Multimedia Tools and Applications
https://doi.org/10.1007/s11042-024-19337-y

Watermarking technique for copyright protection of digital images using coupled differential equations

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Received: 24 March 2023 / Revised: 23 March 2024 / Accepted: 30 April 2024

Abstract
Advancement and expansion of the internet, along with regular software upgrades, have generated many benefits in communication patterns. This rapid growth has also had a negative impact on the security of communicated content, such as images, audio/video messages, and especially on copyright protection for important images. In this paper, a novel watermarking technique is proposed to address the challenge of copyright protection more effectively. Initially, a chaotic dynamical system presented by coupled differential equations is utilized to generate three random, as well as chaotic, sequences of numbers. These sequences are then processed on each channel (red, green, and blue) of a colored image, respectively. Subsequently, the watermark (logo image) is embedded by converting its most significant bits (MSBs) to least significant bits (LSBs). The watermark in the processed image is nearly invisible due to the presence of third-order nonlinearity in the chaotic system and the complexity of the watermarking technique. This technique has demonstrated remarkable resistance to various attacks, such as noise attacks, compression attacks, and more specifically cropping attacks, confirming its applicability in copyright protection for digital images.

Keywords Image watermarking · Frequency domain · Chaotic differential equations

1 Introduction

Today, the contemporary world is a prime example of esteemed development in every sector, yet it has transformed the entire mechanism of communication on a large scale. Rapid and hassle-free communication between two parties is now a matter of mere moments. The emphasis on growth is not only devoted to the invention of computing devices and their software, but also to the rapid evolution of communication methods. The expansion of

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Published online: 20 May 2024

digital data in the form of e-libraries, digital repositories, video and audio messages, etc., has been observed in recent times. The utmost concern associated with this advancement is its security and copyright protection. To address this issue, researchers have utilized cryptosystems for the secure transmission of digital content, particularly encryption and watermarking. The originality of images and their genuine author/creator is securely communicated through the use of watermarking techniques.

The two prominent watermarking techniques used in copyright protection are spatial domain [1] and frequency domain [2]. In spatial domain, the main idea is to substitutes the pixel or set of pixels of host image with the pixels of watermark image directly, which in return produces the watermarked image having great proficiency towards attacks. Whereas in the frequency domain, firstly host image is converted into frequency domain, then watermarking is done on the coefficients' value of the host image to tackle mischievous attacks. The objective in each case is same i.e. to provide copyright protection, integrity, broadcast monitoring, authentication and sturdiness towards mischievous attacks [3].

Literature reveals many techniques of watermarking in both of the aforementioned types [1, 4–7]. Discrete fractional Fourier transform (DFRFT) [8], discrete Fourier transform (DFT) [9], discrete wavelet transform (DWT) [10] and discrete cosine transform (DCT) have been widely used in watermarking [11]. In [12] authors suggest a novel technique of watermarking which uses chaotic Arnold transform and LU decomposition to strengthen the invisibility of watermark. Likewise in [13], the authors proposes two preprocessing steps before embedding of watermark. First is to change the size of watermark and second is to scramble it. Whereas, the security goals for the proposed technique are achieved by a simple, secure and direct approach. In comparison to [13], the inclusion of substitution box (S-box) in watermarking has enhanced the security level and utilization of chaos based differential equations added the strength in the randomness of the substitution box. These distinctive features contribute in uplifting the imperceptibility level of the technique [7].

Similarly, in [10], authors propose a hybrid watermarking technique while remaining in the frequency domain by merging DCT and DWT for embedding and extraction of the watermark. The combination of these two methods yields more benefits and creates one hybrid method. In [11], a different but simpler and robust approach was proposed by the authors, which focuses on the size of the watermarked image and the original image in the embedding process using the DCT approach only. Similarly, some other interesting watermarking techniques have been proposed in the recent past [14–18].

