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A novel approach for high dimension 3D object representation using Multi-Mother Wavelet Network

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Abstract In this paper, we present a novel approach for 3D objects representation. Our idea is to prove that wavelet networks are capable for reconstruction and representing irregular 3D objects used in computer graphics. The major contribution consist to transform an input surface vertices into signals and to provide instantaneously an estimation of the output values for input values. To prove this, we will use a new structure of wavelet network founded on several mother wavelet families. This structure uses several mother wavelet, in order to maximize best wavelet selection probability. An algorithm to construct this structure is presented. First, Data is taken from 3D object. The vertices and their corresponding normal values of a 3D object are used to create a training set. To this stage, the training set can be expressed according to three functions, which interpolates all their vertices. Second we approximate each function using wavelet network. To achieve a better approximation, the network is trained several iterations to optimize wavelet selection for every mother. To guarantee a small error criterion, we adjust wavelet network

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parameters (weight, translation and dilation) by using an improved Orthogonal Least Squares method version. We consider our proposed approach on some 3D examples to prove that the new approach is able to approximate 3D objects with a good approximation ability.

Keywords 3D representation · Multi-Mother Wavelet Networks · OLS algorithm · Approximation · Polywogs

1 Introduction

Much research effort has been made in the area of 3D object modeling over the last decades. As the accuracy of representation increases due to the unceasing advance of the available scanning hardware, 3D models size becomes prohibiting for internet based graphics applications [13] , medical image analysis [20, 32], virtual 3D world modeling, fast rendering and other applications. Classical approaches can be categorized into surface, solid and procedural models. 3D meshes are the most common representation of 3D models. However, surfaces represented by 3D meshes may contain noise or some unrequired details. Multiresolution representations and filtering techniques are very useful in this case, especially in the 3D representation and surface filtering [8, 11, 38]. The most approach used in signal processing field is the construction of frequency spectrum using decompositions in basis functions. The components of the low frequency in the signal correspond to smooth some features, and the components of the high frequency correspond to the thinner details as the creases, the folds and the angles [27, 35]. To carry just lower frequency components suffices to capture and to represent the total perceptual shape of the object. The spherical harmonics have often been used in recent years for the analysis of 3D object [25, 31]. Representation and filtering of 3D models using spherical harmonics are developed using the results of Zhou et al. [38]. There are other techniques using spherical wavelets used to represent the surface at several levels of details [12, 22] that they applied the spherical wavelet transform. The interest in wavelet networks is mainly due to their abilities to produce promising results in many applications such as speech, image and video compression and classification [21], face Recognition [34], denoising, communications, computer and machine vision [9, 29]. Their success stems from the fact that they can simultaneously possess the good properties of orthogonality, symmetry, high modeling order and short support which is not possible in the scalar case. A wavelet network has the capacity to learn high-dimensional, redundant mappings from a limited set of measured data, without a prior mathematical model. They have the builtin capability to adapt their parameters to changes in the environment and are able to provide instantaneously an estimation of the output values for input values. Wavelet networks already have been proven to perform better than scalar wavelets in applications like image compression and denoising [5, 15, 29]. Wavelet networks can be treated as a universal tool for 1D and 2D data modeling [4, 6, 10]. The general problem can be stated as reconstructing a model of an object by approximating a finite set of points in the space belonging to it. With the development of wavelet network, there has been much research on wavelet network algorithms. The error back-propagation may be the most popular algorithm in the learning of wavelet network [23, 36]. In the course of training, the error back-propagation is often combined with Orthogonal Least Square-Backward Elimination [36], which is used in the network structure selection. Other algorithms besides the error back-propagation are also proposed to train the wavelet network, e.g., genetic algorithms [2, 3, 16, 24], fuzzy logic systems [18, 19] and immune algorithms [29]. The above algorithms mostly stem from the typical neural network, so they seldom utilize fully the excellent properties of wavelets [1, 28], in the frequency domain though they have accelerated convergence, avoided local minimum and overcome fitting to some extent. In this work, we present a modeling method that uses Multi-Mother Wavelet Networks (MMWN) for 3D objects modeling. In the first part, we will discribe our new structure in which is in some ways similar to the classic wavelet networks but it admits some originality. Actually, wavelet network basically uses dilations and translations versions of only one mother wavelet to construct the network. The proposed structure uses several mother wavelets, in order to maximize best wavelet selection probability: the nearest wavelet to the input set. Then we will propose a new and compact modeling for 3D mesh irregular surface using wavelet network. First, we generate a set of vertexes with coordinates: x, y and z. Then we associate to every coordinate a 1D function. Therefore, the 3D coordinates of simple object become the wavelet network function inputs. Then we model each of these functions by a wavelet network. For modeling, we use the proposed wavelet network new structure (MMWN) with the training procedure based on an improved version of the Orthogonal least square (OLS) algorithm. This improved OLS is used to select the wavelet network dimension (the number of wavelets in hidden layer) by reducing wavelets number in handling large dimension problem according to the sample data. After the modeling phase, 3D object will be reconstruction by the wavelet network parameters. Finally, the object representation will be compared with the original object to prove the accurateness and the efficiency of this representation approach while using the Multi-Mother Wavelet Network approach.

