



Stability analysis of interconnected complex nonlinear systems using the Lyapunov and Finsler property

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Abstract

This study focuses on the question of the stability analysis of complex interconnected nonlinear systems using the property of Lyapunov and Finsler. The main idea is to minimize the effect of interconnections between the subsystems, for that, we use the Lyapunov function and the H_∞ control, then applying Finsler's lemma to release the conditions of stability, the independent matrices allow to obtain less conservative results. The proposed control approach is formulated in a minimization problem and derived in terms of linear matrix inequalities (LMIs) whose resolution yields the decentralized control gain matrices. All the developed results are tested on two representative examples and compared with some recent previous ones.

Keywords Interconnected system · Lyapunov · H_∞ control · Finsler · Decentralized state feedback · Linear matrix inequality (LMI)

1 Introduction

In the literature the problem of stability analysis of interconnected complex nonlinear systems are using in automatism systems and power electronics, there are many dynamic systems in this domain [4, 14, 18], their complex systems are regrouped in the big area, also their automatic configuration is more complex [8, 12, 16], for the control and the modelization for this complex system, we composite this system in interconnected subsystems, these decompositions facilitate the resolution of the issue of stability [2, 11, 15, 17]. The synthesis of law control and stabilization can be ensuring the global safety of the system and responding

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in a more secure of the system [7, 10]. Moreover, in the phase of the modeling, the classical approach can be very hard and complicated to use to these complex systems in the context of the reliability and practical conception, this method of decentralized control is alternative to assert the stabilization of the complex and big systems [6].

In this work, we studied the stabilization of interconnected subsystems using a decentralized control law by state feedback. These approaches make it possible to stabilize all the interconnected subsystems by taking into account their interconnections. Then, in order to improve this approach, we propose, through a H^∞ criterion, to study the robustness to the effects of the interconnections between subsystems. Pursuing this objective, we propose a new decentralized H^∞ controller based on Finsler's lemma [1, 3] such that all subsystems are asymptotically stable, the new conditions ensuring all the closed-loop stability, are supplied in terms of Linear Matrix Inequalities (LMIs) and the feedback gain matrix of each local controller is obtained by solving the LMIs [5].

The global of this work is organize as follows: in the second section, a state of the art of the system description has been provided. The main results has been introduced in Section 3, after that in Section 4, we show the experimental results. Finally, we give a conclusion of this work in the last section.

2 System description

We consider the following interconnected systems, where the i^{th} subsystem is given by [16]:

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1, j \neq i}^N A_{ij} x_j(t) + B_i u_i(t), \quad i = 1, \dots, N \quad (1)$$

with $x_i(t) \in \mathcal{R}^{n_i}$ and $x_j(t) \in \mathcal{R}^{n_j}$ are the states of the i^{th} and the j^{th} subsystem, $u_i(t) \in \mathcal{R}^{m_i}$ is the control input. The system matrices A_i , A_{ij} and B_i , are known real constant of appropriate dimensions.

$\sum_{j=1, j \neq i}^N A_{ij} x_j(t)$: Represent the influences of $N-1$ subsystems on the i^{th} subsystem.

The closed-loop subsystem is given by:

$$\dot{x}_i(t) = (A_i + B_i k_i) x_i(t) + \sum_{j=1, j \neq i}^N A_{ij} x_j(t), \quad i = 1, \dots, N \quad (2)$$

where $u_i(t) = k_i x_i(t)$.

The following figure (Fig.1) represents the diagram of an interconnected system:

The following Fig. 1 shows the diagram of interconnected complex nonlinear systems, the main idea is to fragment the overall system into several subsystems that are easier to handle.

We then place ourselves in a “decentralized” approach compared to traditional approaches which consist in studying a system as a whole.

The main objective of the decomposition of systems and the decentralization of calculations is: To reduce the calculation time.

The proposed control structure considers the system as a set of interconnected subsystems Fig. 2, each subsystem is controlled by a control unit, i.e. the decentralized controllers partition the information and develop a local control law at level of each subsystem and allows stabilize it.

We end this section with the following lemmas, which will be used for further development.

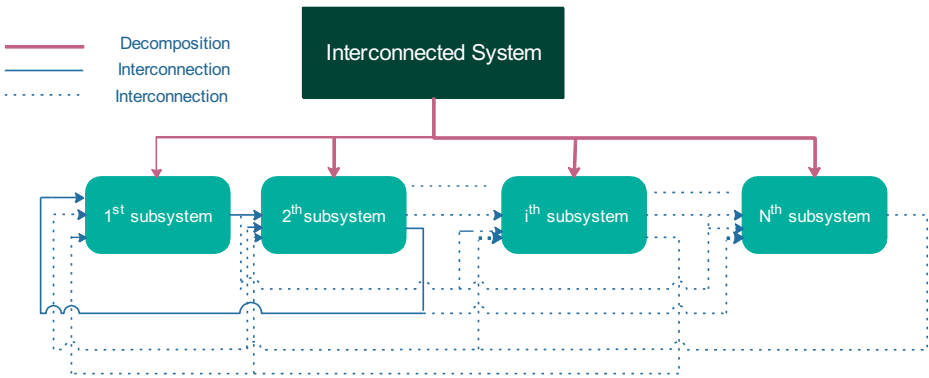


Fig. 1 Diagram of interconnected complex nonlinear systems

Lemma 1 (Schur complement) [9] Given three matrices Q , $R = R^T$ et $S = S^T$, the following statement are equivalent

