

# Dynamic Spectrum Access Algorithm Based on Game Theory in Cognitive Radio Networks

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**Abstract** As an enabling technology for dynamic spectrum access (DSA), cognitive radio (CR) is widely regarded as one of the most promising technologies for future the fifth generation (5G) wireless communications. Although there have been significant prior researches to combat interference on primary users (PUs), the problem of mitigating mutual interference between secondary users (SUs), -which is tightly coupled with SU's spectrum leasing- is still not understood well. This paper proposes a DSA algorithm based on game theory, which jointly performs spectrum leasing and interference mitigation among SUs. The problem is modeled as an oligopolistic competition using Stackelberg model. We have carefully studied the SU's spectrum utilization behavior with respect to various criteria of the proposed game theoretic model. Simulation results shows that, Compared with Cournot game model, the proposed scheme enables SUs to efficiently utilize the licensed spectrum shared with PUs in a dynamic environment while maximizing the spectrum utilization.

**Keywords** Cognitive radio · Dynamic spectrum access · Game theory · Nash equilibrium

## 1 Introduction

With the rapid development of wireless communication technology and fast growth for spectrum demand, spectrum resources become increasingly scarce and one of the bottlenecks, restricting the development of wireless communications [1, 2]. Prior studies show that the fixed spectrum allocation policies result in poor spectrum utilization around 15 % [3, 4]. There are two main reasons for the poor spectrum utilization: i) Spectrum is a limited resource and fixed spectrum policies will naturally lead to scarce spectrum. ii) Licensed spectrum is used very sporadically [1, 3, 4]. Realizing the poor spectrum efficiency of conventional fixed spectrum allocation policies, cognitive radio (CR) is proposed to improve the spectrum utilization [5–7], which alleviates the contradiction between the lack of spectrum resources and the growing demand for wireless access by enabling secondary users (SUs) to intelligently search and utilize the unused spectrum holes in licensed spectrum and thereby increasing the spectrum efficiency [8–10]. CR is also one of the dominant candidate technologies for the fifth generation (5G) wireless communication technologies [11, 12]. 5G systems are envisioned to integrate conventional licensed cellular radio and intelligent dynamic radio into a holistic system, realizing the CR concepts [9, 13, 14]. Unlike traditional radios, CR may change its objectives as radio scenarios vary, which provides a feasible scheme for SUs efficiently using the licensed spectrum in dynamic environment [10, 15].

The benefit of CR will depend on the dynamic spectrum access (DSA) schemes, which can provide a feasible solution for SUs to use the licensed spectrum in a dynamic

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environment efficiently. This requires SUs to detect spectrum holes across three domains; time, space, and frequency [16, 17]. Achieving DSA in cognitive radio networks (CRNs) requires addressing various issues in spectrum access behavior, spectrum sharing mode, and spectrum access mode, which has been well studied in [18–20]. Most of the prior works are devoted to performance analysis of DSA in CRNs [21]; game theory, auction, graph coloring, and mathematical programming are employed to optimize the performance of underlying spectrum access strategy [22, 23]. The urge to adapt to the dynamic environments and challenges associated with providing network-wide information makes centralized approaches less attractive [22]. That calls for tractable approaches to accommodate for distributed nature of CRNs. Game theory has been recognized as an interesting tool to model and analyze the cognitive behaviors in CRNs. Specifically, the distributed nature and both cooperative and non-cooperative DSA behaviors can well be captured by game theory [24, 25].

In order to assess spectrum access in CR environments, one-shot, non-cooperative games based on continuous and discrete reformulations of the Cournot game model was explored in [26], which reveals equilibrium situations that may be reached in simultaneous access scenarios of two and three users. A collision-constrained network selection (CCNS) method was proposed based on Markov queuing mode in [27], which maximizes allowable arrival rate of SUs subject to a target collision probability for the primary users (PUs). In the work, SUs select the shortest time delay to access network through the analysis of the SUs waiting transmission delay. In [28], a cooperative asynchronous multi-channel spectrum access MAC protocol was proposed, which uses a single transceiver and is fully asynchronous. However, it does not consider control frame constraints. In [23], simulation study based on M/G/1 queuing model was explored to evaluate the performance of PUs without closed attention to specifics of primary users and licensed channels. An intervention framework was proposed in [29], which protects the spectrum owner's interests and enforces market regulations. In the framework, secondary networks offer offload services to a primary network (PN) in exchange for temporary access to the PN's spectrum. To allocate unused frequency bands efficiently, two non-cooperative games named interference minimization game and capacity maximization game were proposed [30], which reflect the target of data radios and voice radios, respectively. The problem of spectrum pricing in a CRN is explored in [31], which formulates it as an oligopoly market consisting of a few firms and a consumer. A market equilibrium was explored in multichannel sharing CRNs [32]: it is assumed that every subchannel is orthogonally licensed to a single primary user (PU), and can be shared with multiple secondary users (SUs). A distributed and coordinated radio resource allocation algorithm was proposed for orthogonal frequency division multiple access (OFDMA)-based cellular networks to self-organize

efficient and stable frequency reuse patterns [33], which can mitigate interference in self-organizing LTE networks. A simple self-organization rule was proposed based on minimizing cell transmit power [34], following which a distributed cellular network is able to converge into an efficient resource reuse pattern, which can mitigate interference in OFDMA femtocell networks. A market-clearing model was proposed for spectrum trade in CRNs [35]. A general soft decision SMSE (SDSMSE) framework was proposed [36], which extends the original SMSE framework to achieve synergistic CR benefits of overlay and underlay techniques.

