

On the Deployment of Antenna Elements in Generalized Multi-User Distributed Antenna Systems

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Abstract In this paper, we focus on a generalized multi-user distributed antenna system (DAS), where the antenna elements (AEs) are divided into antenna clusters and the antenna clusters are randomly deployed in the coverage area. The mobile terminals equipped with M AEs each are supposed to be uniformly distributed in the coverage area. We are motivated to study the impact of the deployment of antenna elements on the system performance. In the model of consideration, the deployment of antenna elements is characterized by the antenna cluster size V , i.e., the number of AEs within each antenna cluster, and the distribution of the antenna clusters. With the assumption that the antenna clusters are uniformly deployed in the coverage area, the impact of the antenna cluster size V on the uplink sum rate capacity is particularly investigated. The mean square access distance (MSAD), a function of V , is proposed as a reasonable metric instead of the uplink sum rate capacity. From the analysis of the asymptotic

behavior of MSAD, we derive an approximate closed-form expression for the expectation of MSAD over system topologies. Then, it is concluded that the ergodic uplink sum rate capacity can be improved due to access distance reduction by scattering AEs further only when $V > M$. An approximate closed-form expression for the relative variance of MSAD is also derived. And we conclude that the outage uplink sum rate capacity can be improved due to macro-diversity by scattering AEs further only when $V \leq M$. In other words, when $V \leq M$, the ergodic uplink sum rate capacity can not be improved by scattering AEs further, when $V > M$, the outage uplink sum rate capacity can not be improved by scattering AEs further. Finally, our analysis is well verified by Monte Carlo simulations.

Keywords distributed antenna systems (DAS) · antenna cluster size · mean square access distance (MSAD) · access distance reduction · macro-diversity

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1 Introduction

The future wireless networks should provide data services at a high bit rate for a large number of users. Distributed antenna system (DAS), as a promising technique, has attracted worldwide research interest [1–6]. In a DAS, the antenna elements (AEs) are scattered around in the coverage area, the optical fiber is employed to transfer information and signaling between the distributed AEs and a central processor, where all signals are jointly processed. The concept of DAS was originally introduced to simply cover the dead spots in indoor wireless communications [7]. Recent

studies have identified other potential advantages of DASs in terms of access distance reduction and macro-diversity [8–10] in addition to coverage improvement. As a consequence, the ergodic system capacity and the outage system capacity can be markedly improved due to DAS techniques [11–13].

In a DAS, the deployment of distributed AEs has great impact on the system performance. Many recent studies have focused on this topic, which can be roughly sorted into two kinds: works for single user scenarios and works for multi-user scenarios.

H. Zhuang [8] proposed a single user DAS with random antenna layout, where the AEs were randomly and absolutely scattered in the coverage area around the mobile terminal. H. Zhuang then investigated the downlink capacity of the proposed system and concluded that an \mathcal{A} -DAS¹ outperformed a corresponding co-located antenna system (CAS) due to the access distance reduction and macro-diversity. W. Roh [9, 10] defined the concept of generalized distributed antenna systems (GDAS) with the single user assumption, where the distributed AEs were deployed as antenna clusters. W. Roh further identified the advantage of DASs in terms of macro-diversity, which could enormously reduce the outage probability of the system.

These works have characterized quantitatively the advantages of single user DASs, which were quite helpful for the studies on DASs in multi-user scenarios. Moreover, since the demand for high bit rate services will be dominant in the multi-user environment in future wireless networks, the multi-user assumption should be almost the essential constraint for future academic research on DASs.

In [13], J. Gan proposed a multi-user circular antenna layout DAS (CL-DAS), where the distributed AEs were randomly and absolutely scattered on a circle in the coverage area. The uplink sum capacity of the system was analytically derived, and the optimal radius of the circle to deploy AEs was obtained in [13]. However, J. Gan did not consider the per-user transmit power constraints and adopted a quite special power control scheme. Subsequently, as a developed study, in [14], the authors provided the analysis of the uplink sum capacity of the multi-user CL-DAS with per-user power constraints and derived the updated optimal radius of the circle to deploy distributed AEs. However, CL-DAS is quite special, the discussions on the deployment of AEs based on this model are too particular to be generalized to other DASs. In [15], a

multi-user DAS with random antenna layout (RAL-DAS) was proposed, where the AEs were divided into clusters and the antenna clusters were assumed to be randomly deployed in the coverage area. A good approximation for the uplink sum capacity of RAL-DAS was analytically derived in [15]. This model can be regarded as the most general model of multi-user DASs. Since any other DAS can be treated as a special sample of RAL-DAS. In [16], the DAS with random antenna layout was studied in a multi-cell environment, which is more practical. Then, a good approximation for the downlink capacity was analytically derived in [16]. These two results are important for the more practical and more general research about multi-user DASs. However, these achievements are too complicated to be used to derive further conclusions on the deployment of AEs.

1.1 Motivation

Based on the aforementioned studies, we can find that from CAS to \mathcal{A} -DAS, the ergodic system capacity is improved due to the access distance reduction and the outage system capacity is improved due to the macro-diversity for both the single user scenario and the multi-user scenario. But most current studies directly focused on the ideal DAS model (say \mathcal{A} -DAS), and then characterized the advantages of the corresponding \mathcal{A} -DAS with different system topologies such as the single user DAS with random antenna layout [8] and the multi-user DAS with circular antenna layout [13]. On the other hand, there are few studies on the DASs with intermediate states² between the two extremes (CAS and \mathcal{A} -DAS). Though [9, 10] began to discuss the GDAS and argued the antenna cluster size, but in those works the distances between the considered mobile terminal and the distributed AEs were assumed to be fixed, which can not be generalized to multi-user scenarios. For a multi-user scenario, some studies have proposed more general and more practical DAS models and derived the system performance evaluation results [15, 16], however the general study on the deployment of distributed AEs is still vacant.

