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Methodology for Parameter Identification in Nonlinear Viscoelastic Material Model

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Abstract. Two methodologies for identification of material functions in Schapery's nonlinear viscoelastic material model are compared in context to their ability to deal with deviations from Heaviside stepwise load application and unloading in real test conditions where the time intervals for load increase to plateau value and to unloading to zero are finite. In the first method the description of the whole loading, creep, unloading and recovery process is given by one-step load application and one-step unloading whereas in the second method the load increase and decrease intervals are approximated by two-step load application with 0.5 of the load applied in the increase region. Vinyl ester with known viscoelastic properties and incremental form of Schapery's constitutive equation is used to simulate "experimental data" for several length of load application and unloading. The two data reduction methodologies are applied to these "data" and the accuracy of identified material functions is compared with the true values (input data).

Key words: creep, strain recovery, ramp loading, incremental, Vinyl ester, nonlinear viscoelasticity

1. Introduction

The most general thermodynamically consistent theory of nonlinear viscoelastic and nonlinear viscoplastic materials was developed by Schapery (1997). It has been used in simulations by several authors; see for example work by Guedes et al. (1998) where laminate theory is developed for layers obeying the above material model. The still unsolved problem is the definition of the set of needed experiments to determine the stress dependent functions in the material model and the development of reliable methodology for data reduction. User has to choose between a general approach and material specific submodels which are efficient but applicable only for a particular type of material.

One special branch of Schapery's theory is based on so called free volume approach introduced by Knaus et al. (1981). The free volume which controls the molecular mobility is a parameter describing the nonlinearity through the reduced time. All other stress state dependent parameters in the constitutive law are assumed equal to 1 as in linear case. Popelar et al. (1997) generalized this approach by introducing distortional term additionally to dilatational term in the free volume. The methodology requires testing in the linear viscoelastic range to determine parameters in

Prony series and linear loading and unloading ramp in tension and shear to determine the parameters in the expression relating the shift factor to the hydrostatic stress and effective stress expressed through invariant of the deviatoric stress.

Experimental methodology for time dependent material characterization in case of linear viscoelasticity and nonlinear viscoplasticity was given by Megnis et al. (2003). It was demonstrated that the creep tests with a following strain recovery tests render the necessary information. The observation that for the used material the viscoelastic response is linear significantly simplified the data reduction.

Methodology to determine the viscoelastic nonlinearity parameters in Schapery's constitutive law for materials which obey the power law time dependence was described by Lou et al. (1971). Both creep and strain recovery data at different load levels are required in analysis. For materials with nonlinear time dependence of creep strain in logarithmic axes the power law is not applicable and expansion in Prony series has to be used. The method for data reduction in this case assuming that the load is in form of a Heaviside step function was developed and applied in Nordin et al. (2005). However, in real test conditions the load change time intervals (time interval during which the load increases to the plateau value and the time interval for load removal (unloading)) are not zero. For long creep tests with evolving viscoelasticity during the whole creep interval the perturbations due to abovementioned factors are considered to have a negligible effect on the creep behavior in instants of time far away from these intervals. However, it is not clear how these changes affect the accuracy of obtained nonlinearity parameters.

The objective of this paper is to inspect in a real test conditions the accuracy of the methodology based on one-step loading as affected by the finite length of the load change time intervals. In the analysis we will use the nonlinear viscoelastic material model with parameters for vinyl ester obtained previously (Nordin et al., 2005; Stahlberg et al., 2005) in compression tests. Material does not obey the power law and the creep interval is rather short (1 hour). Even if the procedure which was used for identification of these parameters is questioned here, we will consider them as true material properties which will be used to simulate the strain response in creep and strain recovery tests with a varying length of the load change intervals. Incremental form of Schapery's equation will be used in simulations. The results will be in following treated as "experimental" data and used in data reduction. The results of data reduction will be compared with input material data (true properties) and the accuracy of the method will be evaluated. An explanation for deviations will be given and a more accurate routine of data reduction based on two-step approximation of the loading and unloading will be suggested.

2. Material Model

The nonlinear viscoelastic nonlinear viscoplastic response of the material to applied stress σ_k, $k = 1,2,\ldots 6$ may be described by the very general thermodynamically consistent material model presented by Schapery (1997). We will use this constitutive law in form presented in Megnis et al. (2003). Since for the analyzed material irreversible strains after recovery period were not observed we neglect the viscoplastic term in the constitutive equation and obtain

$$\varepsilon_i = \varepsilon_i^{el} + b \int_o^{\psi} \Delta S_{ik}(\psi - \psi') \frac{d}{d\psi'} (a_{42}\sigma_k) d\psi' \tag{1}$$

Repeating indexes here and in following mean summation. In (1) the first term represents the elastic strain which may be nonlinear with respect to stress. Integration in the second term is over "reduced time" ψ which is introduced as follows

$$\psi = \int_0^t a_{21} dt' \quad \text{and} \quad \psi' = \int_0^{\tau} a_{21} dt' \tag{2}$$

Here a_{21} is function dependent on stress invariants ($= 1$ in the linear region). The stress dependent function b in the front of the integral depends on the current stress level. Analyzing creep and following strain recovery test this function becomes equal to one in the strain recovery part. Function a_{42} is also stress dependent ($= 1$ in the linear region).

The viscoelastic time dependence is given by functions $\Delta S_{ik}(\psi)$ which are usually chosen in form of Prony series with unknown coefficients C_{ik}^m

$$\Delta S_{ik}(\psi) = \sum_m C_{ik}^m \left(1 - \exp\left(-\frac{\psi}{\tau_m}\right)\right) \tag{3}$$

The same set of retardation times τ_m is used for all strain components.

3. Material Properties

The intrinsic time dependent material properties of the vinyl-ester were obtained using data reduction according to the methodology described in Appendix. Analysis was performed using the experimental results from the compressive creep-recovery tests on cylindrical specimens with the axial size of 25 mm and radius of 6 mm; see Nordin et al. (2005) and Stahlberg et al. (2005) for details. The coefficients in Prony series are given in Table I. Since the material is isotropic and strain response only in load direction in uniaxial loading will be considered, only C_{11}^m are of relevance for the following.

The dimensionless stress dependent functions in the material model have following approximations (stress is in MPa)

$$\varepsilon_i^{el} = 3.539 \cdot 10^{-7} \sigma_1^2 + 2.465 \cdot 10^{-4} \sigma_1 \tag{4}$$

Table I. Coefficients C_{11}^{m} in the Prony series

m	τ_m (s)	C_{11}^{m} (1/Pa)
1	3	$8.14\,10^{-13}$
2	10	$5.38\,10^{-12}$
3	30	$-7.50\,10^{-13}$
4	100	$8.63\,10^{-12}$
5	300	$1.90\,10^{-12}$
6	1000	$7.43\,10^{-12}$
7	3000	$-1.10\,10^{-12}$
8	10000	$1.81\,10^{-11}$

$$b = \begin{cases} 1 & \sigma_1 \leq 30\,\text{MPa} \\ 0.01912\sigma_1 + 0.4208 & \sigma_1 > 30\,\text{MPa} \end{cases} \tag{5}$$

$$a_{21} = \left(\begin{array}{ll} 1 & \sigma_1 \leq 30\,\text{MPa} \\ 2.657 \cdot 10^{-4}\sigma_1^2 - 0.04777\sigma_1 + 2.195 & \sigma_1 > 30\,\text{MPa} \end{array} \right. \tag{6}$$

$$a_{42} = \left(\begin{array}{ll} 1 & \sigma_1 \leq 30\,\text{MPa} \\ 1.171 \cdot 10^{-6}\sigma_1^4 - 2.25 \cdot 10^{-4}\sigma_1^3 & \\ \quad +1.631 \cdot 10^{-2}\sigma_1^2 - 0.5095\sigma_1 + 6.755 & \sigma_1 > 30\,\text{MPa} \end{array} \right. \tag{7}$$

4. Simulation of Strain Response in Creep-Strain Recovery Test

To reduce the number of used indexes we will in following take $k = 1$ (thus assuming that only σ_1 acting). Two loading cases shown in Figure 1 will be considered.