(i)
$$\begin{bmatrix} R & Q \\ Q^T & S \end{bmatrix} > 0 \tag{3}$$

(ii)
$$S > 0 \text{ and } R - QS^{-1}Q^T > 0, \tag{4}$$

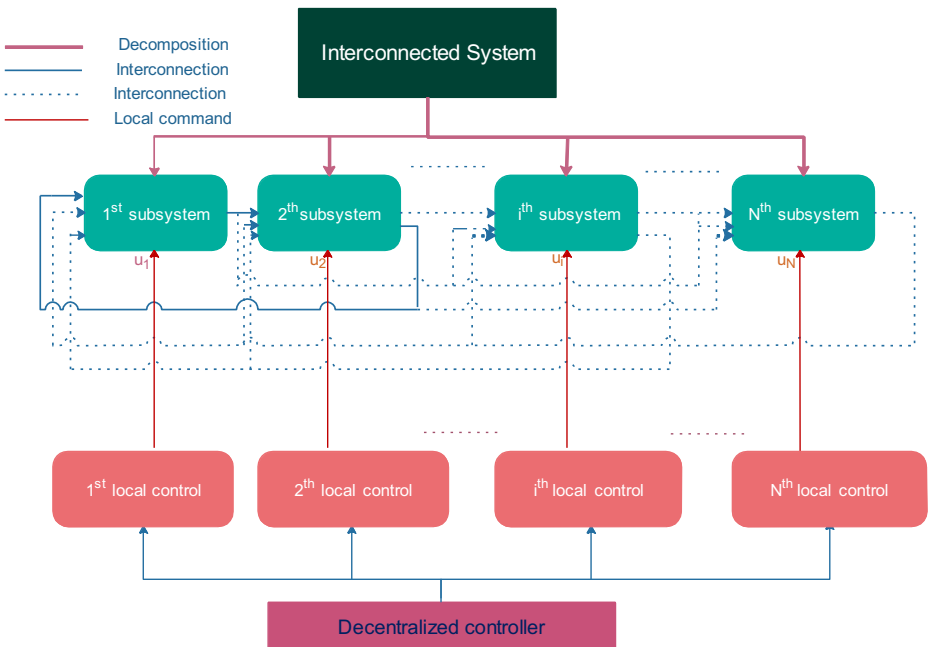


Fig. 2 Diagram of decentralized controller

Lemma 2 [13] For any $n \times n$ matrix Q , any constant $\varepsilon > 0$ and any symmetric positive matrix P ,

$$2x^T Qy \leq \varepsilon x^T QP^{-1} Q^T x + \varepsilon^{-1} y^T P y \tag{5}$$

For all $x, y \in \mathfrak{R}^n$.

Lemma 3 (Finsler Lemma) [3]: Consider $\xi \in \mathfrak{R}^n$, $Q = Q^T \in \mathfrak{R}^{n \times n}$ and $\beta \in \mathfrak{R}^{m \times n}$ such that $rank(\beta) < n$, and β^\perp a basis for the null-space of β (i.e. $\beta^\perp \beta = 0$). The following propositions are equivalents:

- (i) $\xi^T Q \xi < 0$, for all $\xi \neq 0$, $\beta \xi = 0$;
- (ii) $\beta_\perp^T Q \beta_\perp < 0$;
- (iii) $\exists \chi \in \mathfrak{R}^{n \times m} : Q + \chi \beta + \beta^T \chi^T < 0$

This lemma is used to introduce new matrix variables, which are exploited for:

- Increase the degree of freedom of the system.
- separate the Lyapunov matrices from the system parameters.

This allows to release the conditions of stability.

3 Main results

In this section, the main goal is to design a decentralized controller ensuring the stability of closed loop interconnected systems without and with delay.

3.1 New decentralized H ∞ control for interconnected systems using Finsler lemma

In this subsection, a new decentralized H ∞ control conditions using Finsler Lemma for the interconnected closed-loop system (2) is developed. For our work, we are interested in minimizing the influences of interconnections between subsystems, for this purpose and in order to improve the robustness of the proposed control law with respect to the interconnections between the subsystems, we propose a criterion H ∞ allowing to minimize the influence of the N-1 subsystems on the ith subsystem. This criterion is given by the inequality (6):

For $i = 1, \dots, N$,

$$\int_0^\infty x_i^T(t) x_i(t) dt < \gamma_i^2 \int_0^\infty \varphi_i^T(t) \varphi_i(t) dt \tag{6}$$

where

$$\varphi_i(t) = \sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \tag{7}$$

$\gamma_i > 0$ are the H ∞ performances level.

Note that, the vector $\varphi_i(t)$ reflects the influence of a jth subsystem on ith and, in this case, the performance rate H ∞ appropriate to minimize.

The main result is summarized in the following theorem:

Theorem 1 The set S of the N interconnected subsystems S_i described by (2) is globally asymptotically stabilized in closed loop via the network of N decentralized control laws with respect to the criterion H_∞ described by (6), if it exists for all the combinations $i, j = 1, \dots, N, j \neq i$, positive matrices $P_i = P_i^T > 0, Y_i, F_i$ and positive scalars γ_i, a_i, b_i verifying the LMI conditions given by:

Minimize γ_i such as:

$$\begin{bmatrix} Y_{i11} & Y_{i12} & Y_{i13} & I \\ * & \frac{-a_i}{N-1}(F_i + F_i^T) & a_i F_i A_{ij} - \frac{b_i}{N-1} F_i^T & 0 \\ * & * & Y_{i33} & 0 \\ * & * & * & \frac{-I}{N-1} \end{bmatrix} < 0 \tag{8}$$

The new local gains matrices is $k_i = y_i(F_i^T)^{-1}, i = 1, \dots, N$

Where

$$\begin{aligned} Y_{i11} &= \frac{1}{N-1} (F_i A_i^T + A_i F_i^T + B_i y_i + y_i^T B_i^T) \\ Y_{i12} &= \frac{1}{N-1} (P_i^{-1} - F_i - a_i A_i F_i^T + a_i B_i y_i) \\ Y_{i13} &= F_i A_{ij} + \frac{b_i}{N-1} (A_i F_i^T + B_i y_i) \\ Y_{i33} &= -\gamma_i^2 (2N - 3) A_{ij}^T A_{ij} + b_i F_i A_{ij} + b_i A_{ij}^T F_i^T \end{aligned}$$

We notice that the lyapunov matrices P_i are totally separated from the parameters of the system A_i, B_i . This allows to release the stability conditions obtained.