The prior work mainly considers the SUs interference to the PUs and not the mutual interference among SUs. SUs could potentially interfere with each other in attempt to simultaneously access the channel in CRNs. In this work, we propose a spectrum access algorithm based on game theory, which overcomes the problem of mutual interference among SUs. We have modeled the spectrum access problem as an oligopolistic competition model in game theory. We have performed an intensive analysis to characterize the SUs' spectrum utilization behavior with respect to various criteria of the proposed game theoretic model. In particular we have carefully studied the existing of Nash equilibrium, the game process, the convergence of SUs, the total spectrum amount rented by SUs, and SU's revenue.

The remainder of this paper is organized as follows. Section 2 illustrates the proposed DSA algorithm. In Section 3, our intensive simulation study validates the efficiency of the proposed scheme. Section 4 concludes the paper.

## 2 The proposed DSA algorithm

With intelligent sensing to obtain available spectrum information, CRNs can monitor spectrum environment and adjust system parameters effectively. In this section, a general model of DSA is presented based on game theory in CRNs; then the system model of SUs dynamic access is explored. The DSA algorithm based game theory is analyzed and secondary user's utility function is given. And the Nash equilibrium solution of the proposed algorithm is deduced by maximizing the utility of SUs, and obtaining spectrum bandwidth rented by all SUs. And the notations of the proposed algorithm are shown in Table 1.

### 2.1 System model

The DSA system in CRNs consists of three parts: PUs, SUs, and spectrum pool. In this paper, we only consider the game process among SUs. And the system model is shown in Fig. 1.

Considering there are  $N$  game players, the strategic space of the players is  $S_1, \dots, S_N$ , the utility function is  $\mu_1, \dots, \mu_N$ . Therefore, a general model of game theory can be expressed as:

**Table 1** Notations

Notation	Description
$N$	Number of game participants
$S_i$	Strategy set that game player $i$
$\mu_i$	Utility function for game player $i$
$s_i$	Particular strategy
$u_s(b_i)$	Utility function of $SU_i$ with bandwidth $b_i$
$u_p(B)$	Total utility function of PUs
$C(B)$	Rental costs of PUs
$k_i^{(s)}$	Transmission efficiency
$\nu$	Spectrum alternative factor
$\gamma$	Signal-to-noise ratio (SNR)
$\theta_{BER}$	Threshold of the bit error rate
$b_i$	Spectrum bandwidth of $SU_i$ share
$p$	Spectrum price that $SUs$ pay to PU
$x, y, \tau$	Non-negative constant

$$G = \{N, S_1, \dots, S_N; \mu_1, \dots, \mu_N\}, \tag{1}$$

where the strategic space for each game player  $i$  is  $S_i$ ,  $s_i$  is strategy and  $s_i \in S_i$ ,  $\{s_1, \dots, s_N\}$  is the strategy combination for all players. The utility function of each game player  $i$  is  $\mu_i$ , and  $\mu_i(S)$  represents the utility of game player  $i$  choosing strategic component  $\{s_1, \dots, s_N\}$ .

The general game model satisfies three basic elements: 1) the number  $N$  of game players; 2) strategy space  $S_i$  of each player  $i$ ; 3) the strategic combination  $\{s_1, \dots, s_N\}$ , and the utility  $\mu_i(S)$  of each game player  $i$ .

PUs include a base station and some users. Spectrum pool will accomplish centralized distribution of available spectrum within a specific area or location, adjust the process of spectrum sharing between communication devices, and monitor

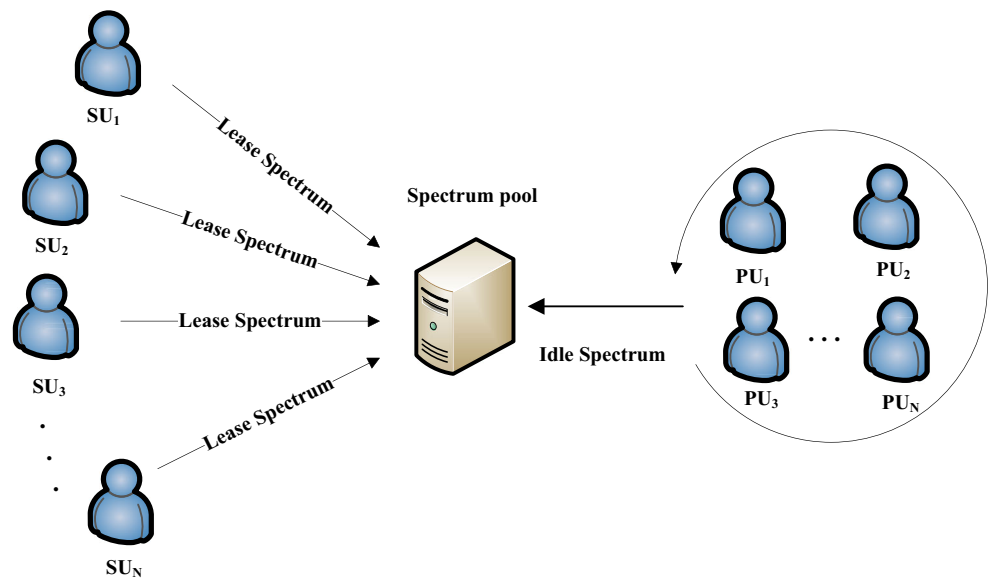
the server of the relevant spectrum. Spectrum rented by SUs can be used for communication between SU and PU, SU and access point, and can also be used for ad hoc network of SUs. The system workflow is shown in Fig. 2.