Literature reveals the utilization of many simple chaotic systems in image encryption and watermarking techniques [19–22]. However, complex chaotic dynamical systems have not been widely used in such areas. Moreover, chaotic systems with higher-order nonlinearities and dimensions are rarely seen in this field. Since such nonlinear multi-dimensional chaotic systems exhibit complex, unpredictable, and random trajectories, this gap motivated us to use a complex chaotic dynamical system for copyright protection. Keeping this aspect in mind, a third-order nonlinear chaotic dynamical system is used in this article for the design of the watermarking technique.

In this article, we will utilize DCT to propose a robust and secure technique of watermarking. It offers a high degree of freedom, due to the multi-resolution attribute. Furthermore, this transformation is not block-based. DCT can also be amalgamated with other transforms to utilize their benefits as well like in [2, 10] and [14]. The theme behind the application of differential equations (DE) in watermarking schemes is to introduce continuous chaotic systems in such schemes. It is a well-established and fostered division of mathematics which is used to represent dynamical systems in terms of mathematical equations. Chaos generating DE have strong and complex random behavior extremely suitable for the secure design of cryptosystem. Recent developments in multimedia applications have revealed productive outcomes from such systems. But there are very few applications of such systems in image watermarking techniques.

The remaining article is organized as follows, Section 2 gives the detailed description of the coupled chaotic system of DE and the watermarking technique. Section 3 presents the complete statistical analyses for the host image and watermarked images. Several attacks of image processing are applied to test the robustness of the suggested technique in Section 4. The whole article is concluded in Section 5.

2 System of coupled differential equations and watermarking technique

A physical system depending upon two or more variables is modelled in terms of ordinary differential equations (ODEs) to exhibit its bifurcation pattern completely in near future. A coupled differential equation is a system of equations where the dependent variables in each equation depend on more than one independent variable. This concept was used to measure randomness appearing in non-equilibrium frictional medium from modulation instability. It involves three variables and hence generates a system of chaotic coupled differential equations involving three ODEs. This system was initially modelled by Rabinovich-Fabricant (RF) [23, 24], whose formation in mathematical form is given below.

$$\begin{aligned} \frac{dx}{dt} &= yz - y + y^3 + \sigma x \\ \frac{dy}{dt} &= 3xz + x - x^3 + \sigma y \\ \frac{dz}{dt} &= -2\beta z - 2xyz \end{aligned} \tag{1}$$

The system (1) is chaotic and highly sensitive towards initial input and parameters. Slight variation in the system generates complex, chaotic and completely different dynamical behavior. Fixing parameters and initial conditions at one stage can result in generation of pseudorandom sequences of excellent cryptographic properties. In Fig. (1), different trajectories are plotted for slightly varied parameters and initial conditions.

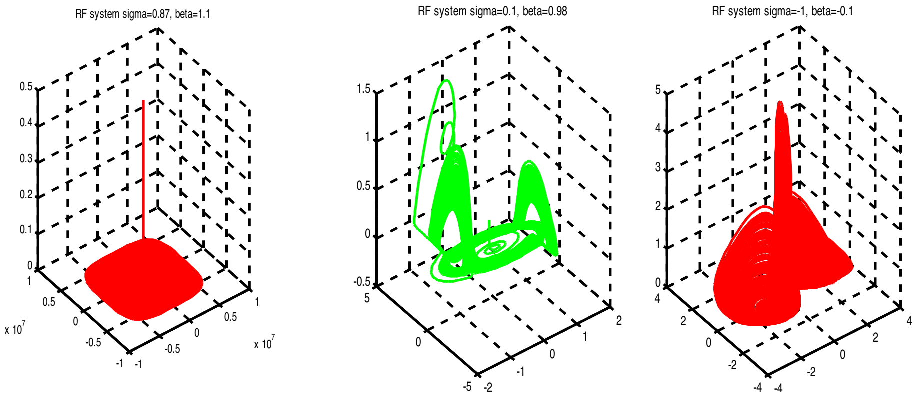

Fig. 1 RF system with Different chaotic behavior for change in β only

2.1 Watermarking technique

The purpose of the watermarking technique is to acquire copyright protection of valuable digital content. Both types of visible and invisible watermarks are usually inserted in the plaintext images. The theme of inserting an invisible watermark is used to protect the ownership of content circulated using insecure channels. The procedure of embedding and extraction of watermark in frequency domain using DE is explained hereafter.