Proof The H_∞ synthesis by state feedback consists in looking for a decentralized control law by state feedback k_i such as $u_i = k_i * x_i$; and ensures both:

- The stability of the interconnected system in closed loop.
- The verification of the H_∞ criterion given by inequality (6).

The closed-loop interconnected system (2) is stable according to the H_∞ criterion if condition (9) is verified

$$\sum_{i=1}^N [\dot{V}_i(t) + x_i^T(t) x_i(t) - \gamma_i^2 \phi_i^T(t) \phi_i(t)] < 0 \tag{9}$$

The above inequality can be reformulated as

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N [A+B-C] < 0 \tag{10}$$

With:

$$\begin{aligned} A &= \frac{1}{N-1} \left(x_i^T(t) P_i^{-1} x_i(t) + x_i^T(t) P_i^{-1} \dot{x}_i(t) \right) \\ B &= x_i^T(t) x_i(t) \\ C &= \gamma_i^2 (2N-3) x_j^T(t) A_{ij}^T A_{ij} x_j(t) \end{aligned}$$

we obtained

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \zeta_i^T Q_i \zeta_i < 0 \tag{11}$$

where

$$\begin{aligned} \zeta_i &= \begin{bmatrix} x_i^T(t) & x_i^T(t) x_j^T(t) \end{bmatrix}^T, \\ Q_i &= \begin{bmatrix} \frac{1}{N-1} & \frac{1}{N-1} P_i^{-1} & 0 \\ * & 0 & 0 \\ * & * & -\gamma_i^2 (2N-3) A_{ij}^T A_{ij} \end{bmatrix}, \end{aligned}$$

Expression (2) can be written as:

$$\sum_{j=1, j \neq i}^N \begin{bmatrix} \frac{1}{N-1} A_{ci} & \frac{-1}{N-1} I & A_{ij} \end{bmatrix} \begin{bmatrix} x_i(t) \\ \dot{x}_i(t) \\ x_j(t) \end{bmatrix} = 0 \tag{12}$$

Where $A_{ci} = A_i + B_i k_i$

Considering

$$\chi_i = \begin{bmatrix} F_i \\ a_i F_i \\ b_i F_i \end{bmatrix}, \tag{13}$$

$$\beta_i = \begin{bmatrix} \frac{1}{N-1} A_{ci} & \frac{-1}{N-1} I & A_{ij} \end{bmatrix}, \tag{14}$$

with

$$\beta_{iL} = \begin{bmatrix} I & 0 \\ A_{ci} & (N-1)A_{ij} \\ 0 & I \end{bmatrix} \tag{15}$$

and using condition (ii) of lemma 3, we obtained the equality between $\beta_{iL}^T Q_i \beta_{iL}$ and the LMIs in (12) [16].

Now, Applying Finsler’s Lemma 3 in condition (6), with (11), (13) and (14) writes

$$\begin{bmatrix} \frac{1}{N-1} & \frac{1}{N-1}P_i^{-1} & 0 \\ * & 0 & 0 \\ * & * & -\gamma_i^2(2N-3)A_{ij}^T A_{ij} \end{bmatrix} + \begin{bmatrix} F_i \\ a_i F_i \\ b_i F_i \end{bmatrix} \begin{bmatrix} \frac{1}{N-1}A_{ci} & \frac{-1}{N-1}I & A_{ij} \end{bmatrix} + (*) < 0 \quad (16)$$

Thus, (16) is equivalent to (8). This completes the proof of the theorem 1.

3.2 Decentralized state feedback H∞ control for interconnected systems with time-delay

We consider the state representation of the interconnected closed loop system with time-delay can be expressed as [7]:

$$\dot{x}_i(t) = (A_i + B_i K_i)x_i(t) + A_{di}x_i(t-\tau_i(t)) + \sum_{j=1, j \neq i}^N A_{ij}x_j(t-\eta_{ij}(t)), \quad i = 1, \dots, N \quad (17)$$

The τ_i, η_{ij} are unknown time-delay factors satisfying the following conditions:

$$\begin{aligned} 0 \leq \tau_i(t) \leq \rho_i, \quad \dot{\tau}_i(t) \leq \mu_i \\ 0 \leq \eta_{ij}(t) \leq \rho_{ij}, \quad \dot{\eta}_{ij}(t) \leq \mu_{ij} \end{aligned} \quad (18)$$

where the bounds ρ_i, ρ_{ij}, μ_i and μ_{ji} are known constants in order to guarantee smooth growth of the state trajectories.

In this section, we are interested in designing a decentralized state feedback H∞ by ensuring the stabilization of interconnected systems with time delay (18).

Our goal is to minimize the effects of interconnections between subsystems by guaranteeing the stabilization of the subsystems (17).

We introduce a criterion H∞ allowing to minimize the influence of the j subsystems ($j = 1, \dots, N$, and $j \neq i$) on the ith subsystem. This criterion is given by:

$$\text{For } i = 1, \dots, N, \int_0^\infty x_i^T(t)x_i(t)dt < \gamma_i^2 \int_0^\infty \left(\sum_{j=1, j \neq i}^N A_{ij}x_j(t-\eta_{ij}(t)) \right)^T \sum_{j=1, j \neq i}^N A_{ij}x_j(t-\eta_{ij}(t))dt \quad (19)$$

where

$$\sum_{j=1, j \neq i}^N A_{ij}x_j(t-\eta_{ij}(t)) \text{ Represent the influences of } N-1 \text{ subsystems on the } i^{\text{th}} \text{ subsystem.}$$

$\gamma_i > 0$ are the H∞ performances level which should be minimized.