From CAS to GDAS, and finally to \mathcal{A} -DAS, we should find out whether the ergodic system capacity can be improved all along due to the access distance reduction, and whether the outage system capacity can be improved all along due to the macro-diversity. In

¹In this paper, we denote the DAS where the distributed AEs are absolutely scattered as “ \mathcal{A} -DAS”.

²The AEs are scattered around to some extent, but not scattered absolutely as an “ \mathcal{A} -DAS”. The antenna cluster size is an important parameter to characterize the DASs with intermediate states.

other words, we should pursue the detailed benefits from scattering distributed AEs from CAS toward A-DAS.

1.2 Contribution

In this paper, we focus on a generalized multi-user DAS, and study the impact of the deployment of distributed AEs on the system performance. Our main contributions can be summarized as follows.

- A generalized system model is proposed, where the deployment of antenna elements is characterized by the antenna cluster size V , i.e., the number of AEs within each antenna cluster,³ and the distribution of the antenna clusters. The mobile terminals with M AEs each are assumed to be uniformly distributed in the coverage area.
- With the assumption that the antenna clusters are deployed with uniform distribution in the coverage area, the impact of the antenna cluster size V on the uplink sum rate capacity is particularly investigated. The mean square access distance (MSAD), a function of V , is proposed as a reasonable metric instead of the system uplink sum rate capacity.
- From the analysis of the asymptotic behavior of MSAD, we derive an approximate closed-form expression for the expectation of MSAD over system topologies. Then, it is concluded that the ergodic uplink sum rate capacity can be improved due to access distance reduction by scattering AEs further only when $V > M$. An approximate closed-form expression for the relative variance of MSAD is also derived. And we conclude that the outage uplink sum rate capacity can be improved due to macro-diversity by scattering AEs further only when $V \leq M$. In other words, when $V \leq M$, the ergodic uplink sum rate capacity can not be improved by scattering AEs further, when $V > M$, the outage uplink sum rate capacity can not be improved by scattering AEs further.

1.3 Organization

The rest of this paper is organized as follows. The system model is described in the next section. The discussions on the impact of antenna cluster size on the

³If V is maximized, i.e., all the antenna elements are deployed together, we can obtain a traditional co-located antenna system (CAS). If $V = 1$, we can obtain an absolute DAS (A-DAS), where the antenna elements are absolutely scattered.

system performance are carried out in Section 3. The simulation results are shown in Section 4. Finally, conclusions are given in Section 5.

1.4 Notation

Lower case and upper case boldface symbols denote vectors and matrices, respectively. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the transpose conjugate, respectively. $\mathbb{C}^{M \times N}$ represents the complex matrix space composed by all $M \times N$ matrices and \mathcal{CN} denotes a complex gaussian distribution. $\mathbf{E}(\cdot)$ represents the expectation operator and $\mathbf{RV}(\cdot)$ denotes the relative variance operator. \mathbf{I}_n is an identity matrix with the dimension equal to n .

2 System model

2.1 Generalized multi-user DAS

The system model of a generalized multi-user DAS with AEs deployed as distributed antenna clusters is illustrated in Fig. 1. The distributed AEs are grouped into antenna clusters, which are geographically separated from each other in the circular coverage area with a radius of R . The optical fiber is employed to transfer information and signaling between the distributed AEs and a central processor, where all signals are jointly processed. We use the notation (N, V, K, M) DAS to describe the system of consideration, which consists of N distributed antenna clusters (DACs) with V AEs

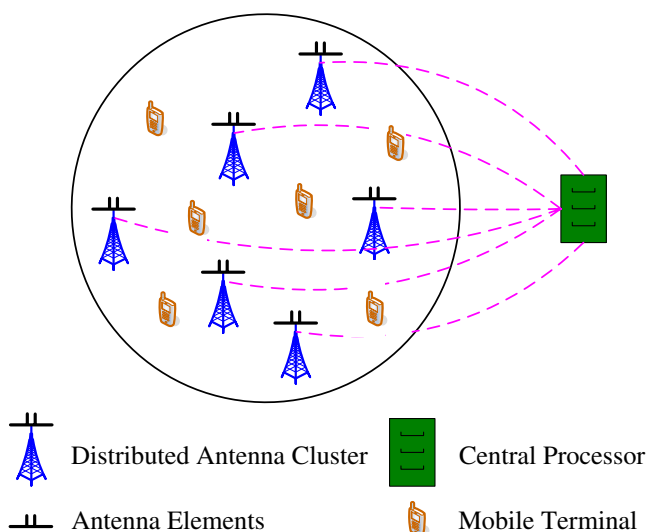


Fig. 1 Illustration of a multi-user DAS with random antenna cluster layout

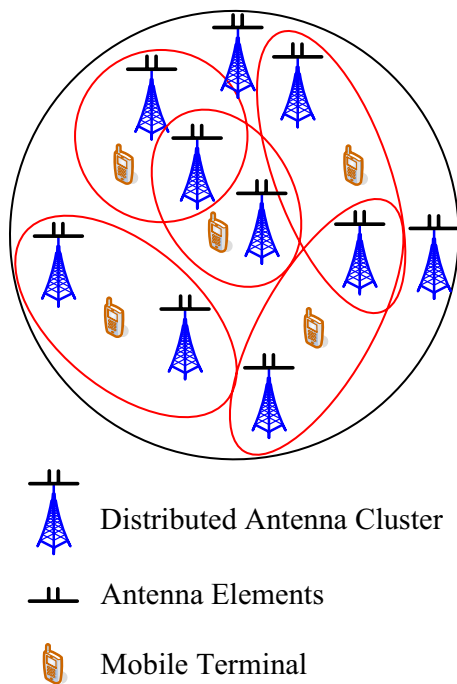


Fig. 2 Illustration of the antenna selection scheme for (10, 2, 5, 4) DAS. Since the number of AEs per MT is 4, the 4 nearest AEs should be selected, thus, the 2 nearest DACs are selected to serve the corresponding MT