2.1.1 Embedding procedure

The following steps describes the embedding procedure of the watermark logo into the plaintext image.

- A plaintext image having size $M \times N$ is selected and segmented into three layers i.e. blue, green and red for the utilization of embedding procedure of watermark logo.
- The solutions of RF system (given in Eq. 1) are then used to generate sequences $X, Y\ and\ Z$. The red, green and blue layers are being permuted by sequences $X, Yand\ Z$ respectively by making use of Eq. 2.

$$\begin{cases} S^{R,G,B}(i,j) = I^{R,G,B}\left(i - x_i, j - x_j\right) & if\ i - x_i \geq 1; j - x_j \geq 1 \\ S^{R,G,B}(i,j) = I^{R,G,B}\left(M + i - x_i, N + j - x_j\right) & if\ i - x_i < 1; j - x_j < 1 \\ S^{R,G,B}(i,j) = I^{R,G,B}\left(i + x_i, j + x_j\right) & if\ i + x_i \leq M; j + x_J \leq N \\ S^{R,G,B}(i,j) = I^{R,G,B}\left(i + x_i - M, j + x_j - N\right) & if\ i + x_i \geq M; i + x_i \geq N \end{cases} \tag{2}$$

- A watermark logo image of dimension $(K \times K)$ is taken into account and obtained $(2K \times 2K)$ LSBs through a shift procedure by altering its MSBs to LSBs.
- Next, a block of dimension $(2K \times 2K)$ is selected from the image in 1st step and replaced its LSBs by the LSBs obtained in previous step.
- Finally, the processed image from the previous step undergoes inverse permutations utilizing the corresponding inverse equations to obtain the watermarked image. The complete watermarking technique is elucidated in Fig. 2.

2.1.2 Extraction procedure

To maintain the productivity of the technique and ensure copyright protection, the extraction procedure is outlined as follows:

- The permutations to each layer using the generated sequences X, Y, and Z is applied after separating the watermarked image into three channels.
- A block of dimension $(2K \times 2K)$ is selected from permuted layer and splited into the MSBs and LSBs.
- $(K \times K)$ MSBs and LSBs are being extracted from the previous step.
- As a final point, inverse permutations are applied on the $(K \times K)$ LSBs to extract the watermark logo from the watermarked image.

