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DEFORMATION FEATURES AND MODELS OF $[\pm 45]_{2s}$ CROSS-PLY FIBER-REINFORCED PLASTICS IN TENSION

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Series of experiments on $[\pm 45]_{2s}$ cross-ply carbon-fiber-reinforced plastic specimens were carried out in tension with various loading programs. In analyzing stress–strains relations, the material was considered homogeneous. The total axial strain is presented as the sum of instantaneous residual (irreversible), nonlinear reversible, irreversible creep, and reversible creep strains. To separate the last two components, the hypothesis that their rates at different instants of time are different is used. Together with a generalized Kachanov hypothesis, this allowed us first to obtain equations for increments of only the viscoelastic strain. Further, equations in which only the viscoplastic strain is unknown are written, and only then the secant elastic modulus is determined. Questions of the choice of relations for describing strain components and the problem on identification of parameters of the relations are considered. Experimental data and results of their processing are presented, and they testify to the acceptability of the assumptions used and the efficiency of the approaches proposed.

Introduction

To problems connected with creep strains, much attention has been given by many researchers (see, in particular, [1-20]). A number of different theories for describing the rheological properties of materials (Voigt, Maxwell, and Kelvin models and theories of aging, flow, hardening, and linear and nonlinear heredity) have been proposed. To predict creep

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strains over long enough intervals of time, various methods based on mathematical analogies (temperature−time, stress−time, etc. ones, see, for example, [5]) have been developed.

An analysis of works devoted to experimental investigations into the deformation processes of fibrous composites shows that the creep in shear arises already at times measurable in minutes and even seconds. Specimens of reinforced fibrous composites made from carbon fiber tapes with the lay-up $[\pm 45]_{2s}$ (2s is the number of monolayers), in tension and compression, obtain axial strains, which, in orthotropy axes, are mostly caused by shear strains. Therefore, it suffices to use exactly this type of testing for a qualitative shear analysis, instead of complex experiments on cross-shaped specimens.

To describe the creep strains of fibrous composites, the model of a viscoelastic (hereditary elastic) material is considered as the most adequate for experiments, which has been confirmed by experimental results. The weakly singular Abel creep kernel is often used to describe them. A peculiarity of the hereditary elastic model is that, after unloading, the creep strains disappear in the limit. However, our experiments showed that a part of the creep strains still remained. This part of creep strains has to be described not by relations of the hereditary elasticity theory, but by an incremental creep theory of the type of aging theory (i.e., in the differential form), according to which, after unloading, the accumulated creep strains are irreversible.

To determine the creep strains and parameters of its models, most convenient is an experiment carried out over a long period of time under a constant load. In this case, the time-dependent part of the total strain can be separated out. However, it is impossible to separate the hereditary elastic and irreversible creep parts of the strain.

Further, one of approaches to solving the problem of separation of these strains is proposed. The question how to determine the initial elastic modulus is considered, and the problem on isolation of the nonlinear reversible part of strains is discussed.

1. Experimental results

To clarify the structure of creep strains, in addition to the results of found in [21-24], two series of experiments in tension of test specimens made of cross-ply reinforced fibrous composites were carried out. Specimens with the lay-up $[\pm 45]_{2s}$ ($2s = 4$) were made from a unidirectional fibrous composite based on an ELUR-P carbon fiber tape and an HT-118 binder of cold curing. Their average thickness $h = 0.56$ mm, width $b = 24.6$ mm, and length of the working part $l = 110$ mm. In such specimens, in tension (compression), most pronounced are creep properties, and, therefore, they were subjected to testing in two loading programs.

The first program consisted of three stages: tension to a maximum stress $\sigma_{\max}$ by a kinematic loading with a rate of 0.68 MPa/s, free unloading to $\sigma = 0$ or 1.5 MPa (in order to exclude the possible bending strains and occurrence of noise in tensodynameter readings), and subsequent holding for 24 h at $\sigma = 1.5$ MPa. The experiments were carried out on different specimens at $\sigma_{\max} = 35$, 45, and 55 MPa.

The second loading program consisted of four stages: tension to $\sigma_{\max} =$ 45 MPa by a kinematic loading at a rate 0.68 MPa/s, holding during t_h h, free unloading to $\sigma = 1.5$ MPa, and subsequent holding for three days. These tests were also carried out on four specimens, with $t_h = 0$, 2, 5, and 10 h.