2 Multi-Mother Wavelet Network

The wavelet theory is a rapidly developing branch of mathematics which has found many applications, for instance, in numerical analysis and signal processing. Though this attractive theory has offered very efficient algorithms for analyzing, approximating, estimating and compressing functions or signals, the applications of such algorithms are usually limited to problems of small input dimension 1D and 2D, because constructing and storing wavelet basis of large input dimension are of prohibitive cost. In order to handle problems of large input dimension, it is desirable to find some techniques whose complexity is less sensitive to the input dimension. It seems that neural networks are promising candidates for such purpose. Wavelet network (WN) has been introduced as a special feed-forward neural network supported by the wavelet theory, and is fast emerging as an efficient tool for function approximation. It combines the property of wavelet transform of analysing non-stationary signal and the classification capability of artificial neural network. Based on the work of Zhang and Benveniste [37], the multidimensional input and single output wavelet network structure proposed in this work is given by the following equation:

$$f(x) = \sum_{i=1}^{N} \omega_i \psi(d_i(x - t_i) + \overline{f}, \tag{1}$$

where the network parameters are $\omega_i \in \Re$, $d_i \in (\Re^d)^*$ and $t_i \in \Re^d$, correspond respectively to the wavelet coefficients, dilation parameters and translation parameters. $\overline{f}$ is introduced in order to make it easier to approximate functions with non-zero average. N is a variable, corresponding to the number of wavelets in hidden layer.

ψ_i , called wavelets, are dilated and translated versions of a single function ψ called the mother wavelet.

The network proposed by our method is a Multi-Mother Wavelet Network structure(MMWN) [7, 30]. It is used for the approximation of 1D and 2D data. It is similar to the classic network, but it possesses some differences; the classic network uses dilation and translation versions of only one mother wavelet, while the new version constructs the network by the implementation of several mother wavelets in the hidden layer. The objective is to maximize the potentiality of the right selection of the wavelet that approximates better the signal. The wavelets network model is given by the following equation:

$$f(x) = \sum_{i=1}^{N_1} \omega_i^1 \psi_i^1(d_i(x - t_i) + ... + \sum_{i=1}^{N_M} \omega_i^M \psi_i^M(d_i(x - t_i) + \overline{f} \tag{2}$$

This equation is equivalent to

$$f(x) = \sum_{j=1}^{M} \sum_{i=1}^{N_j} \omega_i^j \psi_i^j(d_i(x - t_i) + \overline{f}, \tag{3}$$

where ψ_i^j, are wavelet versions of a several mother wavelets ψ^j. The number of selected wavelets N_j, the mother wavelet family ψ_j, depends on the wavelet family and the choice of the mother wavelet.

3 3D object representation procedure

The vertices of a 3D object data are often irregularly and sparsely distributed so that 3D object may not readily give an accurate modeling. There is also one more advantage with the 3D mesh representation. That is, vertex normals are often contained or implied in the 3D mesh data. Based on this observation, we propose a simple approach to represent 3D object from 3D meshes. First data is taken from 3D object. Then we consider a separate function for each coordinate component x, y and z of the 3D objects vertex. We approximate each function using a multi mother wavelet network, independently of the others without considering the dependencies between these three functions on the object. For the approximation, the process of training is used to estimate these three functions on the object in the reconstruction process. The 3D coordinates of simple object have become the inputs for the functions in wavelet network, and produces output that approximate either a point in an object space belongs to the object after training procedure. After modeling procedure, 3D object is represented using wavelet network parameters. Finally, the object is compared with the original object from 3D file to prove its accuracy and efficiency using wavelet network. A block diagram of the 3D representation procedure is shown in Fig. 1.

3.1 Acquisition

In the acquisition stage data is obtained from 3D irregular object. Object in VRML file is represented using triangular meshes. The entire vertices x, y, z and the vertex indices of triangular faces are extracted from VRML file and organized into triangular form. Besides extraction of surface points, the vertex normal value for each point is also very important. In the data acquisition process, the magnitude and vector of each point is calculated. Basically, face normals are first calculated, and then the average of all the normals around each vertex is taken. This can produce a better approximation for that vertex.