each⁴ and K mobile terminals (MTs) with M AEs each. The DACs and the MTs are randomly deployed in the coverage area with uniform distribution, respectively. Thus, any other DAS can be regarded as a special sample of this model, which implies that this study is applicable for any DAS with different system topologies. The distributed AE density λ_a and the MT density λ_u can be obtained

$$\lambda_a = \frac{NV}{\pi R^2}, \quad (1)$$

$$\lambda_u = \frac{K}{\pi R^2}. \quad (2)$$

2.2 Assumptions

As to describe the advantages of DASs as clear as possible, we consider the uplink and assume ideal multi-user joint signal processing at the central processor. Thus, the interference from and to other MTs can be eliminated absolutely. As shown in Fig. 2, in order to maximize the spacial multiplexing of the MTs, we adopt a simple antenna selection scheme, where the M nearest AEs are selected to serve the corresponding

MT. At the same time, in order to maximize the spacial multiplexing of the whole network, we can set

$$NV = KM. \quad (3)$$

The channel state information (CSI) is assumed to be only perfectly available at the receiver (CSIR). Hence, equal power allocation is employed at the MTs with per-MT power constraints.

2.3 Signal model

For an arbitrary MT indexed with i , based on the proposed antenna selection scheme, the received signal vector at the selected AEs in the uplink can be expressed as

$$\mathbf{y}_i = \mathbf{H}_{i,i} \mathbf{P}_i^{1/2} \mathbf{x}_i + \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_i, \quad (4)$$

where $\mathbf{H}_{i,i} \in \mathbb{C}^{M \times M}$, is the channel matrix between the MT i and its M selected AEs, $\mathbf{H}_{k,i} \in \mathbb{C}^{M \times M}$, denotes the channel matrix between the k th MT and the M selected AEs for MT i , the diagonal matrix $\mathbf{P}_k \in \mathbb{C}^{M \times M}$, denotes the transmit power matrix of the k th MT, and $\mathbf{x}_k \in \mathbb{C}^{M \times 1}$ is the transmitted signal vector of the k th MT, $\mathbf{n}_i \in \mathbb{C}^{M \times 1}$, denotes the white noise vector with Gaussian distribution $\mathcal{CN}(0, \sigma_{n,i}^2 \mathbf{I}_M)$. Without loss of generality, we assume $\sigma_{n,1}^2 = \sigma_{n,2}^2 = \dots = \sigma_{n,K}^2 = \sigma_n^2$. Since equal power allocation is employed at the MTs with per-MT power constraints, we have

$$\mathbf{P}_k = \frac{P_k}{M} \mathbf{I}_M, \quad k = 1, 2, \dots, K, \quad (5)$$

where P_k is the transmit power constraint of the k th MT.

Encompassing not only small-scale fading but also large-scale fading, the channel matrix $\mathbf{H}_{k,i}$ can be modeled as [13]

$$\mathbf{H}_{k,i} = \mathbf{L}_{k,i} \mathbf{H}_w^{k,i} = \begin{bmatrix} d_1^{k,i-\gamma/2} & & & \\ & \ddots & & \\ & & d_M^{k,i-\gamma/2} & \\ & & & \end{bmatrix} \times \begin{bmatrix} h_{11}^{k,i} & \dots & h_{1M}^{k,i} \\ \vdots & \ddots & \vdots \\ h_{M1}^{k,i} & \dots & h_{MM}^{k,i} \end{bmatrix}, \quad k = 1, 2, \dots, K, \quad (6)$$

where $\mathbf{H}_w^{k,i}$ reflects the small-scale fading, whose entries are independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance, and $\mathbf{L}_{k,i}$ represents the large-scale fading, i.e., the path loss, $\{d_m^{k,i}\}_{m=1}^M$ denote the distances between the

⁴ V is the antenna cluster size.

k th MT and the selected AEs for MT i , γ is the path loss exponent, which usually takes value from 3 to 6.

Especially, we have

$$\mathbf{H}_{i,i} = \mathbf{L}_{i,i} \mathbf{H}_w^{i,i} = \begin{bmatrix} d_1^{i,i-\gamma/2} & & \\ & \ddots & \\ & & d_M^{i,i-\gamma/2} \end{bmatrix} \begin{bmatrix} h_{11}^{i,i} & \dots & h_{1M}^{i,i} \\ \vdots & \ddots & \vdots \\ h_{M1}^{i,i} & \dots & h_{MM}^{i,i} \end{bmatrix}, \quad (7)$$

where $\{d_m^{i,i}\}_{m=1}^M$ denote the access distances between MT i and its M selected AEs.⁵ Thus, $\{d_m^{i,i}\}_{m=1}^M$ are the M shortest distances between MT i to all the distributed AEs. We can set

$$d_1^{k,k} \leq d_2^{k,k} \leq \dots \leq d_M^{k,k}, \quad k = 1, 2, \dots, K. \quad (8)$$

3 Discussions on antenna cluster size

In order to analyze the impact of the antenna cluster size on the system performance, we should characterize the system performance at first, or at least present a fungible metric. In this section, the mean square access distance (MSAD) is proposed as a reasonable metric instead of the system uplink sum rate capacity. And based on the study on MSAD, the discussions on the impact of antenna cluster size on the system performance are carried out.