The interconnected variable-delay system (17) is stable according to the H∞ criterion if the inequality (20) is verified:

$$\sum_{i=1}^N \int_0^\infty \left(\dot{V}_i(t) + x_i^T(t)x_i(t) - \gamma_i^2 \left(\sum_{j=1, j \neq i}^N A_{ij}x_j(t-\eta_{ij}(t)) \right)^T \sum_{j=1, j \neq i}^N A_{ij}x_j(t-\eta_{ij}(t)) \right) dt < 0 \quad (20)$$

The approach ensuring simultaneously stabilizing the system (17), via the decentralized controller $\mu_i(t) = k_i x_i(t)$ and reduce the effect of interconnections between subsystems is summarized in the following theorem.

Theorem 2 Given $\rho_i > 0$, $\mu_i > 0$ and $\mu_{ij} > 0$, the decentralized H_∞ control problem for the system (17) is solvable if there exist symmetric positive definite matrices $P_i, Q_i, Z_{ij}, W_i, i, j = 1, \dots, N, i \neq j$ and the positive scalars γ_i for any time delays $\tau_i(t), \eta_{ij}(t)$ satisfying (18), satisfying the following conditions LMI:

Minimizing γ_i such as

$$\begin{bmatrix} \varrho_{11} & \frac{1}{N-1} A_{di} X_i & A_{ij} & 0 & \rho_i (X_i A_i^T + y_i^T B_i^T) & X_i \\ * & \frac{1-\mu_i}{N-1} R_i & 0 & 0 & \rho_i X_i A_{di}^T & 0 \\ * & * & \Theta_{33} & 0 & (N-1)\rho_i A_{ij}^T & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} & 0 & 0 \\ * & * & * & * & -(N-1)\rho_i S_i & 0 \\ * & * & * & * & * & -(N-1)I \end{bmatrix} < 0 \tag{21}$$

where

$$\begin{aligned} \vartheta_{11} &= R_{ij} + \frac{1}{N-1} (A_i X_i + X_i A_i^T + B_i y_i + y_i^T B_i^T + R_i) \\ \Theta_{33} &= -\gamma_i^2 (2N-3) A_{ij}^T A_{ij} - (1-\mu_{ij}) Z_{ij} \end{aligned}$$

Moreover, the decentralized state feedback gain matrix is given by:

$$k_i = y_i X_i^{-1}, i = 1, \dots, N \tag{22}$$

Proof The closed loop system (17) is stable under the criterion H_∞ if:

$$\sum_{i=1}^N \left[\dot{V}_i(t) + x_i^T(t) x_i(t) - \gamma_i^2 \left(\sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \right)^T \sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \right] < 0 \tag{23}$$

The inequality (23) may be increased by (see Appendix):

$$\sum_{i=1}^N \left[\dot{V}_i(t) + x_i^T(t) x_i(t) - \gamma_i^2 (2N-3) \sum_{j=1, j \neq i}^N \left(x_j^T(t - \eta_{ij}(t)) A_{ij}^T A_{ij} x_j(t - \eta_{ij}(t)) \right) \right] < 0 \tag{24}$$

with

$$\begin{aligned} \sum_{i=1}^N \dot{V}_i(t) &\leq \sum_{i=1}^N \left\{ \frac{1}{\tau_i(t)} \int_{t-\tau_i(t)}^t \left[2x_i^T(t) P_i \left(A_{ci} x_i(t) + A_{di} x_i(t - \tau_i(t)) + \sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \right) \right. \right. \\ &+ x_i^T(t) Q_i x_i(t) - (1-\mu_i) x_i^T(t - \tau_i(t)) Q_i x_i(t - \tau_i(t)) + \rho_i x_i^T(t) W_i \dot{x}_i(t - \tau_i(t)) x_i^T(s) W_i \dot{x}_i(s) \\ &\left. \left. + \sum_{i=1, j \neq i}^N x_i^T(t) Z_{ij} x_i(t) - \sum_{i=1, j \neq i}^N (1-\mu_{ij}) x_j^T(t - \eta_{ij}(t)) \right] ds \right\} \end{aligned} \tag{25}$$

Let

$$\zeta_i^T(t) = \left[x_i^T(t) x_i^T(t-\tau_i(t)) x_j^T(t-\eta_{ij}(t)) \dot{x}_i^T(s) \right]^T \tag{26}$$

we obtain

$$\sum_{i=1}^N \left[\dot{V}_i(t) + x_i^T(t) x_i(t) - \gamma_i^2 (2N-3) \sum_{j=1, j \neq i}^N \left(x_j^T(t-\eta_{ij}(t)) A_{ij}^T A_{ij} x_j(t-\eta_{ij}(t)) \right) \right] \leq \sum_{i=1}^N \sum_{j=1, j \neq i}^N \zeta_i^T(t) \Xi_i \zeta_i(t) \tag{27}$$

with

$$\Xi_i = \begin{bmatrix} \Lambda_{11} & \frac{1}{N-1} (\rho_i A_{ci}^T W_i A_{di} + P_i A_{di}) & P_i A_{ij} + \rho_i A_{ci}^T W_i A_{ij} & 0 \\ * & \frac{1}{N-1} (\rho_i A_{di}^T W_i A_{di} - (1-\mu_i) Q_i) & \rho_i A_{di}^T W_i A_{ij} & 0 \\ * & * & \Lambda_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} \tag{28a}$$

$$\Lambda_{11} = Z_{ji} + \frac{1}{N-1} (p_i A_{ci} + A_{ci}^T P_i + \rho_i A_{ci}^T W_i A_{ci} + Q_i + I) \tag{28b}$$

$$\Lambda_{33} = -\gamma_i^2 (2N-3) A_{ij}^T A_{ij} - (1-\mu_{ij}) Z_{ij} + (N-1) A_{ij}^T \rho_i W_i A_{ij} \tag{28c}$$

That is to say, for all $i, j = 1, \dots, N$ and $j \neq i$

$$\begin{bmatrix} \Lambda_{11} & \frac{1}{N-1} (\rho_i A_{ci}^T W_i A_{di} + P_i A_{di}) & P_i A_{ij} + \rho_i A_{ci}^T W_i A_{ij} & 0 \\ * & \frac{\rho_i A_{di}^T W_i A_{di}}{N-1} - (1-\mu_i) Q_i & \rho_i A_{di}^T W_i A_{ij} & 0 \\ * & * & \Lambda_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} < 0 \tag{29}$$

