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RESEARCH

Nonlocal and micropolar effects in a transversely isotropic functionally graded thermoelastic solid under an inclined load

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Abstract
The objective of this study is to analyze the thermo-mechanical interactions occurring in a nonlocal transversely isotropic functionally graded (nonhomogeneous) micropolar thermoelastic half-space when subjected to an inclined load, based on the Lord and Shulman (LS) theory. The material properties are assumed to be graded exponentially along the z-direction. Utilizing the normal mode technique, the exact expressions for physical fields such as normal displacement, normal stress, shear stress, temperature field, and couple stress are derived. Numerical computation of the derived results is performed for a material resembling a magnesium crystal, and graphical representations are presented to illustrate the impacts of nonhomogeneity parameter, material's anisotropy, time, nonlocal parameter, microinertia, and the inclination angle of the applied load on the variations of different physical fields. Some specific cases of interest have been deduced from the present investigation.

Keywords Nonlocal · Transversely isotropic · Micropolar · Functionally graded · Lord and Shulman theory

1 Introduction

The process of thermal energy transfer in solids is intricately linked to temperature differentials and is described by Fourier's law of heat conduction. Biot (1956) introduced the coupled theory of thermoelasticity, which combines the equations governing both heat conduction and elasticity. However, a notable limitation of Biot's theory lies in its prediction of an infinite speed of propagation for thermal signals, attributed to the parabolic nature of the heat transport equation, rendering it physically implausible. To overcome this limitation, Lord and Shulman (1967) presented a significant and comprehensive generalization of thermoelasticity theory. They introduce a crucial modification by incorporating a single relaxation time into the heat conduction equation. This modification

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ensures that heat signals now exhibit a finite speed of propagation, rectifying the previously unacceptable prediction of an infinite speed of thermal signals in Biot's theory. This development signifies a noteworthy evolution in the understanding and modeling of heat conduction in thermoelastic materials. Some important research work of wave propagation are performed by Zheng et al. (2021) and Yu et al. (2022) among others.

Functionally graded materials (FGMs) are materials characterized by a gradual variation in elastic and thermal properties corresponding to changes in spatial coordinates, making them nonhomogeneous in nature. FGMs find substantial applications in structures subjected to intense thermal gradients, including combustion chambers in space vehicles, thermal barrier structures in aircraft, the inner walls of nuclear furnaces in nuclear reactors, and fins in heat exchangers, among others. Aboudi et al. (1996) presented an extension of FGMs that involves development of a two-dimensional framework to enable modeling of materials functionally graded in two directions. Lotfy and Tantawi (2020) examined the interactions of photothermal elastic waves in a nonhomogeneous medium with magnetic effects. Barak and Dhankhar (2022) addressed a problem involving thermoelastic interactions in a functionally graded nonhomogeneous fiber-reinforced thermoelastic material with temperature-dependent properties. Subsequently, Barak and Dhankhar (2023) conducted an analysis of disturbances induced in a rotating functionally graded transversely isotropic nonlocal thermoelastic half-space within the framework of the LS theory. Recently, Wang et al. (2023) studied the nonlocal effect on attenuation and phase velocity of thermoelastic Lamb waves in functionally graded nanoplates using an improved Legendre polynomial approach.

Eringen and Suhubi (1964), Suhubi and Eringen (1964) and Eringen (1966) proposed the theory of micropolar elasticity by assuming that micropolar solids are a collection of interconnected particles in the form of small rigid bodies and can undergo macrodeformations and microrotations. This theory was further extended to include the thermal effects by Nowacki (1966a, 1966b, 1966c) and Eringen (1970). The microcontinuum field theories especially theories of micromorphic, microstretch, and micropolar continua are discussed in detail by Eringen (1999). Abbas and Kumar (2014) determined the interaction in transversely isotropic micropolar media due to a mechanical source. Kalkal et al. (2020) studied the reflection of plane waves at the free surface of a rotating nonlocal micropolar isotropic thermoelastic medium. Abouelregal et al. (2023) presented a study of two-dimensional deformations in a micropolar thermoelastic medium whose surface is influenced by a transverse magnetic field and heated by a thermal source.

The nonlocal theories of thermoelasticity allow a characteristic length scale of the medium which is very useful for many physically acceptable situations, where the coupled thermoelasticity theory (1956) is found to be quite inefficient. In nonlocal theory, the value of stress at any spatial reference point inside a continuum does not depend upon the value of strain at that spatial point but also depends on the values of strain fields at other points of the continuum. The presence of nonlocality residuals of mass, internal energy, body force, and entropy, etc., in nonlocal elasticity models has been analyzed by Edelen and Laws (1971), Edelen et al. (1971) and Eringen and Edelen (1972). Eringen extended his research work on the notion of nonlocality to several different fields such as polar elastic, thermoelastic, and micropolar elastic continua, etc., as are mentioned in Eringen (1972, 1974, 1984, 2002). Khurana and Tomar (2013) conducted a study on the reflection of plane longitudinal waves in a nonlocal micropolar elastic half-space. A nonlocal Fourier's law and its application to the conduction of heat in two-dimensional thermal lattices are presented by Challamel et al. (2016). Sarkar and Tomar (2019) investigated the propagation of plane waves in an isotropic nonlocal thermoelastic material with voids. Kalkal et al. (2023) investigated the reflection of time-harmonic plane waves from the stress free surface boundary of a nonlocal fiber-reinforced transversely isotropic rotating thermoelastic half-space in the context of LS theory. Poonia et al. (2023) analyzed the wave propagation in a nonlocal transversely isotropic rotating voided thermoelastic half-space in the context of LS theory. In the recent times, Barak et al. (2024) examined the effect of nonlocal parameter in a transversely isotropic exponentially graded thermoelastic voided medium in the context of dual-phase lag theory.

The investigation aims to examine the thermo-mechanical interactions in a nonlocal transversely isotropic functionally graded micropolar thermoelastic medium under an inclined load within the framework of LS theory. Although, numerous research problems have addressed local isotropic functionally graded micropolar thermoelastic media, homogeneous nonlocal transversely isotropic thermoelastic media and nonlocal functionally graded transversely isotropic thermoelastic media, no prior study has explored thermo-mechanical disturbances in a nonlocal transversely isotropic functionally graded (nonhomogeneous) micropolar thermoelastic medium under an inclined load in the context of LS theory. This study examines the impact of the nonhomogeneity parameter, material anisotropy, time, nonlocal parameter, microinertia, and load inclination angle on various physical fields. The observed phenomena find practical applications in automobiles, nuclear reactors, atomic physics, aerospace, industrial engineering, and thermal power plants etc.

2 Basic equations

Following Eringen (1970, 1984) and Abbas and Kumar (2014), the constitutive relations in a nonlocal functionally graded (nonhomogeneous) transversely isotropic micropolar thermoelastic centrosymmetric medium under the purview of LS theory are given as:

$$(1-\epsilon^2\nabla^2)\sigma_{ij} = \sigma_{ij}^L = A_{ijkl}(u_{l,k} + \mathcal{E}_{lkr}\Phi_r) - \beta_{ij}\theta, \tag{1}$$

$$(1-\epsilon^2\nabla^2)m_{ij} = m_{ij}^L = B_{ijkl}\Phi_{k,l}, \tag{2}$$

$$(1-\epsilon^2\nabla^2)\rho T_0\eta = (\rho T_0\eta)^L = \rho C_E\theta + T_0\beta_{ij}e_{ij}, \tag{3}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{4}$$

where $i, j, k, l, r = 1, 2, 3$, ∇^2 denotes the Laplacian operator, $\epsilon = e_0 s$ is nonlocal parameter, s being the internal characteristic length, e_0 is the corresponding material constant, σ_{ij} are the components of stress tensor, m_{ij} are the components of couple stress tensor, u_i are the components of displacement vector $\boldsymbol{u}$, $\mathcal{E}_{lkr}$ is the permutation symbol, Φ_i are the components of microrotation vector $\boldsymbol{\Phi}$, A_{ijkl} and B_{ijkl} are characteristic constants of the considered micropolar material, $\beta_{ij} = \beta_{ii}\delta_{ij}$ are the components of thermal elastic coupling tensor, δ_{ij} is Kronecker delta, η is the specific entropy, C_E is the specific heat at constant strain, e_{ij} are the components of strain tensor, θ is temperature deviation from the reference temperature given by $\theta = T - T_0$, T is absolute temperature, T_0 is temperature of the material in its natural state assumed to be $|\frac{\theta}{T_0}| \ll 1$. The quantities σ_{ij}^L, m_{ij}^L, and $(\rho T_0\eta)^L$ correspond to local thermoelastic solid.

The stress equation of motion for a nonlocal nonhomogeneous transversely isotropic micropolar thermoelastic medium in the absence of body forces is given as

$$\sigma_{ji,j} = \rho \ddot{u}_i. \tag{5}$$

Following Challamel et al. (2016), the nonlocal generalization of the modified Fourier's law of heat conduction for thermoelastic solids is described as

$$\left(1-\epsilon^2\nabla^2\right)\left(1+\tau_0\frac{\partial}{\partial t}\right)q_i = -K_{ij}\theta,_j\,, \tag{6}$$

where q_i are the components of heat flux vector, τ_0 is the relaxation time, and K_{ij} is thermal conductivity such that $K_{ij} = K_{ii}\,\delta_{ij}$.

Within the framework of linear theory of nonlocal thermoelastic materials developed by Eringen (1974), the energy equation for the considered model takes the following form:

$$\rho T_0 \dot{\eta} = -q_{i,i}. \tag{7}$$

Using equations (2) and (6) in equation (7), one can obtain the following form of heat conduction equation:

$$(K_{ij}\theta,_j),_i = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}\right)\left[\rho C_E\theta + T_0\,\beta_{ij}e_{ij}\right]. \tag{8}$$

The couple stress equation of motion for the considered model is described as

$$m_{ik,i} + \mathcal{E}_{ijk}\sigma_{ij} = \rho J\ddot{\Phi}_k, \tag{9}$$

where J is the microinertia.

In the above relations, a comma denotes material derivative, dot indicates partial temporal derivative, and the summation convention is used.