The first case shown in Figure l(a) corresponds to real loading conditions in compression test when the load is increasing linearly during time interval t_l until the plateau value for the creep test is reached. Then follows the creep period of length t_c. Unloading is also linear with respect to time and takes time t_u. After the unloading the strain recovery is analyzed ($t > 0t_l + t_c + t_u$).

The second case is a simplified description of the real test assuming that the load application and unloading time periods (t_l and t_u) are small as compared to the length of the creep test and the strain recovery. Then it is assumed that the strain perturbations caused by the particular time dependence of load increase and decrease may be neglected and the loading and unloading is described by Heaviside step function, see Figure (1b). This case corresponds to very fast load change during loading and unloading. The 3rd case is a two-step approximation of the linear stress change.

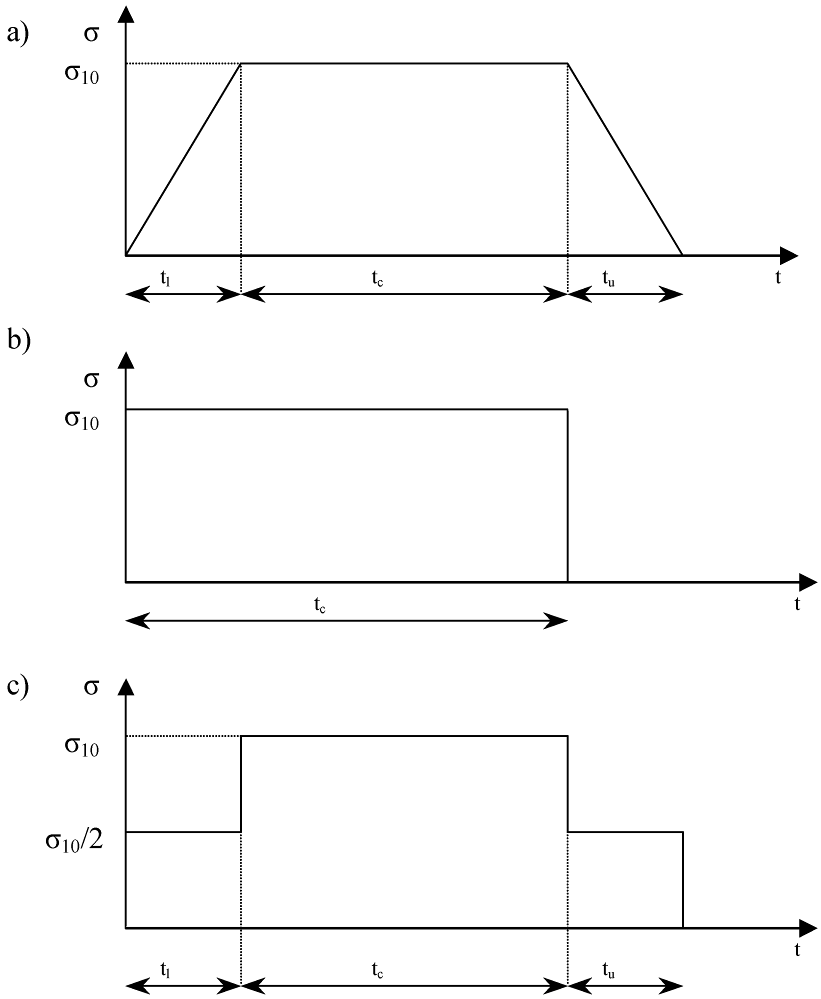

Figure 1. Schematic showing of the three cases of load application and removal in creep-strain recovery test: (a) real case with linear increase and decrease of stress; (b) one-step Heaviside loading and unloading; (c) two-step loading and unloading.

4.1. Linear Loading and Unloading

Stress is considered as a known function of time $\sigma_1 = \sigma_1(t)$.

$$\sigma_1 = \begin{cases} \sigma_{10}\dfrac{t}{t_l}, & t < t_l \\ \sigma_{10}, & t_1 < t < t_l + t_c \\ -\sigma_{10}\dfrac{t - t_l - t_c - t_u}{t_u}, & t_1 + t_c < t < t_l + t_c + t_u \\ 0, & t > t_l + t_c + t_u \end{cases} \tag{8}$$

In case of uniaxial loading in direction 1, considering only the strain component 1 constitutive Equations (1) and (3) may be rewritten in a different form more suitable for calculations. Substitute (3) in (1) and integrate. The result is

$$\varepsilon_1(t) = \varepsilon_1^{el}(\sigma) + b(\sigma_1)a_{42}\sigma_1 \sum_m C_{11}^m - b(\sigma_1)\sum_m \varepsilon_1^m(t) \tag{9}$$

where

$$\varepsilon_1^m(t) = \int_0^{\psi} C_{11}^m e \frac{\psi - \psi'}{\tau_m} \frac{d}{d\psi'}(a_{42}\sigma_1)d\psi' \tag{10}$$

The integral (10) is calculated in a time instant t_k using the value of this function in the time instant t_{k-1} in a recursive expression. Both time instants are related by $t_k = t_{k-1} + \Delta t$. The increment of time may be related to the increment of ψ using differential of (2) $d\psi = a_{21}dt$ which we replace by a finite difference $\Delta\psi = a_{21}\Delta t$.

The recursive expression correlating $\varepsilon_1^m(t_k)$ to $\varepsilon_1^m(t_{k-1})$ is as follows

$$\begin{aligned}\varepsilon_1^m(t_k) &= e\frac{\Delta\psi}{\tau_m}\varepsilon_1^m(t_{k-1})\\ &\quad + C_{11}^m\left(1 - e\frac{\Delta\psi}{\tau_m}\right)\frac{\tau_m}{\Delta t}\frac{d(a_{42}\sigma_1)^{k-1}}{d\sigma_1}\frac{1}{a_{21}^{k-1}}\Delta\sigma_1^{k-1}\\ k &= 1, 2 \ldots.\end{aligned} \tag{11}$$

with $\boldsymbol{t}_0 = 0$, $\varepsilon_1^m(0) = 0$.