Inequality (29) can be rewritten as:

$$\Pi_i + \begin{bmatrix} \rho_i A_{ci}^T W_i \\ \rho_i A_{di}^T W_i \\ (N-1) \rho_i A_{ij}^T W_i \\ 0 \end{bmatrix} ((N-1) \rho_i W_i)^{-1} \times [\rho_i W_i A_{ci} \quad \rho_i W_i A_{di} \quad (N-1) \rho_i W_i A_{ij} \quad 0] < 0 \tag{30}$$

where

$$\Pi_i = \begin{bmatrix} \Theta_{11} & \frac{1}{N-1} P_i A_{di} & P_i A_{ij} & 0 \\ * & -(1-\mu_i) Q_i & 0 & 0 \\ * & * & \Theta_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} \tag{31a}$$

$$\Theta_{11} = Z_{ji} + \frac{1}{N-1} (P_i A_{ci} + A_{ci}^T P_i + Q_i + I) \tag{31b}$$

$$\Theta_{33} = -\gamma_i^2(2N-3)A_{ij}^T A_{ij} - (1-\mu_{ij})Z_{ij} \tag{31c}$$

Applying the Schur complement in lemma 1, we obtain

$$\begin{bmatrix} \Theta_{11} & \frac{1}{N-1}P_i A_{di} & P_i A_{ij} & 0 & \rho_i A_{ci}^T W_i \\ * & -\frac{1-\mu_i}{N-1}Q_i & 0 & 0 & \rho_i A_{di}^T W_i \\ * & * & \Theta_{33} & 0 & (N-1)\rho_i A_{ij}^T W_i \\ * & * & * & -\frac{\rho_i W_i}{N-1} & 0 \\ * & * & * & * & -(N-1)\rho_i W_i \end{bmatrix} < 0 \tag{32}$$

(32) is satisfied if:

$$\psi = \begin{bmatrix} \psi_{11} & \frac{1}{N-1}P_i A_{di} & P_i A_{ij} & 0 & \rho_i A_{ci}^T W_i & I \\ * & -\frac{1-\mu_i}{N-1}Q_i & 0 & 0 & \rho_i A_{di}^T W_i & 0 \\ * & * & \Theta_{33} & 0 & (N-1)\rho_i A_{ij}^T W_i & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} & 0 & 0 \\ * & * & * & * & -(N-1)\rho_i W_i & 0 \\ * & * & * & * & * & -(N-1)I \end{bmatrix} < 0 \tag{33}$$

Where $\psi_{11} = Z_{ji} + \frac{1}{N-1}(P_i A_{ci} + A_{ci}^T P_i + Q)$

Multiplying ψ left and right respectively by

$\text{diag}[P_i^{-T}, P_i^{-T}, I, I, W_i^{-T}, I]$ and $\text{diag}[P_i^{-1}, P_i^{-1}, I, I, W_i^{-1}, I]$,

and let $X_i = P_i^{-1}, y_i = k_i X_i, S_i = W_i^{-1}, R_{ji} = X_i Z_{ji} X_i, R_i = X_i Q_i X_i,$

Finally, we obtain the LMI conditions (21) of theorem 2. This completes the proof.

4 Experimental result

In this section, two numerical examples are supplied to show the advantage of the proposed approaches.

Example 1 We give the decentralized H ∞ control issue of interconnected system (2) with

$$\begin{aligned} A_1 &= \begin{bmatrix} -7 & 0 \\ -6 & 0.1 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, A_{13} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix} \\ A_2 &= \begin{bmatrix} 5 & 0 \\ 1 & -1 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, A_{23} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix} \\ A_3 &= \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}, A_{31} = \begin{bmatrix} 0.01 & 0 \\ 0.1 & 0.1 \end{bmatrix}, A_{32} = \begin{bmatrix} 0.01 & 0.1 \\ 0 & 0.1 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 0.47 \\ 1.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.47 \\ 0.8 \end{bmatrix}, B_3 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} \end{aligned}$$

Table 1 Minimum performance bound H_∞

Methods	γ_{\min}		
	First subsystem	Second subsystem	Third subsystem
Decentralized H_∞ control [16]	$\gamma_1 = 1.5134$	$\gamma_2 = 4.4897$	$\gamma_3 = 1.3708$
H_∞ +Finsler [Theorem 1]	$\gamma_1 = 0.2574$	$\gamma_2 = 0.4783$	$\gamma_3 = 1.0725$

Applying theorem 1 to the system (2) and minimizing γ_i , it is found that the feasible solution is attained at:

In order to show the efficiency of our approach, we will solve the LMI stability conditions proposed in theorem 1 and by minimizing γ_i .

The results obtained of our approach as well as those in the literature are summarized in Tables 1 and 2.

Table 1 shows the comparison of the minimal of H_∞ performance for the three subsystems.

According to the results obtained, we notice that γ_{\min} obtained by application of theorem 1 are smaller than [16].

We note that our approach is more efficient than that of [16].

Table 2 represent the gain matrices obtained for the three subsystems.

We notice that the controller parameters obtained by applying theorem 1 are smaller than those in [16].

The three local controllers are capable of stabilizing the three subsystems.

In this numerical result, the initial conditions are given as:

$$x_1(0) = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad x_3(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

The evolution of the control system and these dynamics in a closed-loop for every each subsystem are shown in Figs. 3, 4 and 5.

We performed a simulation test for the 3 subsystems:

The Fig. 3a represent the comparison of the dynamics of the 1st subsystem by our method and that of [16]; we notice that the latter requires more time before stability is restored. On the other hand, the proposed control ensures a better convergence with a very reduced response time with fewer oscillations.

Similarly for the 2th subsystem, we notice that the proposed control ensures a better convergence with a very reduced response time.