For a functionally graded, i.e., nonhomogeneous material, the parameters A_{ijkl}, β_{ij}, B_{ijkl}, ρ, and K_{ij} are no longer constant but become space-dependent. Hence we consider

$$[A_{ijkl},\ \beta_{ij},\ B_{ijkl},\ \rho,\ K_{ij}] = f(\boldsymbol{x})\,[A'_{ijkl},\ \beta'_{ij},\ B'_{ijkl},\ \rho',\ K'_{ij}], \tag{10}$$

where A'_{ijkl}, β'_{ij}, B'_{ijkl}, ρ', and K'_{ij} are constants and $f(\boldsymbol{x})$ is a given dimensionless function of the space variable $\boldsymbol{x} = (x,\ y,\ z)$. Keeping in view the nonhomogeneity of the parameters defined in (10), the equations (1), (2), (5), (8), and (9) take the following forms:

$$\left(1-\epsilon^2\nabla^2\right)\sigma_{ij} = \sigma^L_{ij} = f(\boldsymbol{x})\left[A'_{ijkl}(u_{l,k}+\mathcal{E}_{lkr}\Phi_r) - \beta'_{ij}\theta\delta_{ij}\right], \tag{11}$$

$$(1-\epsilon^2\nabla^2)m_{ij} = m^L_{ij} = f(\boldsymbol{x})B'_{ijkl}\Phi_{k,l}, \tag{12}$$

$$\left(1-\epsilon^2\nabla^2\right)f(\boldsymbol{x})\rho'\ddot{u}_i = \sigma^L_{ji,j}, \tag{13}$$

$$\left[f(\boldsymbol{x})K'_{ij}\theta,_j\right],_i = \left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}f(\boldsymbol{x})\left(\rho' C_E\theta + \beta'_{ij}T_0u_{i,j}\right), \tag{14}$$

$$\left(1-\epsilon^2\nabla^2\right)f(\boldsymbol{x})\rho' J\ddot{\Phi}_k = m^L_{ik,i} + \mathcal{E}_{ijk}\sigma^L_{ij}. \tag{15}$$

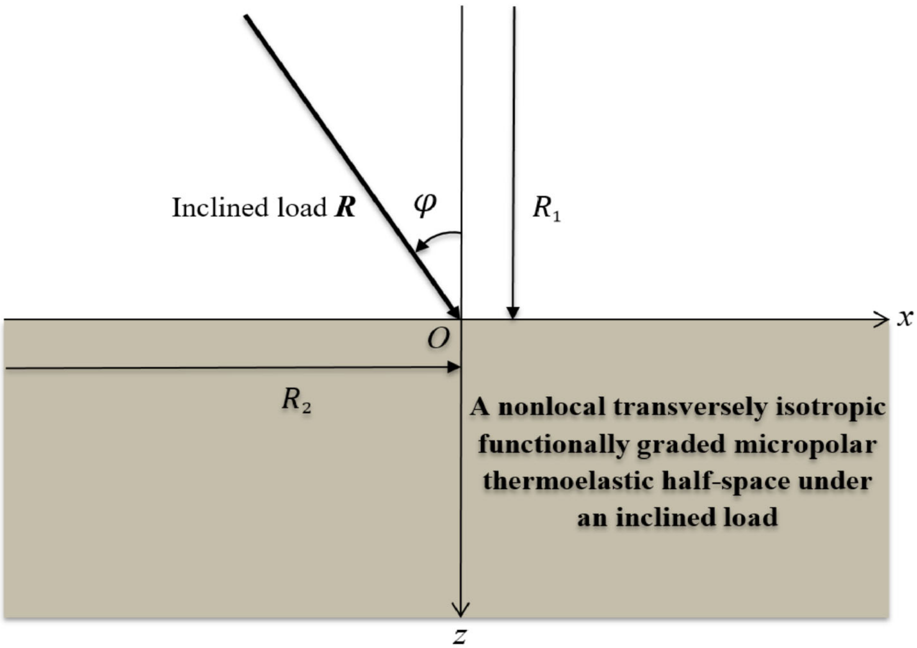

Fig. 1 Geometry of the problem

3 Formulation of the problem

A model made up of a nonlocal functionally graded transversely isotropic micropolar thermoelastic medium under an inclined load in the context of LS theory is considered. The present formulation is restricted to the xz-plane with the z-axis pointing vertically downwards into the medium, as shown in Fig. 1. With the consideration of the xz-plane, all the field quantities are independent of the space variable y. The displacement vector $\boldsymbol{u}$ and microrotation vector $\boldsymbol{\Phi}$ are taken as

$$\boldsymbol{u} = (u, 0, w) \text{ such that } u = u(x, z, t),\ w = w(x, z, t) \text{ and } \boldsymbol{\Phi} = (0, \Phi, 0). \tag{16}$$

The material properties are assumed to be graded in z-direction only, so we take $f(\boldsymbol{x})$ as $f(z)$. Along with these assumptions, the mechanical stresses and couple stresses arising from relations (11) and (12) in the xz-plane can be expressed as

$$(1-\epsilon^2\nabla^2)\sigma_{xx} = \sigma^L_{xx} = f(z)\left(A'_{11}\frac{\partial u}{\partial x} + A'_{13}\frac{\partial w}{\partial z} - \beta'_{11}\theta\right), \tag{17}$$

$$(1-\epsilon^2\nabla^2)\sigma_{zx} = \sigma^L_{zx} = f(z)\left(A'_{56}\frac{\partial w}{\partial x} + A'_{55}\frac{\partial u}{\partial z} + S_1\Phi\right), \tag{18}$$

$$(1-\epsilon^2\nabla^2)\sigma_{xz} = \sigma^L_{xz} = f(z)\left(A'_{66}\frac{\partial w}{\partial x} + A'_{56}\frac{\partial u}{\partial z} + S_2\Phi\right), \tag{19}$$

$$(1-\epsilon^2\nabla^2)\sigma_{zz} = \sigma^L_{zz} = f(z)\left(A'_{13}\frac{\partial u}{\partial x} + A'_{33}\frac{\partial w}{\partial z} - \beta'_{33}\theta\right), \tag{20}$$

$$(1-\epsilon^2\nabla^2)m_{zy}=m^L_{zy}=f(z)\left(B'_{66}\frac{\partial\Phi}{\partial z}\right), \tag{21}$$

$$(1-\epsilon^2\nabla^2)m_{xy}=m^L_{xy}=f(z)\left(B'_{77}\frac{\partial\Phi}{\partial x}\right), \tag{22}$$

where $S_1=A'_{56}-A'_{55}$ and $S_2=A'_{66}-A'_{56}$.

In view of restrictions (16) and components of stresses defined in equations (17)–(22), the stress equation of motion (13), the heat conduction equation (14), and the couple stress equation of motion (15) yield the following equations:

$$\begin{aligned}(1-\epsilon^2\nabla^2)f(z)\rho'\frac{\partial^2 u}{\partial t^2}&=\frac{\partial f(z)}{\partial z}\left[A'_{55}\frac{\partial u}{\partial z}+A'_{56}\frac{\partial w}{\partial x}+S'_1\Phi\right]+f(z)\left[A'_{11}\frac{\partial^2 u}{\partial x^2}\right.\\&\left.+(A'_{13}+A'_{56})\frac{\partial^2 w}{\partial x\partial z}+A'_{55}\frac{\partial^2 u}{\partial z^2}-\beta'_{11}\frac{\partial\theta}{\partial x}+S'_1\frac{\partial\Phi}{\partial z}\right],\end{aligned} \tag{23}$$

$$\begin{aligned}(1-\epsilon^2\nabla^2)f(z)\rho'\frac{\partial^2 w}{\partial t^2}&=\frac{\partial f(z)}{\partial z}\left[A'_{13}\frac{\partial u}{\partial x}+A'_{33}\frac{\partial w}{\partial z}-\beta'_{33}\theta\right]+f(z)\left[A'_{66}\frac{\partial^2 w}{\partial x^2}\right.\\&\left.+(A'_{13}+A'_{56})\frac{\partial^2 u}{\partial x\partial z}+A'_{33}\frac{\partial^2 w}{\partial z^2}-\beta'_{33}\frac{\partial\theta}{\partial z}+S'_2\frac{\partial\Phi}{\partial x}\right],\end{aligned} \tag{24}$$

$$\begin{aligned}&\frac{\partial f(z)}{\partial z}K'_{33}\frac{\partial\theta}{\partial z}+f(z)\left[K'_{11}\frac{\partial^2\theta}{\partial x^2}+K'_{33}\frac{\partial^2\theta}{\partial z^2}\right]\\&\quad=\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}f(z)\left[\rho' C_E\theta+\beta'_{11}T_0\frac{\partial u}{\partial x}+\beta'_{33}T_0\frac{\partial w}{\partial z}\right],\end{aligned} \tag{25}$$

$$\begin{aligned}(1-\epsilon^2\nabla^2)f(z)\rho' J\frac{\partial^2\Phi}{\partial t^2}&=\frac{\partial f(z)}{\partial z}B'_{66}\frac{\partial\Phi}{\partial z}+f(z)\left[B'_{77}\frac{\partial^2\Phi}{\partial x^2}+B'_{66}\frac{\partial^2\Phi}{\partial z^2}\right.\\&\left.-S'_3\Phi-S'_1\frac{\partial u}{\partial z}-S'_2\frac{\partial w}{\partial x}\right],\end{aligned} \tag{26}$$

where $S_3=S_2-S_1$.

For convenience, the governing field equations can be normalized by introducing the following set of nondimensional quantities:

$$\begin{aligned}&(\widehat{x},\ \widehat{z},\ \widehat{u},\ \widehat{w},\ \widehat{\epsilon})=\frac{\omega^*}{c_1}(x,\ z,\ u,\ w,\ \epsilon),\ \ (\widehat{m_{ij}},\ \widehat{m^L_{ij}})=\frac{c_1}{\omega^*\, B'_{66}}(m_{ij},\ m^L_{ij}),\\&(\widehat{\sigma_{ij}},\ \widehat{\sigma^L_{ij}})=\frac{1}{\rho' c_1^2}(\sigma_{ij},\ \sigma^L_{ij}),\ \ (\widehat{t},\ \widehat{\tau_0})=\omega^*(t,\ \tau_0),\ \ \widehat{\Phi}=\frac{A'_{55}}{S_1}\,\Phi,\\&\widehat{\theta}=\frac{1}{T_0}\theta,\end{aligned} \tag{27}$$

where

$$c_1^2=\frac{A'_{11}}{\rho'},\ \omega^{*2}=\frac{S_3}{\rho'\,J}.$$

4 Exponential variation of nonhomogeneity

To account for the nonhomogeneity of the model, let us consider $f(z) = e^{-nz}$, where n is the nonhomogeneity parameter. Using this expression of $f(z)$ and the dimensionless quantities (27), the governing equations (17)–(26) transform to the following forms (ignoring the hats):

$$(1-\epsilon^2\nabla^2)\sigma_{xx} = \sigma_{xx}^L = e^{-nz}\left(\frac{\partial u}{\partial x} + J_1\frac{\partial w}{\partial z} - J_2\theta\right), \tag{28}$$

$$(1-\epsilon^2\nabla^2)\sigma_{zx} = \sigma_{zx}^L = e^{-nz}\left(J_5\frac{\partial w}{\partial x} + J_6\frac{\partial u}{\partial z} + J_7\Phi\right), \tag{29}$$

$$(1-\epsilon^2\nabla^2)\sigma_{xz} = \sigma_{xz}^L = e^{-nz}\left(J_8\frac{\partial w}{\partial x} + J_5\frac{\partial u}{\partial z} + J_9\Phi\right), \tag{30}$$

$$(1-\epsilon^2\nabla^2)\sigma_{zz} = \sigma_{zz}^L = e^{-nz}\left(J_1\frac{\partial u}{\partial x} + J_3\frac{\partial w}{\partial z} - J_4\theta\right), \tag{31}$$

$$(1-\epsilon^2\nabla^2)m_{zy} = m_{zy}^L = e^{-nz}\left(J_{11}\frac{\partial \Phi}{\partial z}\right), \tag{32}$$

$$(1-\epsilon^2\nabla^2)m_{xy} = m_{xy}^L = e^{-nz}\left(J_{10}\frac{\partial \Phi}{\partial x}\right), \tag{33}$$