3.1 MSAD

The essential system performance should be sum rate capacity. With the assumption of CSIR, the sum rate capacity in the uplink is expressed as [17]

$$C = \log_2 \det \left(\mathbf{I}_{NV} + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{P} \mathbf{H}^H \right), \quad (9)$$

where C denotes the instantaneous uplink sum rate capacity, \mathbf{H} is the total channel matrix between all the MTs and all the AEs, and \mathbf{P} represents the total transmit power matrix. Based on C , we can further derive the ergodic uplink sum rate capacity and the outage uplink sum rate capacity. However, by summarizing the current works, we can observe that it is difficult to find a compact expression for the sum rate capacity and then obtain the relationship between the antenna cluster size and the sum rate capacity.

Therefore, we turn to investigate another important indicator of the system performance, received signal to noise ratio (SNR), which has direct impact on the sum rate capacity. For MT i , the total power of the received signal can be derived as

$$\begin{aligned} \mathbf{E}(\mathbf{y}_i^H \mathbf{y}_i) &= \mathbf{E} \left[\left(\mathbf{H}_{i,i} \mathbf{P}_i^{1/2} \mathbf{x}_i + \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_i \right)^H \right. \\ &\quad \left. \times \left(\mathbf{H}_{i,i} \mathbf{P}_i^{1/2} \mathbf{x}_i + \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_i \right) \right] \\ &= \mathbf{E} \left\{ \text{tr} \left[\left(\mathbf{H}_{i,i} \mathbf{P}_i^{1/2} \mathbf{x}_i + \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_i \right) \right. \right. \\ &\quad \left. \left. \times \left(\mathbf{H}_{i,i} \mathbf{P}_i^{1/2} \mathbf{x}_i + \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_i \right)^H \right] \right\} \\ &= \mathbf{E} \left\{ \text{tr} \left[\mathbf{H}_{i,i} \mathbf{P}_i \mathbf{H}_{i,i}^H + \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{P}_k \mathbf{H}_{k,i}^H \right. \right. \\ &\quad \left. \left. + \mathbf{n}_i \mathbf{n}_i^H \right] \right\} \\ &= P_i \sum_{m=1}^M d_m^{i,i-\gamma} + \sum_{k=1, k \neq i}^K P_k \sum_{m=1}^M d_m^{k,i-\gamma} + M \sigma_n^2. \end{aligned} \quad (10)$$

Obviously, the total received power consists of the useful signal power, the interference power from other MTs and the noise power. Since the ideal multi-user joint signal processing at the central processor is assumed, the interference can be perfectly estimated and then be eliminated. Consequently, the total signal to noise ratio (SNR) for MT i is derived

$$SNR_i = \frac{P_i \sum_{m=1}^M d_m^{i,i-\gamma}}{M \sigma_n^2}. \quad (11)$$

If

$$d_{max}^i = \max\{d_m^{i,i} | m = 1, 2, \dots, M\}, \quad (12)$$

$$d_{min}^i = \min\{d_m^{i,i} | m = 1, 2, \dots, M\}, \quad (13)$$

⁵The M nearest AEs of MT i .

we directly have the upper bound and the lower bound of the SNR_i as

$$SNR_i^{upper} = \frac{P_i d_{min}^i^{-\gamma}}{\sigma_n^2}, \tag{14}$$

$$SNR_i^{lower} = \frac{P_i d_{max}^i^{-\gamma}}{\sigma_n^2}. \tag{15}$$

Define the mean square access distance of MT i (MSAD- i) as

$$\bar{d}_i^2 = \frac{1}{M} \sum_{m=1}^M (d_m^{i,i})^2, \tag{16}$$

and the corresponding SNR_i^* can be obtained

$$SNR_i^* = \frac{P_i \bar{d}_i^{-\gamma}}{\sigma_n^2}. \tag{17}$$

From Eqs. 12, 13 and 16, we can find

$$d_{min}^i \leq \bar{d}_i \leq d_{max}^i, \tag{18}$$

as a consequence, we have

$$SNR_i^{lower} \leq SNR_i^* \leq SNR_i^{upper}. \tag{19}$$

Intuitively, we can directly use SNR_i^* to evaluate the received SNR for MT i . Since SNR_i^* is a monotone decreasing function with respect to \bar{d}_i , we can use \bar{d}_i to indicate the performance of MT i .

As to the whole system, by averaging all the single-MT MSADs, we can define the network MSAD $\bar{d}^2(N, V, K, M)$ as

$$\bar{d}^2(N, V, K, M) = \frac{1}{K} \sum_{k=1}^K [\text{MSAD-}k]. \tag{20}$$

By plugging Eqs. 16 to 20, we have

$$\bar{d}^2(N, V, K, M) = \frac{1}{K} \sum_{k=1}^K \left[\frac{1}{M} \sum_{m=1}^M (d_m^{k,k})^2 \right]. \tag{21}$$

And $d_m^{k,k}$ is the m th shortest distance from the k th MT to the distributed AEs.

Because the distributed AEs are deployed as antenna clusters, it is more straightforward to calculate the distances from the MTs to their respective nearest DACs. For an arbitrary MT, the number of nearest DACs to be selected can be derived

$$M_c = \left\lceil \frac{M}{\min(M, V)} \right\rceil, \tag{22}$$

where $\lceil \cdot \rceil$ is the integer ceiling operator. The selected DACs should totally include M AEs based on the proposed antenna selection scheme.