Table 2 Decentralized control laws

Methods	k_i		
	First subsystem	Second subsystem	Third subsystem
Decentralized H_∞ control [16]	$k_1 = [10.674 \quad -10.552]$	$k_2 = [-85.729 \quad 36.663]$	$k_3 = [-6.4300 \quad -4.628]$
H_∞ +Finsler [Theorem 1]	$k_1 = [10.0746 \quad -1.8061]$	$k_2 = [-32.5773 \quad 6.7252]$	$k_3 = [-0.2800 \quad -0.3621]$

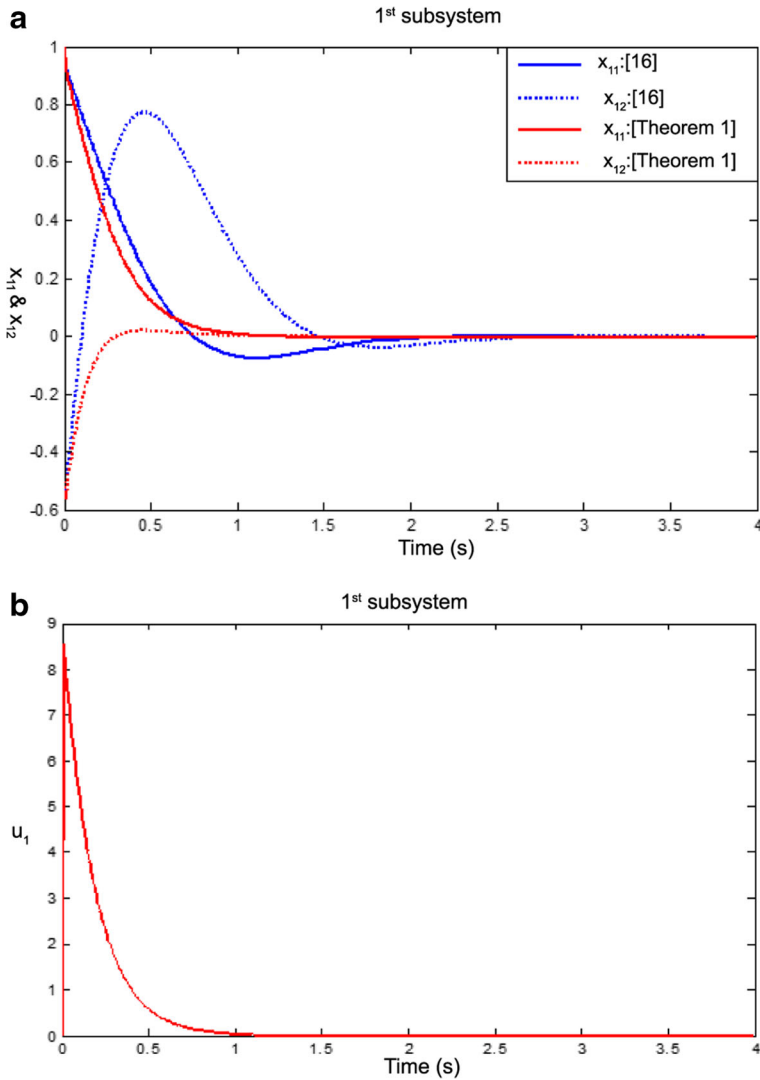


Fig. 3 **a** Dynamics of the 1st closed-loop subsystem according to [16] and [Theorem 1]. **b** Evolution of the control law of the 1st subsystem by the proposed method

Also for the 3th subsystem, we can always notice that the proposed control ensures a better convergence with a very reduced response time of about 1 s.

We note that all the decentralized controllers stabilize the global interconnected system after a time of the order of 1 s. While each subsystem has its own convergence time, each of the subsystems has its own convergence time.

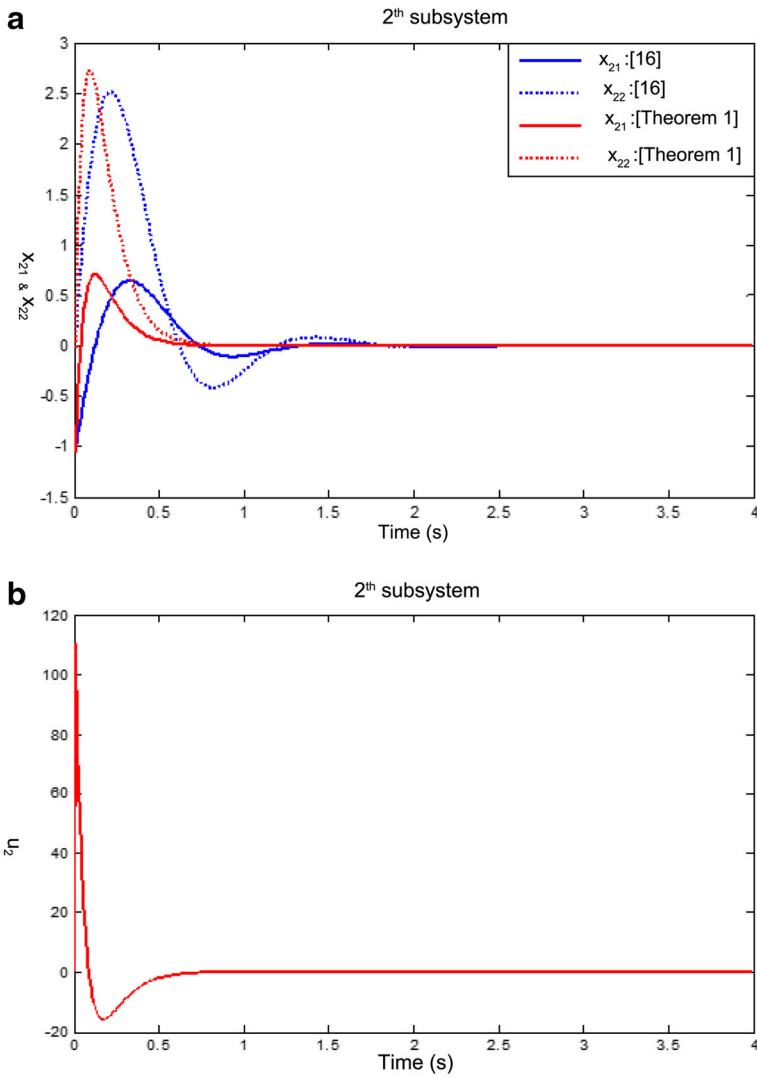


Fig. 4 **a** Dynamics of the 2th closed-loop subsystem according to [16] and [Theorem 1]. **b** Evolution of the control law of the 2th subsystem by the proposed method