The experimental results obtained in the first loading program were presented in the form of superimposed stress–strain diagrams $\sigma = \sigma(\varepsilon)$ (Fig. 1a). At all loading stages, the corresponding branches of the diagrams, for all three specimens, with a high degree of accuracy, turned out to lie on the same curve, but at the subsequent stages, after unloading and holding up to the total disappearance of hereditary elastic strains, different values of residual strains were recorded (Table 1). From this fact, it can concluded that, in addition to the elastic and hereditary elastic components, irreversible, although small, residual strains ε^r had actually developed. They could consist of both irreversible creep and

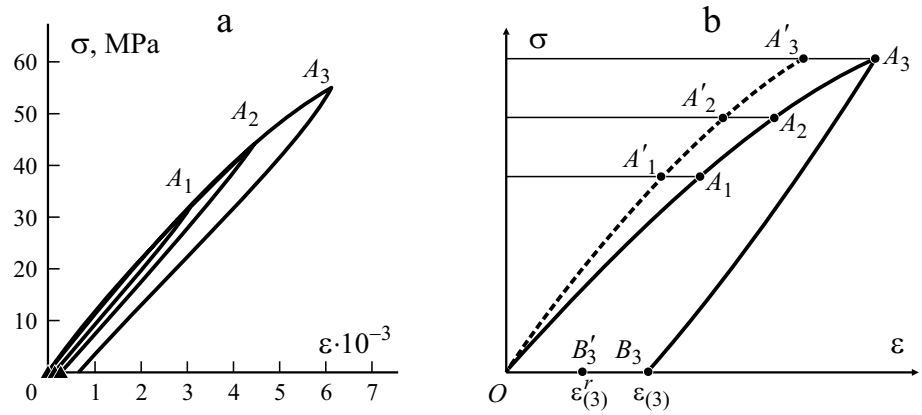

Fig. 1. Combined diagrams of loading $\sigma = \sigma(\varepsilon)$ at three values of $\sigma_{\max}$ (a) and a schematic for determining the residual strains (b).

TABLE. 1.

$\sigma_{\max}$, MPa	$\varepsilon^r_{(i)} \cdot 10^4$
35	0.44
45	1.13
55	2.61

instantaneous irreversible strains caused by various reasons (plasticity, microdamage, etc.). In cyclic loadings, as it was established earlier in [21-23], the residual strains, at each *i*th loading cycle, obtained increments tending to a constant value by the end of some *N*th cycle (also see [25, 26]).

In Fig. 1b, schematically repeating Fig. 1a in changed scales, shown are the points A_1, A_2, and A_3 corresponding to the end of the first loading stages, the points B_3 and B_3' corresponding to the end of the second and third stages, and the segments A_1A_1', A_2A_2', and A_3A_3' corresponding to the indicated $\sigma_{\max}$ and found from the recorded experimental values of $\varepsilon_{(i)}$ and $\varepsilon^r_{(i)}$ in accordance with the equalities $A_iA_i' = \varepsilon^r_{(i)}$, where $i = 1,\ 2,$ and 3 correspond to $\sigma^{(1)}_{\max} = 35$ MPa, $\sigma^{(2)}_{\max} = 45$ MPa, and $\sigma^{(3)}_{\max} = 55$ MPa. Connecting the origin *O* of coordinates and the points A_1', A_2', and A_3' by some approximation curve, an experimental deformation diagram for a specimen, without account of its residual strains, can be obtained. Let us describe it with the relation (the superscript "+" indicates the loading stage)

$$\varepsilon^+ = \varepsilon^+(\sigma, t). \tag{1.1}$$

By virtue of the fact that, after unloading of the specimen to $\sigma = 0$ and holding it for a long time, a residual irreversible strain ε^r developed in it, which, according to the data of Table 1, could be approximated as

$$\varepsilon^r = \varepsilon^r(\sigma, \dot{\varepsilon}), \tag{1.2}$$

the total strain ε arising in the specimen at the loading stage could be represented as the sum

$$\varepsilon^+ = \varepsilon^r(\sigma) + \frac{\sigma(t)}{E(\sigma, \dot{\varepsilon})}. \tag{1.3}$$

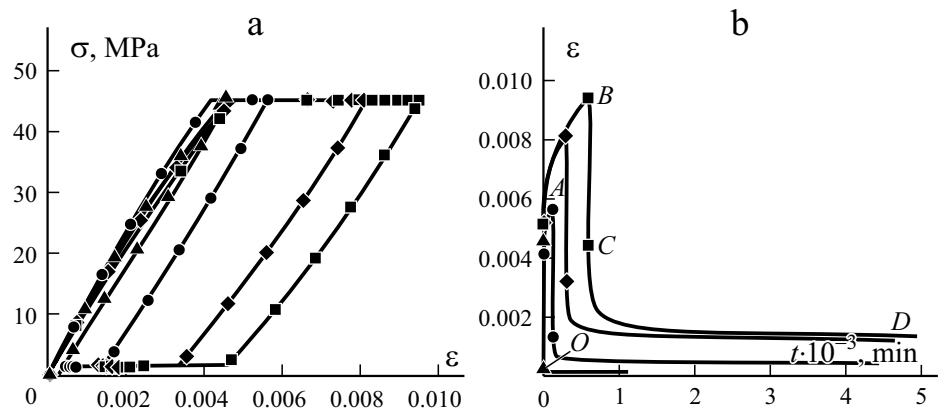

Fig. 2 Deformation diagrams $\sigma = \sigma(\varepsilon)$ (a) and $\varepsilon = \varepsilon(t)$ (b) obtained in the second loading program at t_h = 0 (▲), 2 (●), 5 (♦), and 10 h (■). Explanations in the text.