3.2 Separation

Let O denote an irregular triangulated object. $O = \{v = (x, y, z) \in \Re^3\}$. After the acquisition procedure, the object O can be expressed using three functions, which interpolates all vertex of O. We go treated these functions as signals. Taking into consideration the x, y, and z coordinates of v in object space, the each vertex $v = (x, y, z)^T$ can be represented as: $V(x, y, z) = (S_x(x), S_y(y), S_z(z))^T$, where V denotes the set of points of O. We calculate the new position V of each vertex v of the object O using a multi mother wavelet network. So we approximate each function of S_x, S_y and S_z independently of the others without considering the dependencies between these three functions on the object. Therefore the modeling problem objects 3D can be transformed in the approximation of these functions using wavelet network. For that, we must determine the parameters of the these wavelet networks according to (4–6):

$$X(\omega_x, t_x, d_x) = \sum_{j=1}^{M} \sum_{i=1}^{N_j} \omega_{i_x}^j \psi_{i_x}^j (d_{i_x}(x - t_{i_x}) + \overline{f_x} \tag{4}$$

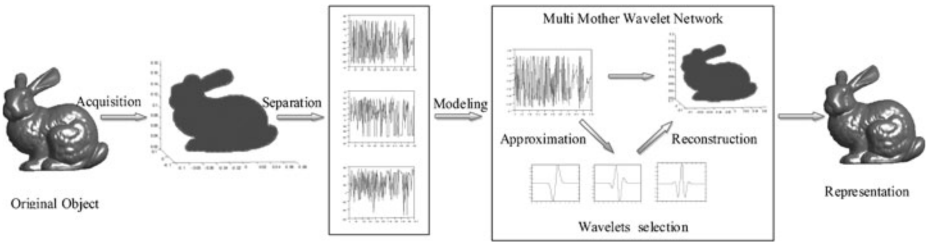

Fig. 1 3D object representation procedure

$$Y(\omega_y, t_y, d_y) = \sum_{j=1}^{M} \sum_{i=1}^{N_j} \omega_{i_y}^j \psi_{i_y}^j (d_{i_y}(y - t_{i_y}) + \overline{f_y} \quad (5)$$

$$Z(\omega_z, t_z, d_z) = \sum_{j=1}^{M} \sum_{i=1}^{N_j} \omega_{i_z}^j \psi_{i_z}^j (d_{i_z}(z - t_{i_z}) + \overline{f_z} \quad (6)$$

3.3 Wavelet network modeling

3.3.1 Network settings

The wavelet network can be considered as constituted of three layers: A first layer with N_i input. A hidden layer constituted by N_j wavelets choosing M mother wavelets. A linear output neuron receiving the pondered wavelets outputs and a linear part. There are five main parameters to construct a wavelet network: the type of basis function (mother wavelet) for each wavelet, the number of wavelets in the hidden layer, the two parameters (translation, dilation) of the mother wavelet and the weights of connections between wavelet in hidden layer and output layer. Changing one of these parameters leads a change of network behavior. To perform the approximation ability, each 1D signal is divided to k subset input patterns.

3.3.2 Training procedure

In this section, we present our approach for the modeling of the 3D object using Multi-Mother Wavelet Network. The idea is to approximate approximate each function using a wavelet network. To achieve a better approximation, the network is trained several iterations to optimize wavelet selection for every mother, the same for the wavelet network parameters, until the error criterion is small enough. There exist many different approaches for modeling. Orthogonal least square (OLS) algorithm [33] is to find the regressors, which provide the most significant contribution to approximation error reduction. It has been widely accepted as an effective method for regressor selection in RBF network [15]. The advantage of employing OLS is that the responses of the hidden layer wavelets are decorrelated so that the contribution of individual candidate wavelets to the modeling error reduction can be evaluated independently. For the best selection, we propose an improved version of OLS procedure for subset model selection. Contrary to the OLS algorithm in which the best regressors are first selected then optimized to the network, the optimal OLS algorithm presented in this work integrates in every iteration an operation of selection, optimization and orthogonalization. The algorithm is summarized as follows:

Step 1 Initialization: The multi wavelet families library consists in constructing a several mother wavelets family for the network construction. To every wavelet ψ_i^j we associate a regressor whose components are the values of this wavelet according to the examples of the training sequence. We constitute a matrix that is constituent of the blocks of the regressors representing the wavelets of every mother wavelet where the expression is given by:

$$V_{Mw} = \{V_i^j\}_{i=[1..N],\, j=[1..M]} \quad (7)$$

This choice presents the advantage to enrich the library, and to get a better performance for a given wavelet number. The inconvenience introduces by this choice concerns the size of the library. A wavelet library having several wavelet families is more voluminous than the one that possesses the same wavelet mother. It implies a more elevated calculation cost during the stage of selection. Nevertheless, using classic algorithms optimization, the selection of wavelets is often shorter than the training of the dilations and translations; the supplementary cost introduced by different dilations can be therefore acceptable.

Step 2 Selection: At the first iteration, we must find the input vector which explains the better the network output. For this, a selection method is applied in order to determine the most meaningful regressor for modeling the considered signal. Generally, the regressors in V_{Mw} are not all meaningful to estimate the signal. The regressor that provides the best combination with the output to model the signals S_x, S_y and S_z will be picked to form the new output X, Y and Z. In this part we goes treated the case of the signal approximation S_x by the output wavelet network X. The same ways for the other signals S_y and S_z. The selected wavelet is the one for which the absolute value of the cosine with $f(x, y, z)$ is maximal:

$$i_{pert}(i, j) = arg \max_{i,j} \frac{\overrightarrow{S_x} \cdot \overrightarrow{V_i^j}}{\|\overrightarrow{S_x}\| \cdot \|\overrightarrow{V_i^j}\|} \quad (8)$$

$with\ i = [1..N],\ j = [1..M]$

The regressor that provides the minimum error is selected as the first regressor of V_{Mw} of the blocks of the vectors representing the wavelets of every mother wavelet. It can be considered like a parametrable temporal function used for modeling signal.