$$\begin{aligned}(1-\epsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2} = {}&\left[\frac{\partial^2 u}{\partial x^2} + J_{20}\frac{\partial^2 w}{\partial x\partial z} + J_6\frac{\partial^2 u}{\partial z^2} - J_2\frac{\partial \theta}{\partial x} + J_7\frac{\partial \Phi}{\partial z}\right]\\ &-n\left[J_5\frac{\partial w}{\partial x} + J_6\frac{\partial u}{\partial z} + J_7\Phi\right],\end{aligned} \tag{34}$$

$$\begin{aligned}(1-\epsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2} = {}&\left[J_8\frac{\partial^2 w}{\partial x^2} + J_{20}\frac{\partial^2 u}{\partial x\partial z} + J_3\frac{\partial^2 w}{\partial z^2} - J_4\frac{\partial \theta}{\partial z} + J_9\frac{\partial \Phi}{\partial x}\right]\\ &-n\left[J_1\frac{\partial u}{\partial x} + J_3\frac{\partial w}{\partial z} - J_4\theta\right],\end{aligned} \tag{35}$$

$$J_0\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2} - n\frac{\partial\theta}{\partial z} = \left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left[J_{12}\theta + J_{13}\frac{\partial u}{\partial x} + J_{14}\frac{\partial w}{\partial z}\right], \tag{36}$$

$$(1-\epsilon^2\nabla^2)\frac{\partial^2\Phi}{\partial t^2} = J_{15}\frac{\partial^2\Phi}{\partial x^2} + J_{16}\frac{\partial^2\Phi}{\partial z^2} - J_{17}\Phi - J_{18}\frac{\partial u}{\partial z} - J_{19}\frac{\partial w}{\partial x} - nJ_{16}\frac{\partial\Phi}{\partial z}, \tag{37}$$

where

$$J_0 = \frac{K'_{11}}{K'_{33}}, [J_1,\ J_3,\ J_5,\ J_6,\ J_8] = \frac{1}{A'_{11}}[A'_{13},\ A'_{33},\ A'_{56},\ A'_{55},\ A'_{66}],$$

$$[J_2,\ J_4] = \frac{T_0}{A'_{11}}[\beta'_{11},\ \beta'_{33}], [J_7,\ J_9] = \frac{S'_1}{A'_{11}A'_{55}}[S'_1,\ S'_2],$$

$$[J_{10},\ J_{11}] = \frac{S'_1}{B'_{66}A'_{55}}[B'_{77},\ B'_{66}], [J_{12},\ J_{13},\ J_{14}] = \frac{c_1^2}{\omega^* K'_{11}}[\rho' C_E,\ \beta'_{11},\ \beta'_{33}],$$

$$[J_{15},\ J_{16},\ J_1] = \frac{1}{J\ A'_{11}}[B'_{77},\ B'_{66},\ \frac{S'_3 c_1^2}{\omega^{*2}}], [J_{18},\ J_{19}] = \frac{A'_{55}}{J\omega^{*2}\rho S'_1}[S'_1,\ S'_2],$$

$$[J_{18},\ J_{19}] = \frac{1}{\omega^* K_{11}}[K^*_{11},\ K^*_{33}], J_{20} = J_1 + J_5.$$

5 Solution methodology

The exact solutions without any assumed constraints on the physical quantities are obtained in this section using normal mode analysis. Therefore, the physical quantities under discussion can be decomposed in terms of normal modes in the following form:

$$[u,\ w,\ \sigma_{ij},\ \sigma_{ij}^L,\ m_{ij},\ m_{ij}^L,\ \theta,\ \Phi](x,z,t) = [u^*,\ w^*,\ \sigma^*_{ij},\ \sigma_{ij}^{L*},\ m^*_{ij},\ m_{ij}^{L*},\ \theta^*,\ \Phi^*](z)\exp(\omega t + \iota m x), \quad (38)$$

where m is the wave number in the x-direction, ι is the imaginary unit, ω is the frequency, and u^*, w^*, σ^*_{ij}, σ_{ij}^{L*}, m^*_{ij}, m_{ij}^{L*}, θ^*, and Φ^* are the amplitudes of the functions u, w, σ_{ij}, σ^L_{ij}, m_{ij}, m^L_{ij}, θ, and Φ, respectively.

Introducing expression (38) to equations (32)–(35), one can get

$$(I_{11}D^2 + I_{12}D + I_{13})\,u^*(z) + (I_{14}D + I_{15})\,w^*(z) + I_{16}\,\theta^*(z) + (I_{17}D + I_{18})\,\Phi^*(z) = 0, \quad (39)$$

$$(I_{14}D + I_{21})\,u^*(z) + (I_{22}D^2 + I_{23}D + I_{24})\,w^*(z) + (I_{25}D + I_{26})\theta^*(z) + I_{27}\Phi^*(z) = 0, \quad (40)$$

$$I_{31}\,u^*(z) + I_{32}D\,w^*(z) + (D^2 + I_{33}D + I_{34})\,\theta^*(z) = 0, \quad (41)$$

$$I_{41}D\,u^*(z) + I_{42}\,w^*(z) + (I_{43}D^2 + I_{44}D + I_{45})\,\Phi^*(z) = 0, \quad (42)$$

where

$$D = \frac{d}{dz}, I_{11} = J_6 + \epsilon^2\omega^2, I_{12} = -nJ_6, I_{13} = -[m^2 + \omega^2(1+\epsilon^2 m^2)],$$

$$I_{14} = \iota m J_{20}, I_{15} = -\iota n m J_5, I_{16} = -\iota m J_2, I_{17} = J_7, I_{18} = -nJ_7,$$

$$I_{21} = -nmJ_1\iota, I_{22} = J_3 + \epsilon^2\omega^2, I_{23} = -nJ_3, I_{24} = -J_8m^2 - \omega^2(1+\epsilon^2 m^2),$$

$$I_{25} = -J_4, I_{26} = nJ_4, I_{27} = J_9 m\iota, I_{28} = \omega(1+\tau_0\omega), I_{31} = -\iota m J_{13}I_{28},$$

$$I_{32} = -J_{14}I_{28}, I_{33} = -n,\ \ I_{34} = -J_0 m^2 - J_{12}I_{28}, I_{41} = -J_{18},$$

$$I_{42} = -J_{19}m\iota, I_{43} = J_{16} + \epsilon^2\omega^2, I_{44} = -J_{18} - nJ_{16},$$

$$I_{45} = -[\omega^2(1+\epsilon^2 m^2) + J_{15}m^2 + J_{17}].$$

A system of four linear differential equations in the physical quantities $u^*(z)$, $w^*(z)$, $\theta^*(z)$, and $\Phi^*(z)$ are formed by the equations (39) through (42). The following differential equation of order eight is derived by using the elimination method:

$$\left[D^8 + A_1D^7 + A_2D^6 + A_3D^5 + A_4D^4 + A_5D^3 + A_6D^2 + A_7D + A_8\right]$$

$$(u^*,\ w^*,\ \theta^*,\ \Phi^*)(z) = 0, \quad (43)$$

where A_i $(i = 1, 2, 3, \ldots, 8)$ are listed in the Appendix.

The solution of equation (43), which is bounded as $z \to \infty$, is given by

$$(u^*, w^*, \theta^*,\ \Phi^*)(z) = \sum_{j=1}^{4} (H_j, H'_j, H''_j, H'''_j)(m, \omega) e^{-\lambda_j z}, \ \text{ for } \operatorname{Re}(\lambda_j) > 0, \quad (44)$$

where H_j, H'_j, H''_j, and H'''_j are expressions which depend upon ω and m. Using the solutions (44) in the system of equations (39)–(42), one can get the following expressions:

$$[u^*,\ w^*,\ \theta^*,\ \Phi^*](z) = \sum_{j=1}^{4} [1,\ N_{1j},\ N_{2j},\ N_{3j}] H_j(m,\ \omega) e^{-\lambda_j z}, \ \text{ for } \operatorname{Re}(\lambda_j) > 0, \ (45)$$

where

$$N_{1j} = \frac{-(H_{11}\lambda_j^4 - H_{21}\lambda_j^3 + H_{22}\lambda_j^2 - H_{23}\lambda_j + H_{24})}{(-H_{14}\lambda_j^5 + H_{25}\lambda_j^4 - H_{26}\lambda_j^3 + H_{27}\lambda_j^2 - H_{28}\lambda_j + H_{29})},$$

$$N_{2j} = \frac{(H_{11}\lambda_j^2 - H_{12}\lambda_j + H_{13}) + (-H_{14}\lambda_j^3 + H_{15}\lambda_j^2 - H_{16}\lambda_j + H_{17}) N_{1j}}{(-H_{18}\lambda_j^2 + H_{19}\lambda_j - H_{20})},$$

$$N_{3j} = \frac{(I_{11}\lambda_j^2 - I_{12}\lambda_j + I_{13}) + (-I_{14}\lambda_j + I_{15}) N_{1j} + I_{16} N_{2j}}{(I_{17}\lambda_j - I_{18})},$$

$$H_{11} = (I_{27}I_{11} - I_{17}I_{14}), \quad H_{12} = (I_{27}I_{12} - I_{14}I_{18} - I_{21}I_{17}),$$

$$H_{13} = (I_{13}I_{27} - I_{21}I_{18}), \quad H_{14} = (-I_{22}I_{17}), \quad H_{15} = (-I_{22}I_{18} - I_{23}I_{17}),$$

$$H_{16} = (I_{14}I_{27} - I_{23}I_{18} - I_{24}I_{17}), \quad H_{17} = (I_{27}I_{15} - I_{24}I_{18}), \quad H_{18} = (-I_{25}I_{17}),$$

$$H_{19} = (-I_{25}I_{18} - I_{26}I_{17}), \quad H_{20} = (I_{16}I_{27} - I_{18}I_{26}), \quad H_{21} = H_{11}I_{33} + H_{12},$$

$$H_{22} = (H_{11}I_{34} + H_{12}I_{33} + H_{13} - H_{18}I_{31}),$$

$$H_{23} = (H_{12}I_{34} + H_{13}I_{33} - H_{19}I_{31}),$$

$$H_{24} = (H_{13}I_{34} - H_{20}I_{31}), \quad H_{25} = (H_{14}I_{33} + H_{15}),$$

$$H_{26} = (H_{14}I_{34} + H_{15}I_{33} + H_{16} - H_{18}I_{32}),$$

$$H_{27} = (H_{15}I_{34} + H_{16}I_{33} + H_{17} - H_{19}I_{32}),$$

$$H_{28} = (H_{16}I_{34} + H_{17}I_{33} - H_{20}I_{32}),$$

$$H_{29} = (H_{17}I_{34}).$$

In view of solution equation (45), normal stress (31), shear stress (29), and couple stress (32) take the form

$$[\sigma_{zz}^*,\ \sigma_{zx}^*,\ m_{zy}^*](z) = \sum_{j=1}^{4} [N_{4j},\ N_{5j},\ N_{6j}] H_j(m,\ \omega) e^{-\lambda_j z - nz}, \ \text{ for } \operatorname{Re}(\lambda_j) > 0, \quad (46)$$

where

$$N_{4j} = \frac{\left(\iota m J_1 - J_3\lambda_j N_{1j} - J_4 N_{2j}\right)}{\left[1-\epsilon^2(\lambda_j^2 - m^2)\right]}, \quad N_{5j} = \frac{\left(-J_6\lambda_j + J_5\iota m N_{1j} + J_7 N_{3j}\right)}{\left[1-\epsilon^2(\lambda_j^2 - m^2)\right]},$$

$$N_{6j} = \frac{\left(-J_{11}\lambda_j N_{3j}\right)}{\left[1-\epsilon^2(\lambda_j^2 - m^2)\right]}.$$