TABLE. 2.

$\varepsilon^r_{(T)}$	t_h, h			
	0	2	5	10
$\varepsilon^r_{(10)} \cdot 10^3$	0.11	0.53	1.46	1.71
$\varepsilon^r_{(24)} \cdot 10^3$	0.11	0.50	1.34	1.53
$\varepsilon^r_{(48)} \cdot 10^3$	0.11	0.47	1.30	1.39
$\varepsilon^r_{(72)} \cdot 10^3$	0.11	0.46	1.25	1.35
$\varepsilon_{\max} \cdot 10^3$	4.56	5.64	8.15	9.46

Here, the function $E(\sigma, \dot{\varepsilon})$, having the meaning of secant elastic modulus, can be determined on the basis of experimental relation (1.1). It depends not only on the stress level, but also, as established in [21-23], on the strain rate $\dot{\varepsilon}$.

Experiments showed (see [21-23] and Fig. 1b) that the relation $\sigma^+ = \sigma^+(\varepsilon^+)$ inverse to (1.1) had the first derivative that decreased with increasing ε^+. Therefore, even without account of creep strains, the tangent modulus of elasticity (and hence the secant one), which can be determined from the diagram $\sigma^+ = \sigma^+(\varepsilon^+)$, decreased with increasing σ. Such a decrease in the elastic modulus can be explained not only by the rearrangement of composite structure caused by binder cracking (such models are considered, for example, in [13, 14, 27-29]). It can also be caused by the strains ε^{nel} of microrearrangement due to the realization and continuous changes of internal buckling modes [30, 31] with permanent variations in the parameters of wave formation at the loading stage. The diagram of such strains is nonlinear at the loading stage, but the strains ε^{nel} disappear after unloading. The residual strains ε^r can contribute to the reduction in the secant modulus due to the break of bonds between the fibers and matrix (degradation at the weakest points of composite), which are not recovered at the unloading stage.

The test results obtained in the second loading program are presented in Fig. 2a in the form of stress–strain diagrams $\sigma = \sigma(\varepsilon)$, and values of the residual strains $\varepsilon^r_{(T)}$ are given in Table 2 for $t_h = 0$, 2, 5, and 10 h. The values

of $\varepsilon^r_{(T)}$ were recorded for each specimen at t = 10, 24, 48, and 72 h after their unloading in the conditions of holding at $\sigma = 1.5$ MPa.

The characteristic relation $\varepsilon = \varepsilon(t)$, with various holding times t_h (in hours) under the stress $\sigma = 45$ MPa, is shown in Fig. 2b, where the sections *OA*, *AB*, *BC*, and *CD* of the curve correspond to the loading stage, holding at the second stage, free unloading to the stress $\sigma = 1.5$ MPa at the third stage, and holding more than a day after unloading, respectively. Values of the strains developed at the end of the second loading stage (corresponding to the point *B* in Fig. 2b) are shown in the last line of Table 2. These results also indicate the presence of irreversible creep strains of the composite.

An analysis of the results obtained shows that the values of residual strains satisfy the inequalities $\varepsilon^r\big|_{t_h=0} < \varepsilon^r\big|_{t_h=2} < \varepsilon^r\big|_{t_h=5} < \varepsilon^r\big|_{t_h=10}$ for all values of holding time at the second loading stage (the value $\varepsilon^r_{(1)} = 0.11 \cdot 10^{-3}$ corresponding to t_h = 0 h was found by the method described in [21-23]). Consequently, the quantity ε^r, in contrast to the assumption accepted and representation (1.2), in the loading program considered, was a function not only of the stress level σ and strain rate $\dot{\varepsilon}$, but also of time *t*, i.e.,

$$\varepsilon^r = \varepsilon^r(\sigma, \dot{\varepsilon}, t). \tag{1.4}$$

Let us assume that this type of function ε^r is caused not only by degradation of the material due to the break of bonds between the fibers and matrix at the weakest points in the first loading stage, but also by the formation of irreversible creep strains of matrix material of the composite. Such assumptions allow us to represent relation (1.6) as

$$\varepsilon^r = \varepsilon^R(\sigma) + \varepsilon^r_\partial(\sigma, t). \tag{1.5}$$

Here, $\varepsilon^R(\sigma)$ is the irreversible instantaneous strain and ε^r_∂ is the irreversible creep strain.

2. Identification of the parameters of rheological models at long-term loading

In the general case, the governing relations (written, for example, in the form of relations between the components of strain and stress tensors) will contain, as arguments, three invariants of the stress tensor even in the case of plane stress state. Their construction requires a large number of different experiments on specimens with different layer stacking angles and special methods for their processing. There are works in which various hypotheses are used to reduce the number of arguments, or, for example, their simplification on the basis of an analysis using the features of material properties. For example, this was done in [12, 32, 33], where a review of studies in this direction is also given. Some other variants of such relations can be found, for example, in [13-15, 34-38]. In this case, for determining the parameters of deformation models from experiments, it is sometimes necessary to use various methods of regularization of the identification problem (see, for example, [15, 39, 40]).