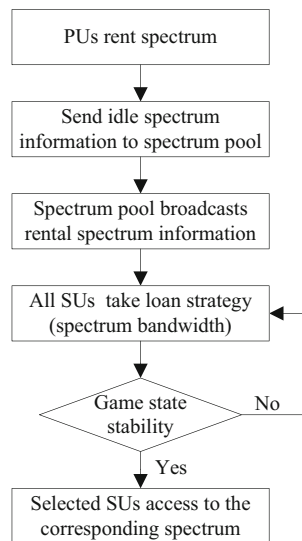
The system works as follows:

- 1) In order to rent spectrum, PUs establishes the connection to spectrum pool and send their request information including idle spectrum and the utility function.
- 2) Spectrum pool will broadcast rental spectrum request information and rules to all SUs with the same price, and the price will change along with the market supply and demand.
- 3) Different SUs rent a certain spectrum bandwidth, and observe the market supply and demand, then adjust their rental spectrum bandwidth to achieve maximum benefit.
- 4) Spectrum pool determines whether game among the SUs reaches a steady state, namely whether the amount of rental spectrum bandwidth on the market and spectrum bandwidth that all SUs rent are stable. If the game does not reach steady state, all SUs continue to wait for the end of the game. If the steady state is reached, then the game will stop and the selected SU rents corresponding spectrum.
- 5) The SUs establish communication connection to the PUs, and then decide to loan spectrum to the PUs, in accordance with spectrum price of the market stability when the game among the SUs reaches a steady state.
- 6) The SU uses the spectrum of PUs for packets transmission, and then will return spectrum to the PUs after task ending.

In 4), spectrum pool judges whether the game reaches steady state by observing whether game results are the same three times in a row. If game results of three times are the

**Fig. 1** System model





**Fig. 2** Workflow of the system

same, it shows that game is in stable equilibrium, and all SUs do not need to make decisions to carry out the game.

Spectrum access in CRNs is related to the selected strategies of SUs in game. Spectrum access game includes game process among SUs, game process among PUs and the united game process between PUs and SUs. In the game among SUs, the participants are SUs, and action strategy is selected to loan spectrum bandwidth. The decision-making process is that SUs select corresponding spectrum bandwidth for data transmission. While in the game among PUs, participants are the PUs with licensed spectrum, and action strategy is to select the number of rental spectrum. The decision-making process is to determine the number of PUs selecting spectrum on loan to SUs. For the united game process between PUs and SUs, the participants are PUs and SUs, where action strategy of the PUs is to select the number of rental spectrum, and the action strategy of the SUs is to select loan spectrum bandwidth. The decision-making process of the PUs is to determine the number of rental spectrum, and the decision-making process of the SU is to decide leasing spectrum bandwidth.

In order to depict the spectrum competition accurately, the DSA problem is modeled as an oligopolistic competition using Stackelberg model, which aims to improve spectrum bandwidth that the SUs loan, while improving the spectrum utilization of the PUs. When game is stable, the utility of SUs reaches the maximum, thus the enthusiasm of leased spectrum is highest. And then SUs will communicate with other ends by the leased spectrum, which improves the spectrum utilization of idle licensed spectrum efficiently.

## 2.2 Stackelberg game algorithm

In the Stackelberg game algorithm, all SUs rent the same quality spectrum, and the same price fluctuates with the

market demand. The behavior of SU is selfish and uncooperative, and it carries out the game among the SUs according to the loaned spectrum bandwidth. Due to some factors, such as the different network delay between each SU and spectrum pool server, all SUs do not take loan strategy simultaneously. According to Stackelberg game model, all participants will be separated into two groups. And a group of SUs take the strategy firstly, which will be the leaders in the game. And then the other group of SUs will take suitable game strategy according to the prior SU's strategy, which will be the followers. According to the historical strategy, SUs will optimize their behaviors and select the most suitable strategy to gain more benefits. After some rounds of repeated game, the spectrum bandwidth leased by SUs will reach Nash equilibrium.

The goal of the game algorithm is to maximize the spectrum bandwidth leased by SUs, while the goal of the proposed solution is to maximize the utility of the SUs. Hence, the following analysis will consider the utility of SUs.

We can get the utility function  $u_s(b_i)$  of  $SU_i$  with the benefit of bandwidth  $b_i$  for data transmission:

$$u_s(b_i) = k_i^{(s)} b_i - \frac{1}{2} \left( N b_i^2 + 2\nu b_i \sum_{j \neq i}^N b_j \right) - p b_i, \quad (2)$$

where  $\nu$  is spectrum alternative factor, which means the size of the bandwidth requested by  $SU_i$  is affected by other SUs. When  $\nu = 0$ ,  $SU_i$  is not affected by other SUs; when  $\nu = 1$ , other SUs have maximum influence on  $SU_i$ . We only consider the utility of the SUs when  $\nu = 0$ , and therefore according to Eq. (2), we can obtain the utility function  $u_s(b_i)$  of the SUs:

$$u_s(b_i) = k_i^{(s)} b_i - \frac{1}{2} N b_i^2 - p b_i. \quad (3)$$

To maximize the utility of the PUs, the utility of the PUs can be set as a function of bandwidth price, therefore, the total utility function  $u_p(B)$  of the PUs is:

$$u_p(B) = \sum_{i=1}^N p b_i - \sum_{i=1}^N C(B), \quad (4)$$

where  $p$  is the spectrum price that SUs pay to the PUs,  $b_i$  is the spectrum bandwidth that the SUs loan, and  $C(B)$  is rental costs of the PUs. Rental costs are defined as follows:

$$C(B) = x + y \left( \sum_{i=1}^N b_i \right)^\tau, \quad (5)$$

where  $x, y, \tau$  are non-negative constant,  $\tau \geq 1$  (ensure the pricing function is a convex function).