Step 3 Optimization: To estimate the network parameters, we need an optimization algorithm. It then implements iterative methods such as back propagation. Choosing a selection method is made, it remains to choose the optimization algorithm to optimize the parameters. The LevenbergMarquardt algorithm is used to optimize the wavelet selection. It applied to minimize the total sum of differences between the training object and the selected wavelet reconstruction:

$$MSE = \frac{1}{N} \sum_{k=1}^{N} (S_x(x_k) - \sum_{j=1}^{M} \sum_{i=1}^{N_j} \omega_i^j \psi_i^j(x_k) + \overline{f})^2 \quad (9)$$

After optimization, the parameters ω_i, d_i and t_i of the optimal regressor $V_{i_{pert}}$ are adjusted.

Step 4 Orthogonalization: The regressor selected and optimized substitutes the old and the orthogonalization will be done by using the new regressor. The vectors V_i^j are always linearly independent and non orthogonal. The orthogonalization procedure is made very efficient by employing orthogonalization of the Gram-Shmidt [14]. We orthogonalize the remaining regressors according to the adjusted regressor $V_{i_{pert}}$:

$$V_i^{j\perp} = V_i^j - \overrightarrow{V_i^j} \cdot \overrightarrow{V_{i_{pert}}} \cdot V_{i_{pert}} \quad (10)$$

The regressors $V_i^{j\perp}$ are what remains from the regressors V_i^j in the orthogonal space to $V_{i_{pert}}$.

Step 5 updating: At each orthogonalization, all the unused regressor are studied to determine how each regressor will contribute to fit the desired signal with the current subset selected and optimized regressors, therefore, we make the library updating which will be used for the next iteration. The library will have:

$$V_{Mw} = \left\{V_i^{j\perp}\right\}_{i=[1..N],\, j=[1..M]\backslash\{i_{pert}\}} \tag{11}$$

The training procedure repeats itself until the selection of N wavelets in the hidden layer network.

3.4 Representation

In this phase, 3D object is approximated by 3 wavelet networks,each one can be represented by its own wavelet network. The new position object vertex become therefore: $V_N((\omega, t, d)) = (X(\omega_x, t_x, d_x), Y(\omega_y, t_y, d_y), Z(\omega_z, t_z, d_z))^T$. The new represented object can be represented as follows:

$$O_N = \{V_N = (x_N, y_N, z_N) \in \Re^3\} \tag{12}$$

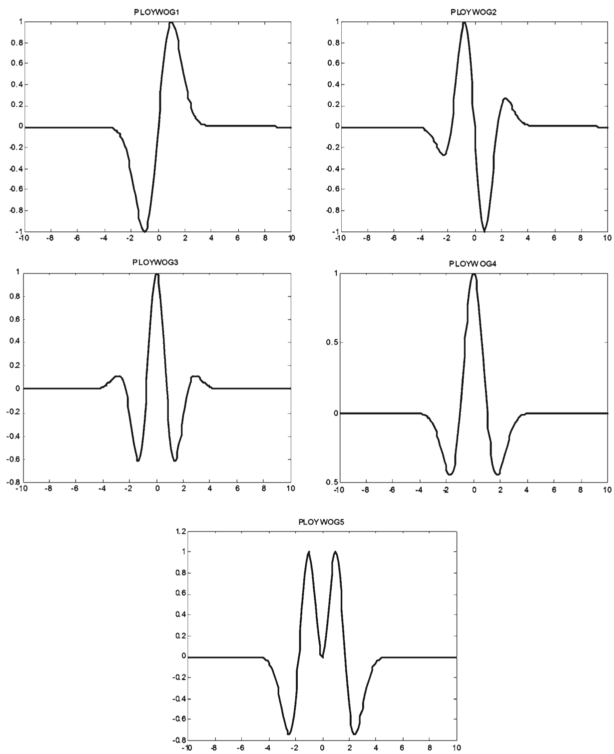

Fig. 2 POLYWOGs wavelets

4 Experimental results

To evaluate the performance with this approach, we used a wavelet network whose library consists of several POLYWOGs wavelet mothers. A POLYnomials WindOwed with Gaussians (POLYWOG) wavelets are defined and illustrated in what follow (Fig. 2) as a mother wavelets and used to construct the MMWN. The approximation object complexity is directly related to the number of selected wavelet and iterations to construct the network. As a performance index, we used the Mean Square Error (MSE) and the Normalized Square Root of the Mean Square Error (NSRMSE) which is defined as:

$$MSE = \frac{1}{N_i} \sum_{k=1}^{N_i} \left(O_N(x_{Nk}, y_{Nk}, z_{Nk}) - O(x_k, y_k, z_k) \right)^2 \tag{13}$$

$$NSRMSE = \sqrt{\frac{\sum_{k=1}^{N} \left(O_N(x_{Nk}, y_{Nk}, z_{Nk}) - O(x_k, y_k, z_k) \right)^2}{\sum_{k=1}^{N} (\overline{O} - O(x_k, y_k, z_k)))^2}} \tag{14}$$

O is the object to be approximated, K is the number of observations and $\overline{O} = \sum_{k=1}^{N} O(x_k, y_k, z_k)$. The results of the 3D representation, issued from the multi-mother wavelet network are based on an improved version of the OLS.