Further, we will consider only some features of the problem on constructing physical relations and only for the simplest, one-dimensional case, namely, the problem on constructing relations connecting the axial stress and the total longitudinal strain ε (relative elongation of specimen) with the following components: the hereditary elastic (viscoelastic) reversible ε^{v}, irreversible creep ε^r_∂, nonlinear reversible ε^{nel}, and instantly irreversible ε^R strains. To solve this problem, we introduce a generalization of Kachanov hypothesis [41], according to which the creep, viscoelastic, instantly reversible, and irreversible parts of strain develop independently of each other and depend only on the level of stresses, i.e.,

$$\varepsilon^r_\partial = \varepsilon^r_\partial(\sigma, t),\ \varepsilon^{\mathrm{v}} = \varepsilon^{\mathrm{v}}(\sigma, t),\ \varepsilon^{nel} = \varepsilon^{nel}(\sigma),\ \varepsilon^R = \varepsilon^R(\sigma).$$

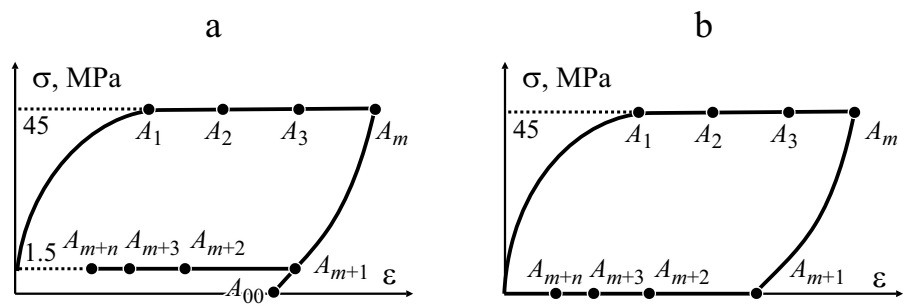

Fig. 3. Deformation diagrams $\sigma = \sigma(\varepsilon)$ at the fourth stage (in changed scales): at $\sigma = 1.5$ (a) and 0 MPa (b).

Based on this hypothesis and on an analysis of the above-mentioned experimental results, it can be assumed that, in tension according to the second loading program, the total axial strain includes the sum of the components

$$\varepsilon = \frac{\sigma}{E_0} + \varepsilon_{\partial}^{r}(\sigma,t) + \varepsilon^{\mathrm{v}}(\sigma,t) + \varepsilon^{nel}(\sigma) + \varepsilon^{R}(\sigma)\,, \tag{2.1}$$

$$\varepsilon^{\mathrm{v}} = \int_0^t f(\sigma)\, H(\sigma, t-\tau)\, d\tau\,, \tag{2.2}$$

$$d\varepsilon_{\partial}^{r} / dt = F(\sigma, \varepsilon_{\partial}^{r})\,, \tag{2.3}$$

where H is the creep kernel and E_0 is the initial elastic modulus.

Thus, within the framework of representation (2.1), it can be assumed that, after complete unloading (at $\sigma = 0$), the accumulated creep strains $\varepsilon_{\partial}^{r}(\sigma,t)$ remain unchanged, while the hereditary elastic strains $\varepsilon^{\mathrm{v}} = \varepsilon^{\mathrm{v}}(\sigma,t)$ decrease and disappear in the limit after an unlimited period of time.

Let us now turn to the identification of parameters of the models of hereditary deformation and irreversible creep. Further reasonings will be based on an analysis of results of the experiments carried out according to the second loading program (see Fig. 3).

First, in time t_1, specimens are loaded up to the stress $\sigma = \sigma_{\max}$ (in Fig. 3, this corresponds to the point A1), and then it is held under this stress during the time $\Delta t_{\mathrm{m}} = t_{\mathrm{m}} - t_1$.

Let us denote the experimental values of the strains $\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots$ at the points $A_1, A_2, A_3, \ldots$ obtained at the instants of time $t_1, t_2, t_3, \ldots$ under the stress $\sigma = \sigma_{\max}$. Then, at $t = t_m$, specimens are unloaded to $\sigma = \sigma_{\min}$ during the time $\Delta t_n = t_{m+n} - t_{m+1}$.

Formally, relations in which only the strains of hereditary elasticity ε^{v} remain can be obtained. For this purpose, the specimen has to be completely unloaded to $\sigma_{\min} = 0$ (see Fig. 3b) and expressions for strain increments have to be written. Since, after complete unloading at zero stresses, the strains ε^{nel}, $\varepsilon^{R}(\sigma)$, and $\varepsilon_{\partial}^{r}(\sigma,t)$ remain constant in time, we have

$$\Delta_{ij}\varepsilon = \varepsilon_i - \varepsilon_j = \varepsilon^{\mathrm{v}}(0,t_i) - \varepsilon^{\mathrm{v}}(0,t_j), \quad i,j > m\,. \tag{2.4}$$