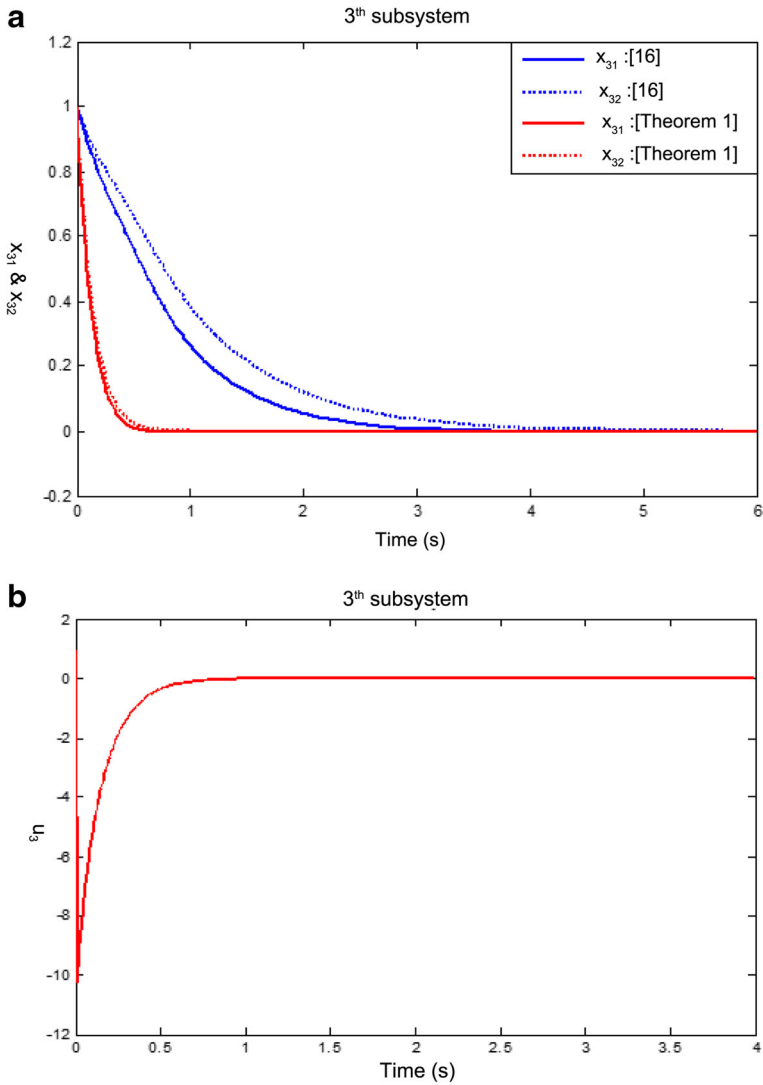


Fig. 5 **a** Dynamics of the 3th closed-loop subsystem according to [16] and [Theorem 1]. **b** Evolution of the control law of the 3th subsystem by the proposed method

Example 2 Consider the interconnected closed loop system with time-delay [7] given by:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -2 & 0 \\ -2 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, A_{13} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\
 A_2 &= \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix}, A_{d2} = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}, A_{21} = \begin{bmatrix} -1 & -2 \\ 3 & 6 \end{bmatrix}, A_{23} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \\
 A_3 &= \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, A_{d3} = \begin{bmatrix} 0 & 0 \\ -1 & -2 \end{bmatrix}, A_{31} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, A_{32} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, B_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}
 \end{aligned}$$

The interconnections of the subsystems influence the stabilization of the system as an overall. Applying theorem 2, for:

$$\rho_1 = 2 ; \mu_1 = 0.6 ; \rho_2 = 2; \quad \mu_2 = 0.3 ; \rho_3 = 3; \mu_3 = 0.2;$$

$$\mu_{12} = 0.5; \mu_{13} = 0.7; \mu_{23} = 0.2; \mu_{21} = 0.9 ; \mu_{31} = 0.6; \mu_{32} = 0.3;$$

We obtain $P_1 = \begin{bmatrix} 1.4664 & -0.1928 \\ -0.1928 & 0.1505 \end{bmatrix}; Q_1 = \begin{bmatrix} 28.4358 & -4.0533 \\ -4.0533 & 0.7780 \end{bmatrix};$

$$\begin{aligned}
 W_1 &= \begin{bmatrix} 0.1334 & -0.0020 \\ -0.0020 & 0.1343 \end{bmatrix}; \\
 P_2 &= \begin{bmatrix} 2.6855 & 1.0542 \\ 1.0542 & 0.5788 \end{bmatrix}; Q_2 = \begin{bmatrix} 116.5229 & 48.1746 \\ 48.1746 & 20.2471 \end{bmatrix}; W_2 = \begin{bmatrix} 0.1313 & 0.0116 \\ 0.0116 & 0.10118 \end{bmatrix}; \\
 P_3 &= \begin{bmatrix} 0.7291 & 1.6097 \\ 1.6097 & 4.9738 \end{bmatrix}; Q_3 = \begin{bmatrix} 46.8429 & 137.7018 \\ 137.7018 & 409.9437 \end{bmatrix}; W_3 = \begin{bmatrix} 0.1942 & -0.0143 \\ -0.0143 & 0.1883 \end{bmatrix};
 \end{aligned}$$

Since $P_i, Q_i, W_i > 0, i = 1, 2, 3$

Then, the conditions provided by Theorem 2 are satisfied.