6 Application: inclined mechanical load is subjected to the surface boundary of the medium

The surface of the nonlocal transversely isotropic micropolar functionally graded thermoelastic half-space, i.e., $z=0$, is subjected to a mechanical load $\boldsymbol{R}(R_1, R_2, 0)$, having an inclination angle φ with the negative x-axis, as shown in Fig. 1. The applied load $\boldsymbol{R}$ is decomposed as a normal load $R_1 = R\cos\varphi$ and shear load $R_2 = R\sin\varphi$, where $|\boldsymbol{R}| = R$. Temperature field and couple stress are assumed to be zero at the surface of the half-space, therefore the boundary conditions can be written as

$$\sigma_{zz}(x,0,t) = -R_1, \tag{47}$$

$$\sigma_{zx}(x,0,t) = -R_2, \tag{48}$$

$$\theta(x,0,t) = 0, \tag{49}$$

$$m_{zy}(x,0,t) = 0, \qquad \text{at } z=0. \tag{50}$$

Using the normal mode technique (38), expressions (45) and (46), the boundary conditions (47)–(50) yield a nonhomogeneous system of four linear equations, which can be written in matrix form as

$$\begin{bmatrix} N_{41} & N_{42} & N_{43} & N_{44} \\ N_{51} & N_{52} & N_{53} & N_{54} \\ N_{21} & N_{22} & N_{23} & N_{24} \\ N_{61} & N_{62} & N_{63} & N_{64} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} -R_1^* \\ -R_2^* \\ 0 \\ 0 \end{bmatrix}, \tag{51}$$

where $R_1^* = R^*\cos\varphi$, $R_2^* = R^*\sin\varphi$, and R^* is defined by the expression $R = R^*\exp(\omega t + imx)$.

The expressions for $H_j (j = 1,2,3,4)$ obtained by solving the system (51) are:

$$H_1 = \frac{\Delta_1}{\Delta}, \quad H_2 = \frac{\Delta_2}{\Delta}, \quad H_3 = \frac{\Delta_3}{\Delta}, \quad H_4 = \frac{\Delta_4}{\Delta}, \tag{52}$$

where

$$\Delta = N_{41}r_1 - N_{42}r_2 + N_{43}r_3 - N_{44}r_4, \Delta_1 = -R_1^* r_1 + R_2^* o_1,$$

$$\Delta_2 = R_1^* r_2 - R_2^* o_2, \Delta_3 = -R_1^* r_3 + R_2^* o_3, \quad \Delta_4 = R_1^* r_4 - R_2^* o_4,$$

$$r_1 = N_{52}(N_{63}N_{24} - N_{23}N_{64}) - N_{53}(N_{62}N_{24} - N_{22}N_{64}) + N_{54}(N_{62}N_{23} - N_{22}N_{63}),$$

$$r_2 = N_{51}(N_{63}N_{24} - N_{23}N_{64}) - N_{53}(N_{61}N_{24} - N_{21}N_{64}) + N_{54}(N_{61}N_{23} - N_{21}N_{63}),$$

$$r_3 = N_{51}(N_{62}N_{24} - N_{22}N_{64}) - N_{52}(N_{61}N_{24} - N_{21}N_{64}) + N_{54}(N_{61}N_{22} - N_{21}N_{62}),$$

$$r_4 = N_{51}(N_{62}N_{23} - N_{22}N_{63}) - N_{52}(N_{61}N_{23} - N_{21}N_{63}) + N_{53}(N_{61}N_{22} - N_{21}N_{62}),$$
$$o_1 = N_{42}(N_{63}N_{24} - N_{23}N_{64}) - N_{43}(N_{62}N_{24} - N_{22}N_{64}) + N_{44}(N_{62}N_{23} - N_{22}N_{63}),$$
$$o_2 = N_{41}(N_{63}N_{24} - N_{23}N_{64}) - N_{43}(N_{61}N_{24} - N_{21}N_{64}) + N_{44}(N_{61}N_{23} - N_{21}N_{63}),$$
$$o_3 = N_{41}(N_{62}N_{24} - N_{22}N_{64}) - N_{42}(N_{61}N_{24} - N_{21}N_{64}) + N_{44}(N_{61}N_{22} - N_{21}N_{62}),$$
$$o_4 = N_{41}(N_{62}N_{23} - N_{22}N_{63}) - N_{42}(N_{61}N_{23} - N_{21}N_{63}) + N_{43}(N_{61}N_{22} - N_{21}N_{62}).$$

Substitution of (52) into (45) and (46) provides us the following expressions of physical fields:

$$u^*(z) = \frac{1}{\Delta}[\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z} + \Delta_4 e^{-\lambda_4 z}], \tag{53}$$

$$w^*(z) = \frac{1}{\Delta}[N_{11}\Delta_1 e^{-\lambda_1 z} + N_{12}\Delta_2 e^{-\lambda_2 z} + N_{13}\Delta_3 e^{-\lambda_3 z} + N_{14}\Delta_4 e^{-\lambda_4 z}], \tag{54}$$

$$\theta^*(z) = \frac{1}{\Delta}[N_{21}\Delta_1 e^{-\lambda_1 z} + N_{22}\Delta_2 e^{-\lambda_2 z} + N_{23}\Delta_3 e^{-\lambda_3 z} + N_{24}\Delta_4 e^{-\lambda_4 z}], \tag{55}$$

$$\Phi^*(z) = \frac{1}{\Delta}[N_{31}\Delta_1 e^{-\lambda_1 z} + N_{32}\Delta_2 e^{-\lambda_2 z} + N_{33}\Delta_3 e^{-\lambda_3 z} + N_{34}\Delta_4 e^{-\lambda_4 z}], \tag{56}$$

$$\sigma^*_{zz}(z) = \frac{1}{\Delta}[N_{41}\Delta_1 e^{-\lambda_1 z} + N_{42}\Delta_2 e^{-\lambda_2 z} + N_{43}\Delta_3 e^{-\lambda_3 z} + N_{44}\Delta_4 e^{-\lambda_4 z}]e^{-nz}, \tag{57}$$

$$\sigma^*_{zx}(z) = \frac{1}{\Delta}[N_{51}\Delta_1 e^{-\lambda_1 z} + N_{52}\Delta_2 e^{-\lambda_2 z} + N_{53}\Delta_3 e^{-\lambda_3 z} + N_{54}\Delta_4 e^{-\lambda_4 z}]e^{-nz}, \tag{58}$$

$$m^*_{zy}(z) = \frac{1}{\Delta}[N_{61}\Delta_1 e^{-\lambda_1 z} + N_{62}\Delta_2 e^{-\lambda_2 z} + N_{63}\Delta_3 e^{-\lambda_3 z} + N_{64}\Delta_4 e^{-\lambda_4 z}]e^{-nz}. \tag{59}$$

7 Particular cases

7.1 Neglecting nonlocality effect

To discuss the problem in a local transversely isotropic functionally graded micopolar thermoelastic medium under an inclined load in the context of LS theory, it is sufficient to set the value of nonlocal parameter ϵ as $\epsilon = 0$ in the basic field equations. Furthermore, by setting the values of stiffness parameters in the constitutive relations as $A'_{11} = A'_{33} = \lambda + 2\mu + K$, $A'_{13} = \lambda$, $A'_{56} = \mu$, $A'_{55} = A'_{66} = \mu + K$, $-S'_1 = S'_2 = K$, $B'_{77} = B'_{66} = \gamma$, $K_{11} = K_{33}$, $\beta'_{11} = \beta'_{33} = (3\lambda + 2\mu)\alpha_t$, $\alpha_{11} = \alpha_{33} = \alpha_t$, one can investigate the interactions in a local isotropic functionally graded micropolar thermoelastic medium under an inclined load in the context of LS theory. Along with this modification, the results obtained match with the limiting case (without magnetic field) of Kalkal et al. (2020).

7.2 Without micropolarity effect

To discuss the problem in a nonlocal transversely isotropic functionally graded thermoelastic medium without micropolarity effect under an inclined load in the context of LS theory, it is sufficient to set the values of parameters in basic equations as $A_{ij} = C_{ij}$, $B_{ij} = J = m_{ij} = \mathcal{E}_{ijk} = 0$. Along with these modifications, by setting $\varphi = 0°$, the results obtained match with the limiting case (without rotation) of Barak and Dhankhar (2023).

Table 1 Physical values of the material parameters

Parameter	Unit	Value	Parameter	Unit	Value
A'_{11}	$\mathrm{N\,m^{-2}}$	17.80×10^{10}	A'_{13}	$\mathrm{N\,m^{-2}}$	7.59×10^{10}
A'_{33}	$\mathrm{N\,m^{-2}}$	1.843×10^{10}	A'_{55}	$\mathrm{N\,m^{-2}}$	4.357×10^{10}
A'_{56}	$\mathrm{N\,m^{-2}}$	1.89×10^{10}	A'_{66}	$\mathrm{N\,m^{-2}}$	4.42×10^{10}
K'_{11}	$\mathrm{W\,m^{-1}\,deg^{-1}}$	1.7×10^{2}	K'_{33}	$\mathrm{W\,m^{-1}\,deg^{-1}}$	1.73×10^{2}
B'_{66}	N	5.648×10^{9}	B'_{77}	N	2.63×10^{9}
ρ'	$\mathrm{kg\,m^{-3}}$	1.74×10^{3}	T_0	K	298
J	$\mathrm{m^2}$	0.2×10^{-2}	C_E	$\mathrm{J\,kg^{-1}\,deg^{-1}}$	1.04×10^{3}
β'_{11}	$\mathrm{N\,m^{-2}\,deg^{-1}}$	2.68×10^{6}	β'_{33}	$\mathrm{N\,m^{-2}\,deg^{-1}}$	2.61×10^{6}
n	$\mathrm{m^{-1}}$	0.1×10^{-1}	ω	$\mathrm{s^{-1}}$	1.0
t	s	0.1×10^{-1}	φ	degree (°)	30
λ'	$\mathrm{N\,m^{-2}}$	7.59×10^{9}	μ'	$\mathrm{N\,m^{-2}}$	1.89×10^{9}
γ'	N	2.63×10^{9}	K'	$\mathrm{N\,m^{-2}}$	1.49×10^{9}
ϵ	m	0.39×10^{-2}	τ_0	s	0.02

7.3 Homogeneous transversely isotropic medium without micropolarity effect

By setting $n = 0$, i.e., nonhomogeneity function $f(\boldsymbol{x}) = 1$ in formulation of without micropolarity effect case of this model, we shall be dealing with a nonlocal homogeneous transversely isotropic thermoelastic medium under an inclined mechanical load in the context of LS theory. In addition, by setting inclination angle of the load as nonlocal parameter as $\varphi = 0°$, the governing equations and formulation of this particular case match exactly with those of Sheoran et al. (2021) and hence, the results obtained in this particular case match with the limiting case (without rotation) of Sheoran et al. (2021).