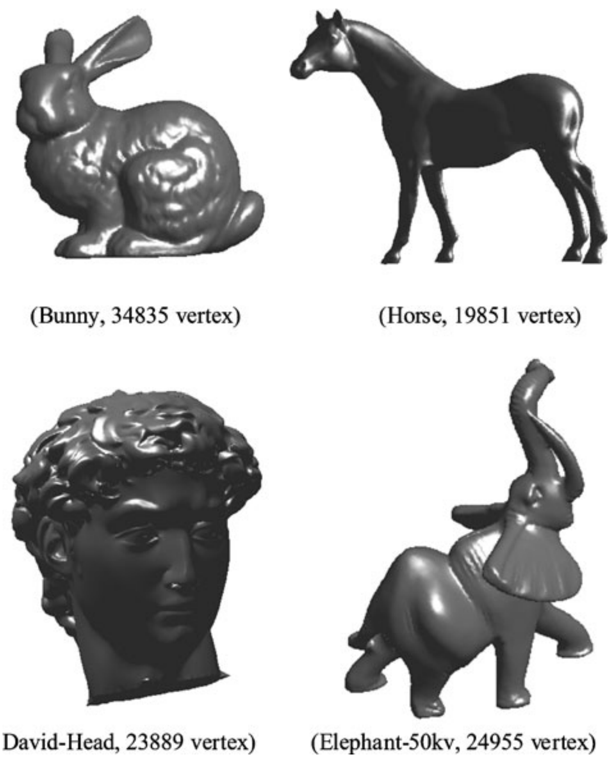

Fig. 3 Original 3D objects representation

4.1 Mother wavelet functions

Numerous useful mother wavelets arise from POLYnomials WindOwed types of functions. All derivatives of a Gaussian function are POLYWOGs and so are admissible mother wavelets. The following simulations will describe the results of the MMWN performance employing several types of POLYWOGs [26] : Polywog1