In the experiment, know usually is the law of stress variation in the specimen,

$$\sigma = \varsigma(t)\,. \tag{2.5}$$

Then, expressions for $\varepsilon_i^{\mathrm{v}}$ can be presented as

$$\varepsilon_i^{\mathrm{v}} = \int_0^{t_i} f(\varsigma(\tau))\, H(\varsigma(\tau), t_i - \tau)\, d\tau \,. \tag{2.6}$$

Knowing the values of strains measured at $\sigma_{\min} = 0$ at the instants of time $t_i, t_j \geq t_m$, we obtain relations containing only the creep kernel, namely,

$$\begin{aligned} \Delta_{ij}\varepsilon^{\exp} &= \varepsilon_i^{\exp} - \varepsilon_j^{\exp} = \varepsilon_i^{\mathrm{v}} - \varepsilon_j^{\mathrm{v}} \\ &= \int_0^{t_i} f(\varsigma(\tau))\, H(\varsigma(\tau), t_i - \tau)\, d\tau - \int_0^{t_j} f(\varsigma(\tau))\, H(\varsigma(\tau), t_j - \tau)\, d\tau, \qquad t_i, t_j \geq t_m. \end{aligned} \tag{2.7}$$

Further, some assumptions about the form of governing relations (2.2) have to be accepted. Many studies have shown that, at moderate operating stresses and their small variations, the linear theory of hereditary elasticity can be used (in particular, from milliseconds to thousands of hours by using the Abel kernel [42]). Then

$$f(\sigma) = \sigma, \qquad H(\sigma, t-\tau) = H(t-\tau) \,. \tag{2.8}$$

We approximate $H(t-\tau)$ by some system of functions H_k:

$$H(t-\tau) = \sum_{k=1}^{K} H_k(\alpha_k, t-\tau) \,. \tag{2.9}$$

Let the number K be not greater than the number of different equations (2.7). Then, to determine the mechanical characteristics α_k, an overdetermined system of algebraic equations follows from Eq. (2.7) in the general case. Formally, the constants α_k can be found minimizing the quadratic residual of this system of equations, i.e., the creep kernel H can be determined.

However, experiments have shown that, performing complete unloading, it is not possible to obtain sufficiently accurate values of strains on the section A_{m+1}, A_{m+2}, owing to the presence of various "noises" (backlash of equipment, residual bending strains of specimens owing to the misalignment of grips). At the same time, the values of ε^{v} after complete unloading can formally be obtained by extrapolation (i.e., by finding the strain at the point A_{00} in Fig. 3a). However, this would require repeated testing of the same specimen many times in the second program at different exposure times $\Delta t_{\mathrm{m}} = t_{\mathrm{m}} - t_1$, which can lead to a considerable scatter in experimental data. Therefore, the next challenge in identifying the rheological characteristics of the material studied is how to separate the viscoelastic ε^{v} and creep $\varepsilon_{\partial}^{r}(\sigma, t)$ strains in one experiment at $\sigma = \sigma_{\max}$. To do this, let us accept the experimentally confirmed hypothesis that the attenuation rate of irreversible creep strain increments are much higher than that of the hereditary elastic ones. This means that, at a constant stress $\sigma = \sigma_{\max}$, the increment of strains after a long period of time consists only of the increment of hereditary elastic strains, $\Delta\varepsilon^{\mathrm{v}}$. Therefore, at $\sigma = \sigma_{\max}$, with a small error, a relation coinciding with Eq. (2.7) can be used, but only at $t_m > t_i, t_j \gg t_1$, namely,

$$\Delta_{ij}\varepsilon^{\exp} = \varepsilon_i^{\exp} - \varepsilon_j^{\exp} = \varepsilon_i^{\mathrm{v}} - \varepsilon_j^{\mathrm{v}}, \quad t_i, t_j < t_m \tag{2.10}$$

We should emphasize that here, in contrast to Eq. (2.7), the integration times t_i and t_j correspond to holding the specimen under the maximum stress, which makes it possible to determine, with a sufficient accuracy, the left side of

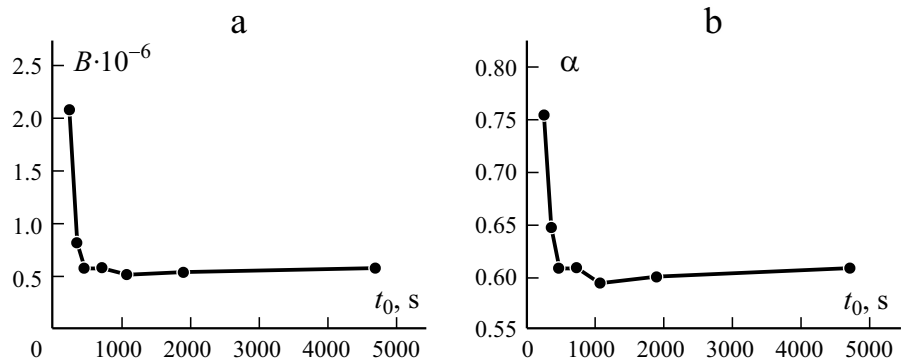

Fig. 4. Values of the parameters B (a) and α (b) identified on the interval of time [t_0, $t_{\max}$] at $\sigma_{\max} = 45$ MPa ($t_{\max}$ = 18,000 s = 5 h).

relations (2.10). Writing them for different times t_i and t_j, we obtain a system of equations. Minimizing its quadratic residual, the constants α_k of approximating expressions (2.9) for the creep kernel H can be found.