Hence, by focusing on the distances from the MTs to their respective nearest DACs, an approximation for Eq. 21 can be derived

$$\bar{d}^2(N, V, K, M) \approx \frac{1}{K} \sum_{k=1}^K \left[\frac{1}{M_c} \sum_{m=1}^{M_c} (d_m^{k,k})^2 \right], \tag{23}$$

where $d_m^{k,k}$ represents the m th shortest distance from the k th MT to the DACs.

3.2 Statistics of MSAD

Because the topology of the considered system is a random variable, the system MSAD in (23) is also a random variable. Therefore, we are interested in the first-order and the second-order statistics of the MSAD.

Defining the single-MT MSAD as

$$\mathcal{D}_k^2 = \frac{1}{M_c} \sum_{m=1}^{M_c} (d_m^{k,k})^2, \quad k = 1, 2, \dots, K, \tag{24}$$

we have

$$\bar{d}^2(N, V, K, M) \approx \frac{1}{K} \sum_{k=1}^K \mathcal{D}_k^2. \tag{25}$$

For an arbitrary MT i , while keeping the distributed AE density λ_a as a finite constant, as R goes to infinity, we derive the asymptotic probability density function (PDF) [8] of $d_1^{i,i}$ as

$$f_{d_1^{i,i}}(x) = \frac{2\pi\lambda_a x}{V} e^{-\frac{\pi\lambda_a x^2}{V}}, \quad 0 < x \leq R. \tag{26}$$

Similarly, we can derive the conditional PDF of $d_m^{i,i}$ ($m \geq 2$) as

$$f_{d_m^{i,i}}(x | d_{m-1}^{i,i}) = \frac{2\pi\lambda_a x}{V} e^{-\frac{\pi\lambda_a(x^2 - (d_{m-1}^{i,i})^2)}{V}}, \quad d_{m-1}^{i,i} \leq x \leq R, \tag{27}$$

$$2 \leq m \leq M.$$

Since \mathcal{D}_i^2 is a function with respect to $d_1^{i,i}, d_2^{i,i}, \dots$, and $d_M^{i,i}$, with some further algebraic manipulations, the following asymptotic expectation and relative variance of \mathcal{D}_i^2 can be obtained

$$\mathbf{E}(\mathcal{D}_i^2) = \frac{V(M_c + 1)}{2\pi\lambda_a}, \tag{28}$$

$$\mathbf{RV}(\mathcal{D}_i^2) = \frac{2(2M_c + 1)}{3M_c(M_c + 1)}. \tag{29}$$

Based on the assumptions of the considered system model, we can find that $\{\mathcal{D}_k^2 | k = 1, 2, \dots, K\}$ can be regarded as i.i.d. random variables. Thus, from Eqs. 25,

28 and 29, the expectation and relative variance of the system MSAD $\bar{d}^2(N, V, K, M)$ can be derived

$$\mathbf{E}(\bar{d}^2) = \frac{V(\lceil \frac{M}{\min(M,V)} \rceil + 1)}{2\pi\lambda_a}, \tag{30}$$

$$\mathbf{RV}(\bar{d}^2) = \frac{2(2\lceil \frac{M}{\min(M,V)} \rceil + 1)}{3K\lceil \frac{M}{\min(M,V)} \rceil(\lceil \frac{M}{\min(M,V)} \rceil + 1)}. \tag{31}$$

Utilizing the symmetry characteristics of the DACs and MTs in the proposed system model, by ignoring the integer ceiling operator, we can further have

$$\mathbf{E}(\bar{d}^2) \approx \frac{\frac{M}{\min(M,V)}\lambda_u V + \lambda_a}{2\pi\lambda_a\lambda_u}, \tag{32}$$

$$\mathbf{RV}(\bar{d}^2) \approx \frac{2(2\frac{M}{\min(M,V)}\lambda_u V + \lambda_a)}{3\pi R^2\lambda_u \frac{M}{\min(M,V)}(\frac{M}{\min(M,V)}\lambda_u V + \lambda_a)}. \tag{33}$$

3.3 Impact of antenna cluster size on system performance

In order to observe the relationship between V and the two obtained statistics of MSAD, we first derive the difference of the expectation with respect to V as

$$\nabla_V [\mathbf{E}(\bar{d}^2)] = \begin{cases} 0, & V \leq M; \\ \frac{1}{2\pi\lambda_a}, & V > M. \end{cases} \tag{34}$$

And then the difference of the relative variance with respect to V is derived as

$$\begin{aligned} &\nabla_V[\mathbf{RV}(\bar{d}^2)] \\ &= \begin{cases} \frac{1}{6\pi R^2\lambda_u} \left[\frac{2M\lambda_u + \lambda_a}{\frac{M}{V}(M\lambda_u + \lambda_a)} - \frac{2M\lambda_u + \lambda_a}{\frac{M}{V-1}(M\lambda_u + \lambda_a)} \right] \\ 0 \end{cases} \\ &= \begin{cases} \frac{2M\lambda_u + \lambda_a}{6\pi R^2 M\lambda_u(M\lambda_u + \lambda_a)}, & V \leq M; \\ 0, & V > M. \end{cases} \end{aligned} \tag{35}$$

From Eqs. 34 and 35, the following remarks can be given with the same AE density and the same MT density.