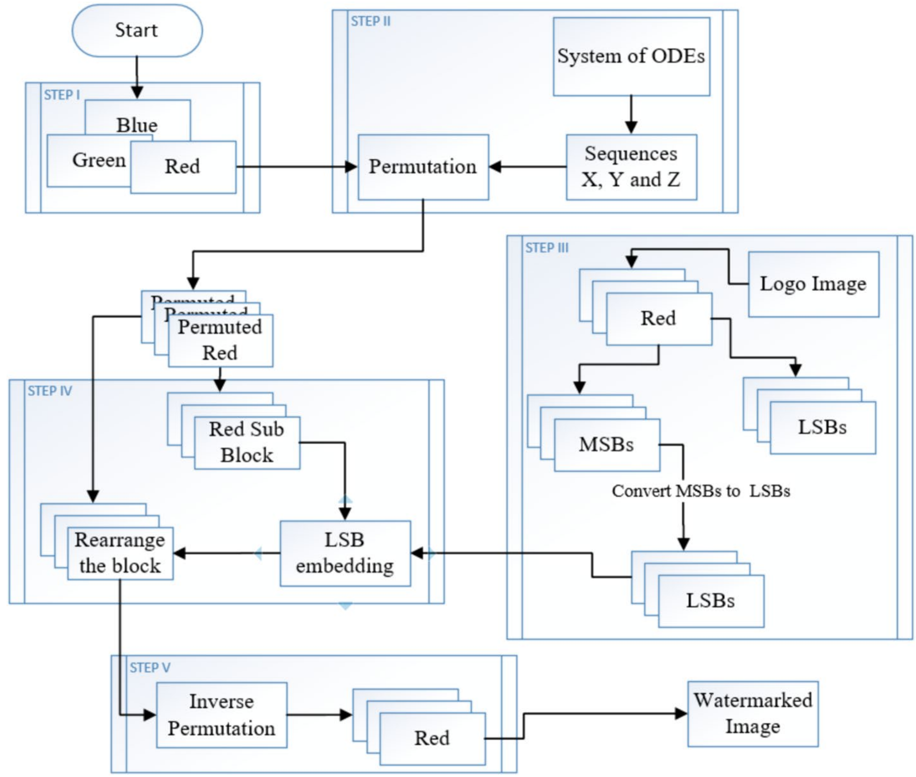

Fig. 2 Structural demonstration for the proposed watermarking technique

3 Statistical analysis and simulation results

This section of the paper presents comprehensive details of essential analyses aimed at validating the efficacy of the proposed watermarking technique. These analyses encompass contrast, correlation, homogeneity, energy, peak signal-to-noise ratio, and mean square error. The evaluation of these metrics is conducted by examining the Gray Level Co-occurrence Matrix (GLCM) of both the watermarked and host images. It is noteworthy that the size of the watermarked and original images remains consistent. The strength of the watermarking technique is established through a thorough assessment of robustness, invisibility, security, and capacity.

3.1 Homogeneity

GLCM is used to measure the homogeneity of an image. The diagonal closeness distribution is measured in homogeneity. If $p(i,j)$ represents pixel position at i^{th} row and j^{th} column then formula is

$$Hom = \sum \frac{p(i,j)}{1 + |i - j|} \tag{3}$$

3.2 Contrast

In response to intensity modifications the sensitivity of the image textures is calculated in doing the analysis of contrast. It is mathematically defined as

$$Contrast = \sum |i - j|^2 p(i,j) \tag{4}$$

3.3 Energy

Here the sum of squares of entire values of gray pixels of each row and column is calculated, mathematically,

$$Energy = \sum p(i,j)^2 \tag{5}$$

3.4 Entropy

The amount of randomness in a digital image is measured in entropy. It is used to quantify the uncertainty in digital images. Mathematically it is given as

$$Entropy = -\sum p\left(y_i\right) log_2 p\left(y_i\right) \tag{6}$$

The entropy results in Table 6 for the host and the watermarked images suggest that the two images are quite the same also depicted in the Fig. 6.

3.5 Correlation

Pixels of a plaintext image are highly correlated to its neighboring pixels. This pattern in a watermarked image of pixels should also remain approximately same if the watermarking technique is fine enough. The similarity of original and watermarked images is evaluated by this analysis. Mathematical representation is given as follows.

$$Corr = \sum \frac{(i - \mu i)(j - \mu j)p(i,j)}{\sigma_i \sigma_j} \tag{7}$$

Where σ and μ are standard deviation and mean respectively.

The results of these analyses for both the host and watermarked images exhibit a high degree of similarity, as summarized in Table 6. Moreover, the comparison indicates that the watermarked image with proposed technique is more similar to the original image as compared to the [7] and [26]. Achieving such congruence in outcomes is the ultimate objective for a watermarking technique (Table 1).

Table 1 Analysis Host image and watermark image QAU

Salt & Pepper	PSNR	MSE	NPCR	UACI
2%	25.5882	179.579	0.04846	0.009022
10%	18.3747697	945.37	0.2312	0.045311

3.6 Mean squared Error

How dissimilar are the two images i.e. watermarked and the original image, are examined by the Mean Square Error (MSE). Its mathematical expression is given in Eq. 8 and the values are being tabulated in Tables 2, 3, 4, 5, and 6.

$$MSE = \frac{1}{n}\sum (x_i - x_i^*)^2 \tag{8}$$

3.7 Peak signal to noise ratio

The Peak Signal to Noise Ratio (PSNR) is defined by the given expression.

$$PSNR = 10log_{10}\frac{MAX_I^2}{MSE} \tag{9}$$

Table 2 Analysis Host image and watermark image QAU

Scaling	MSE	UACI	NPCR	PSNR
Half Lower Cut	6064.5375	20.438	78.2958	10.3028
Half upper cut	5976.7818	19.6887	78.5400	10.36612
Mid Cut	2415.27	10.3383	50.87	14.3011
Half Right Cut	5876.355	20.078	79.7668	10.43972
Half Left Cut	5864.324	20.046	79.44335	10.44862