In the analysis of our experiments, the Abel kernel

$$H(t-\tau) = \frac{B}{(t-\tau)^{\alpha}}, \quad 0 < \alpha < 1, \quad B > 0, \tag{2.11}$$

was used in the governing relations (2.2).

The parameters of relation (2.11) were identified using test results at $\sigma = \sigma_{\max}$ = 45 MPa. In Eqs. (2.11), it was assumed that $t_j = \text{const} = t_0$ and $t_i > t_0$.

Different selection variants of t_0 in Eqs (2.10) were considered. Identification results for the parameters B and α are shown in Fig. 4. It follows from them that, indeed, at long holding times, the behavior of the material studied can be described by the model of a hereditary elastic material (2.11) alone, because the parameters B and α of the creep kernel are stabilized.

As already mentioned above, irreversible strains, comparable with the hereditary elastic ones, were revealed in long-term experiments. We will describe them by relations (2.3), taking into account that, as it follows from an analysis of the results shown in Fig. 4, their growth attenuated after $t = t_0 \sim 1000$ s. A number of different functions $F(\sigma, \varepsilon_{\partial}^{r})$ were analyzed. However, it was not an easy task to obtain a model that would give an attenuation rate of the strain ε_0^r much greater than that of ε^{v}. Since the variant with only one value of $\sigma_{\max}$ for the holding stress was considered, the aging theory in the simplest version with respect to stresses was chosen, namely,

$$d\varepsilon_{\partial}^{r} / dt = \sigma\, \psi(\varepsilon_{\partial}^{r}). \tag{2.12}$$

Further, numerical experiments were carried out using various kinds of functions $\psi(\varepsilon_{\partial}^{r})$, one of which was adopted in the form

$$d\varepsilon_{\partial}^{r} / dt = \chi_0 \sigma / (1 + \chi_1 (\varepsilon_{\partial}^{r})^{m}). \tag{2.13}$$

Here, m, χ_0, and χ_1 are the sought-for constants. Since the strains ε^{nel} and ε^{R} are constant at the points $A_1, A_2, A_3, \ldots$, relations similar to (2.7), but taking into account the irreversible creep strains, can be used, namely,

$$\Delta_{ij}\varepsilon = [\varepsilon^{\text{v}}(\sigma_{\max}, t_i) + \varepsilon_{\partial}^{r}(\sigma_{\max}, t_i)] - [\varepsilon^{\text{v}}(\sigma_{\max}, t_j) + \varepsilon_{\partial}^{r}(\sigma_{\max}, t_j)]. \tag{2.14}$$

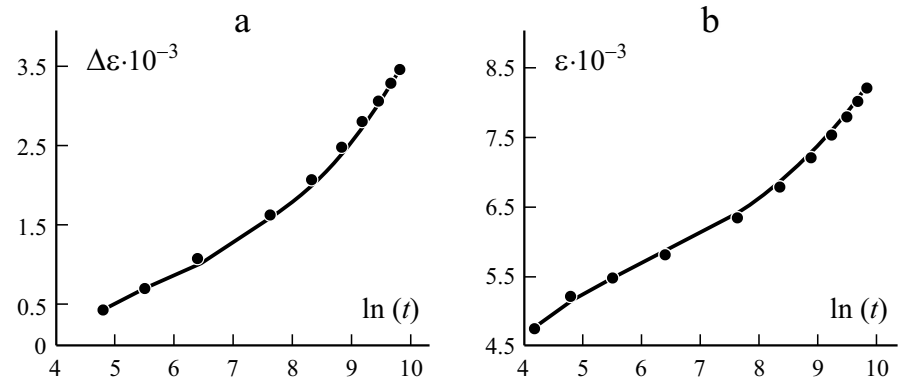

Fig. 5. Values of $\Delta_{i1}\varepsilon^{\exp}$ and $\Delta_{i1}\varepsilon$ (a) and ε_i (b). Explanations in the text.

Here, the strains $\varepsilon^{\mathrm{v}}(\sigma_{\max}, t_j)$ can be calculated by relations (2.2) and (2.11).

Minimizing the quadratic residual between the experimental values of $\Delta_{ij}\varepsilon^{\exp}$ and $\Delta_{ij}\varepsilon$ calculated by Eqs. (2.14) for all holding times t_i of specimen at the stress $\sigma_{\max}$, the parameters m, χ_0, and χ_1 appearing in relations (2.13) for the irreversible creep strains can be determined.

Processing results for experiments at $\sigma = \sigma_{\max}$ = 45 MPa on the segment A_1–A_m (see Fig. 5) confirmed the acceptability of the assumptions used and the efficiency of the approach proposed for determining the mechanical characteristics of the composite studied.