In order to obtain  $k_i^{(s)}$ , the SUs adopt adaptive modulation technology, and the transmission rate can dynamically adjust according to the channel quality. By adopting multiple

quadrature amplitude modulations (MQAM), bit error rate (BER) single-input single-output Gaussian noise channel can be approximated expressed as:

$$BER \approx 0.2 \exp\left(-1.5\gamma / (2^k - 1)\right), \tag{6}$$

where  $\gamma$  is receiver signal-to-noise ratio,  $k$  is the transmission efficiency, namely  $k_i^{(s)}$  in expression (2).  $\theta_{BER}$  is the threshold of BER, and then the transmission efficiency can be formulated as:

$$k = \log_2(1 + K\gamma), \tag{7}$$

where  $K$  is:

$$K = \frac{1.5}{\ln(0.2 / \theta_{BER})}. \tag{8}$$

According to Eqs. (6), (7) and (8), we can obtain:

$$k_i^{(s)} = \log_2\left(1 + \frac{1.5}{\ln(0.2 / \theta_{BER})} \gamma\right). \tag{9}$$

The deduction of the utility function of PUs is with respect to spectrum bandwidth, which can maximize utility. At the same time, we can get the relationship between spectrum price and spectrum bandwidth:

$$u_p(B) = \sum_{i=1}^N p b_i - \sum_{i=1}^N C(B), \tag{10}$$

$$\frac{\partial u_p(b)}{\partial b_i} = p - \tau y \left(\sum_{i=1}^N b_i\right)^{\tau-1} = 0, \tag{11}$$

$$p = \tau y \left(\sum_{i=1}^N b_i\right)^{\tau-1}, \tag{12}$$

according to the expressions (10), (11) and (12), we can get the utility function of the SUs as:

$$u_s(b_i) = \log_2\left(1 + \frac{1.5}{\ln(0.2 / \theta_{BER})} \gamma\right) b_i - \frac{1}{2} N b_i^2 - \tau y \left(\sum_{i=1}^N b_i\right)^{\tau-1} b_i \tag{13}$$

### 2.3 Algorithm analysis

In the proposed algorithm, we assume that the  $SU_i$  can obtain its utility function  $u_s(b_i)$  with information of spectrum pool. All SUs do not take loan strategy simultaneously. But a group

of SUs take the strategy firstly. And then another group of the SUs will take corresponding suitable strategy according to the previous SUs' strategy. Based on the historical strategies, the SUs will select the optimized strategy to guide their game behaviors. With some round of repeated games, the spectrum bandwidth leased by the SUs will reach Nash equilibrium. The flow chart of Stackelberg game algorithm is shown in Fig. 3. And DSA algorithm based on Stackelberg game model is shown in Algorithm 1.

#### Algorithm 1: DSA algorithm

**Input:**  $N, \theta_{BER}, \gamma, x, y, \tau$

**Output:**  $b_i, u_s(b_i)$

1. Calculate  $u_s(b_i)$  according to (13) with the initial value.
2. Calculate spectrum bandwidth  $b_i$  with (14) when utility function of  $SU_{i+1}$  obtain the maximum value.
3. According to the formula (15), obtain strategy summation value  $\sum_{i=1}^n b_i$ .
4. Calculate  $u_s(B_i)$  of the prior  $SU_i$  according to (13) with the initial value.
5. Substitute (16) into (17), calculate  $B_i$  when utility function of the prior  $SU_i$  obtain the maximum value.
6. According to (19), calculate summation value  $\sum_{i=1}^m B_i$ .
7. Substitute (19) into (16), calculate strategy values of  $n$  following  $SUs \sum_{i=1}^n b_i$ .

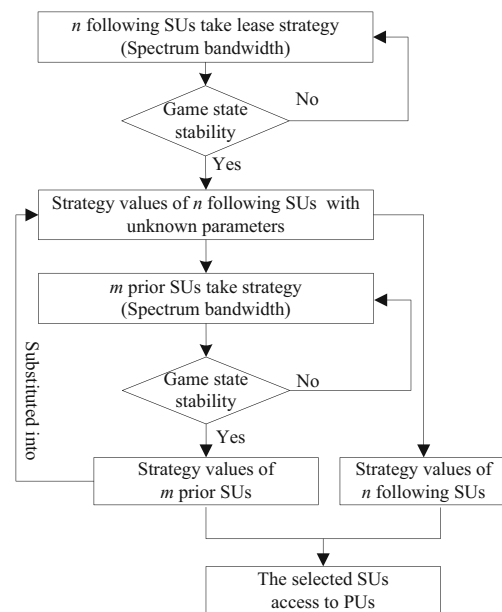


Fig. 3 Flow chart of Stackelberg game algorithm

The proposed algorithm works as follows:

Step 1: With the initial value of  $\theta_{BER}$ ,  $\gamma$ ,  $\tau$  and  $y$ , for example,  $\theta_{BER}=10^{-4}$ ,  $\gamma=15.4\text{dB}$ ,  $\tau=2$ ,  $y=1/2$ , we can calculate the utility function  $u_s(b_i)$  of  $SU_i$  according to expression (13), in the case the strategy value of the prior  $SU_i$  is  $B_i$ , and the summation is  $\sum_{i=1}^m B_i$ , where  $m$  is the number of the prior SUs,  $n$  is the number of the following SUs. Then we can get the utility function of the following  $SU_i$  is:

$$u_s(b_i) = \left(2 - \sum_{i=1}^m B_i - \sum_{j \neq i}^n b_j - b_i\right) b_i - \frac{1}{2}(n+m)b_i^2. \tag{14}$$

Step 2: Equation (14) is a quadratic function, and solve the value of  $b_i$  when the function obtain the maximum value:

$$b_i = \frac{1}{2+n+m} \left(2 - \sum_{i=1}^m B_i - \sum_{j \neq i}^n b_j\right). \tag{15}$$

Step 3: According to the Eq. (15), we obtain strategy values of  $n$  following SUs:

$$\sum_{i=1}^n b_i = \frac{n \left(2 - \sum_{i=1}^m B_i\right)}{1+2n+m}. \tag{16}$$