Table 3 JPG images with their complexity analyses

	Proposed Technique		Ref. [2]	
Dimensions	256 × 256	512 × 512	256 × 256	512 × 512
Lena Image	1.5318	5.4129	1.7533	5.6291
Pepper Image	1.1914	5.0134	1.2096	5.3579

Table 4 Analysis Original and watermark image

	Blue	Green	Red
AD Average Difference	0.091163	-0.29938	-0.43899
MD Mean Difference	15	13	10
MSE	3.4207	0.1291	4.6660
NAE Normalized Absolute Error	0.003575	0.003647	0.002490
NCC	0.99921	1.0023760	1.002282
NPCR	0.05837	0.587	0.5767
PSNR	42.7895	43.2050	41.4413
SC Structural Content	1.001285	0.9950236	0.9953
UACI	0.01484	0.014134	0.01763

Table 5 Analysis of the Original Host image and Extracted image QAU

	Red	Green	Blue
AD	0	0	0
MD	0	0	0
MSE	1	1	1
NAE	0	0	0
NCC	1	1	1
NPCR	0	0	0
PSNR	99	99	99
SC	1	1	1
UACI	0	0	0

Table 6 Statistical Analysis for the Host and processed images

		Correlation	Contrast	Entropy	Energy	Homogeneity
Lena Image	Original	0.9700	0.1465	7.2743	0.1678	0.9315
	Watermarked	0.9705	0.1465	7.2399	0.1678	0.9314
	Ref. [7]	0.9707	0.0695	7.2705	0.1753	0.9119
	Ref. [26]	0.9257	0.1506	7.2401	0.1670	0.9289
Pepper Image	Original	0.9615	0.1633	7.3802	0.1526	0.9282
	Watermarked	0.9615	0.1633	7.3707	0.1528	0.9281
	Ref. [7]	0.9617	0.1634	7.3790	0.1530	0.9290
	Ref. [26]	0.9595	0.1707	7.3700	0.1503	0.9244

In the above equation, the denominator represents the dissimilarity between the images, specifically the MSE, while the numerator corresponds to the signal strength. The evaluation of invisibility is determined using metrics such as PSNR and Structural Similarity Index (SSIM), which measure the similarity between the watermarked and the original image [17, 25].

3.8 Complexity and differential analysis

One very important factor in analyzing the different algorithms of information security is the complexity analyses. The speed of embedding and extraction of watermarks in the era of speedy communication matters a lot. The speed of extraction and embedding for the proposed technique is very close to the other similar techniques as one given in Table 3, whereas the level of security attained by this technique is much superior than [26]. The entire procedure is executed using MATLAB (R2013a) on a laptop running Windows 8, equipped with an Intel(R) Core(TM) i5-4300U CPU @ 1.90 GHz (2.50 GHz) and 4 GB RAM.

The unified average changing intensity (UACI) ensures the efficiency of cryptosystem to stand against the differential attacks with an optimal value about 33.333%. The resistance against the plaintext attack improves as number of pixels change rate (NPCR) approaches 100%. These expressions are expressed as follows.

$$UACI = \frac{1}{L \times M} \sum_{j,k} \left(\frac{C_1(j,k) - C_2(j,k)}{255} \right) \times 100\% \tag{10}$$

$$NPCR = \frac{\sum_{j,k} D(j,k)}{L \times M} \times 100\% \tag{11}$$

where $C_1$and C_2 are the watermarked images before and after one pixel of the host image is altered and D is their difference. If $C_1 \neq C_2$, $D = 1$ else $D = 0$. The results of these analyses are calculated in Tables 1, 2, 4 and 5. Figures 3, 4, 5, 6, 7, 8 and 9 showcase the respected images, illustrating the effectiveness of the proposed watermarking technique.