The simulations were made by implementing the Equations (9)–(11) and the nonlinearity functions ε_1^{el}, b, a_{21}, a_{42} from Section 3 in MATLAB functions. The output of data was with a step of 1s, whereas Δt in (11) was reduced until converging results were obtained. The value of 0.001s was found as the most suitable for accurate calculation.

4.2. Heaviside Step Loading and Unloading

4.2.1. *Creep Test*

In creep test, the stress σ_1 is applied as a step function at $t = 0$, see Figure l(b)

$$\sigma_1 = \sigma_{10}H(\psi - 0) \tag{12}$$

In creep test from (2) follows

$$\psi = a_{21}t \tag{13}$$

Using (12) and (13) in (1) and using the property of Dirac delta function we obtain

$$\varepsilon_i^{\text{creep}}(t) = \varepsilon_i^{el} + b\, a_{42}\sigma_{10}\Delta S_{i1}(a_{21}t) \tag{14}$$

Using (3) Equation (14) can be rewritten in form

$$\varepsilon_i^{\text{creep}}(t) = \varepsilon_i^{el} + b\, a_{42}\sigma_{10}\sum_m C_{i1}^m\left(1 - \exp\left(-a_{21}\frac{t}{\tau_m}\right)\right) \tag{15}$$

4.2.2. *Strain Recovery After Creep Test*

Simulating the strain recovery, the removal of the load in the time instant $t_1 = t_c$, see Figure 1, can be described as a superposition of the previously applied step function and a step function with an opposite sign applied at instant of time t_1. Corresponding value of ψ for this time instant is denoted ψ_1.

$$\sigma_1 = \sigma_{10}(H(\psi - 0) - H(\psi - \psi_1)) \tag{16}$$

Substituting (16) in (1) and considering time region $t > t_1$ we obtain

$$\varepsilon_i^{rec} = 0 + \Delta S_{i1}(\psi)\, a_{42}\sigma_{10} - \Delta S_{i1}(\psi - \psi_1)\, a_{42}\sigma_{10} \tag{17}$$

We remind that a_{42} as well as a_{21} are unknown functions of stress level σ_{10} applied in the creep test. Since stress is constant during the loading period, ψ in strain recovery stage is easily obtained using (2)

$$\psi = (a_{21} - 1)_{t_1} + t \tag{18}$$

Using (3) and (18) in Equation (17) we obtain

$$\varepsilon_i^{rec} = a_{42}\sigma_{10}\sum_m C_{i1}^m \exp\left(-\frac{t - t_1}{\tau_m}\right)\left(1 - \exp\left(-a_{21}\frac{t_1}{\tau_m}\right)\right) \tag{19}$$

Introducing strains normalized with respect to stress σ_{10} (notation $\tilde{\varepsilon}_i^{rec}$) and introducing new unknown time independent but stress dependent parameters

$$A_{ik}^m = a_{42}C_{ik}^m\left(1 - \exp\left(-a_{21}\frac{t_1}{\tau_m}\right)\right) \tag{20}$$

expression (19) when rewritten for normalized strains becomes

$$\tilde{\varepsilon}_i^{rec} = \sum_m A_{i1}^m \exp\left(-\frac{t - t_1}{\tau_m}\right) \tag{21}$$

4.3. Two-Step Loading and Unloading in Creep Test

In this case we assume that the specimen loading to the stress level σ_{10} used in the creep test is in two steps. First constant stress $\sigma_{10}/2$ is applied and t_l seconds later an additional stress $\sigma_{10}/2$. After the creep period t_c the load is removed also in two steps: first the load is reduced to $\sigma_{10}/2$ and t_u seconds later even this load is removed in one step. The strain response during creep and strain recovery for two-step loading can be described by simple expressions. The described sequence may correspond to real load application scenario in creep test. However, here we want to analyze this case as a possible analytical approximation of the linear load application case. In this model the creep strain at $t > t_l$ may be expressed as

$$\begin{aligned}\varepsilon_i^{\text{creep}}(t,\sigma_{10}) &= \varepsilon_i^{el}(\sigma_{10}) + b(\sigma_{10})\sigma_{10}\left\{\Delta S(\psi)\frac{1}{2}a_{42}(\sigma_{10}/2)\right.\\ &\left.+\Delta S(\psi-\psi_l)\left[a_{42}(\sigma_{10}) - \frac{1}{2}a_{42}(\sigma_{10}/2)\right]\right\} \end{aligned} \tag{22}$$

which for the simplified case $t_l = t_u$ may be rewritten as

$$\begin{aligned}\varepsilon_i^{\text{creep}}(t,\sigma_{10}) &= \varepsilon_l^{el}(\sigma_{10}) + b(\sigma_{10})\sigma_{10}\\ &\times\sum_m C_{i1}^m\left\{a_{42}(\sigma_{10})\left[1-\exp\left(-\frac{a_{21}(\sigma_{10})(t-t_1)}{\tau_m}\right)\right]\right.\\ &\left.+\frac{a_{42}(\sigma_{10}/2)}{2}\left[1-\exp\left(-\frac{a_{21}(\sigma_{10}/2)t_1}{\tau_m}\right)\right]\exp\left(-\frac{a_{21}(\sigma_{10})(t-t_1)}{\tau_m}\right)\right\}\end{aligned} \tag{23}$$

The strain recovery ($t > t_l + t_c + t_u$) can be described as

$$\begin{aligned}\varepsilon_i^{rec} &= \frac{1}{2}\sigma_{10}a_{42}(\sigma_{10}/2)\Delta S(\psi) + \sigma_{10}\left[a_{42}(\sigma_{10}) - \frac{1}{2}a_{42}(\sigma_{10}/2)\right]\Delta S(\psi-\psi_l)\\ &-\sigma_{10}\left[a_{42}(\sigma_{10}) - \frac{1}{2}a_{42}(\sigma_{10}/2)\right]\Delta S(\psi-\psi_c)\\ &-\frac{1}{2}\sigma_{10}a_{42}(\sigma_{10}/2)\Delta S(\psi-\psi_u)\end{aligned} \tag{24}$$

Here ψ_l, ψ_c and ψ_u correspond to time instants t_l, $t_l + t_c$ and $t_1 = t_l + t_c + t_u$ respectively. The last equation may be rewritten as

$$\varepsilon_i^{rec} = \sigma_{10}\sum_m A_{i1}^m \exp\left(-\frac{t-t_1}{\tau_m}\right) \tag{25}$$

where

$$A_{i1}^{m} = C_{i1}^{m}\phi_m \tag{26}$$

and assuming $t_l = t_u$

$$\begin{aligned}\phi_m = {}& a_{42}(\sigma_{10}) \exp\left(-\frac{a_{21}(\sigma_{10}/2)t_u}{\tau_m}\right)\left[1 - \exp\left(-\frac{a_{21}(\sigma_{10})t_c}{\tau_m}\right)\right] \\ & + \frac{1}{2}a_{42}(\sigma_{10}/2)\left[1 - \exp\left(-\frac{a_{21}(\sigma_{10}/2)t_u}{\tau_m}\right)\right] \\ & \times \left[1 + \exp\left(-\frac{a_{21}(\sigma_{10})t_c + a_{21}(\sigma_{10}/2)t_u}{\tau_m}\right)\right]\end{aligned} \tag{27}$$

In the linear region

$$\phi_m = \frac{1}{2}\left[1 + \exp\left(-\frac{t_u}{\tau_m}\right)\right]\left[1 + \exp\left(-\frac{t_c + t_u}{\tau_m}\right)\right] \tag{28}$$

5. Results and Discussion

The incremental formulation of the stress – strain relationship (1)–(3) for nonlinear viscoelastic materials given by Equation (9) to (11) was used together with material data for the considered material given in Section 3 to generate creep–strain recovery test data.