Step 4: Then the utility function of the prior  $SU_i$  is:

$$u_s(B_i) = \left(2 - \sum_{i=1}^n b_i - \sum_{j \neq i}^m B_j - B_i\right) B_i - \frac{1}{2}(n+m)B_i^2. \tag{17}$$

Step 5: Substitute (16) into the Eq. (17), and then we can calculate spectrum bandwidth  $B_i$  when utility function of the prior  $SU_i$  obtain the maximum value.

$$B_i = \frac{1+n+m}{2n^2+m^2+3nm+3n+3m+2} \left(2 - \sum_{j \neq i}^m B_j\right). \tag{18}$$

Step 6: According to the Eq. (18), we obtain strategy values of  $m$  prior SUs.

$$\sum_{i=1}^m B_i = \frac{2m(1+n+m)}{2n^2+2m^2+4mn+2n+3m+1}. \tag{19}$$

Step 7: Substitute expression (19) into the Eq. (16), then we can get strategy values of  $n$  following SUs as:

$$\sum_{i=1}^n b_i = \frac{n(4n^2+2m^2+6mn+4n+4m+2)}{(1+2n+m)(2n^2+2m^2+4mn+2n+3m+1)}. \tag{20}$$

**Lemma 1** For the proposed DSA algorithm based on Stackelberg model, there are  $N$  game players, the strategic space of the players is  $S_1, \dots, S_N$ , the utility function is  $\mu_1, \dots, \mu_N$ , the Nash equilibrium is existence.

**Proof** According to the game model defined in (1), in the proposed DSA algorithm, there are  $N$  SUs, which indicates the set of participants is limited.

And the strategic space for each game player  $i$  is  $S_i$ ,  $s_i$  is strategy and  $s_i \in S_i$ ,  $\{s_1, \dots, s_N\}$  is the strategy combination for all players. The strategic set is closed and bounded convex set.

According to the description of the utility function  $u_s(b_i)$  of  $SU_i$  with the benefit of bandwidth  $b_i$  as shown in expression (2) and (13), we can see that the utility function  $u_s(b_i)$  in action space is continuous, quasi-concave function.

The analysis mentioned above shows that game process of the proposed scheme satisfies the conditions of the existence of Nash equilibrium.

This completes the proof.

### 3 Performance analysis

In order to evaluate the performance of the proposed scheme, the DSA algorithm is implemented in Matlab simulation platform. In the simulation, all SUs are with same capacity. In order to analyze competition among SUs, the simulation considers the spectrum access times during the user choice, mutual interference between users and the scale of secondary CRNs. And the performance of the proposed scheme will be evaluated in the terms of Nash equilibrium, convergence of the game process, and the total amount of leased spectrum of SUs.

#### 3.1 Effectiveness of the proposed algorithm

In this scenario, we consider there are two SUs, namely  $SU_1$  and  $SU_2$ . The leased bandwidth of two SUs are  $b_1$  and  $b_2$  respectively, and the values of  $b_1$  and  $b_2$  are initialized stochastically. The response function curves of two SUs are shown in Fig. 4, respectively.

According to the expression (15), the values of  $b_1$  and  $b_2$  are in the range  $[0, 2]$ . Assume that  $b_1$  increases 0.1 by step, the value of  $b_2$  gradually decreases until 0 with the increasing of  $b_1$ . On the contrary, if  $b_2$  increases 0.1 by step, the value of  $b_1$  gradually decreases until 0 with the increasing of  $b_2$ , and

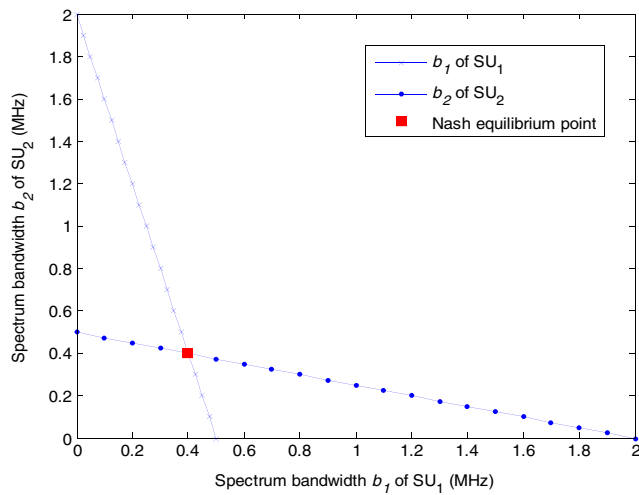


Fig. 4 Nash equilibrium

the intersection of two curves is the Nash equilibrium point, where both  $b_1$  and  $b_2$  are 0.4. The results show that the proposed DSA is convergent and will reach the steady state.

In the second scenario, we will focus on the detailed game process of SUs and the convergence of the proposed game algorithm with varying number of SUs. And the results are shown in Figs. 5, 6 and 7, with 4, 6 and 8 SUs respectively. When the game reaches steady state, the strategy values of SUs converge to their Nash equilibrium points respectively.