4 Robustness test

The resemblance between the extracted and the inserted watermark is the mathematical calculation of two watermarks [27]. Robustness is achieved when these two watermarks exhibit high correlation, which translates to a similarity outcome on the higher side. If the

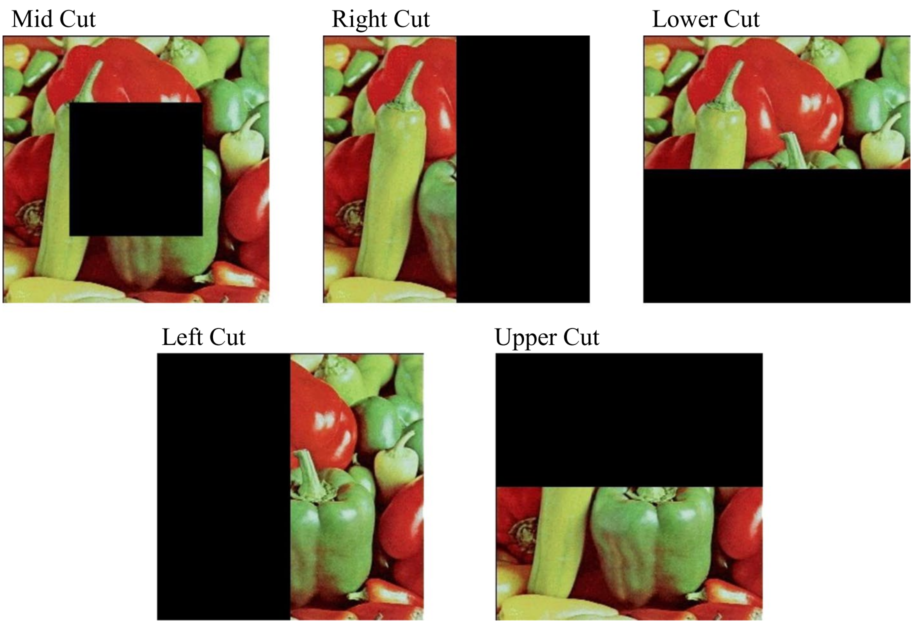

Fig. 3 Watermarked images with various cuts

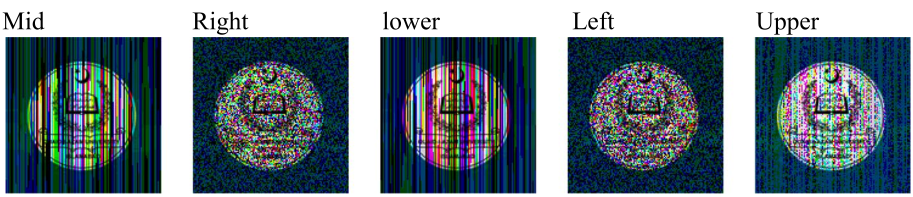

Fig. 4 Extracted logo from various watermarked cut images

Fig. 5 Original images, Pepper and Lena

Fig. 6 Processed (watermarked) Images, Pepper and Lena

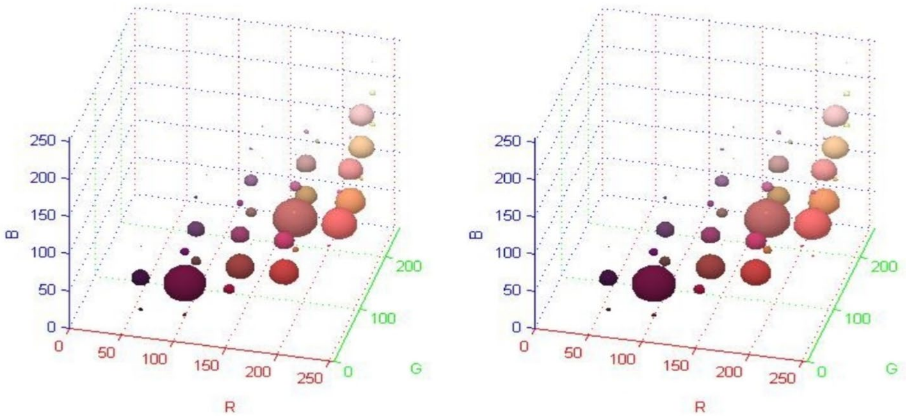

Fig. 7 Histogram of original and watermarked Lena image

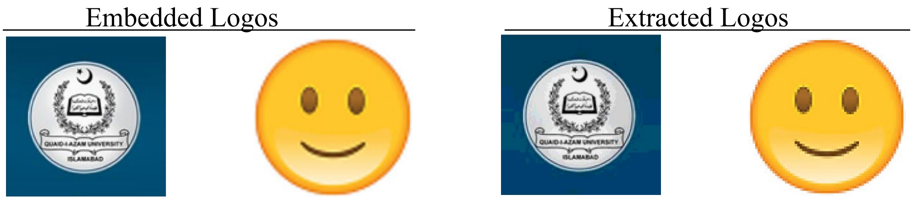

Fig. 8 QAU and emoji logos

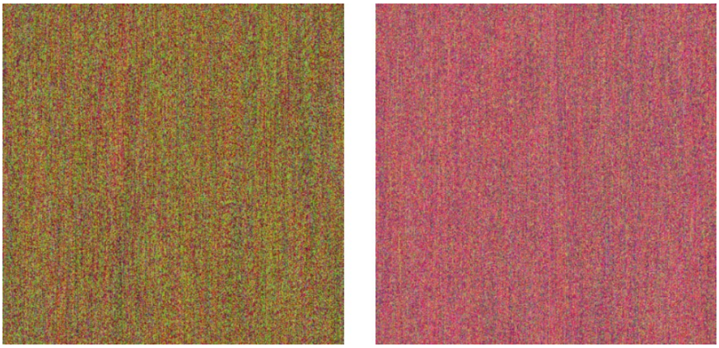

Fig. 9 Disrupted Pepper and Lena images

kth term of the extracted and original watermark is e_k and o_k, respectively then the similarity is mathematically given as follows:

$$Sim = \frac{\sum e_i.o_i}{\sqrt{\sum e_i^2 \sum o_i^2}} \tag{12}$$

The optimal value of similarity index for strong watermarking should be approximately 100. It turns out to be 99.92 for the proposed technique that is almost the optimal value. The similarity values for the Lena and Pepper images are tabulated and compared with those in [26] in Table 7, demonstrating the validity of the proposed technique. Noise, compression and cropping attacks are deliberated in the upcoming subsections.

Table 7 Robustness Analysis for the Host and processed images

Images	SSIM	SSIM Ref [26]	Speakel (1%)	Salt & Pepper 1%	Average Filter (3*3)	Histogram Equalization	JPEG 20	JPEG 30
Lena	0.9989	0.9937	0.0765	0.9872	0.0824	0.0321	0.4234	0.5234
Pepper	0.9992	0.9964	0.0864	0.0988	0.0487	0.0421	0.3982	0.4654

4.1 Noise attack

For the resistance against several noise attacks, 1% Speckle noise and 1% salt noise have been added to the images. Table 7 shows the great resistance against noise attack.

4.2 Compression attack

For the compression attack of the proposed watermarking technique, the Joint Photographic Experts Group (JPEG) is examined and presented in Table 7.

4.3 Cropping attack

Either the extracted image is deliberately distorted or less material than the host image is provided in this attack. Tables 1, 2, and 7 comprises the results of nearly all data processing attacks.

5 Conclusion

Digital watermarking is indeed an appropriate choice for the fortification of copyrights of multimedia data. In this article an innovative watermarking technique is obtained which operates on the solution space of ordinary coupled differential equations. Initially, from system of ODE three random sequences are being generated contrary to the usual sequences obtained from Galois field/ring. The encouraging outcomes of these sequences suggested their applications in multimedia security. Secondly, a watermarking technique based on DE is devised for the copyright protection of the data. The watermark becomes nearly imperceptible due to the permutation process and frequency domain make it more secure by its specific features. Furthermore, the results of the robustness test and statistical analyses substantiate our assertion that this technique exhibits semi-fragility in the context of copyright protection.

Data availability The datasets generated during the current study are not publicly available due to privacy but are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that there is no conflict of interests regarding the publication of this paper.

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