Four cases with different load change intervals were considered: (a) infinite rate which corresponds to Heaviside step function type of load application, $t_l = t_u = 0s$; (b), with $t_l = t_u = 10s$ and (c) with $t_l = t_u = 20s$ and with $t_l = t_u = 30s$, see Figure (1a) for used notation. The generated data are in following considered as test data.

5.1. Accuracy of the Methodology Based on One-Step Loading Assumptions

In this subsection the methodology described in Appendix is used to obtain material properties from these tests. Comparing the results with the true values we will draw conclusions regarding the applicability and accuracy of the data reduction methodology which is based on one-step load application expressions. An example of the nonlinear creep and strain recovery curves is shown in Figure 2.

The one-step model exactly describes strain development in test with Heaviside step type of load application and removal, see Figure 1(b). Hence, the results in this loading case exactly coincide with the input data.

The elastic strain ε_1^{el} which according to Appendix is equal to the total strain in the time instant $t = t_l$ is given in Table II. The error is increasing with t_l and stress.

Table II. ε_1^{el} (%) determined as $\varepsilon_1(t_l)$ and its accuracy with varying rates of load increase. One-step model.

$\sigma_1 0\,(MPa)$	Input	$t_l = t_u = 0$		$t_l = t_u = 10s$		$t_l = t_u = 20s$		$t_l = t_u = 30s$	
			(%)		(%)		(%)		(%)
30	0.771	0.771	0.0	0.780	1.1	0.785	1.8	0.788	2.2
60	1.606	1.606	0.0	1.632	1.6	1.646	2.5	1.656	3.1
90	2.505	2.505	0.0	2.546	1.7	2.569	2.6	2.589	3.4

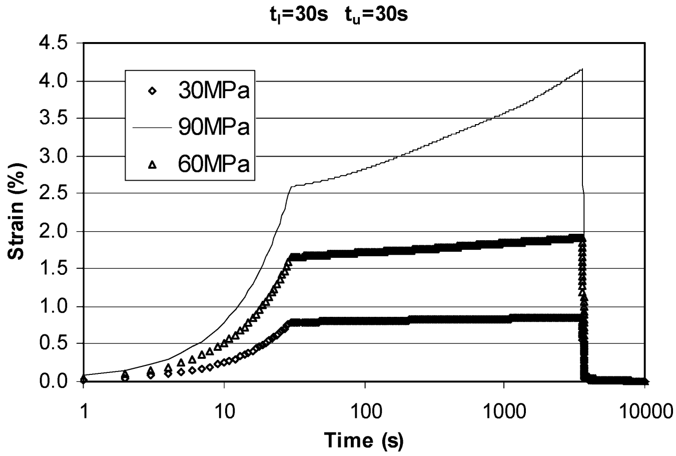

Figure 2. Creep and strain recovery curves at three load levels.

The difference is due to viscoelastic strains developing during the load increase interval t_l. The viscoelastic constants C_{11}^m from test at $\sigma_{10} = 30$ MPa are presented in Table III.It appears that only constants corresponding to τ_m which are at least 10 times larger than t_u have sufficient accuracy and hence the viscoelastic description in short time region is inaccurate. The obtained results for $b(\sigma_{10})$ are presented in Table IV. The error is rather large even if t_l, is only 10s (about 20% at 90 MPa). It rapidly increases with increasing t_l and stress. The calculated a_{21} and a_{42} in Table V and Table VI tab demonstrate that the values may be rather erroneous at higher loads and increasing t_l. The obtained values are higher than the input data. The error in a_{42} values is not that big as for a_{21}. The trend in differences is opposite than for a_{21}: the values of a_{42} determined from tests with a finite loading rate are lower than the input data.

The difference between the total strain just before and after the unloading is sometimes used for elastic strain determination. The introduced error is shown in Table VII.

The results presented in this section convincingly prove that the simple methodology based on one-step model in many cases fails to give true material properties. It does not correspond to the real test conditions. The common assumption that this

Table III. C_{11}^{m} values from tests with varying rates of load increase. One-step model.

τ_m (s)	Input (1/Pa)	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
3	$8.145\ 10^{-13}$	$8.156\ 10^{-13}$	$3.293\ 10^{-13}$	$1.706\ 10^{-13}$	$1.143\ 10^{-13}$
10	$5.379\ 10^{-12}$	$5.378\ 10^{-12}$	$3.757\ 10^{-12}$	$2.570\ 10^{-12}$	$1.882\ 10^{-12}$
30	$-7.458\ 10^{-13}$	$-7.449\ 10^{-13}$	$-6.547\ 10^{-13}$	$-5.624\ 10^{-13}$	$-4.869\ 10^{-13}$
100	$8.630\ 10^{-12}$	$8.630\ 10^{-12}$	$8.295\ 10^{-12}$	$7.900\ 10^{-12}$	$7.531\ 10^{-12}$
300	$1.900\ 10^{-12}$	$1.900\ 10^{-12}$	$1.875\ 10^{-12}$	$1.844\ 10^{-12}$	$1.814\ 10^{-12}$
1000	$7.429\ 10^{-12}$	$7.428\ 10^{-12}$	$7.397\ 10^{-12}$	$7.358\ 10^{-12}$	$7.324\ 10^{-12}$
3000	$-1.076\ 10^{-12}$	$-1.076\ 10^{-12}$	$-1.073\ 10^{-12}$	$-1.070\ 10^{-12}$	$-1.070\ 10^{-12}$
10000	$1.807\ 10^{-11}$	$1.807\ 10^{-11}$	1.80210^{-11}	$1.797\ 10^{-11}$	$1.800\ 10^{-11}$

Table IV. $b(\sigma_{10})$ values from tests with varying rates of load increase. One-step model.

σ_{10} (MPa)	Input	Heaviside $t_l = t_u = 0$	$t_l = t_u = 10s$	$t_l = t_u = 20s$	$t_l = t_u = 30s$
30	1.000	1.000	0.979	0.983	0.986
60	1.568	1.568	1.654	1.735	1.783
90	2.142	2.142	2.554	2.967	3.283

Table V. a_{21} values from tests with varying rates of load increase. One-step model.

σ_{10} (MPa)	Input	Heaviside $t_l = t_u = 0$	$t_l = t_u = 10s$	$t_l = t_u = 20s$	$t_l = t_u = 30s$
30	1.000	1.000	1.000	1.000	1.000
60	0.285	0.285	0.325	0.360	0.385
90	0.048	0.048	0.065	0.080	0.088

Table VI. a_{42} values from tests with varying rates of load increase. One-step model.