- When $V \leq M$, the relative variance decreases with the decreasing of V , while the expectation remains a constant that equals to the value at $V = M$.
- When $V > M$, the expectation decreases with the decreasing of V , while the relative variance remains a constant that equals to the value at $V = M$.

Intuitively, with consideration of the relationship between MSAD and the system SNR, and the uplink sum rate capacity of the system, we can conclude as follows.

- When $V \leq M$, the outage uplink sum rate capacity can be improved due to macro-diversity by scattering AEs further.
- When $V \leq M$, the ergodic uplink sum rate capacity can not be improved by scattering AEs further.
- When $V > M$, the ergodic uplink sum rate capacity can be improved due to access distance reduction by scattering AEs further.
- When $V > M$, the outage uplink sum rate capacity can not be improved by scattering AEs further.

Accordingly, as illustrated in Fig. 3, when $V \leq M$, benefiting from only the macro-diversity, a DAS outperforms a CAS in terms of outage system performance; when $V > M$, benefiting from only the access distance reduction, a DAS outperforms a CAS in terms of ergodic system performance.

4 Simulation results

In this section, we will verify our analysis via Monte Carlo simulations. The sum rate capacity normalized by the number of MTs in the uplink will be presented to illustrate the system performance.

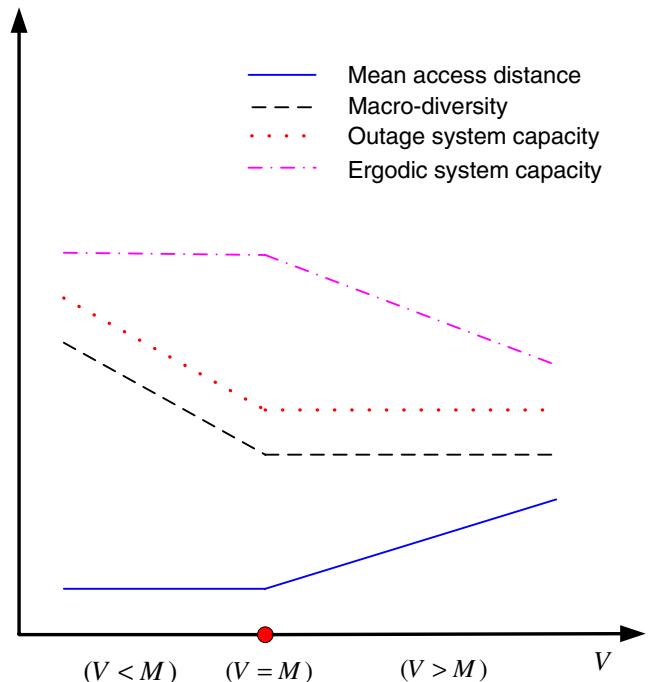


Fig. 3 Illustration of the impact of V on system performance

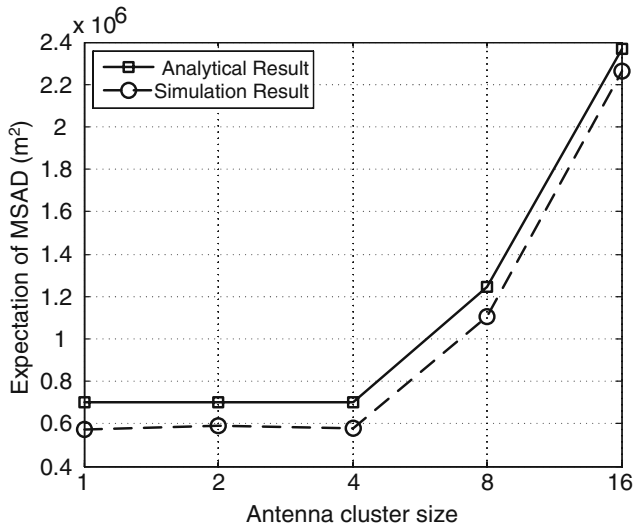


Fig. 4 The expectation of MSAD ($NV = 128, K = 32, M = 4$)

In our simulations, a circular coverage area with a radius of $6000m$ is considered. The path loss exponent is set to be 4 [18], and the noise power σ_n^2 is assumed to be -107 dBm given 5 MHz bandwidth and -174 dBm/Hz thermal noise as in the universal mobile telecommunications system (UMTS). NV and KM are both assumed to be 128. The antenna cluster size V takes value in $\{1, 2, 4, 8, 16\}$, thus, the number of DACs N takes value in $\{128, 64, 32, 16, 8\}$. Without loss of generality, the same transmit power constraint of MTs are assumed, which takes value from 0dBm to 30dBm in our simulations.

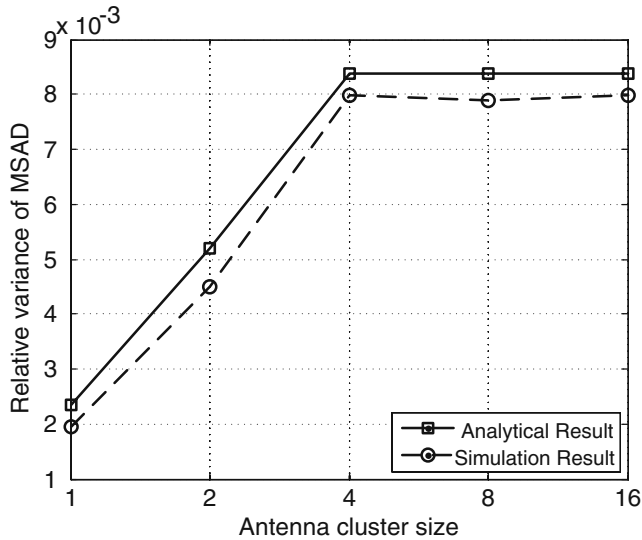


Fig. 5 The relative variance of MSAD ($NV = 128, K = 32, M = 4$)

