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THE ONSET OF MIXED-MODE INTRALAMINAR CRACKING IN A CROSS-PLY COMPOSITE LAMINATE

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The intralaminar fracture toughness of a unidirectionally reinforced glass/epoxy composite is determined experimentally at several mode I and mode II loading ratios. The crack propagation criterion, expressed as a quadratic form in terms of single-mode stress intensity factors (alternatively, linear in terms of energy release rates), approximates the test results reasonably well. The mixed-mode cracking criterion obtained is used to predict the intralaminar crack onset in a cross-ply glass/epoxy composite under off-axis tensile loading.

1. Introduction

The failure process of continuous fiber-reinforced composite laminates in tension usually commences with the formation of intralaminar cracks. The cracks run along the fibers of the plies subjected to transverse tensile stresses. A particular case of cracking of transverse plies in a cross-ply laminate has already been studied in detail, see, e.g., the recent reviews [1-3]. The composite lay-ups typically considered are of the type $[0_n/90_m]_s$, in which, when loaded in tension along, e.g., the reinforcement direction of the outer plies, cracks normal to the loading direction develop in the inner transverse plies. The initiation of the transverse cracks is governed by the ply stress, whereas their propagation is determined by the mode I energy release rate (ERR) in the plies [4, 5]. Therefore, a strength criterion of failure is applied to thick transverse plies, where the cracking is initiation-controlled, and the critical ERR criterion is used for thin transverse plies, where crack propagation controls the cracking [6-10].

In composite laminates with more complex lay-ups and/or under combined loading, intralaminar cracks may develop in plies with different reinforcement directions. A review of experimental and modeling activities related to the off-axis ply damage in laminates is presented in [11]. Models for the stiffness reduction of a laminate with a complex crack system have recently been derived [12-19] and verified [20-23]. The development of intralaminar cracking of a unidirectionally reinforced (UD) ply under a complex stress state, comprising the in-plane tensile and shear stresses, in a composite laminate is considered in [12, 15, 16, 24-27]. An indispensable part of mixed-mode cracking models is the criterion of cracking, usually formulated in terms of mode I and II ERRs, G_{I} and G_{II}, and the corresponding critical ERRs, G_{Ic} and G_{IIc}. A number of such empirical crite-

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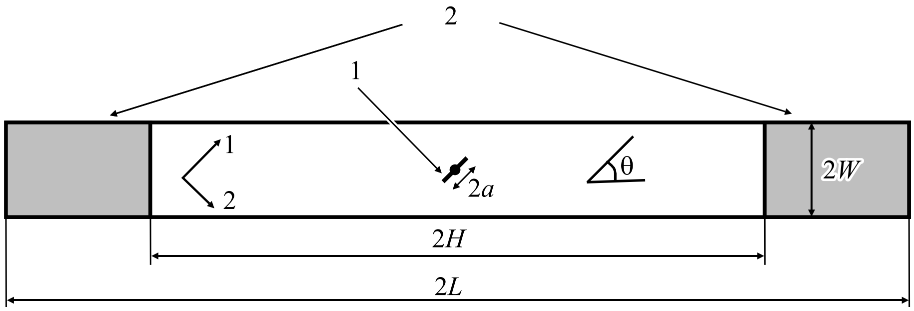

Fig. 1. Schematic of a UD specimen with a precrack parallel to the fiber direction (1) and end tabs (2).

ria have been proposed, for both the inter- and the intralaminar cracking, see, e.g., [28] and the references therein. Specifically, the criterion

$$\frac{G_{\mathrm{I}}}{G_{\mathrm{Ic}}}^{N} \qquad \frac{G_{\mathrm{II}}}{G_{\mathrm{IIc}}}^{M} \qquad 1 \tag{1}$$

with $N = 1$ and $M = 2$ has been applied in [25-27] to predict the crack onset in $[0/\ ]_s$ laminates. Using typical critical ERR values, a good agreement with the test results reported in [29] was obtained for the ply orientation angles 75° 90, while for 45 75 the model prediction markedly underestimated the crack onset strain (COS).