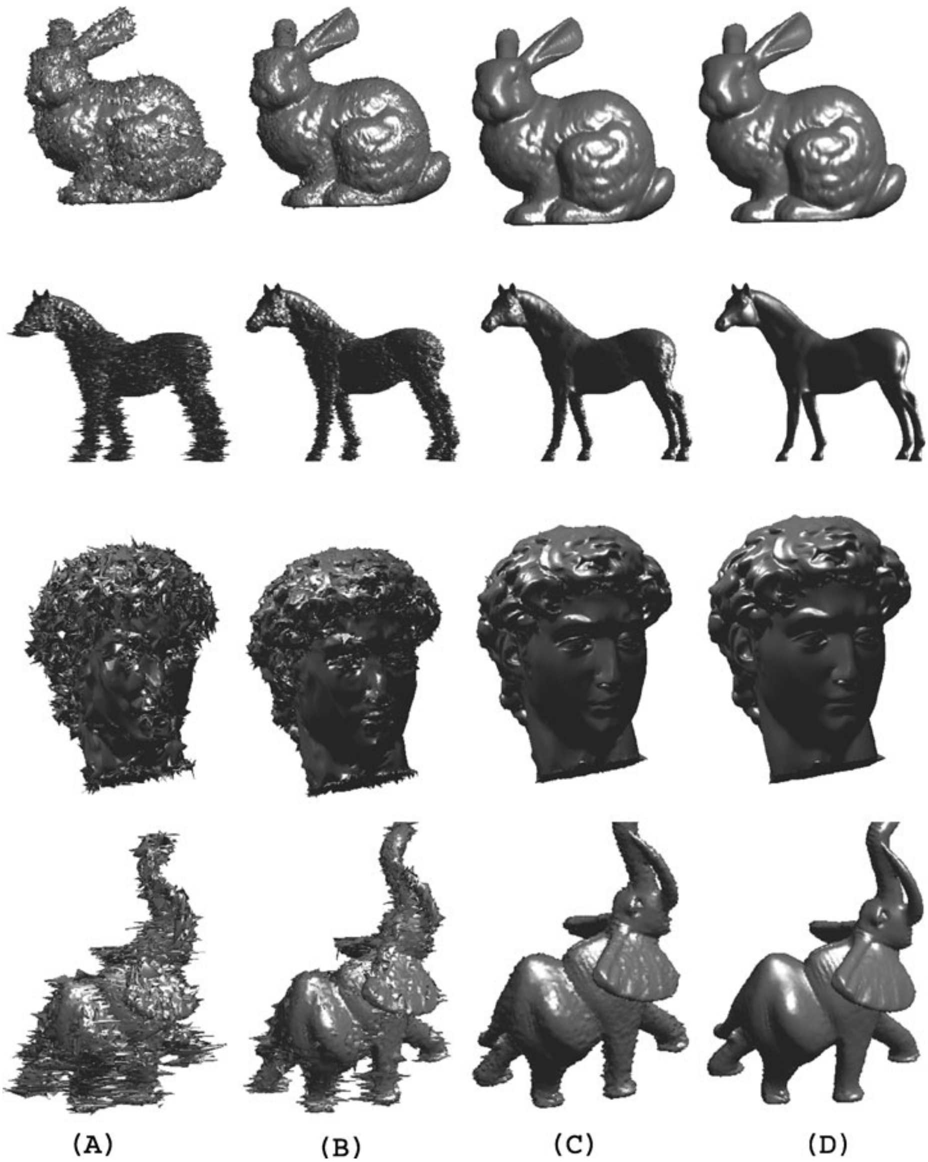

Fig. 4 The original objects and the Results of 3D representation with the following number of POLYWOGs wavelets (from **a** to **d**) : 200, 250, 300 and 350

(15), Polywog2 (16), Polywog3 (17), Polywog4 (18), Polywog5 (19) as super-mother wavelets:

$$\psi_{Polywog1}(x) = x \exp\left(-\frac{x^2}{2}\right) \tag{15}$$

$$\psi_{Polywog2}(x) = \left(x^3 - 3x\right) \exp\left(-\frac{x^2}{2}\right) \tag{16}$$

$$\psi_{Polywog3}(x) = \left(x^4 - 6x^2 + 3\right) \exp\left(-\frac{x^2}{2}\right) \tag{17}$$

$$\psi_{Polywog4}(x) = \left(1 - x^2\right) \exp\left(-\frac{x^2}{2}\right) \tag{18}$$

$$\psi_{Polywog5}(x) = \left(3x^2 - x^4\right) \exp\left(-\frac{x^2}{2}\right) \tag{19}$$

Figure 2 shows curves of the five examples of Several types of POLYWOGs.

4.2 3D objects representation

The 3D object input to the acquisition procedure is provided by VRML files [17]. We use VRML because there is an abundance of VRML models available on Internet. VRML models of common objects are easily located on Internet. This allows us to simulate the network without having to create our own visual representations. So, the vertices and their corresponding normal values of a 3D object model given by the VRML are used to create a training set. The network we use has 5 Mother wavelets. We proved that the multi-mother wavelet network is adequate for approximate complicated objects by the criteria of the error [30]. The only unknown variable is the number of hidden wavelets. There is no known efficient, fail proof way of determining this number for the specific problem. Depending on the complexity of the object 200 to 350 hidden wavelet are used for modeling. For higher accuracy the number of hidden wavelet used for modeling the network should be increased. During the training procedure we observe NSRMSE and MSE, based on its values we determine if the training is succeeding or not and whether we need to increase or decrease the number of hidden wavelet.

Table 1 MSE and NSRMSE for the 3D test objects

Object	200 wavelets	250 wavelets	300 wavelets	350 wavelets	
Bunny	MSE	9.01638e-7	1.48429e-7	1.12097e-8	1.26555e-10
	NSRMSE	1.50221e-2	6.09500e-3	1.67499e-3	1.77973e-4
Horse	MSE	4.75790e-4	9.83144e-5	8.90879e-6	1.05340e-7
	NSRMSE	5.64909e-2	2.56791e-2	7.73002e-3	8.40560e-4
David-Head	MSE	8.43023e-4	1.74466e-4	1.64645e-5	1.94173e-7
	NSRMSE	5.31016e-2	2.41571e-2	7.42101e-3	8.05903e-4
Elephant	MSE	2.91680e-4	5.80565e-5	6.52525e-6	6.73467e-7
	NSRMSE	8.95486e-2	3.99513e-2	1.33938e-2	4.30292e-3

Figure 3 illustrate the 3D representation of several objects (Bunny: Consisting of 34,835 vertex, Horse: Consisting of 19,851 vertex, David-Head: Consisting of 23,889 vertex and Elephant: Consisting of 24,995), obtained by the MMWN.

In this simulation results each subset input pattern contains 500 points (Fig. 4). Table 1 gives the the mean square error and final normalized square root of the mean