7.4 Local isotropic nonhomogeneous medium without micropolarity effect

To discuss the problem in an isotropic nonlocal functionally graded micropolar thermoelastic medium in the context of LS, it is sufficient to set the values of stiffness parameters A_{ij} in the constitutive relations as $A_{11} = A_{33} = \lambda + 2\mu + K$, $A_{13} = \lambda$, $A_{56} = \mu$, $A_{55} = A_{66} = \mu + K$, $-S_1 = S_2 = K$, $B_{77} = B_{66} = \gamma$, $K_{11} = K_{33}$, $\beta_{11} = \beta_{33} = (3\lambda + 2\mu)\alpha_t$, $\alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha_t$. Furthermore, by setting $\varphi = 0°$, $\epsilon = 0$, and neglecting micropolarity effect along with these modifications, if we assimilate GN theory of type III instead of LS theory, our results match with the limiting case (without magnetic field) of Gunghas et al. (2019).

8 Numerical results and discussion

In support of the theoretical approach presented earlier, a numerical computation has been carried out using MATLAB software to illustrate the problem in greater details. For the purpose of simulation, we are considering a model made up of a magnesium crystal-like material whose relevant parameters are listed in Table 1 (Eringen (1984) and Kumar and Gupta (2010)). Utilizing the numerical values of the parameters mentioned in Table 1, the values of the dimensionless physical field variables have been computed and results are shown as graphs at various positions along the z-axis with $m = 1.1$, $R^* = 1.0$, and $x = 1.0$.

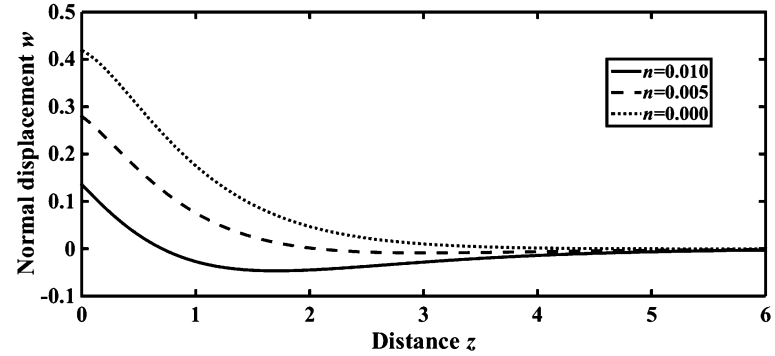

Fig. 2 Effect of nonhomogeneity parameter on normal displacement

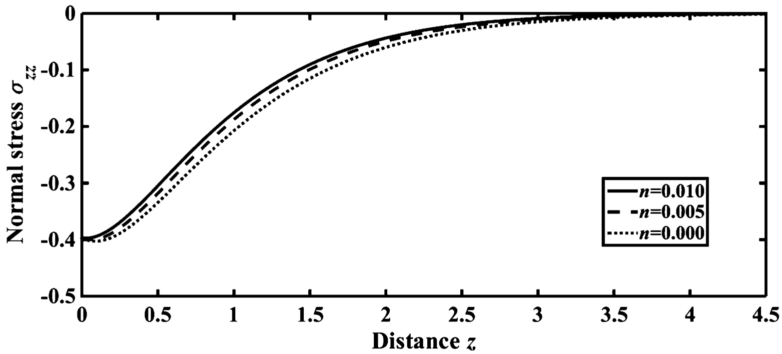

Fig. 3 Effect of nonhomogeneity parameter on normal stress

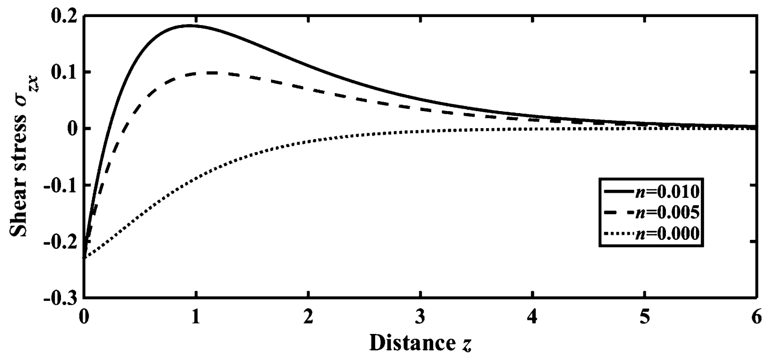

Fig. 4 Effect of nonhomogeneity parameter on shear stress

Figures 2–6 illustrate the effect of nonhomogeneity parameter n on physical fields, i.e., normal displacement, normal stress, shear stress, temperature field, and couple stress, respectively, for three different values of n (0.010, 0.005, and 0.000). The influences of material's anisotropy and time on normal displacement, normal stress, shear stress, temperature field, and couple stress for transversely isotropic and isotropic media at two different values

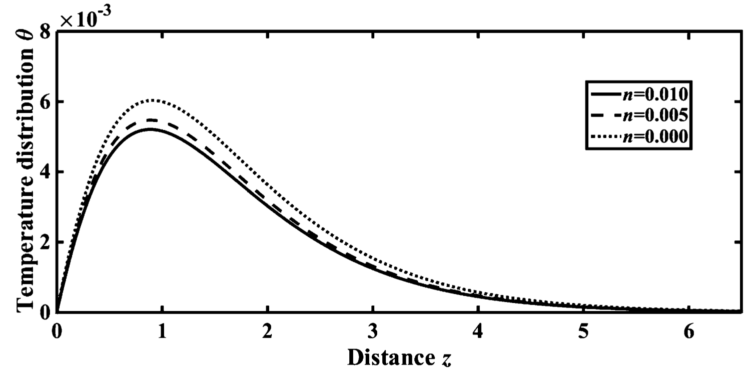

Fig. 5 Effect of nonhomogeneity parameter on temperature field

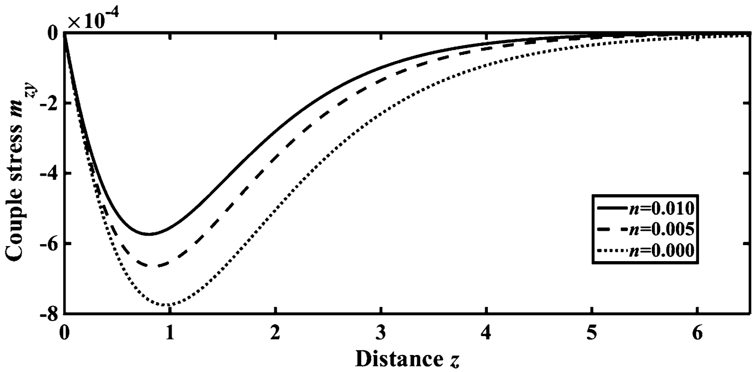

Fig. 6 Effect of nonhomogeneity parameter on couple stress

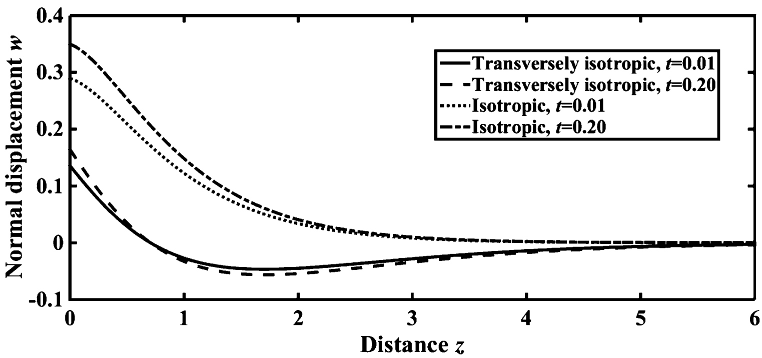

Fig. 7 Influences of material's anisotropy and time on normal displacement

of time t (0.01 and 0.20) are shown in Figs. 7–11, respectively. Figures 12–16 offer the graphic details about the effect of nonlocal parameter on the physical fields for three different values of ϵ (0.39×10^{-2}, 0.39×10^{-3}, and 0.00). To analyze the effect of microinertia J

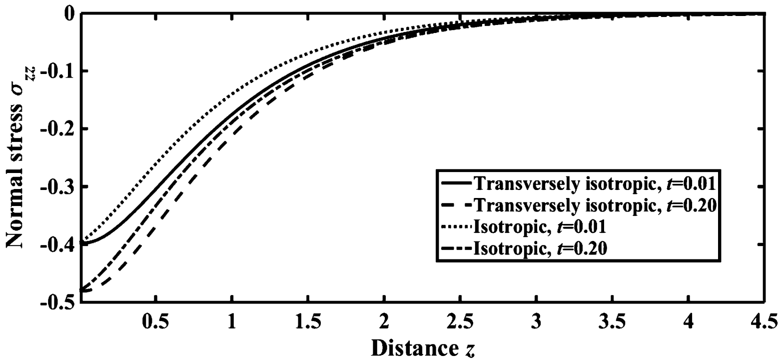

Fig. 8 Influences of material's anisotropy and time on normal stress

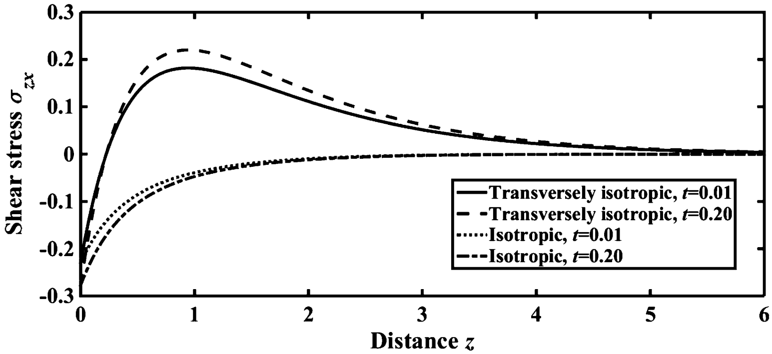

Fig. 9 Influences of material's anisotropy and time on shear stress

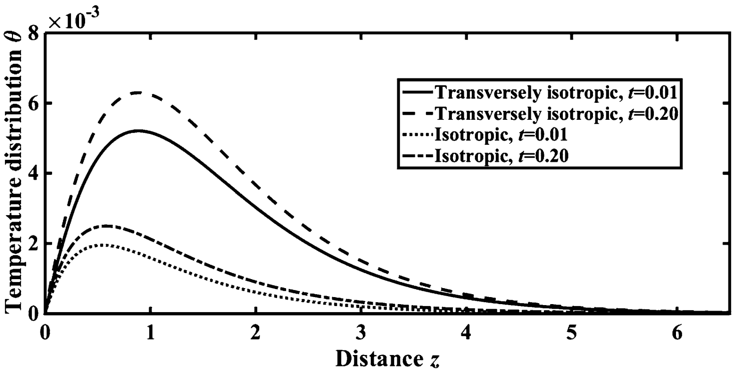

Fig. 10 Influences of material's anisotropy and time on temperature field

on the physical fields, Figs. 17–21 are presented for three different values of J (0.2×10^{-2}, 0.4×10^{-2}, and 0.6×10^{-2}). Figures 22–26 are plotted to analyze the effect of inclination angle φ of the load on all the physical fields for three different values of φ (0°, 30°, and 60°).

