.124×10^4
Lower Bounds	7928.0	1.5760×10^5	5.4960×10^5	3.416×10^4	3.416×10^4
Cp Optimized	8300.0	2.3410×10^5	5.5713×10^5	4.148×10^4	3.586×10^4
Vc Optimized	10164.0	2.3440×10^5	7.7381×10^5	5.115×10^4	3.620×10^4

Primary Suspension Parameters: k_{1x}: Longitudinal Stiffness; k_{1y}: Lateral Stiffness; k_{1z}: Vertical Stiffness; c_{1x}: Longitudinal Damping; c_{1y}: Lateral Damping; c_{1z}: Vertical Damping.
Secondary Suspension Parameters: k_{2y}: Lateral Stiffness; k_{2z}: Vertical Stiffness; c_{2y}: Lateral Damping; c_{2z}: Vertical Damping.

except for the lateral spring stiffness and damping coefficients, k_{1y} and c_{1y}, the other parameters either take higher values than the corresponding nominal values or take the corresponding upper bound values. Thus, the lateral stability and curving performance have conflicting requirements on the primary suspension design variables. These optimization results are consistent with previous observations by Wickens [21] that suspensions that are soft in the lateral and longitudinal direction tend to hunt more readily on tangent track and become unstable even at low speeds. However, such suspensions allow the wheelsets to follow curved track with decreased wheel wear and flange forces. The exception, i.e., the lateral spring stiffness and damping coefficients (k_{1y} and c_{1y}), to the observation by Wickens may be interpreted by the fact that at values above certain values, the lateral stability becomes relatively insensitive to these parameters. This exception was once reported by Hedrick *et al*. [22].

Besides the above conflicting requirements on suspension parameters, the lateral stability and curving performance also have conflicting requirements on geometric, inertial, or even active design variables. We will see in the following subsection that the hybrid MDO optimization approach offers an effective way to resolve these conflicting requirements.

6.4.2. *Results of the Hybrid MDO Problem*

The combined vehicle model is optimized with respect to three criteria, lateral stability, curving performance, and vertical ride quality as shown in the objective function (17). The constants ξ and η are both set to 1.0. To facilitate the implementation of the optimization problem, $\max(|\frac{A_{\mathrm{ani}}}{\tilde{A}_{\mathrm{ani}}}|) + \max(|\frac{L_{vk}}{\tilde{L}_{vk}}|)$ ($i = 1, 2, 3, 4$, and $k = 1, 2, \ldots, 8$), $\frac{A_{\mathrm{R}}}{\tilde{A}_{\mathrm{R}}}$, and $\frac{\tilde{V}_{\mathrm{c}}}{V_{\mathrm{c}}}$ are defined as curving performance index, lateral stability performance index, and vertical ride quality index, respectively. To obtain a whole picture of the EP-optimal set, three sets of weighting factors ($\{\nu_1, \nu_2, \nu_3\}$) are selected and the corresponding optimizations are carried out. The three selected sets of weighting factors take the values of $\{1, 1, 1\}$, $\{1, 1, 2\}$, and $\{1, 1, 4\}$. A total of 29 parameters including geometric parameters, inertial property parameters, and suspension stiffness and damping coefficients are chosen as design variables. These design variables are permitted to vary by $\pm 20\%$ from their nominal values.

In the study, all computations were carried out on a Silicon Graphics Indigo 2XZ workstation. In the numerical experiments of the hybrid MDO method that combines IDF and A-i-O, it was found by trial and error that consistent results were obtained for the GA using a crossover probability of 100%, a mutation probability of 1.0%, and a population size (the number of design variable sets) of 160. The maximum number of generations is set to 260. For each set of weighting factors ($\{\nu_1, \nu_2, \nu_3\}$) provided above, the elapsed time for one operation of the hybrid MDO method shown in Figure 8 is approximately 182.6 h.

Figures 9–11 illustrate selected results from the hybrid MDO method. Note that the results offered in each of Figures 9–11 are derived from three operations of

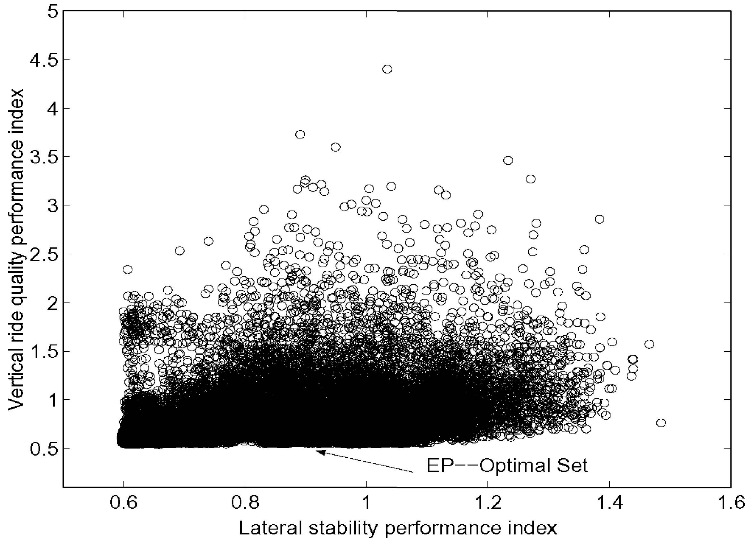

Figure 9. Relationship between lateral stability and vertical ride quality.