After identification of the rheological characteristics of the material, the secant elastic modulus $E(\sigma, \dot{\varepsilon})$ can be found from the condition of closeness of experimental and calculated values of total strains.

For the material considered, identification of the rheological characteristics and the secant elastic modulus for models (2.11)-(2.13) by the above-described method gave the following results:

$$B = 6.0878 \cdot 10^{-7}\ \mathrm{s}^{\alpha-1}/\mathrm{MPa},\ \alpha = 0.6151,\ E(\sigma_{\max}, \dot{\varepsilon}) = 10{,}700\ \mathrm{MPa},$$

$$\chi_0 = 1.2485 \cdot 10^{-7}\ (\mathrm{s} \cdot \mathrm{MPa})^{-1},\ \chi_1 = 1.8764 \cdot 10^{4},\ m = 21.1. \tag{2.15}$$

For the purposes of illustration, the values of $\Delta_{i1}\varepsilon^{\exp}$ obtained in the experiment (indicated by markers) and their calculated values $\Delta_{i1}\varepsilon = \varepsilon_i - \varepsilon_1$ are presented in Fig. 5a, but the values of total strains are shown in Fig. 5b.

To determine the initial elastic modulus E_0, it is necessary first to find the law of variation in the instantaneous irreversible strain $\varepsilon^R(\sigma)$. To do this, the strain ε_{00} at the point A_{00} at the instant of complete unloading has to be found at various levels of the maximum stress. This can be done by extrapolating the curve $A_m - A_{m+1}$ (see Fig. 3). Then, the irreversible creep strains $\varepsilon^r_{\partial}(0,\ t_{00})$ and the hereditary elastic $\varepsilon^{\mathrm{v}}(0,\ t_{00})$, which can be calculated at any instant of time by relations (2.2), (2.11), and (2.13), have to be subtracted from it. This difference is equal to $\varepsilon^R(\sigma)$,

$$\varepsilon^R(\sigma) = \varepsilon_{00} - \varepsilon^r_{\partial}(0, t_{00}) - \varepsilon^{\mathrm{v}}(0, t_{00}). \tag{2.16}$$

The results of an analysis of experimental data by relation (2.16) showed that, for the material studied, the values of $\varepsilon^R(\sigma)$ were by two orders of magnitude smaller than the total strains at the beginning of holding specimens under the maximum stress. They were comparable with the accuracy of strains measurements, which led to their wide scatter, even to negative values. From this fact, we can conclude that the instantaneous irreversible strains $\varepsilon^R(\sigma)$ in relation (2.1) can be neglected.

TABLE. 3.

σ_{max}, MPa	$\varepsilon^{nel} \cdot 10^4$
35	2.356
45	6.883
55	10.105

Therefore, the elastic modulus E_0 can be determined from the increments of stresses and of the elastic part of strains on a small initial interval of time. To determine the latter ones, the viscoelastic $\varepsilon^{v}(\sigma,t)$ and irreversible creep $\varepsilon_{\partial}^{r}(\sigma,t)$ strains were subtracted from the total strains. They were calculated by relations (2.2), (2.3), (2.6), (2.11), (2.13), and (2.15), which were found from an analysis of the results obtained at long holding of specimens under the maximum stress.

We should note the following fact observed in numerical experiments performed using parameters (2.15) of the creep kernel. For viscoelastic bodies described by models with weakly singular creep kernels, the increment $\Delta\varepsilon^{v}$ of viscoelastic strain, even at rather high loading rates, can make several tens of percent of the total strain increment $\Delta\varepsilon$. Therefore, determination of the elastic modulus E_0 from the relation (in accordance with known standards, for example, GOST 25.601, 25.603, 9550, 23805)

$$E_0 = \frac{\Delta\sigma}{\Delta\varepsilon} \tag{2.17}$$

gives underestimated values. In particular, it turned out that, after 2s in a full-scale experiment, (the loading rate was 0.68 MPa/s), a growth in the stress by only 1 MPa increased the viscous component by $\Delta\varepsilon^{v}$, which made 18% of the total strain increment $\Delta\varepsilon$.

In addition, the value of $\Delta\mu_{\partial}^{r}$ could also be considerable. Therefore, it is advisable to carry out experiments so that to obtain first the mechanical characteristics of relations (2.2) and (2.3). For viscoelastic bodies, such approaches to determining relations (2.2) are described, for example, in [4].

Further, it was assumed that, at low stresses, the elasticity relations are close to linear ones, i.e., the strain $\varepsilon^{nel}(\sigma)$ can be neglected, and the initial elastic modulus was obtained from relation (2.17):

$$E_0 = \Delta\sigma / (\Delta\varepsilon - \Delta\varepsilon^{v} - \Delta\varepsilon_0^{r}) = 12{,}410 \text{ MPa}. \tag{2.18}$$

Let us consider the problem on constructing a relation for the reversible strains ε^{nel} as functions of stresses. The strains ε^{nel} can be caused, for example, by the occurrence of one or another buckling mode of phases of a fiber composite [30, 31] or by the initiation of microcracks [27, 28, 29]. In both cases, significant strains can arise only when stresses exceed a certain critical value. An analysis of experimental data confirms this assumption.