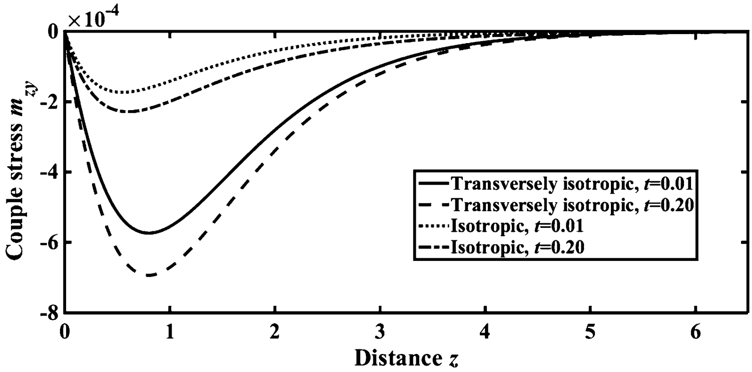

Fig. 11 Influences of material's anisotropy and time on couple stress

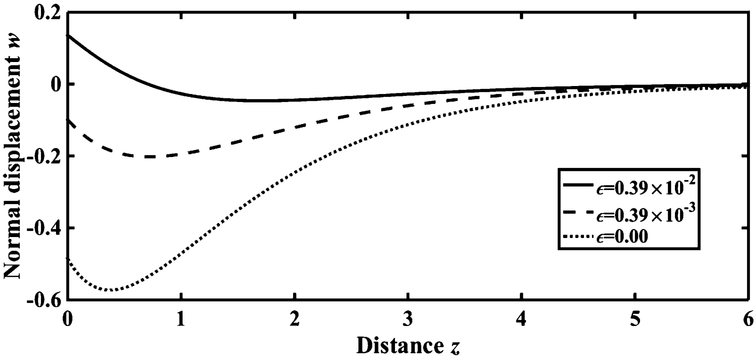

Fig. 12 Effect of nonlocal parameter on normal displacement

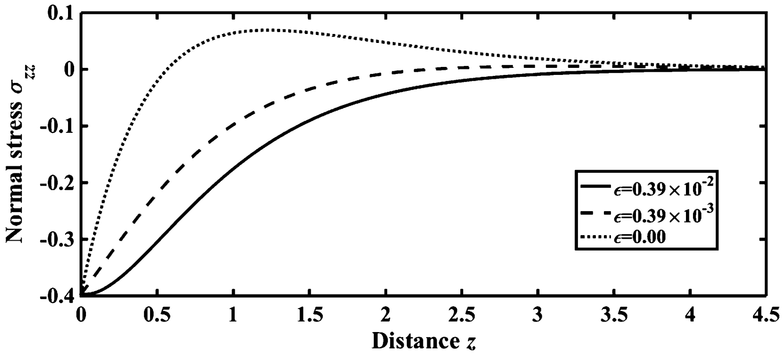

Fig. 13 Effect of nonlocal parameter on normal stress

Figure 2 illustrates that corresponding to three different values (0.01, 0.005, and 0.000) of nonhomogeneity parameter n, all three curves of normal displacement w exhibit same pattern of distribution with difference in magnitudes. It is observed from the figure that the nonhomogeneity parameter has a mixed influence on the profile of normal displacement. It

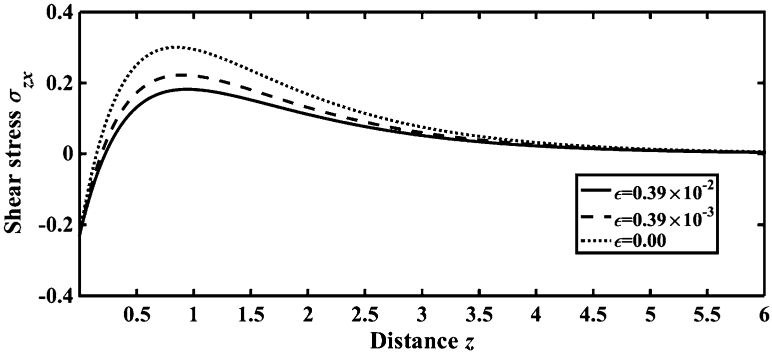

Fig. 14 Effect of nonlocal parameter on shear stress

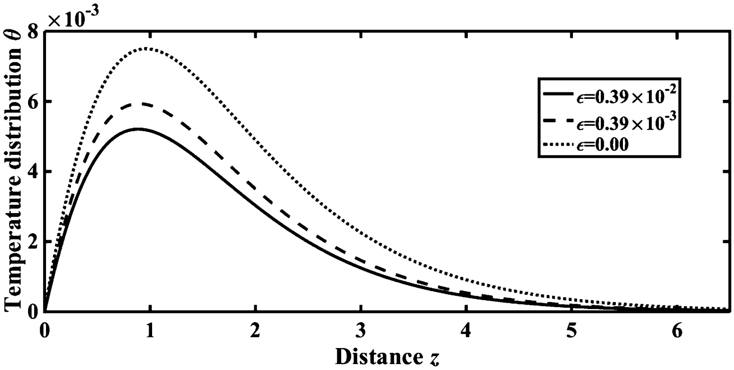

Fig. 15 Effect of nonlocal parameter on temperature field

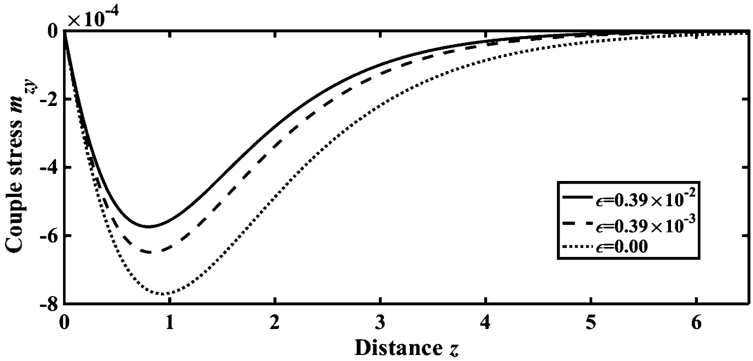

Fig. 16 Effect of nonlocal parameter on couple stress

is seen from Fig. 3 that normal stress is compressive in nature and all three curves of normal stress start with a coinciding nonzero value at the boundary surface of the medium, which satisfies the boundary condition as an inclined mechanical load is imposed on the half-space. It is noted from Fig. 4 that the nonhomogeneity parameter has a mixed effect on shear stress.

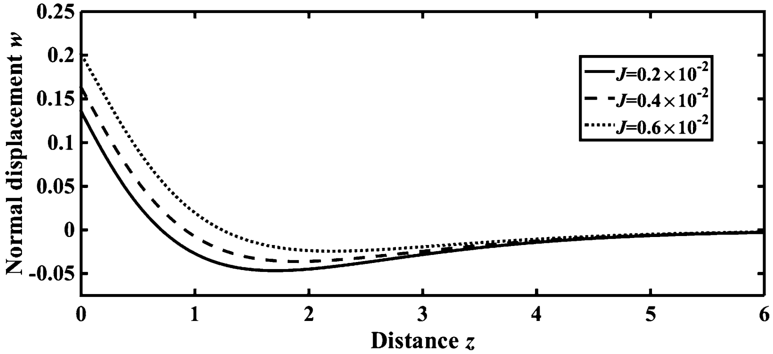

Fig. 17 Effect of microinertia J on normal displacement

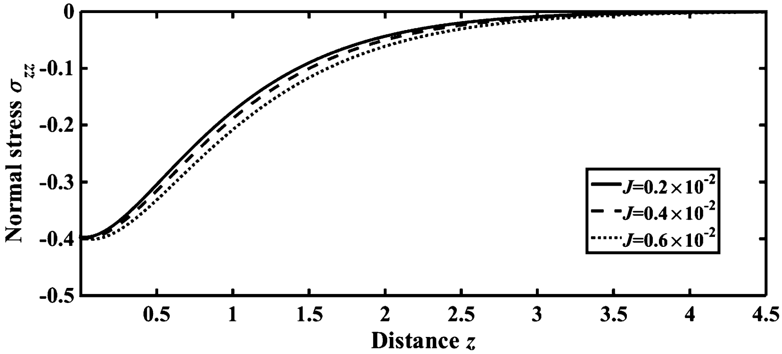

Fig. 18 Effect of microinertia J on normal stress

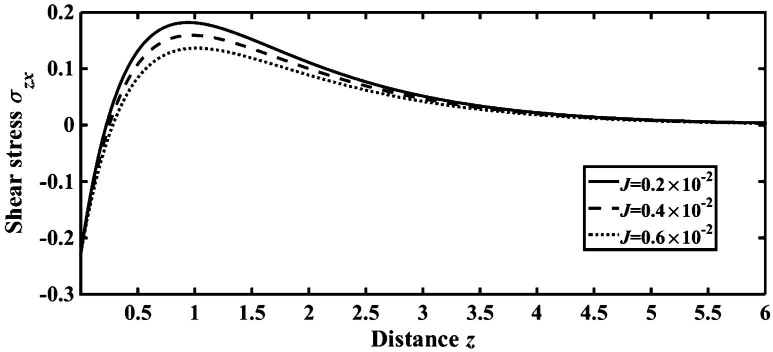

Fig. 19 Effect of microinertia J on shear stress

In Figs. 5–6, there is significant decrement in the modulus values of temperature distribution and couple stress for increasing values of nonhomogeneity parameter n. Therefore, the nonhomogeneity parameter has a decreasing effect on temperature distribution and couple stress.