σ_{10} (MPa)	Input	Heaviside $t_l = t_u = 0$	$t_l = t_u = 10s$	$t_l = t_u = 20s$	$t_l = t_u = 30s$
30	1.000	1.000	1.000	1.000	1.000
60	1.460	1.460	1.389	1.342	1.311
90	5.781	5.779	5.067	4.561	4.296

discrepancy may affect the creep behavior only in the short time region appears to be incorrect when dealing with nonlinear materials, because not only the constants related to first terms of Prony series are affected but also the whole set of stress dependent material functions.

In the following subsection we will inspect the details of the strain variations just after the load has reached its plateau value and also in the region just after the load is removed considering the loading rate as a variable. This discussion will motivate an introduction of a more reliable (but also more complex) data reduction methodology.

5.2. Loading Rate Effects on Strain Curves

First we consider in details the simulated strain curves during the load increase from zero to the plateau value and a short time interval after that, shown in Figure 3. Four different load cases specified with the value of t_l as discussed in Section 5.1 are presented.

The values at $t = 0$ in one-step loading test shown as H in Figure 3 correspond to true elastic response of the material. Obviously it is lower than the value of $\varepsilon_1(t_l)$ tests with finite value of t_l. Therefore the value $\varepsilon_1(t_l)$ can not be used as an accurate estimate for elastic strain ε_1^{el} as it was done in Section 5.1. The difference is increasing with an increase of t_l. The difference is due to viscoelastic strain developed during the time interval t_l. However, this strain increment is smaller than the viscoelastic creep strain developed during t_l in the one-step loading with σ_{10} and the latter can not be used as a correction. More likely the average value of the stress during the interval t_l is governing the value of the viscoelastic strain.

Analyzing the strain recovery after the load has been removed ($t = 0$ in Figure 4 corresponds to this time instant) we see large differences in the time dependency for tests with different t_u. First of all the value of strain at $t = 0$ is very dependent on the way of load removal. Since the strain at $t = 0$ in Figure 4 is rather different the "elastic strain" calculated using it leads to errors previously shown in Table VII. The sensitivity of the time dependence to t_u in the time interval shown in Figure 4 unavoidably leads also to non reliable values of first coefficients in Prony series describing the time dependence.

The above analysis shows that the observed differences between strain curves and the resulting error in identified viscoelastic material parameters are due to linear time dependence of the load change in test in contrast to the one-step loading and unloading model used in data reduction.

To understand the nature of the modified data analysis methodology suggested in the next section we have to realize that linear loading ramp as any other ramp can be approximated by a large number of small stepwise load increments. Since an exact solution exists for strain response to each load step, the solution can be written for any number of approximation steps. This more exact description of the loading leads to more exact description of the strain response but on the expense of more complex expressions. A first step in this direction, which still leads to reasonable complexity of the data treatment, is the two-step approximation described in Section 4.3.

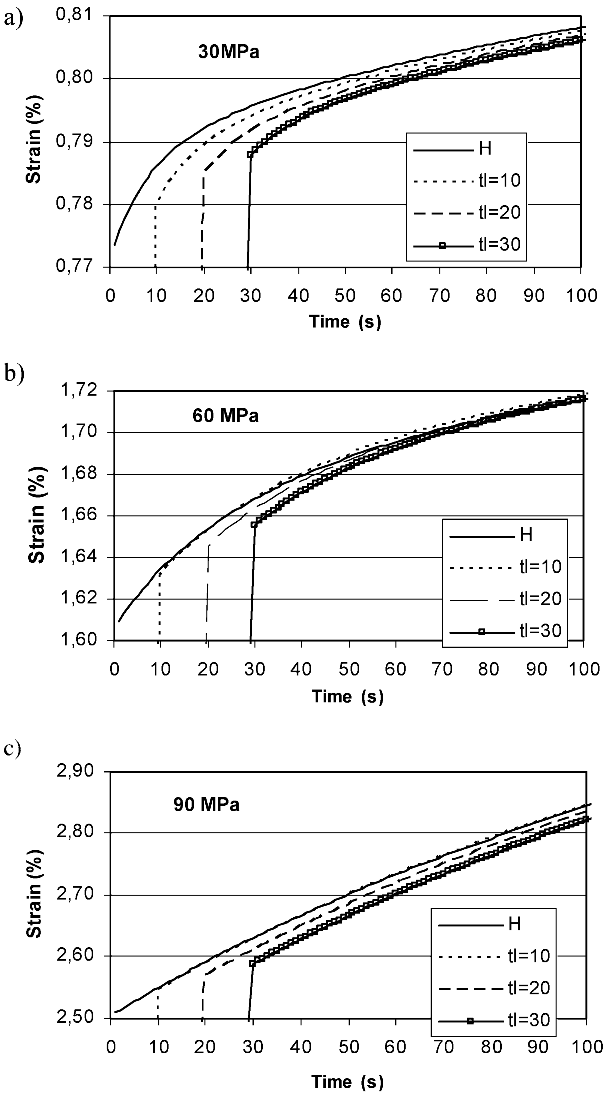

Figure 3. Simulated creep strain curves in the initial part of the creep test considering the time interval t_l as a parameter.

The results presented in Figures 5 ans 6 show that the two-step model is a much better approximation of the linear loading application and unloading than the one-step approximation used before. Considering the linear loading and unloading cases as real test situations, these promising results motivate to develop data reduction methodology based on two-step description which is presented in the next section.

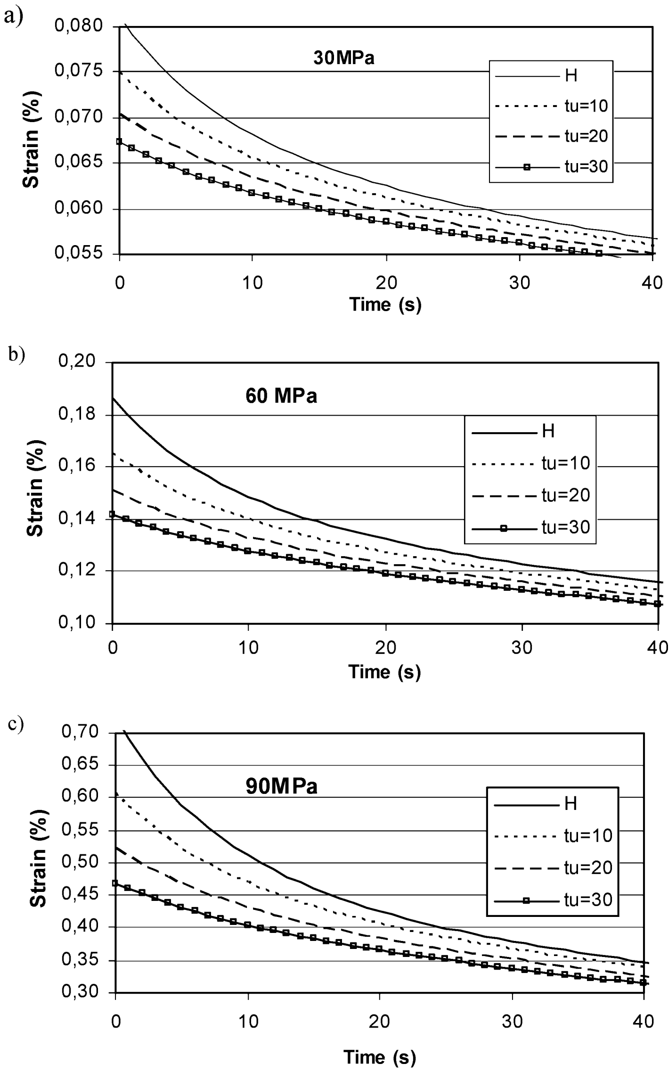

Figure 4. Details of the strain recovery after the load has reached zero value at $t = 0$. Effect of the time interval t_u and the stress level in creep test.