To establish the relation $\varepsilon^{nel}(\sigma)$, the experiments results obtained at stresses of 35, 45, and 55 MPa were used, and the strains ε^{nel} were found from the relation

$$\varepsilon^{nel}(\sigma) = \varepsilon - \left(\frac{\sigma}{E_0} + \varepsilon_{\partial}^{r}(\sigma,t) + \varepsilon^{v}(\sigma,t) \right).$$

Calculation results are presented in Table 3.

Various types of approximations $\varepsilon^{nel} = \varepsilon^{nel}(\sigma)$ were tested. An analysis of the results of numerical calculations led to the conclusion that a piecewise-continuous function, similar to that in the elastic-plastic deformation, can be employed. Namely, up to a certain value $\sigma = \sigma_{kr.}$, these strains are absent, but then they develop as functions of stresses.

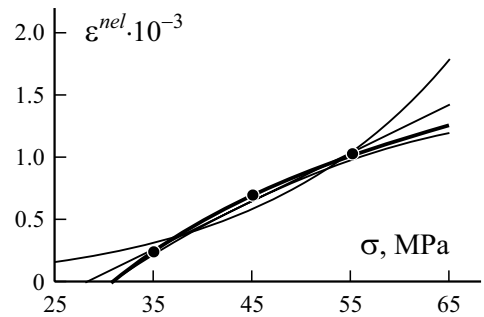

Fig. 6. Combined plots of different approximations $\varepsilon^{nel} = \varepsilon^{nel}(\sigma)$. Explanations in the text.

Of all the functions considered, the most successful was a hyperbolic one (which is shown by the bold line in Fig. 6). Approximation by a second-order polynomial gave a function that began to decrease after $\sigma \approx 70$ MPa (the thin line with a negative curvature in Fig. 6). A similar situation arose in approximation by a third-order polynomial. Linear and exponential functions gave much larger errors than the hyperbolic one (the upper thin lines in Fig. 6), which was found in the form

$$\varepsilon^{nel} = a + c\sigma / (1 + b\sigma), \quad a = -0.006315, \quad c = 0.0006433 \text{ MPa}^{-1}, \quad b = 0.06964 \text{ MPa}^{-1}, \quad \sigma_{kr} \approx 31 \text{ MPa}.$$

Conclusions

An analysis of experimental data corresponding to a long-term holding of specimens after unloading showed they had obtained irreversible residual strains. It was assumed that they had been caused not only by viscoelastic strains, but also, as the experiments showed, by irreversible creep strains. In constructing relations connecting various components of the total axial strain (stretch ratio) to the axial stress, a generalized Kachanov hypothesis was used, according to which the creep, viscoelastic, and instantaneous reversible parts of the total strain develop independently of each other. When developing a method for determining the rheological characteristics of a material, only test results for the specimens held under a constant load for a long time were used. In this case, only the differences of total strains were analyzed, which enabled us to exclude all other components of the strain.

To separate the hereditary elastic reversible strains from the irreversible creep strains, it was assumed that the rate of the latter ones decreased faster with time. Therefore, identifying the parameters of creep kernel of the viscoelastic model was reduced to analyzing test results after a sufficiently long time of holding specimens under a constant load. This assumption was confirmed by processing results of experimental data. The mechanical characteristics of irreversible creep strains were then determined by analyzing test results immediately after the beginning of holding specimens under a constant load. Thereafter, it was possible to find the secant elastic modulus.

A further analysis of experimental results, with regard to the relations obtained for the rheological components, allowed us to believe that the instantaneous irreversible strain was negligible. Therefore, at the next stage, the initial elastic modulus was determined from experimental data found at the initial stage of loading at low stresses. It was found that, even at low stresses and small their increments, it was necessary to subtract the creep and viscoelastic strains from the total ones, because changes in the viscoelastic strains can make several of tens of percent of the increment of total strains even at rather high loading rates if singular creep kernels are used.

At the last stage, after determination of the rheological characteristics and the initial elastic modulu, the nonlinear reversible part of the total strain, whicht can be caused by microrearrangements in the composite structure, for example, owing to the loss of stability of its phases, cracking of binder, etc, can be separated out. Based on an analysis of experimental data, the well-known conclusion that this part of the total strain arises only after reaching a certain critical stress was confirmed. Various forms of approximating functions for this component were considered, and the optimum one was chosen.

If test results for specimens with other layer stacking angles are known, the methods for identifying the mechanical characteristics can be used to determine relations between the parameters of stress strain state of a layer in orthotropy axes. The problem of constructing such governing relations is a separate problem. We should note that the results obtained in this work can also be used directly in calculating composite members (for example, thin-walled tubes of truss systems, in particular, spacecraft structures) manufactured by winding at the angles +45°/–45°.

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