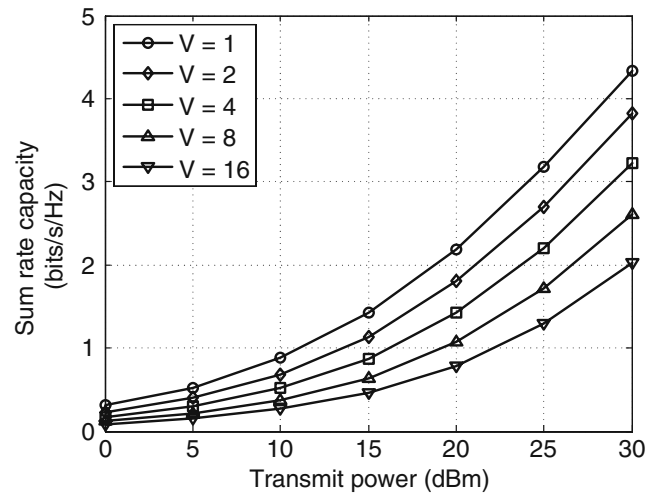


Fig. 6 Sum rate capacity for $M = 1$

Both the analytical and simulation results for the expectation and relative variance of MSAD are presented in Figs. 4 and 5, respectively. We can observe that the analytical results are accordant with the Monte Carlo simulations, which implies the high validity of our analysis on the asymptotic behavior of MSAD. The gap is reasonable since the analytical results are asymptotically derived via some approximations.

Figures 6 and 7 present the Monte Carlo simulation results for the uplink sum rate capacity for $M = 1$. We can find that with the decreasing of antenna cluster size V , the ergodic sum rate capacity is improved, but the

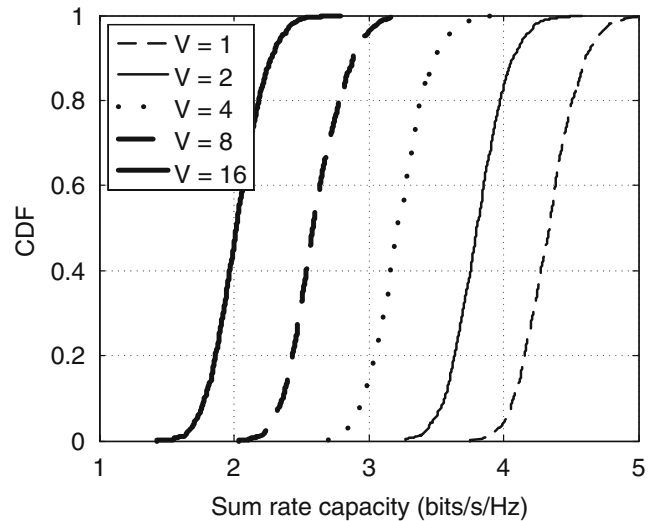


Fig. 7 CDF of the sum rate capacity for $M = 1$

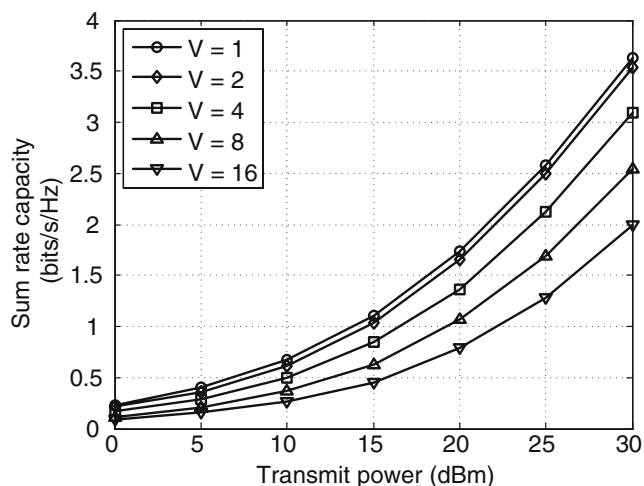


Fig. 8 Sum rate capacity for $M = 2$

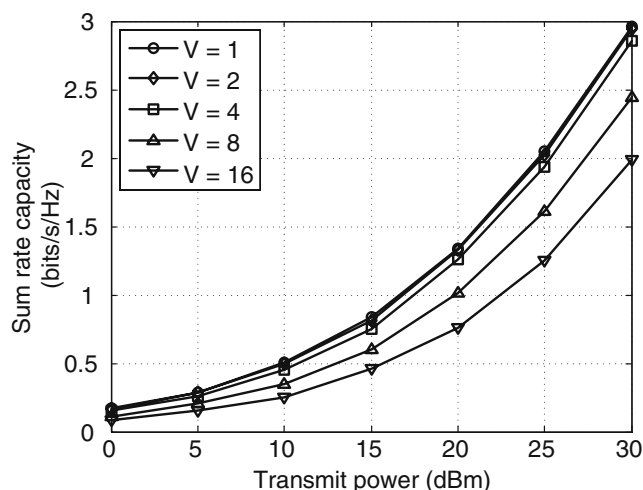


Fig. 10 Sum rate capacity for $M = 4$

CDF curve of the sum rate capacity only shifts rightwards without change of shape. This implies that only the ergodic system performance is enhanced benefiting from the access distance reduction.