5.3. Two-Step Model for Data Reduction

Since most of the parameters are stress dependent functions the procedure described below has to be repeated for all stress levels used in creep and strain recovery tests starting with the lowest one.

(1) We find, using experimentally measured strain ε_i^{rec} and (25), values of A_{i1}^m. Applying first the procedure in the linear region ($a_{42} = a_{21} = 1$) the coefficients in Prony series C_{i1}^m can be directly obtained from (26), (28).

Table VII. Error (%) when $\varepsilon_1(t_l + t_c + t_u)$ is used to determine the elastic strain. One-step model.

σ_{10}/(MPa)	$t_l = t_u = 10s$	$t_l = t_u = 20s$	$t_l = t_u = 30s$
30	1.1	1.8	2.2
60	8.8	9.7	10
90	42	45	47

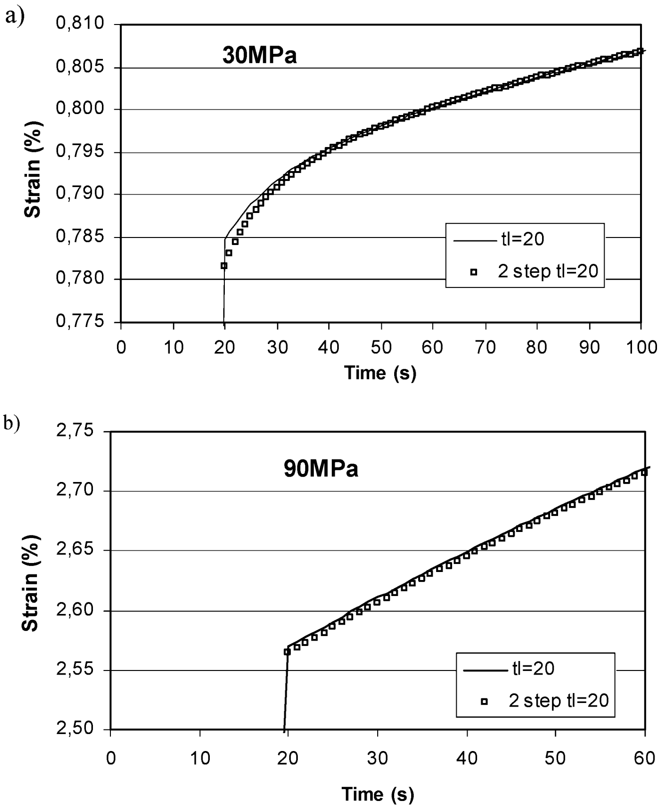

Figure 5. Creep strain development in linear load application case with $t_l = 20s$ and approximation by the two-step model.

(2) Creep strain in two step loading, see (23) in the time instant $t = t_2 = t_l + t_c$ is

$$\varepsilon_i^{\text{creep}}(t_2, \sigma_{10}) = \varepsilon_i^{el}(\sigma_{10}) + b(\sigma_{10})\sigma_{10} \times \sum_m C_{i1}^m \left\{ a_{42}(\sigma_{10}) \left[1 - \exp\left(-\frac{a_{21}(\sigma_{10})(t_c)}{\tau_m} \right) \right] \right.$$

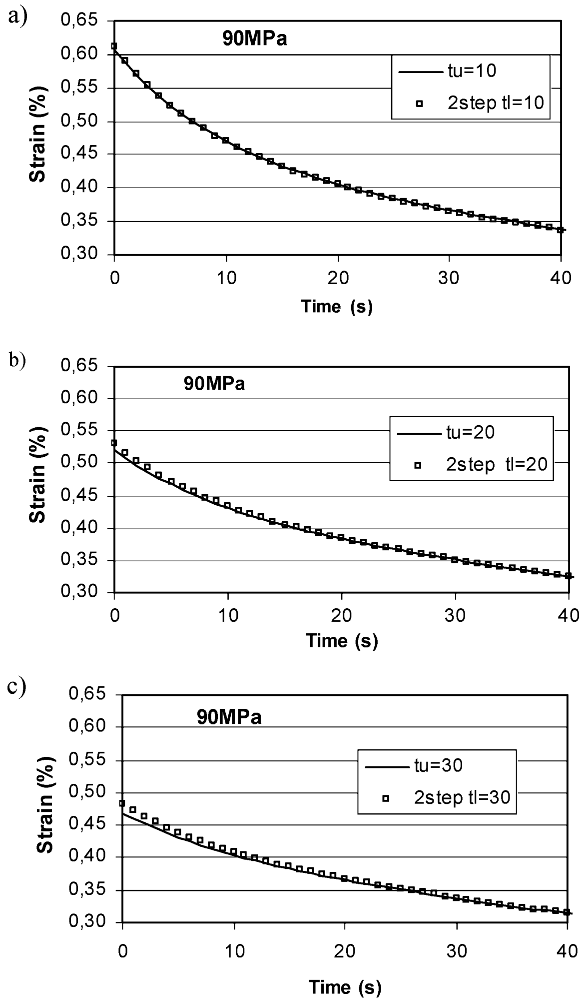

Figure 6. Strain recovery after total removal of load ($\sigma_{10} = 0$ when $t = 0$) and approximation by a two-step unloading model.

$$+\frac{a_{42}(\sigma_{10}/2)}{2}\left[1-\exp\left(-\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right)\right]\exp\left(-\frac{a_{21}(\sigma_{10})t_c}{\tau_m}\right)\Bigg\} \quad (29)$$

Expression in { } in (29) can be expressed as

$$\{\} = \left[\phi_m - \frac{a_{42}(\sigma_{10}/2)}{2}\left(1-\exp\left(-\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right)\right)\right] \times \exp\left(-\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right) \quad (30)$$

After simple rearrangements we obtain expression for the accumulated creep strain

$$\varepsilon_i^{\text{creep}}(t_2,\sigma_{10}) = \varepsilon_i^{el}(\sigma_{10}) + b(\sigma_{10})\sigma_{10} \times \left\{ \sum_m A_{i1}^m \exp\left(\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right) - \frac{a_{42}(\sigma_{10}/2)}{2} \sum_m C_{i1}^m \left[\exp\left(\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right) - 1\right]\right\} \quad (31)$$

The elastic strain ε_i^{el} may be expressed as the difference between the total strain measured in the instant $t = t_l$ and the accumulated viscoelastic strain at that instant. According to (23)

$$\varepsilon_i^{el}(\sigma_{10}) = \varepsilon_i(t_l) - b(\sigma_{10})\sigma_{10}\frac{a_{42}(\sigma_{10}/2)}{2}\sum_m C_{i1}^m \left[1 - \exp\left(-\frac{a_{21}(\sigma_{10}/2)t_l}{\tau}\right)\right] \quad (32)$$

The experimental value of $\varepsilon_i^{\text{creep}}(t_2)$ is known from the creep test. Obviously one can use (31) and (32) to determine parameter b for the considered stress level

$$b(\sigma_{10}) = \frac{\varepsilon_i^{\text{creep}}(t_2,\sigma_{10}) - \varepsilon_i(t_l,t_{10})}{\sigma_{10}\left\{\sum_m A_{i1}^m \exp\left(\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right) - a_{42}(\sigma_{10}/2)\sum_m C_{i1}^m \sinh\left(\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right)\right\}} \quad (33)$$

The accuracy of terms with lowest m in (33) is not good and therefore the first two terms were considered explicitly from (29) without involving procedures given by (30) and (31).