As shown in Fig. 5, the spectrum bandwidth leased by SUs will vary with the game process of 4 SUs competing idle spectrum of PUs as game times increasing. In initialization, we set the initial strategy values of the prior  $SU_1$  and  $SU_2$  as 0.125 MHz and 0.75 MHz respectively, and the initial strategy values of the following  $SU_1$  and  $SU_2$  are 0.08 MHz and 0.5 MHz respectively. Since the initial strategy values of all SUs are not identical and not equal to the Nash equilibrium point, each SU will adjust its strategy. As game times increasing, the strategy values of 4 SUs gradually approach to the

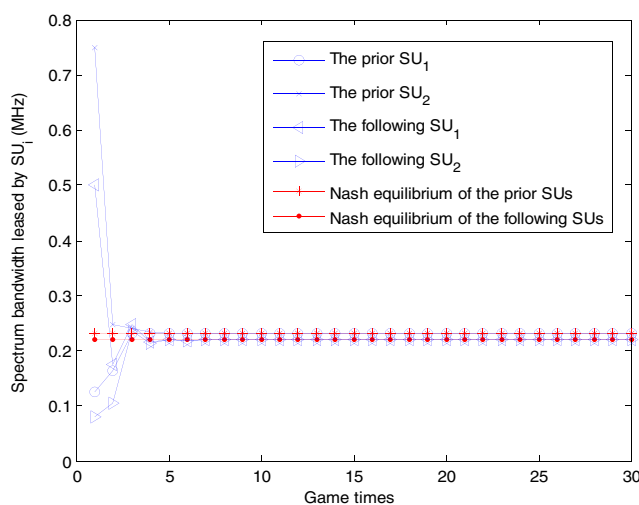


Fig. 5 Game process of 4 SUs

Nash equilibrium point in the game process. And after 5 games, they reach the steady state and converge to their Nash equilibrium points respectively. From Fig. 5, we can see the Nash equilibrium point of the prior SUs is always greater than that of the following SUs, which demonstrates the first-mover advantage in Stackelberg game algorithm.

From Fig. 6, we can see the game results with 6 SUs. In this case, we set that the initial strategy values of the prior  $SU_1$ ,  $SU_2$  and  $SU_3$  as 0.1 MHz, 0.4 MHz and 0.8 MHz, and the initial strategy values of following  $SU_1$ ,  $SU_2$  and  $SU_3$  are 0.05 MHz, 0.25 MHz and 0.6 MHz respectively. In order to obtain more spectrums, all SUs will adjust their game strategies to achieve more benefit. During the repeated games, the strategy values of 6 SUs fluctuate obviously and come to approach to the Nash equilibrium point gradually. After 6 rounds game, they reach the steady state and converge to their Nash equilibrium points. As shown in Fig. 6, Nash equilibrium point of the prior SUs is always greater than that of the following SUs. However, the value of the former is slightly higher than that of the latter.

As shown in Fig. 7, we can see the similar results. And the initial strategy values of the prior  $SU_1$ ,  $SU_2$ ,  $SU_3$  and  $SU_4$  are 0.1 MHz, 0.2 MHz, 0.4 MHz and 0.8 MHz respectively. And the initial strategy values of the following  $SU_1$ ,  $SU_2$ ,  $SU_3$  and  $SU_4$  are 0.05 MHz, 0.3 MHz, 0.5 MHz and 0.75 MHz respectively. Each SU will adjust its strategy to get more bandwidth. With the game times increasing, the strategy values of 8 SUs oscillate obviously in the game process. After 7 games, all SUs get to the steady state and converge to their Nash equilibrium points. However, the difference of Nash equilibrium points between the prior SUs and the following SUs is very slight.

From Figs. 5, 6 and 7, we can see that, with limited idle spectrum limited of PUs, if the number of game participants is small, the strategy value (spectrum bandwidth) of the prior

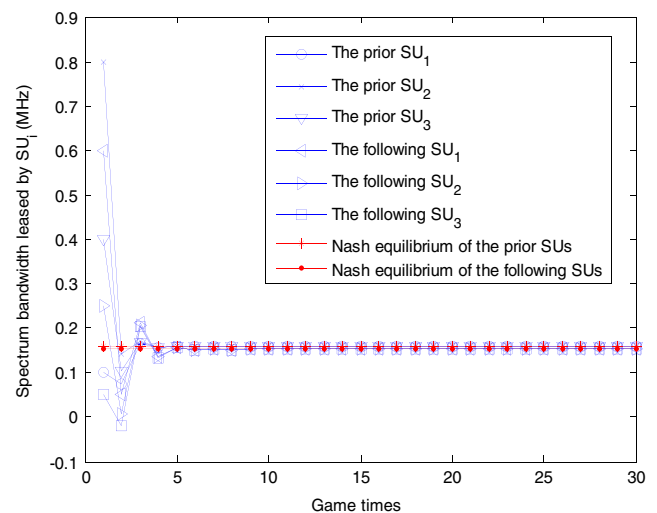
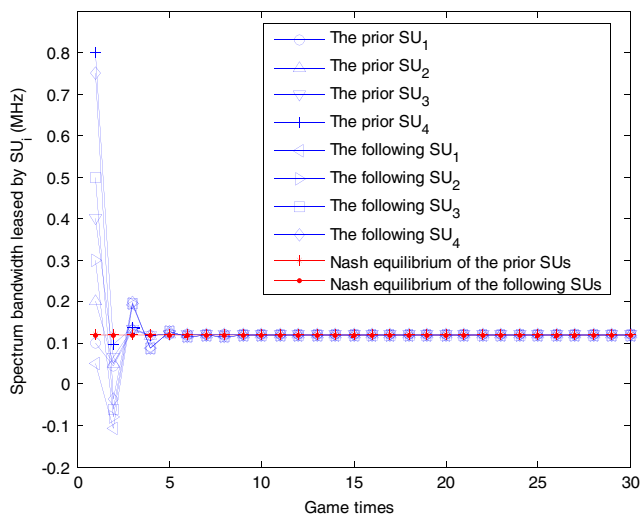