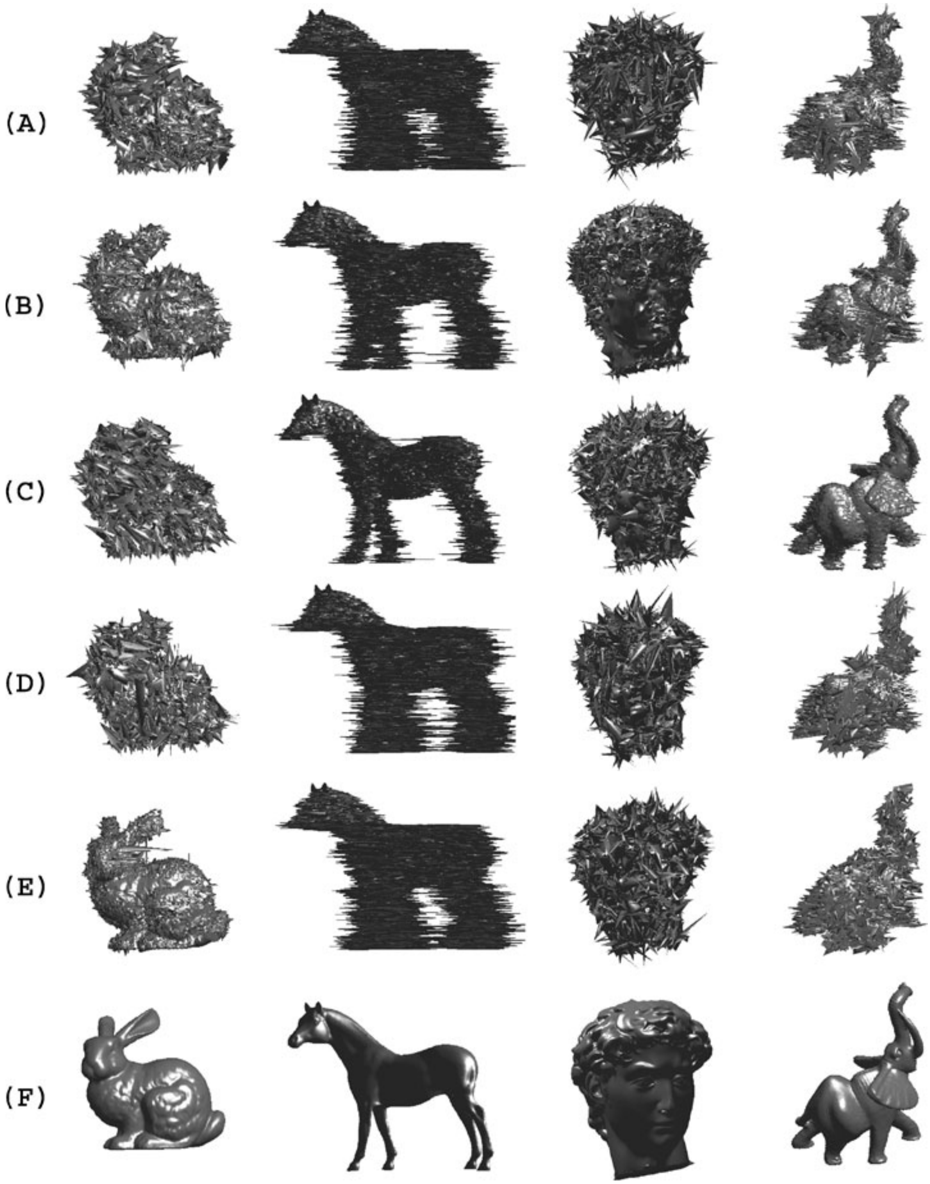

Fig. 5 The Results of 3D modeling with the Polywogs wavelets in the hidden layer (From **a** to **e**: Polywog1 to Polywog5, **f** Multi-Wavelet Polywogs)

Table 2 MSE for different Polywog wavelets using wavelet network constructed with 350 wavelets

Object	Bunny	Horse	David-Head	Elephant
Polywog1	2.34654e-5	7.39727e-3	1.37243e-2	5.50337e-4
Polywog2	5.80666e-6	1.92425e-3	1.06145e-3	1.79214e-4
Polywog3	4.09144e-5	6.50837e-4	3.68749e-3	4.76304e-5
Polywog4	2.14729e-5	5.44032e-3	1.11955e-2	4.86579e-4
Polywog5	1.47262e-6	7.90883e-3	1.47012e-2	7.14737e-4
Multi-Wavelet Polywogs	1.26555e-10	1.05340e-7	1.94173e-7	6.73467e-7

square error after 100 training iterations using MMWN constructed with 200,250,300 and 350 wavelets in hidden layer and based on Polywogs wavelets. For example to model the Bunny object, which is composed of 34,835 vertex, using a wavelet network composed of 350 wavelets we obtained a MSE of 1.26555e-10 and a NSRMSE of 1.77973 e-4. From this table we see clearly that increasing wavelet number in hidden layer increase the approximation capacity. But increasing the wavelet number increases time cost and algorithm complexity. Also to perform these criterions(MSE and NSRMSE) we can increase the iterations number in the training stage, but in the same way as increasing wavelet number, time cost and algorithm complexity increase considerably. To compare the performances of the method we proceed by using the multi-mother wavelet network (MMWN), using a library composed of five mother wavelets (from Polywog1 to Polywog5) against the classical wavelet network (CWN), using a library composed of only mother wavelet, we intend to modeling the four objects using CWN in the same condition as the example of modeling using MMWN with 350 wavelet in hidden layer. For the modeling, we use the training procedure, as presented in Section 3.3.2, that is based on an improved version of the OLS. The selection of wavelets in the Polywogs Family (from Polywog1 to Polywog5) to use to construct the network is automatically determined. The procedure permit to specify the wavelets selected for every mother wavelet. For the classical wavelet network we apply the OLS procedure.