However immediate application of (33) in this form to calculate $b(\sigma_{10})$ is not possible because these first two terms contain an unknown value of $a_{42}(\sigma_{10})$.

(3) In order to determine a_{42} and a_{21} for a given stress level σ_{10} we analyze the creep strain data $\varepsilon_i^{\text{creep}}(t)$. From Equation (23) follows that we have to fit these data with function

$$f_i(t,a_{21}) = \varepsilon_i^{el}(\sigma_{10}) + b(\sigma_{10})\sigma_{10} \times \sum_m C_{i1}^m \left\{ a_{42}(\sigma_{10})\left[1 - \exp\left(-\frac{a_{21}(\sigma_{10})(t-t_l)}{\tau_m}\right)\right]\right.$$

$$+\frac{a_{42}(\sigma_{10}/2)}{2}\left[1-\exp\left(-\frac{a_{21}(\sigma_{10}/2)t_l}{\tau_m}\right)\right]\exp\left(-\frac{a_{21}(\sigma_{10})(t-t_l)}{\tau_m}\right)\Bigg\} \tag{34}$$

The unknown $a_{21}(\sigma_{10})$ is used to optimize the fit. Value of $a_{42}(\sigma_{10})$ corresponding to a given value of $a_{21}(\sigma_{10})$ is calculated using

$$a_{42}(\sigma_{10}) = \frac{\sum_{m=2}^{M} A_{i1}^m - \frac{a_{42}(\sigma_{10}/2)}{2}\sum_{m=2}^{M} C_{i1}^m\left[1-\exp\left(-\frac{a_{21}(\sigma_{10}/2)t_u}{\tau_m}\right)\right]\left[1+\exp\left(-\frac{a_{21}(\sigma_{10})t_c+a_{21}(\sigma_{10}/2)t_u}{\tau_m}\right)\right]}{\sum_{m=2}^{M} C_{i1}^m \exp\left(-\frac{a_{21}(\sigma_{10})t_u}{\tau_m}\right)\left(1-\exp\left(-\frac{a_{21}(\sigma_{10})t_c}{\tau_m}\right)\right)} \tag{35}$$

Thus, for any value of $a_{21}(\sigma_{10})$ we have corresponding values of $a_{42}(\sigma_{10})$, $b(\sigma_{10})$ and $\varepsilon_i^{el}(\sigma_{10})$ given by (32), (33) and (35). To find $a_{21}(\sigma_{10})$ we consider several instants of time denoted t_n $n = 1, 2 \ldots N$. The choice of these points depends on the region in which the simulated time dependence should be adjusted. The difference between the experimental and the calculated value is

$$\Delta_i(t_n) = \varepsilon_i^{\text{creep}}(t_n) - f_i(t_n, \sigma_{21})n = 1, 2 \ldots N \tag{36}$$

The sum of squares of these differences is

$$S_i(a_{21}) = \sum_{n=1}^{N} \Delta_i^2(t_n) \tag{37}$$

It depends on the chosen value of a_{21} and it may be minimized with respect to this parameter. An interesting feature is that calculations require values of material functions at stress level $\sigma_{10}/2$ which must be analyzed before.

5.4. Results Using the Two-Step Model

The two-step model based methodology described in Section 5.3 was applied to "test data" corresponding to the values of t_l and t_u used in Section 5.1. The coefficients C_{11}^m determined using strain recovery data are given in Table VIII. Obviously the accuracy is better than using one-step loading analysis, see Table III. They are satisfactory starting with $m = 3$ even for the slow load increase with $t_u = 30$.

The calculated values of $b(\sigma_{10})$, $\varepsilon_1^{el}(\sigma_{10})$, $a_{21}(\sigma_{10})$ and $a_{42}(\sigma_{10})$ are presented in Tables IX to XII. The b values presented in Table IX are much closer to the true values than using the one-step model, see Table IV. A noticeable error was observed only for 90 MPa load with $t_l = t_u = 20$s and 30s.

The a_{21} values are the most sensitive and unreliable from all characteristics of the nonlinear viscoelastic material. Fortunately the predicted creep strains are rather

Table VIII. C_{11}^m values from tests with varying rates of load increase. Two-step model.

τ_m (S)	Input (1/Pa)	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
3	$8.145\ 10^{-13}$	$8.156\ 10^{-13}$	$6.358\ 10^{-13}$	$3.407\ 10^{-13}$	$2.285\ 10^{-13}$
10	$5.379\ 10^{-12}$	$5.378\ 10^{-12}$	$5.493\ 10^{-12}$	$4.527\ 10^{-12}$	$3.586\ 10^{-12}$
30	$-7.458\ 10^{-13}$	$-7.449\ 10^{-13}$	$-7.629\ 10^{-13}$	$-7.433\ 10^{-13}$	$-7.119\ 10^{-13}$
100	$8.630\ 10^{-12}$	$8.630\ 10^{-12}$	$8.709\ 10^{-12}$	$8.687\ 10^{-12}$	$8.652\ 10^{-12}$
300	$1.900\ 10^{-12}$	$1.900\ 10^{-12}$	$1.906\ 10^{-12}$	$1.906\ 10^{-12}$	$1.904\ 10^{-12}$
1000	$7.429\ 10^{-12}$	$7.428\ 10^{-12}$	$7.436\ 10^{-12}$	$7.436\ 10^{-12}$	$7.436\ 10^{-12}$
3000	$-1.076\ 10^{-12}$	$-1.076\ 10^{-12}$	$-1.076\ 10^{-12}$	$-1.076\ 10^{-12}$	$-1.076\ 10^{-12}$
10000	$1.807\ 10^{-11}$	$1.807\ 10^{-11}$	$1.807\ 10^{-11}$	1.80710^{-11}	$1.807\ 10^{-11}$

Table IX. $b(\sigma_{10})$ values from tests with varying rates of load increase. Two-step model.

σ_{10} (MPa)	Input	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
30	1.000	1.000*	1.000*	1.000*	1.000*
60	1.568	1.569	1.564	1.636	1.686
90	2.142	2.166	2.201	2.516	2.704

*Obtained assuming linearity.

Table X. a_{21} values from tests with varying rates of load increase. Two-step model.