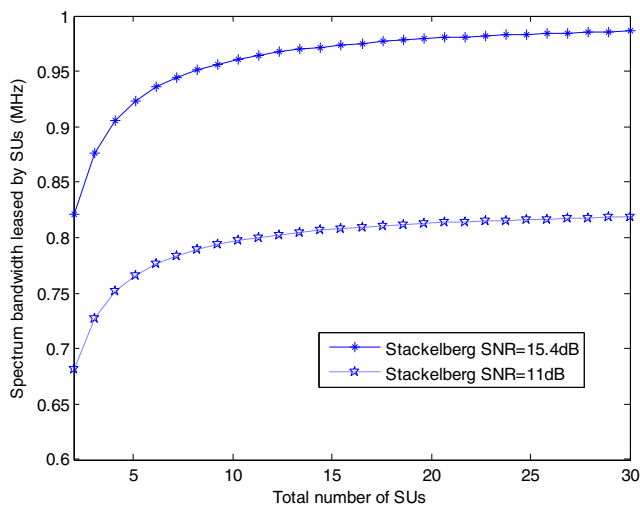
Fig. 6 Game process of 6 SUs



**Fig. 7** Game process of 8 SUs

SUs is greater than that of the following SUs when the game reaches the steady state. The results show the first-mover advantage in Stackelberg game. However with the increasing of the number of SUs, the strategy value of prior SUs is closer to that of following SUs at steady state. And the game times of reaching Nash equilibrium point will not increase greatly, which shows that the greater number of game participants, the less obvious the first-mover advantage in Stackelberg model is. At the same time, free spectrum bandwidth leased by each SU will decrease; while the proposed DSA based Stackelberg game algorithm solves the fairness of leasing idle spectrum resources among SUs. And the total spectrum bandwidth leased by all SUs also increased. The results show that the proposed DSA maximizes the utilization of spectrum resources efficiently.

The total amount of spectrum leased by SUs with different SNR and varying number of SUs is shown in Fig. 8. When the



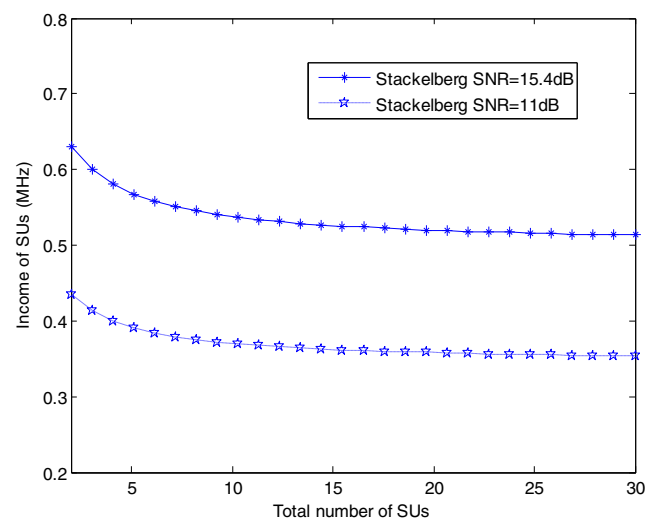
**Fig. 8** Total spectrum bandwidth leased by SUs with varying number of SUs

game reaches the steady state, the spectrum bandwidth leased by SUs will vary with the number of SUs increased from 2 to 30.

As shown in Fig. 8, when the number of SUs is 2, the total spectrum bandwidth leased by SUs is minimum. When SNR is 15.4 dB, the bandwidth is 0.824 MHz, while the bandwidth is 0.682 MHz when SNR is 11 dB. With the number of SUs increasing, the total spectrum bandwidth leased by SUs increases obviously. When the number of SUs is larger than 15, the total spectrum bandwidth leased by SUs will increase slowly. And when the number of SUs is 30, the total spectrum bandwidth leased by SUs approaches to the total idle spectrum bandwidth of PUs, and the bandwidths are 0.984 MHz (SNR=15.4 dB) and 0.821 MHz (SNR=11 dB) respectively. The results show the higher SNR, the more total spectrum bandwidth leased by SUs.

The income of SUs at steady state with different SNR and varying number of SUs is shown in Fig. 9, where the number of SUs increases from 2 to 30.

In Fig. 9, when the number of SUs is 2, the income of SUs is the maximum. The income of SUs is 0.63 when SNR is 15.4 dB, while that is 0.44 when SNR is 11 dB. The reason is spectrum oversupply when the number of SUs is very small, and the price of PUs' idle spectrum is cheap. However, with the number of SUs increasing, the income of SUs decreases significantly, and when the number of SUs is more than 15, the income of SUs will decrease slowly and flat. When there are 30 SUs, the income of SUs is 0.51 as SNR is 15.4 dB, and that is 0.36 as SNR is 11 dB. This is because the limited spectrum resources of PUs, and with the number of SUs increasing, the requirements of leasing idle spectrum increase. Therefore, spectrum price will rise accordingly. The results demonstrate the higher SNR will help SUs to obtain more income. The reason is that SUs with higher SNR will help SUs to detect the idle spectrum accurately, which will increase idle spectrum of PUs.



**Fig. 9** Income of SUs with varying number of SUs



### 3.2 Performance comparison

In order to validate the effectiveness of the proposed scheme, in this scenario, the spectrum bandwidth leased by SUs and income of SUs are explored and compared with the Cournot game model [26].

The amount of spectrum bandwidth leased by SUs at steady state with different algorithms is shown in Fig. 10, where we considers the cases with different SNR, and the number of SUs varying from 2 to 30.