Figure 5 represent the result of training after 100 iterations using 350 in hidden layer for the 3D modeling of the four objects (Bunny, Horse, David-Head and Elephant), obtained by the CWN and MMWN. The error of each modeled object obtained by the training procedure is shown in Tables 2 and 3. When comparing the below figures which are obtained with MMWN and the other one which is obtained with CWN, we can say that the performances obtained when using MMWN approach are always better than the one obtained with CWN. Besides, we can notice, that we cannot achieve the exact and clear form of the object to be model.

Table 3 NSRMSE for different Polywog wavelets using wavelet network constructed with 350 wavelets

Object	Bunny	Horse	David-Head	Elephant
Polywog1	7.66351e-2	2.22744e-1	2.14257e-1	1.23004e-1
Polywog2	3.81221e-2	1.13606e-1	5.95852e-2	7.01926e-2
Polywog3	1.01193e-1	6.60704e-2	1.11059e-1	3.61866e-2
Polywog4	7.33094e-2	1.91022e-1	1.93513e-1	1.15660e-1
Polywog5	1.91981e-2	2.30318e-1	2.21750e-1	1.40178e-1
Multi-Wavelet Polywogs	1.77973e-4	8.40560e-4	8.05903e-4	4.30292e-3

Tables 2 and 3 represents the MSE and NSRMSE of representation of the four considered objects, using classical wavelets network constructed with 350 wavelets in hidden layer and based on Polywog1, Polywog2, Polywog3, Polywog4, Polywog5 and multi-mother wavelet network constructed with Multi-wavelet Polywogs(from Polywog1 to Polywog5). Considering the results mentioned above, we can note that the MSE (and NSRMSE) reaches very weak values of 1.26555e-10 (and 1.77973e-4) for the Bunny object, using multi-mother wavelet network constructed with 5 Polywogs, against an MSE (and NSRMSE) equal to 1.47262e-6 (and 1.91981e-2) with classic wavelet network based on the best mother wavelet which is Polywog5. For Horse objects, we have a MSE of 1.05340e-7 and a NSRMSE of 8.40560e-4 for a MMWN over a MSE 6.50837e-4 and NSRMSE of 6.60704e-2 as the best value for CWN which is obtained with Polywog3. The MSE (and NSRMSE) is equal to 1.94173e-7 (and 8.05903e-4) for the proposed wavelet network structure, for the David-Head object, over 1.06145e-3 (and 5.95852e-2) which is the better error of CWN obtained with Polywog2. Finally, for the Elephant object, we have a MSE of 6.73467e-7 (NSRMSE of 4.30292e-3), for a multi-mother wavelet network, over a MSE of 4.76304e-5 (NSRMSE of 3.61866e-2). Therefore the MSE here with the MMWN network are weaker than those obtained by the classical algorithm. In the case of the classical wavelet network, we can note that the better wavelet term doesn't exist, the better wavelet depends of the object to model. For example, to model the Bunny object, the best mother wavelet is Polywog5. For Horse objects, the best modeling obtained with Polywog3. With David-Head object, the better error of CWN obtained with Polywog2.

To solve the problem of best wavelet selection that well represent the object, and to enhance the modeling quality, we used several wavelets (Multi-Wavelet Polywogs) to construct the wavelet network. So, when comparing results given by Tables 2 and 3, we can say that the performances obtained in term of NRMSE and MSE using the MMWN structure are always very better than the one obtained with CWNN. This shows that the proposed procedure brings effectively a better capacity of modeling using the Polywogs wavelet network.

5 Conclusions

A 3D object representation based on MMWN is presented. This structure which consists to construct the network by several mother wavelets has the advantage to solve high dimensions problem using the best mother wavelet that model the signal the better. For the validation of the 3D representation using multi-mother wavelet network approach (MMWN), we have presented a comparison between the classical wavelet network (CWN) and MMWN approach in the domain of 3D object modeling. Many examples permitted to compare the capacity of representation using MMWN and CWNN. We deduce from these examples that the quality of reconstruction depends a lot on the choice of the activation function (wavelet) used in hidden layer and on their localization. For that we have used a Polywogs wavelets family(Multi-Wavelet Polywogs) that we can see that they are more superior then the classical one in term of modeling and to achieve the performance and to improve the quality of representation. Based on these experiment results, we can say that 3D representation using wavelet network is not always possible, when using the classical wavelet network that cannot achieve the exact and clear form of the object to model, but it offers certain advantages in solving some of the major problems of computer graphics representation, when using the proposed multi-mother wavelet network that achieves a good precision of modeling in terms of error. The obvious application of the suggested method is 3D object modeling using MMWN. The result shows that the proposed procedure brings effectively a better capacity of modeling using the several mother wavelets. Other future applications include: 3D reconstruction from 2D image, compression of 3D object, and finally generation of 3D data for computer aided design based on MMWN.

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