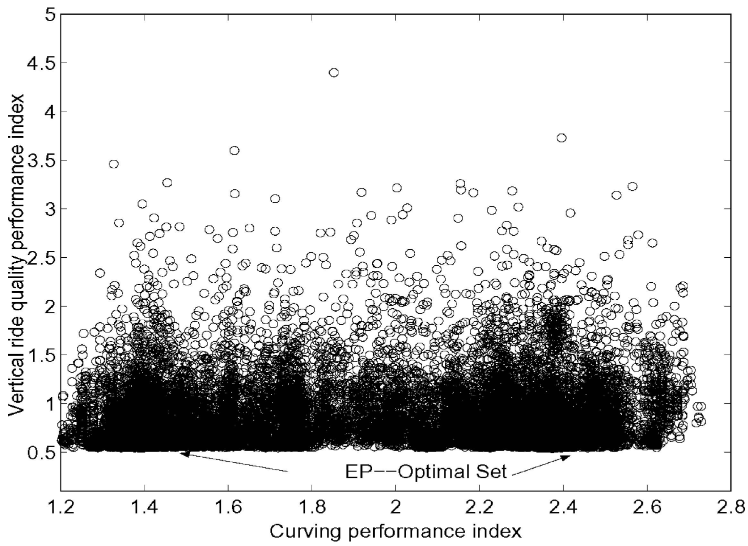

Figure 10. Relationship between curving performance and vertical ride quality.

the hybrid MDO method corresponding to the three sets of values of the weighting factor vector $\{\nu_1, \nu_2, \nu_3\}$ offered above. The individual designs from the GA are represented by circles, which tend to cluster as the GA converges to the optimal design. Plotted in Figure 9 is the vertical ride quality performance index versus lateral stability performance index. The clustered data corresponding to the EP-optimal set

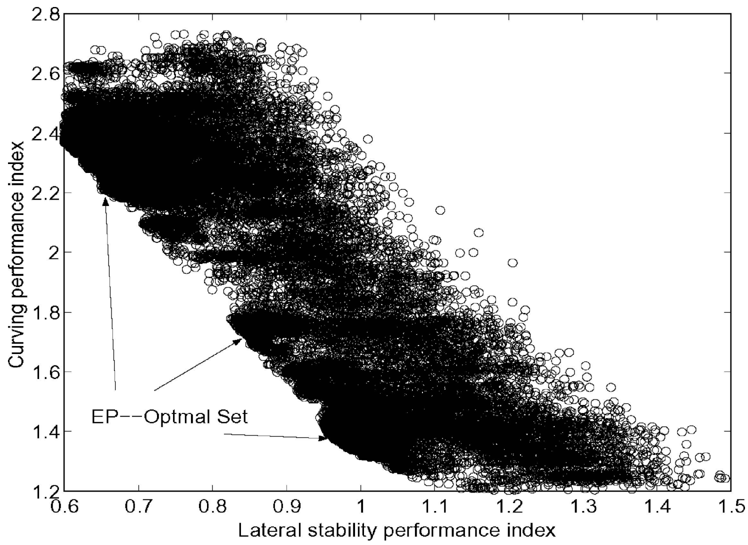

Figure 11. Relationship between lateral stability and curving performance.

is almost horizontal, which shows that the optimized vertical ride quality is mainly independent of lateral stability. This is also true, as shown in Figure 10, for the relationship between vertical ride quality and curving performance. The observation about the relationship between vertical ride quality and lateral stability and that between vertical ride quality and curving performance demonstrates the conclusion [23] that a relatively weak coupling exists between the vertical and lateral motions of a rail vehicle.

However, Figure 11 shows a distinct trade-off in the relationship between lateral stability and curving performance. The EP-optimal set in the densely-clustered region shows that lateral stability can only be improved at the expense of curving performance, and vice-versa. No one criterion is favored over another; instead, the designer obtains explicit information about the trade-offs between lateral stability and curving performance. By running several more optimizations with different sets of weighting factors, one can get an even clearer picture of the EP-optimal set. Although this is a computationally expensive process, the results are of obvious importance to rail vehicle designers.

7. Conclusions

A methodology for the design optimization of multibody systems was developed in the research. The essence of this methodology is that: (1) The effective dynamic system modeling technique (multibody dynamics) is utilized for the generation of complex realistic objective-oriented multibody system models; (2) By means of multidisciplinary optimization methods, these coupled objective-oriented multibody system models and/or additional (e.g., control) systems are integrated as a synergistic whole; (3) With the scalarization technique, a vector optimization problem is converted into a scalar optimization problem; (4) With a genetic algorithm used at system level and the appropriately selected search algorithms used at subsystem level, the coupled systems are optimized simultaneously.

Numerical experiments demonstrated the feasibility and efficacy of the proposed design optimization methodology for resolving conflicting design requirements. This methodology is suitable for complex design optimization problems where: (a) There is interaction between different multibody systems or analysis disciplines; (b) There are multiple design criteria; (c) There are multiple local optima; (d) There are multiple design variables; (e) No matter whether the scalar objective function is continuous or discontinuous, there is no need for sensitivity analysis for the system solver or the GA.

The limitation of the application of the methodology is that the associated computational burden is heavy. However, parallel processing, for which the methodology is ideally suited, could be used for reducing the computer time required for the optimization.

The methodology is applied to the design of a rail vehicle. This methodology is implemented in the form of a hybrid multidisciplinary optimization method. The hybrid MDO method, which is a combination of the individual discipline feasible (IDF) method used at the discipline level and the All-in-One (A-i-O) method used at sub-discipline level, is used to optimize the complex rail vehicle model with respect to lateral stability, curving performance, and vertical ride quality. The hybrid MDO method combines the lateral stability model with 17 DOF, the nonlinear dynamic curving performance model with 21 DOF, the vertical ride quality model with car body flexibility and with 36 DOF, and relevant analysis tools into a synergistic whole.

The trade-off relationship between lateral stability and curving performance are clearly revealed by means of EP-optimal solutions. Moreover, the resulting EP-optimal sets visualize a well-known fact that a relatively weak coupling exists between the vertical and lateral motions of a rail vehicle.

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