As shown in Fig. 10, with the same SNR, the total amount of spectrum bandwidth leased by SUs based on Stackelberg game model is greater than that based on Cournot game model. For example, when the number of SUs is 5, the spectrum bandwidth leased by SUs is 0.924 MHz of the proposed scheme with SNR 15.4 dB, which is higher about 0.014 MHz than that based on Cournot game. The reason is that Cournot game model is complete information static game model, and all game participants take game strategy simultaneously. While the proposed scheme based on Stackelberg game model, is a dynamic game model, and a part of game participants will take the strategy firstly, and then the following game participants will optimize their own game strategies based on the game strategy of the prior game participants. Therefore, such scheme will help the following participants can pursue more spectrum bandwidth and get more benefit. With the number of SUs increasing, the total spectrum leased by SUs of the two game algorithms will increase rapidly. Then the total bandwidth leased by SUs will increase slowly when the number of SUs is larger than 10. This is because that the idle spectrum of PUs is abundant when the number of game participant is small. Therefore, the total bandwidth leased by SUs will increase obviously. As the number of participants increasing, idle spectrum of PUs will not increase accordingly,

which leads to the total bandwidth leased by SUs increasing slowly. However, total spectrum leased by SUs of two algorithms comes to close and tends to coincide when the number of SUs increases to 30. The reason is that first-mover advantage of Stackelberg model will disappear gradually as the number of participants increasing. It's obviously that the higher SNR, the more spectrum bandwidth rented by SUs is. The results show that the proposed scheme can achieve better game performance in term of bandwidth leased by SUs.

The income of SUs at steady state for different algorithms is shown in Fig. 11, where we considers the cases with different SNR, and the number of SUs varying from 2 to 30.

As shown in Fig. 11, with the same SNR, the income of SUs for two game models will decrease obviously, especially for the total number of SUs is less than 10. And then the income will degrade slightly. The reason is the limited idle spectrum resources of PUs. And as the number of SUs increasing, the idle spectrum resources will become scarcity, and spectrum price will correspondingly increase, which will lead to the income of SUs decrease accordingly. In addition, the income of SUs based on Stackelberg game model is lower than that based on Cournot game model. For example, the income of the former is 0.63, and that of the latter is 0.64 when there are 2 SUs and SNR is 15.4 dB. This is because the total amount of spectrum bandwidth leased by SUs based on Stackelberg game model is higher than that of the latter as shown in Fig. 10. However when the number of SUs approaches to 30, the income of SUs of two schemes comes to coincide. When the SNR changes from 15.4 to 11 dB, the incomes of SUs of two game model algorithms will reduce accordingly. The reason is lower SNR will lead to decreasing of the total amount of idle spectrum bandwidth, and the spectrum price is not changed. Hence, the income of SUs will degrade correspondingly.

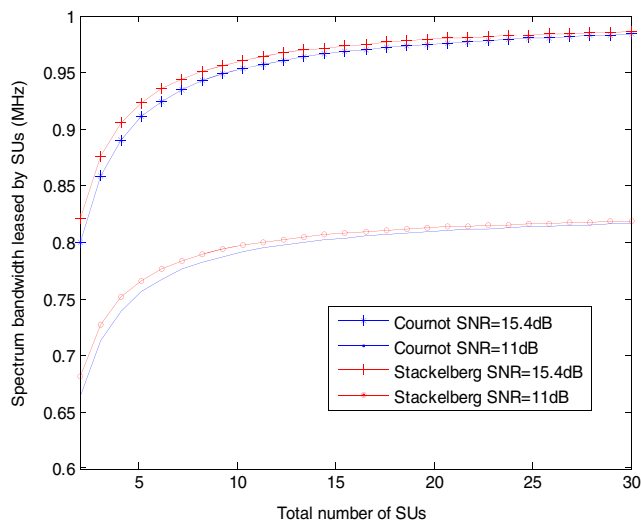


Fig. 10 Spectrum bandwidth leased by SUs of different algorithms

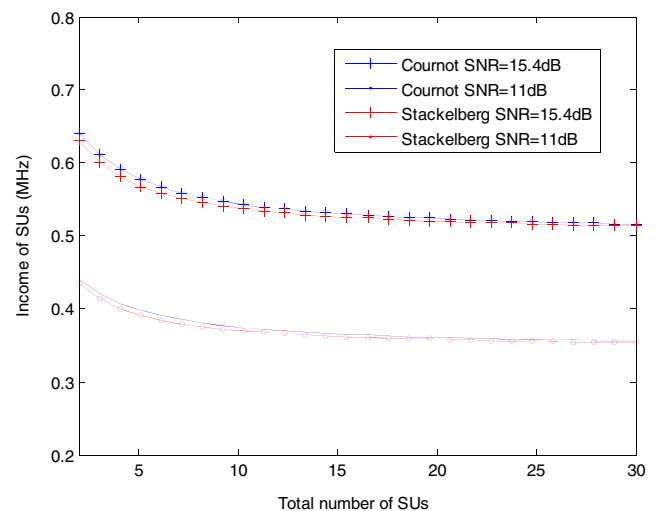


Fig. 11 Income of SUs with different algorithms

## 4 Conclusion

This paper proposed a novel DSA algorithm for CRNs based on game theory, which captures the game behavior among SUs during a specific leased spectrum and efficiently accounts for mutual interference between SUs for various sizes of CRNs. The problem is modeled as an oligopolistic competition model. And we have employed the Nash equilibrium solution of Stackelberg model in order to obtain the bandwidth of SUs' leased spectrum. An intensive analysis has been presented to characterize the SUs' leased spectrum utilization behavior with respect to various criteria of the proposed game theoretic model. In particular, the existing of Nash equilibrium, the game process, the convergence of SUs, the total SUs' leased spectrum, and the SUs' income have been carefully investigated. Simulation study shows that our proposed scheme enables SUs to efficiently utilize the licensed spectrum shared with PUs in a dynamic environment while maximizing the spectrum utilization rate. The performance of proposed DSA algorithm was investigated under different SNR regimes. We have found as SNR increased, the aggregated bandwidth of the SUs' leased spectrum will be increased. Similarly, increasing SNR will increase SUs' income. We have also compared Stackelberg with Cournot model for the case of same SNR to provide additional insights. We have observed that Stackelberg model improves the spectrum utilization by increasing the aggregated amount of SUs leased spectrum. However, the increasing in leased spectrum comes with boosting the spectrum lease price, which results in lower income for Stackelberg model in comparing to Cournot model.

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