In the current study, the fracture toughness of a UD glass-epoxy composite is determined experimentally for mode I and mixed-mode loading conditions, and a mixed-mode fracture criterion is established. The latter is used to predict the COS of a cross-ply laminate under off-axis tension, the test results for which have been reported in [30].

2. Experimental

The material tested was an E-glass fiber/epoxy matrix composite. UD plates were prepared by hand lay-up from a UD prepreg VICOTEX NVE 913/28%/192/EC9756. The curing cycle comprised 30 min at 90°C followed by 60 min at 120°C under a 3- to 7-bar pressure. Rectangular specimens of length 220-240 mm and width 22-26 mm were cut from the plates under different angles to the ply reinforcement direction. Thus, specimens with lay-ups $[20_8]$, $[30_8]$, $[45_8]$, and $[90_8]$ were obtained. The specimen thickness was about 1 mm, and they were provided with GFRP end tabs, leaving a 120 to 140-mm-long test section.

A precrack was made in each specimen according to the procedure described below. First, a small-diameter hole was drilled in the middle of the specimen by using a 0.6-mm drill. Then, a miniature saw blade 0.2 mm thick was inserted in the hole, and a saw cut was made along the fibers in both directions from the hole for about 2.5 mm. As a result, a precrack of about 5 mm length, oriented in the fiber direction, was obtained in the central part of each specimen. A schematic picture of the specimens is shown in Fig. 1.

The specimens were subjected to tensile loading by using a Zwick/Roell Z2.5 testing machine. The load was measured by a 2.5-kN load cell. All the specimens were tested up to failure, with stroke control, at a 2-mm/min rate. In all cases, the failure occurred due to the unstable propagation of the precrack along fibers.

3. Fracture Toughness under Mixed-Mode Loading

In order to evaluate the fracture toughness of a UD composite, the applied stress at the onset of crack propagation has to be related to the mode I and mode II stress intensity factors (SIFs) of the crack. It has been shown in [31] that, for problems involving self-equilibrating loads, the SIFs for anisotropic materials are identical to those of isotropic ones. Hence, the SIFs for a crack of length $2a$ in an infinite anisotropic plate subjected to tension by a stress are

$$K_{\mathrm{I}} \quad \sqrt{\quad a}\sin^2 \quad , \quad K_{\mathrm{II}} \quad \sqrt{\quad a}\sin \quad \cos \quad , \tag{2}$$

where is the angle between the crack line and the loading axis. The effect of the finite size of test specimens necessitates corrections to the SIF values provided by Eq. (2). Such corrections have been obtained by a numerical analysis in [32] for UD glass/epoxy and carbon/epoxy composites having a crack parallel to the reinforcement. They are presented as functions of the reinforcement angle , the crack length to specimen width ratio a/W, and the specimen length to width ratio H/W, both for mode I and mode II SIFs. The limited specimen size leads to higher SIFs compared with those for an infinite plate.

The geometrical factors in the present case amount to $H/W \quad 5$ and $a/W \quad 0.2$. The respective corrections do not exceed a few percent for in the range of 30 to 90 considered in [32]. In the following, we will neglect the size effect, as its influence is small compared with the scatter of experimental data, and evaluate the SIFs by Eq. (2). The SIF values at the onset of crack growth are plotted in Fig. 2. They appear to comply with the quadratic criterion for crack propagation

$$\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^2 \quad \left(\frac{K_{\mathrm{II}}}{K_{\mathrm{IIc}}}\right)^2 \quad 1. \tag{3}$$

$[90_8]$ composite tests yielded $K_{\mathrm{Ic}} = 2.1 \quad 0.2$ MPa m$^{1/2}$. Approximating the data in Fig. 2 by Eq. (3), we obtained $K_{\mathrm{IIc}} = 3.6$ MPa m$^{1/2}$.