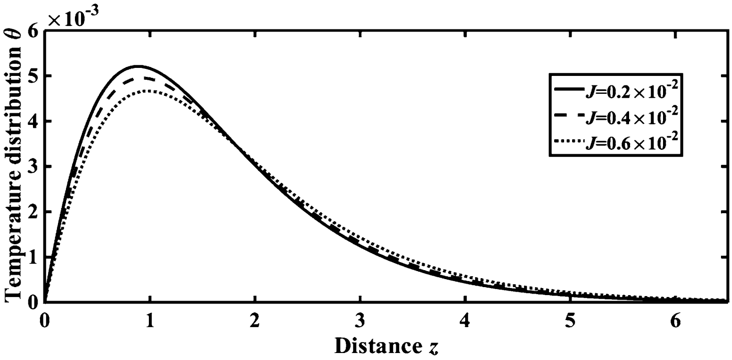

Fig. 20 Effect of microinertia J on temperature field

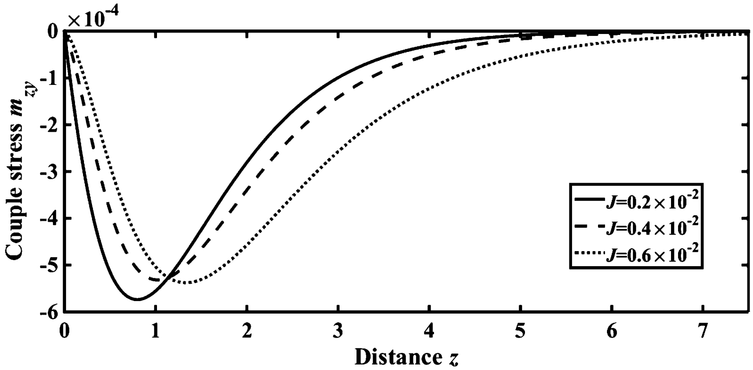

Fig. 21 Effect of microinertia J on couple stress

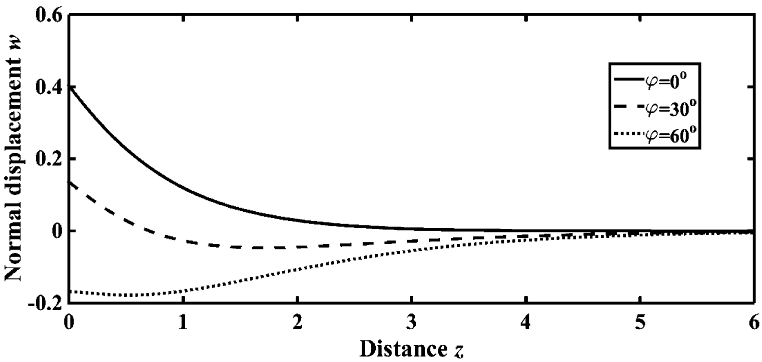

Fig. 22 Effect of inclination angle of the load on normal displacement

Figures 7 and 9 show that all the curves of normal displacement and shear stress start with positive and negative values, respectively, at the boundary surface of the medium. The curves exhibit variations with difference in magnitude to finally approach zero as $z \geq 6$. A careful observation of the figure reveals that the anisotropy of the material has a mixed effect

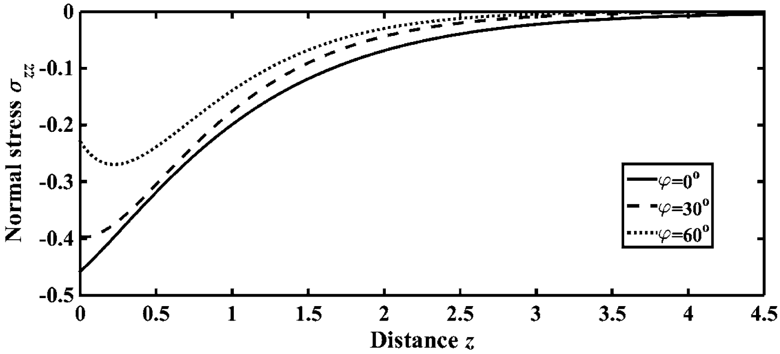

Fig. 23 Effect of inclination angle of the load on normal stress

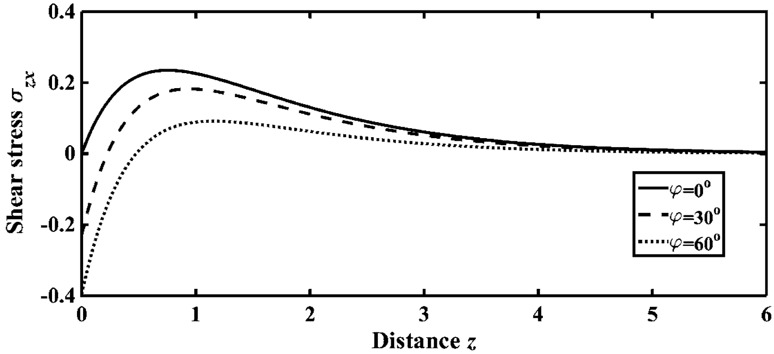

Fig. 24 Effect of inclination angle of the load on shear stress

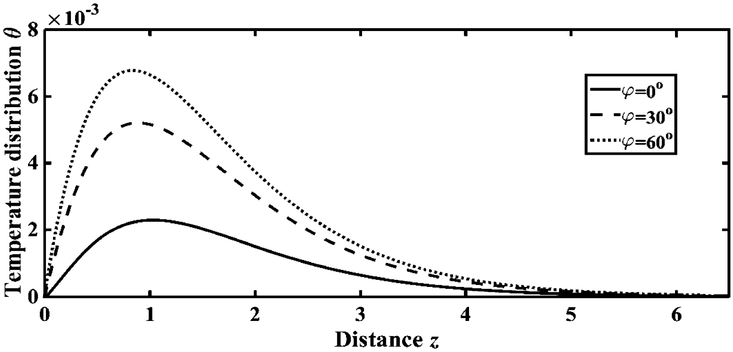

Fig. 25 Effect of inclination angle of the load on temperature field

on normal displacement and shear stress. Figures 8, 10, and 11 show that the magnitude of values of normal stress, temperature field, and couple stress is larger in the transversely isotropic medium than in the isotropic medium. Therefore, material's anisotropy has an increasing effect on the profiles of normal stress, temperature field, and couple stress. In addition, it is also noticed from Figs. 7–11 that all the physical fields are markedly and

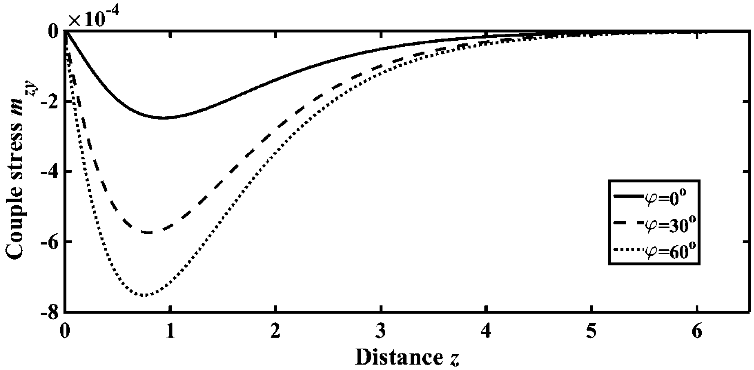

Fig. 26 Effect of inclination angle of the load on couple stress

increasingly influenced by the passage of time t. Figures 12–16 are drawn to observe the graphical details of normal displacement, normal stress, shear stress, temperature field, and couple stress, respectively, against the spatial distance z for three different values of nonlocal parameter ϵ (0.39×10^{-2}, 0.39×10^{-3}, and 0.00). The figures clearly indicate that nonlocal parameter ϵ has a mixed kind of effect on normal displacement, normal stress, and shear stress, whereas it has a decreasing effect on temperature field and couple stress.

Figure 17 is portrayed to examine the influence of micropolar parameter, microinertia J, on the variations of normal displacement for the three different models having $J = 0.2 \times 10^{-2}$, 0.4×10^{-2}, and 0.6×10^{-2}. It is noticed from the figure that the generalized theories have a mixed effect on normal displacement. Figures 18 and 19 reveal the variations of normal stress and shear stress, respectively, for the three specific values of microinertia. A view of the figures emphasizes the point that the normal and shear stresses start with nonzero negative values at the boundary surface of the considered medium, which satisfies the boundary conditions. Also, an increment in value of the microinertia increases the magnitude of normal stress. The variations of temperature distribution and couple stress against distance z are depicted in Figs. 20 and 21, respectively. The figures show that the temperature field and couple stress begin with a zero value at the surface, which is physically plausible and consistent with the theoretical boundary condition because the medium is considered under an inclined mechanical load only. It is observed from these figures that all the physical fields are significantly impacted by the micropolar parameter (microinertia) in a mixed manner, except the normal stress.

Figures 22–26 are drawn to observe the graphical details of normal displacement, normal stress, shear stress, temperature field, and couple stress, respectively, against the spatial distance z for three different values of inclination angle φ of the mechanical load applied at the boundary of the half-space. The figures clearly indicate that inclination angle of the load has a mixed effect on displacement and shear stress, with a decreasing impact on normal stress and an increasing influence on the temperature distribution and couple stress.

9 Concluding remarks

The present study offers a mathematical model for examining the disturbances in a transversely isotropic nonhomogeneous nonlocal micropolar thermoelastic medium due to an inclined mechanical load within the framework of LS theory, utilizing the normal mode technique. Various factors such as nonhomogeneity parameter, material's anisotropy, time, nonlocal parameter, microinertia, and inclination angle influences the distribution of physical fields. The following remarks are concluded from the analysis of this study:

1. The nonhomogeneity parameter has a mixed effect on the normal displacement and shear stress, but it has a decreasing effect on the normal stress, temperature distribution, and couple stress.
2. The anisotropy of the material increasingly affects the profiles of normal stress, temperature field, and couple stress. However, its impact on the profiles of normal displacement and shear stress is mixed. Time has an increasing effect on all the profiles of all the physical fields.
3. The nonlocal parameter exhibits a mixed impact on the normal displacement, normal stress, and shear stress, whereas it consistently increases the profiles of temperature distribution and couple stress.
4. All the physical fields, i.e., normal displacement, normal stress, shear stress, temperature distribution, and couple stress, are significantly impacted by the micropolar parameter, namely microinertia.
5. The inclination angle of the load has a mixed effect on the displacement and shear stress, with a decreasing impact on the normal stress and an increasing influence on the temperature distribution and couple stress.

The aforementioned study examines applications for problems relating to seismology, the second sound effect, developing novel materials, etc.

Appendix

$$A_1 = \frac{L_{12}}{L_{11}},\ A_2 = \frac{L_{13}}{L_{11}},\ A_3 = \frac{L_{14}}{L_{11}},\ A_4 = \frac{L_{15}}{L_{11}},\ A_5 = \frac{L_{16}}{L_{11}},\ A_6 = \frac{L_{17}}{L_{11}},$$

$$A_7 = \frac{L_{18}}{L_{11}},\ A_8 = \frac{L_{19}}{L_{11}},\ L_{11} = I_{11}I_{22}I_{43},$$

$$L_{12} = I_{11}I_{22}I_{44} + I_{11}I_{23}I_{43} + I_{12}I_{22}I_{43} + I_{11}I_{22}I_{33}I_{43},$$

$$L_{13} = T_1 + T_2 + T_3, L_{14} = T_4 + T_5 + T_6 + T_7 + T_8,$$

$$L_{15} = T_9 + T_{10} + T_{11} + T_{12} + T_{13} + T_{14} + T_{15} + T_{16} + T_{17},$$

$$L_{16} = T_{18} + T_{19} + T_{20} + T_{21} + T_{22} + T_{23} + T_{24} + T_{25} + T_{26} + T_{27},$$

$$L_{17} = T_{28} + T_{29} + T_{30} + T_{31} + T_{32} + T_{33} + T_{34} + T_{35} + T_{36},$$

$$L_{18} = T_{37} + T_{38} + T_{39} + T_{40} + T_{41}, L_{19} = +T_{42} + T_{43},$$

$$T_1 = I_{11}I_{22}I_{45} - I_{14}^2 I_{43} + I_{11}I_{23}I_{44} + I_{11}I_{24}I_{43} + I_{12}I_{22}I_{44} + I_{12}I_{23}I_{43},$$