The results obtained of our approach as well as those in the literature are summarized in Tables 1 and 2.

Table 3 shows the comparison of the minimum performance bound H_∞ obtained for the three subsystems.

According to the results obtained, we notice that γ_{\min} obtained by application of theorem 2 are smaller than those of [7]. Thus, our approach is more efficient than that of [7].

The Table 4 shows the gain matrices obtained for the three subsystems. The three local controllers are capable to stabilize the three subsystems.

Table 3 Minimum H_∞ performance γ_{\min} of interconnected system with time-varying delays

Methods	γ_{\min}		
	First subsystem	Second subsystem	Third subsystem
By [7]	$\gamma_1 = 1.8011$	$\gamma_2 = 13.4931$	$\gamma_3 = 1.4746$
By Theorem 2	$\gamma_1 = 1.6725$	$\gamma_2 = 0.5162$	$\gamma_3 = 0.9170$

Table 4 Decentralized control laws k_i

Methods	k_i										
	First subsystem			Second subsystem			Third subsystem				
By [7]	k_1	[1.1187	0.8962]	k_2	[-8.8441	1.3237	-3.9501	-3.3300]	k_3	[-0.6289	-0.2550]
By Theorem 2	k_1	[1.1374	0.4022]	k_2	[0.3479	-1.1906	0.5539	1.2342]	k_3	[-0.0237	-0.6346]

5 Conclusion

In this study, we proposed a new decentralized H_∞ control for interconnected complex nonlinear systems such that the closed-loop feedback subsystems are asymptotically stable. We used a Lyapunov function and H_∞ criterion to reduce and minimize the effect of interconnections between the subsystems, in order to release the conditions of stability, we applied Finsler’s lemma. The sufficient conditions ensuring stability in closed loop were formulated in the terms of LMI. Finally, the obtained result are given to prove the efficiency of the proposed method, and the graphics result shows that the system is well stabilized.

Appendix

Decentralized State Feedback H_∞ Control

In this [Appendix](#), we verify the inequality (19) used in section 3, with:

$$\varphi_i = \sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \tag{A.1}$$

$$\begin{aligned} \varphi_i^T \varphi_i &= \left(\sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \right)^T \sum_{j=1, j \neq i}^N A_{ij} x_j(t - \eta_{ij}(t)) \\ &= \sum_{j=1, j \neq i}^N \sum_{l=1}^N \sum_{\substack{l \neq i, l \neq j}} \left(x_j^T(t - \eta_{ij}(t)) A_{ij}^T A_{il} x_l(t - \eta_{ij}(t)) + x_l^T(t - \eta_{ij}(t)) A_{il}^T A_{ij} x_j(t - \eta_{ij}(t)) \right) \\ &\quad + \sum_{j=1, j \neq i}^N x_j^T(t - \eta_{ij}(t)) A_{ij}^T A_{ij} x_j(t - \eta_{ij}(t)) \\ &= \sum_{j=1, j \neq i}^N \left[x_j^T(t - \eta_{ij}(t)) A_{ij}^T A_{ij} x_j(t - \eta_{ij}(t)) \right. \\ &\quad \left. + \sum_{\substack{l=1 \\ l \neq i, l \neq j}}^N \left(x_j^T(t - \eta_{ij}(t)) A_{ij}^T A_{il} x_l(t - \eta_{ij}(t)) + x_l^T(t - \eta_{ij}(t)) A_{il}^T A_{ij} x_j(t - \eta_{ij}(t)) \right) \right] \end{aligned} \tag{A.2}$$

Apply the lemma of the square matrix, we have:

$$\begin{aligned}
 & \sum_{\substack{l=1 \\ l \neq i, l \neq j}}^N \left(x_j^T(t-\eta_{ij}(t)) A_{ij}^T A_{il} x_l(t-\eta_{ij}(t)) + x_l^T(t-\eta_{ij}(t)) A_{il}^T A_{ij} x_j(t-\eta_{ij}(t)) \right) \\
 & \leq \sum_{\substack{l=1 \\ l \neq i, l \neq j}}^N \left[A_{ij} x_j(t-\eta_{ij}(t)) \right]^T \left[A_{ij} x_j(t-\eta_{ij}(t)) \right] + \left[A_{il} x_l(t-\eta_{ij}(t)) \right]^T \left[A_{il} x_l(t-\eta_{ij}(t)) \right] \\
 & = (N-2) \left[A_{ij} x_j(t-\eta_{ij}(t)) \right]^T A_{ij} x_j(t-\eta_{ij}(t)) + \sum_{\substack{l=1 \\ l \neq i, l \neq j}}^N \left[A_{il} x_l(t-\eta_{ij}(t)) \right]^T \left[A_{il} x_l(t-\eta_{ij}(t)) \right]
 \end{aligned} \tag{A.3}$$

Then

$$\varphi_i^T \varphi_i \leq \sum_{j=1, j \neq i}^N \left((N-1) x_j^T(t-\eta_{ij}(t)) A_{ij}^T A_{ij} x_j(t-\eta_{ij}(t)) + \sum_{\substack{l=1 \\ l \neq i, l \neq j}}^N x_l^T(t-\eta_{ij}(t)) A_{il}^T A_{il} x_l(t-\eta_{ij}(t)) \right) \tag{A.4}$$

Since

$$\sum_{j=1, j \neq i}^N \left(\psi_{ij} + \sum_{\substack{l=1 \\ l \neq i, l \neq j}}^N \psi_{il} \right) = (N-1) \sum_{j=1, j \neq i}^N \psi_{ij} \tag{A.5}$$

Inequality (A.4) can be rewritten as follows:

$$\varphi_i^T \varphi_i \leq (2N-3) \sum_{j=1, j \neq i}^N x_j^T(t-\eta_{ij}(t)) A_{ij}^T A_{ij} x_j(t-\eta_{ij}(t)) \tag{A.6}$$

Finally, the inequality (24) is verified.

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