For a crack lying in the symmetry plane of an orthotropic material, the basic modes are independent [31]. Hence, the mode I and mode II ERRs depend only on the respective SIFs:

$$G_{\mathrm{I}} \quad K_{\mathrm{I}}^2 \left(\frac{a_{11}a_{22}}{2}\right)^{1/2} \left(\frac{a_{22}}{a_{11}}\right)^{1/2} \left(\frac{2a_{12} \quad a_{66}}{2a_{11}}\right)^{1/2},$$

$$G_{\mathrm{II}} \quad K_{\mathrm{II}}^2 \frac{a_{11}}{\sqrt{2}} \left(\frac{a_{22}}{a_{11}}\right)^{1/2} \left(\frac{2a_{12} \quad a_{66}}{2a_{11}}\right)^{1/2}, \tag{4}$$

where a_{ij} are the compliance matrix elements of the UD composite (with axis 1 along the crack, i.e., in the fiber direction, see Fig. 1). Crack propagation criterion Eq. (3), expressed in terms of ERRs, takes the form

$$\frac{G_{\mathrm{I}}}{G_{\mathrm{Ic}}} \quad \frac{G_{\mathrm{II}}}{G_{\mathrm{IIc}}} \quad 1, \tag{5}$$

and the critical ERRs amount to $G_{\mathrm{Ic}} = 300 \quad 56$ J/m^2 and $G_{\mathrm{IIc}} = 500$ J/m^2.

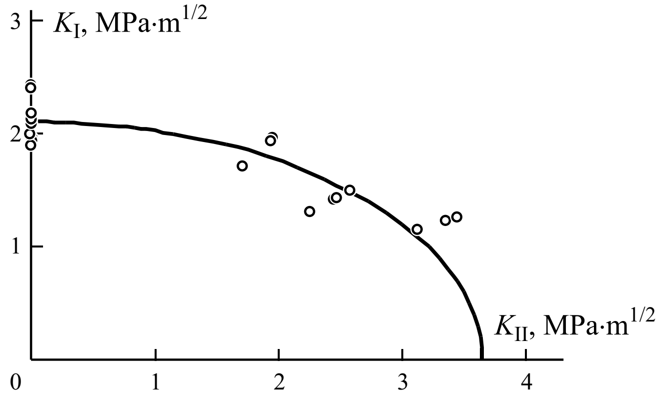

Fig. 2. Fracture toughness of UD glass/epoxy composite under mixed-mode loading: (○) — experiment and (—) — according to Eq. (3).

4. Crack Onset in Cross-Ply Laminates under Off-Axis Loadings

Having established the mixed-mode cracking criterion for the UD glass/epoxy composite, we proceed by applying it to the crack onset prediction in a cross-ply glass/epoxy laminate under uniaxial tensile loadings at different angles to the material orthotropy axis. To this end, we express the mode I and II steady-state ERRs via the corresponding crack face displacements and the far-field ply stresses and relate the stresses to the applied load by using the classical laminate theory.

The intralaminar cracking of $[0_2/90_2]_s$, $[15_2/{-75}_2]_s$, $[30_2/{-60}_2]_s$, and $[45_2/{-45}_2]_s$ laminates under uniaxial tension is reported in [30]. The off-axis lay-ups correspond to the $[0_2/90_2]_s$ cross-ply laminate rotated by 15, 30, and 45°, respectively. The cracks appear in the inner plies of the laminates with the first three lay-ups and in all plies of the $[45_2/{-45}_2]_s$ laminate. The cracks in the plies run along the orthotropy axis of the laminate, therefore, the loading modes are independent. Knowing the crack opening displacement at a given stress, the ERR can be easily evaluated via the work of crack closure (see, e.g., [33]).