$$T_2 = I_{13}I_{22}I_{43} - I_{17}I_{22}I_{41} + I_{11}I_{22}I_{33}I_{44} + I_{11}I_{22}I_{34}I_{43} + I_{11}I_{23}I_{33}I_{43},$$

$$T_3 = I_{12}I_{22}I_{33}I_{43} - I_{11}I_{25}I_{32}I_{43},$$

$$T_4 = I_{11}I_{23}I_{45} - I_{14}I_{15}I_{43} - I_{14}I_{21}I_{43} - I_{14}^2 I_{44} + I_{11}I_{24}I_{44} + I_{12}I_{22}I_{45},$$

$$T_5 = +I_{12}I_{23}I_{44} + I_{12}I_{24}I_{43} + I_{13}I_{22}I_{44} + I_{13}I_{23}I_{43} - I_{17}I_{23}I_{41} - I_{18}I_{22}I_{41},$$

$$T_6 = -I_{14}^2 I_{33}I_{43} + I_{11}I_{22}I_{33}I_{45} + I_{11}I_{22}I_{34}I_{44} + I_{11}I_{23}I_{33}I_{44} + I_{11}I_{23}I_{34}I_{43},$$

$$T_7 = I_{11}I_{24}I_{33}I_{43} + I_{12}I_{22}I_{33}I_{44} + I_{12}I_{22}I_{34}I_{43} + I_{12}I_{23}I_{33}I_{43} + I_{13}I_{22}I_{33}I_{43},$$

$$T_8 = -I_{11}I_{25}I_{32}I_{44} - I_{11}I_{26}I_{32}I_{43} - I_{12}I_{25}I_{32}I_{43} - I_{17}I_{22}I_{33}I_{41},$$

$$T_9 = I_{14}I_{17}I_{42} - I_{14}I_{15}I_{44} - I_{14}^2I_{45} - I_{14}I_{21}I_{44} - I_{15}I_{21}I_{43} + I_{11}I_{24}I_{45},$$

$$T_{10} = -I_{11}I_{27}I_{42} + I_{12}I_{23}I_{45} + I_{12}I_{24}I_{44} + I_{13}I_{22}I_{45} + I_{13}I_{23}I_{44} + I_{13}I_{24}I_{43},$$

$$T_{11} = I_{14}I_{27}I_{41} - I_{17}I_{24}I_{41} - I_{18}I_{23}I_{41} - I_{14}^2I_{33}I_{44} - I_{14}^2I_{34}I_{43} - I_{14}I_{15}I_{33}I_{43},$$

$$T_{12} = I_{14}I_{16}I_{32}I_{43} - I_{14}I_{21}I_{33}I_{43} + I_{11}I_{22}I_{34}I_{45} + I_{11}I_{23}I_{33}I_{45} + I_{11}I_{23}I_{34}I_{44},$$

$$T_{13} = I_{11}I_{24}I_{33}I_{44} + I_{11}I_{24}I_{34}I_{43} + I_{12}I_{22}I_{33}I_{45} + I_{12}I_{22}I_{34}I_{44} + I_{12}I_{23}I_{33}I_{44},$$

$$T_{14} = I_{12}I_{23}I_{34}I_{43} + I_{12}I_{24}I_{33}I_{43} + I_{13}I_{22}I_{33}I_{44} + I_{13}I_{22}I_{34}I_{43} + I_{13}I_{23}I_{33}I_{43},$$

$$T_{15} = -I_{16}I_{22}I_{31}I_{43} - I_{11}I_{25}I_{32}I_{45} - I_{11}I_{26}I_{32}I_{44} - I_{12}I_{25}I_{32}I_{44} - I_{12}I_{26}I_{32}I_{43},$$

$$T_{16} = -I_{13}I_{25}I_{32}I_{43} + I_{14}I_{25}I_{31}I_{43} - I_{17}I_{22}I_{34}I_{41} - I_{17}I_{23}I_{33}I_{41} - I_{18}I_{22}I_{33}I_{41},$$

$$T_{17} = I_{17}I_{25}I_{32}I_{41},$$

$$T_{18} = I_{14}I_{18}I_{42} - I_{14}I_{15}I_{45} - I_{14}I_{21}I_{45} - I_{15}I_{21}I_{44} + I_{17}I_{21}I_{42} + I_{12}I_{24}I_{45},$$

$$T_{19} = -I_{12}I_{27}I_{42} + I_{13}I_{23}I_{45} + I_{13}I_{24}I_{44} + I_{15}I_{27}I_{41} - I_{18}I_{24}I_{41} - I_{14}^2I_{33}I_{45},$$

$$T_{20} = -I_{14}^2I_{34}I_{44} - I_{14}I_{15}I_{33}I_{44} - I_{14}I_{15}I_{34}I_{43} + I_{14}I_{16}I_{32}I_{44} + I_{14}I_{17}I_{33}I_{42},$$

$$T_{21} = -I_{14}I_{21}I_{33}I_{44} - I_{14}I_{21}I_{34}I_{43} - I_{15}I_{21}I_{33}I_{43} + I_{16}I_{21}I_{32}I_{43} + I_{11}I_{23}I_{34}I_{45},$$

$$T_{22} = I_{11}I_{24}I_{33}I_{45} + I_{11}I_{24}I_{34}I_{44} - I_{11}I_{27}I_{33}I_{42} + I_{12}I_{22}I_{34}I_{45} + I_{12}I_{23}I_{33}I_{45},$$

$$T_{23} = I_{12}I_{23}I_{34}I_{44} + I_{12}I_{24}I_{33}I_{44} + I_{12}I_{24}I_{34}I_{43} + I_{13}I_{22}I_{33}I_{45} + I_{13}I_{22}I_{34}I_{44},$$

$$T_{24} = I_{13}I_{23}I_{33}I_{44} + I_{13}I_{23}I_{34}I_{43} + I_{13}I_{24}I_{33}I_{43} - I_{16}I_{22}I_{31}I_{44} - I_{16}I_{23}I_{31}I_{43},$$

$$T_{25} = -I_{11}I_{26}I_{32}I_{45} - I_{12}I_{25}I_{32}I_{45} - I_{12}I_{26}I_{32}I_{44} - I_{13}I_{25}I_{32}I_{44} - I_{13}I_{26}I_{32}I_{43},$$

$$T_{26} = I_{14}I_{25}I_{31}I_{44} + I_{14}I_{26}I_{31}I_{43} + I_{15}I_{25}I_{31}I_{43} + I_{14}I_{27}I_{33}I_{41} - I_{17}I_{23}I_{34}I_{41},$$

$$T_{27} = -I_{17}I_{24}I_{33}I_{41} - I_{18}I_{22}I_{34}I_{41} - I_{18}I_{23}I_{33}I_{41} + I_{17}I_{26}I_{32}I_{41} + I_{18}I_{25}I_{32}I_{41},$$

$$T_{28} = I_{18}I_{21}I_{42} - I_{15}I_{21}I_{45} + I_{13}I_{24}I_{45} - I_{13}I_{27}I_{42} - I_{14}^2I_{34}I_{45} - I_{14}I_{15}I_{33}I_{45},$$

$$T_{29} = -I_{14}I_{15}I_{34}I_{44} + I_{14}I_{16}I_{32}I_{45} + I_{14}I_{17}I_{34}I_{42} + I_{14}I_{18}I_{33}I_{42} - I_{14}I_{21}I_{33}I_{45},$$

$$T_{30} = -I_{14}I_{21}I_{34}I_{44} - I_{15}I_{21}I_{33}I_{44} - I_{15}I_{21}I_{34}I_{43} + I_{16}I_{21}I_{32}I_{44} + I_{17}I_{21}I_{33}I_{42},$$

$$T_{31} = I_{11}I_{24}I_{34}I_{45} - I_{11}I_{27}I_{34}I_{42} + I_{12}I_{23}I_{34}I_{45} + I_{12}I_{24}I_{33}I_{45} + I_{12}I_{24}I_{34}I_{44},$$

$$T_{32} = -I_{12}I_{27}I_{33}I_{42} + I_{13}I_{22}I_{34}I_{45} + I_{13}I_{23}I_{33}I_{45} + I_{13}I_{23}I_{34}I_{44} + I_{13}I_{24}I_{33}I_{44},$$

$$T_{33} = I_{13}I_{24}I_{34}I_{43} - I_{16}I_{22}I_{31}I_{45} - I_{16}I_{23}I_{31}I_{44} - I_{16}I_{24}I_{31}I_{43} - I_{12}I_{26}I_{32}I_{45},$$

$$T_{34} = -I_{13}I_{25}I_{32}I_{45} - I_{13}I_{26}I_{32}I_{44} + I_{14}I_{25}I_{31}I_{45} + I_{14}I_{26}I_{31}I_{44} + I_{15}I_{25}I_{31}I_{44},$$

$$T_{35} = I_{15}I_{26}I_{31}I_{43} - I_{17}I_{25}I_{31}I_{42} + I_{14}I_{27}I_{34}I_{41} + I_{15}I_{27}I_{33}I_{41} - I_{16}I_{27}I_{32}I_{41},$$

$$T_{36} = -I_{17}I_{24}I_{34}I_{41} - I_{18}I_{23}I_{34}I_{41} - I_{18}I_{24}I_{33}I_{41} + I_{18}I_{26}I_{32}I_{41},$$

$$T_{37} = I_{14}I_{18}I_{34}I_{42} - I_{14}I_{15}I_{34}I_{45} - I_{14}I_{21}I_{34}I_{45} - I_{15}I_{21}I_{33}I_{45} - I_{15}I_{21}I_{34}I_{44},$$

$$T_{38} = I_{16}I_{21}I_{32}I_{45} + I_{17}I_{21}I_{34}I_{42} + I_{18}I_{21}I_{33}I_{42} + I_{12}I_{24}I_{34}I_{45} - I_{12}I_{27}I_{34}I_{42},$$

$$T_{39} = I_{13}I_{23}I_{34}I_{45} + I_{13}I_{24}I_{33}I_{45} + I_{13}I_{24}I_{34}I_{44} - I_{13}I_{27}I_{33}I_{42} - I_{16}I_{23}I_{31}I_{45},$$
$$T_{40} = -I_{16}I_{24}I_{31}I_{44} - I_{13}I_{26}I_{32}I_{45} + I_{14}I_{26}I_{31}I_{45} + I_{15}I_{25}I_{31}I_{45} + I_{15}I_{26}I_{31}I_{44},$$
$$T_{41} = -I_{17}I_{26}I_{31}I_{42} - I_{18}I_{25}I_{31}I_{42} + I_{15}I_{27}I_{34}I_{41} - I_{18}I_{24}I_{34}I_{41},$$
$$T_{42} = -I_{15}I_{21}I_{34}I_{45} + I_{18}I_{21}I_{34}I_{42} + I_{13}I_{24}I_{34}I_{45} - I_{13}I_{27}I_{34}I_{42} - I_{16}I_{24}I_{31}I_{45},$$
$$T_{43} = I_{16}I_{27}I_{31}I_{42} + I_{15}I_{26}I_{31}I_{45} - I_{18}I_{26}I_{31}I_{42}.$$

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Data Availability No datasets were generated or analysed during the current study.

Declarations

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