For $M = 2$, the Monte Carlo simulation results are presented in Figs. 8 and 9. We can find that as V varies from 16 to 2, the ergodic sum rate capacity increases benefiting from the access distance reduction, while the corresponding CDF curve only shifts rightwards without change of shape. When V changes from 2 to 1, the ergodic sum rate capacity nearly remains a constant that equals to the value at $V = 2$, while the CDF curve becomes steeper instead of shifting. This implies that only the outage system performance is improved benefiting from the macro-diversity.

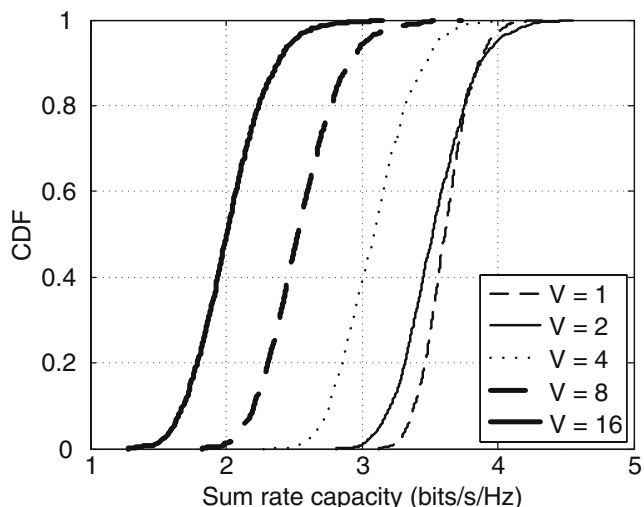


Fig. 9 CDF of the sum rate capacity for $M = 2$

Similar results can be found in Figs. 10 and 11 for $M = 4$. As V takes value from 16 to 4, benefiting from the access distance reduction, only the ergodic uplink sum rate capacity is improved. As V takes value from 4 to 1, benefiting from the macro-diversity, only the outage uplink sum rate capacity is improved.

Since the assumptions and constraints of this work are quit general, our analysis will be applicable for any other related DAS models. For example, considering an aforementioned CL-DAS [13], we can similarly obtain the ergodic uplink sum rate capacity and the corresponding CDF curves via Monte Carlo simulations. Consequentially, the observation will accord with the conclusions derived in this work.

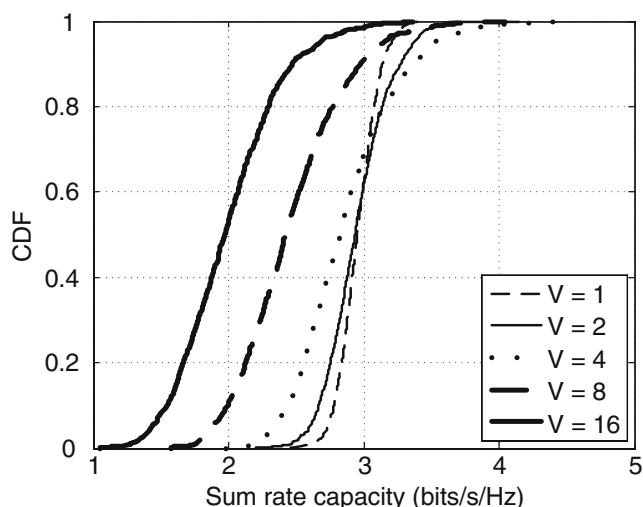


Fig. 11 CDF of the sum rate capacity for $M = 4$

5 Conclusions

In this work, we focus on the impact of the deployment of AEs on the system performance for a multi-user DAS. A generalized system model is proposed, where the AEs are divided into antenna clusters and the antenna clusters are randomly deployed in the coverage area. We characterize the deployment of AEs with two parameters: the antenna cluster size V and the distribution of the antenna clusters. Without loss of generality, the antenna clusters are assumed to be deployed with uniform distribution. Thus, the impact of the antenna cluster size on the system performance is particularly studied. With some algebraic manipulations, the MSAD, a function of the antenna cluster size, is proposed as a reasonable metric instead of the system uplink sum rate capacity. Then, we put our emphasis on the impact of the antenna cluster size on the MSAD. From the analysis of the asymptotic behavior of the MSAD, we derive approximate closed-form expressions for the first-order and the second-order statistics of the MSAD, respectively. Moreover, we find out the semi-quantitative relationship between the antenna cluster size and the statistics of the MSAD, which can directly indicate the impact of the antenna cluster size on the system uplink sum rate capacity.

The conclusions can be summarized as follows.

- (1) when $V \leq M$, the outage uplink sum rate capacity can be improved by scattering AEs further, benefiting from the macro-diversity.
- (2) when $V \leq M$, the ergodic uplink sum rate capacity can not be improved by scattering AEs further.
- (3) when $V > M$, the ergodic uplink sum rate capacity can be improved by scattering AEs further, benefiting from the access distance reduction.
- (4) when $V > M$, the outage uplink sum rate capacity can not be improved by scattering AEs further.

Therefore, when $V \leq M$, benefiting from only the macro-diversity, a DAS outperforms a CAS in terms of outage system performance; when $V > M$, benefiting from only the access distance reduction, a DAS outperforms a CAS in terms of ergodic system performance.

As to realistic networks, if we only care about the ergodic system performance, then, based on our remarks, it is enough to configure the network with $V = M$. We need not scatter the AEs absolutely into an \mathcal{A} -DAS,

which will occupy more geographical sites and increase the system penalty.

Finally, it is worth mentioning that the results derived here are applicable for other related DAS models. In addition, the issues of optimizing the deployment of AEs with the given MTs equipped with different number of antennas, as well as optimizing the deployment of AEs with consideration of network hotspots are interesting possible future extension directions.

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