σ_{10} (MPa)	Input	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
30	1.000	1.000*	1.000*	1.000*	1.000*
60	0.285	0.286	0.290	0.384	0.526
90	0.048	0.050	0.054	0.080	0.106

*Obtained assuming linearity.

insensitive to this parameter. Results presented in Table X have the same degree of accuracy (rather bad) as the ones presented in Table IV. Results for a_{42} in Table XI are approximately with the same accuracy as the ones obtained using one-step loading methodology, see Table VI.

The elastic strain values are obtained from $\varepsilon_1(t_l)$ correcting them for the viscoelastic strain from the two-step model (Equation (32)). They are much closer to the true values, see Table XII, than the values in Table II.

Table XI. a_{42} values from tests with varying rates of load increase. Two-step model.

σ_{10} (MPa)	Input	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
30	1.000	1.000*	1.000*	1.000*	1.000*
60	1.460	1.458	1.452	1.345	1.233
90	5.781	5.630	5.445	4.613	4.229

*Obtained assuming linearity

Table XII. ε^{el} (%) as $\varepsilon(t_l)$ with varying rates of load increase. Two-step model.

σ_{10} (MPa)	Input	$t_l = t_u = 0$ (1/Pa)		$t_l = t_u = 10s$ (1/Pa)		$t_l = t_u = 20s$ (1/Pa)		$t_l = t_u = 30s$ (1/Pa)	
			(%)		(%)		(%)		(%)
30	0.771	0.771	0.0	0.773	0.2	0.776	0.6	0.779	1.0
60	1.606	1.606	0.0	1.609	0.2	1.617	0.7	1.626	1.2
90	2.505	2.505	0.0	2.502	−0.1	2.506	0.0	2.519	0.6

6. Conclusions

Two data reduction methodologies are compared in their ability to handle the "disturbances" in nonlinear creep and recovery strains caused by deviations of real loading conditions from Heaviside stepwise loading and unloading. One method is based on the common one-step loading and unloading model whereas the other one considers loading and unloading as a two-step processes.

The effect of the time interval t_l of the linear load increase and the unloading time interval t_u on the introduced error in results is analysed. Known nonlinear viscoelastic material is used to simulate the strain response in creep-strain recovery tests. The simulated creep – strain recovery curves are used as "experimental data". The results show significant increase of the error with increasing t_l and t_u. It appears that the length of the load increase and decrease interval is effecting not only the accuracy of first terms in Prony series but also the accuracy of the whole set of nonlinearity parameters.

The accuracy of data reduction is higher using two-step approximation data reduction method. This methodology gives much better values of coefficients in Prony series C_{11}^m for small relaxation times and the values of non-linearity function $b(\sigma_{10})$. The elastic strain determined as the strain at the instant $t = t_l$ corrected for viscoelastic strains according to two-step loading methodology is obtained with 1% accuracy as compared with 3% using the old methodology. This difference may be crucial when analysing nonlinearity parameters. Values of the rest of the non-linearity functions are rather sensitive to t_l but the accuracy does not depend on the used data reduction methodology.

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Appendix: Material Properties Derivation Using One-Step Model

The expressions in Section 4.2 for creep and strain recovery test description by one-step loading and unloading model are used. 30 MPa was chosen as the upper stress limit for linear behavior. The methodology has to be used for each considered stress level.

(1) The creep strain at the time instant when the plateau value of load has been reached is used as the elastic strain value ε_1^{el}.
(2) The strain recovery data points $\widetilde{\varepsilon}_1^{rec}$ were fitted by the method of the least squares to obtain the constants A_{11}^m, see Equation (21). The coefficients in Prony series C_{11}^m could be directly obtained from (20) using data in the linearity region.
(3) The creep strain (15) for time instant $t_1 = t_l + t_c$ after simple rearrangements may be written as

$$\varepsilon_1^{\text{creep}}(t_1) = \varepsilon_1^{el} + b\sigma_{10}\sum_m A_{11}^m. \tag{A1}$$

The experimental value of $\varepsilon_1^{\text{creep}}(t_1)$ is known. Hence, (A1) may be used to determine parameter b for the considered stress level

$$b(\sigma_{10}) = \frac{\varepsilon_1^{\text{creep}}(t_1, \sigma_{10}) - \varepsilon_1^{el}(\sigma_{10})}{\sigma_{10}\sum_m A_{11}^m(\sigma_{10})} \tag{A2}$$

(4) To obtain the values of a_{21} and a_{42} for a given stress level we analyze creep strain. Expressing $a_{42}C_{11}^m$ from (20) and substituting in (15), we obtain for the creep strain

$$\varepsilon_1^{\text{creep}}(t) = \varepsilon_1^{el} + b\sigma_{10}\sum_m A_{11}^m \frac{1-\exp\left(-a_{21}\frac{t}{\tau_m}\right)}{1-\exp\left(-a_{21}\frac{t_1}{\tau_m}\right)} \tag{A3}$$

According to (A3) creep strain at the time instant $t'_1 < t_1$ can be written as

$$\varepsilon_1^{\text{creep}}(t'_1) = \varepsilon_1^{el} + b\sigma_{10}\sum_m A_{11}^m \frac{1-\exp\left(-a_{21}\frac{t'_1}{\tau_m}\right)}{1-\exp\left(-a_{21}\frac{t_1}{\tau_m}\right)} \tag{A4}$$

a_{21} is determined by numerically finding the value which best satisfies (A4). Finally a_{42} is calculated as average of values obtained using (20).

References

Guedes, R.M., Marques, A.T., and Cardon, A., 'Analytical and experimental evaluation of nonlinear viscoelastic- viscoplastic composite laminates under creep, creep-recovery, relaxation and ramp loading', *Mechanics of Time-Dependent Materials* **2**, 1998, 113–128.

Knauss, W.G. and Emri, I., 'Non-linear viscoelasticity based on free volume considerations', *Computers and Structures* **13**, 1981, 123–128.

Lou, Y.C. and Schapery, R.A., 'Viscoelastic characterization of a nonlinear fiber-reinforced plastic', *J. of Composite Materials* **5**, 1971, 208–234.

Megnis, M. and Varna, J., 'Nonlinear Viscoelastic, viscoplastic characterization of unidirectional GF/EP composite', *Mechanics of Time-Dependent Materials* **7**, 2003, 269–290.

Megnis, M. and Varna, J., 'Micromechanics based modeling of non-linear viscoplastic response of unidirectional composite', *Composite Science and Technology* **63**, 2003, 19–31.

Nordin, L-O., Marklund, E., Stahlberg, D. and Varna, J., 'Mechanical response of thermoset polymers under high compressive loads, 2', *Macromolecular Materials and Engineering* **290**, 2005, 1073–1082.

Popelar, C.F. and Liechti, K.M., 'Multiaxial nonlinear viscoelastic characterization and modeling of a structural adhesive', *J. of Engineering Materials Technology* **119**, 1997, 205–210.

Schapery, R.A., 'Nonlinear viscoelastic and viscoplastic constitutive equations based on thermodynamics', *Mechanics of Time-Dependent Materials* **1**, 1997, 209–240.

Stahlberg, D., Nordin, L-O., Varna, J., and Johansson, M., 'Mechanical response of thermoset polymers under high compressive loads, 1', *Macromolecular Materials and Engineering* **290**, 2005, 1063–1072.