Specifically, the mode I ERR for a transverse crack in a cross-ply laminate is determined as

$$G_{\rm I} = \frac{h\bar{u}_2\sigma_2^2}{E_2}, \tag{6}$$

where h is the thickness of the transverse ply, σ_2 designates the unperturbed stress in the ply normal to the crack plane, E_2 is the transverse Young's modulus of the UD ply, and $\bar{u}_2$ is the normalized average crack opening displacement, as defined in [17]. The nondimensional displacement $\bar{u}_2$ is evaluated by an approximate relation based on a series of FEM calculations and has the form [17]

$$\bar{u}_2 = A_2 + B_2\left(\frac{E_2}{E_1}\right)^{n_2}. \tag{7}$$

For a crack in the inner transverse layer of a symmetric cross-ply composite with equal thicknesses of the inner and (total) outer plies (i.e., of the lay-up $[0_n/90_n]_s$), the parameters of Eq. (7) are as follows: $A_2 = 0.52$, $B_2 = 0.3075$, and $n_2 = 0.671767$. If an outer ply is cracking, the parameters of Eq. (7) are $A_2 = 1.2$, $B_2 = 0.5942$, and $n_2 = 0.57057$.

Similarly, the mode II ERR for a transverse crack in a cross-ply laminate is evaluated as

$$G_{\rm II} = \frac{h\bar{u}_1\tau_{12}^2}{G_{12}}, \tag{8}$$

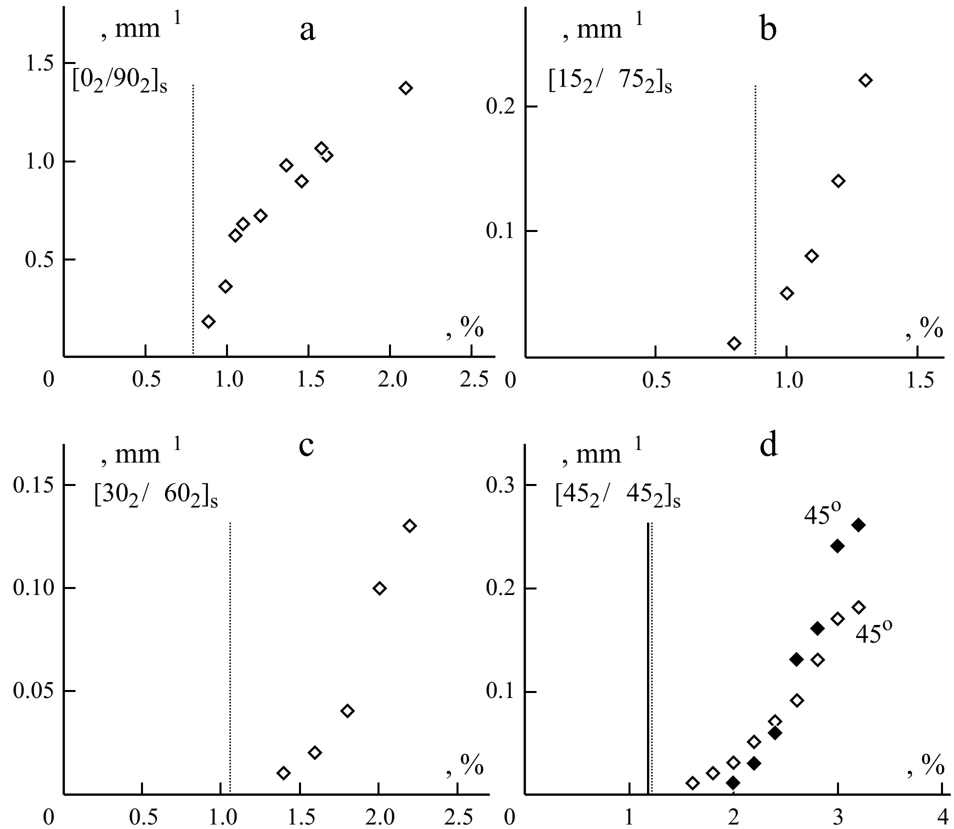

Fig. 3. Evolution of the crack density [30] and the predicted onset strain [Eq. (10)] for the $[0_2/90_2]_s$ (a), $[15_2/–75_2]_s$ (b), $[30_2/–60_2]_s$ (c), and $[45_2/–45_2]_s$ (d) laminates. (- - -) — the crack onset strain of the inner plies; (——) — the crack onset strain of the outer plies (for the $[45_2/–45_2]_s$ laminate).

where $_{12}$ stands for the unperturbed in-plane shear stress in the ply, G_{12} is the in-plane shear modulus of the UD ply, and $\bar{u}_1$ is the normalized average sliding displacement of the crack face. An approximate relation for $\bar{u}_1$, similar to Eq. (7), is derived in [18]. For cross-ply composites with equal total thicknesses of the inner and outer plies, it reduces to a constant: $\bar{u}_1$ = 0.42 for a crack in the inner ply and $\bar{u}_1$ = 0.839 for a crack in an outer ply.

Since the laminates are treated as linear elastic, the ply stresses are linear functions of the applied tensile strain :

$$ _2 \quad _{2r} \quad k_2 \, , \quad _{12} \quad k_{12} \, , \tag{9}$$

where $_{2r}$ is the residual stress, and the factors k_2 and k_{12} are easily obtained by the classical laminate theory for each lay-up and ply considered. Combining Eqs. (5), (6), (8), and (9), the crack onset criterion can be put in the form

$$\frac{h\bar{u}_2(\quad_{2r} \quad k_2 \quad ^2}{G_{\mathrm{Ic}}E_2} \quad \frac{h\bar{u}_1(k_{12}}{G_{\mathrm{IIc}}G_{12}} \quad 1. \tag{10}$$

Solving Eq. (10) for , the applied strain at crack onset in the corresponding ply is obtained. The experimental data, borrowed from [30], are shown in Fig. 3, where the crack density as a function of applied strain for the $[0_2/90_2]_s$, $[15_2/–75_2]_s$, $[30_2/–60_2]_s$, and $[45_2/–45_2]_s$ laminates is presented along with the predicted crack onset strain according to Eq. (10). Note that the theoretical COSs of the inner and outer plies of the $[45_2/–45_2]_s$ laminate are almost equal, because the larger normalized crack face displacements of the outer plies are offset by the smaller ply thickness.

It is seen that the theoretical estimate agrees reasonably well with the test results for the $[0_2/90_2]_s$ and $[15_2/–75_2]_s$ laminates, whereas the COS of $[30_2/–60_2]_s$ and $[45_2/–45_2]_s$ laminates is underestimated. The discrepancy is likely to be caused by the deviation from linearity in the laminate response, apparent in stress–strain diagrams [30], which precedes the cracking onset in these laminates. The nonlinearity is caused, in part, by the nonlinear shear stress–strain response typical of UD polymer matrix composites [34]. Thus, the linear laminate theory may overestimate the shear stresses in the plies of $[30_2/–60_2]_s$ and $[45_2/–45_2]_s$ laminates, which eventually could lead to a conservative COS prediction. A nonlinear laminate analysis, as in, e.g., [35-38] and corresponding corrections to the ERR estimates would be needed to improve the model accuracy.

5. Conclusions

The fracture toughness of UD glass/epoxy composite under mode I and mixed-mode loadings has been determined experimentally. The crack propagation criterion is found to be linear in terms of single-mode energy release rates and the critical ERRs evaluated at G_{Ic} = 300 J/m^2 and G_{IIc} = 500 J/m^2. The mixed-mode cracking criterion was employed to predict the intralaminar crack onset in a $[0_2/90_2]_s$ cross-ply glass/epoxy composite under off-axis tensile loading. The COS predicted agrees with test results at small loading angles (on-axis and 15 off-axis loadings) and underestimates the COS at larger (30 and 45) loading angles. The discrepancy is likely to be caused by the deviation from linearity in the laminate response before cracking onset in these laminates, related to the nonlinear shear characteristics of the UD plies.

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