Mechanics of Composite Materials, Vol. 44, No. 2, 2008

ON THE PROPAGATION OF LAMB WAVES IN A SANDWICH PLATE MADE OF COMPRESSIBLE MATERIALS WITH FINITE INITIAL STRAINS

S. D. Akbarov,[*,] A. D. Zamanov,[*] and E. R. Agasiyev[***]**

Keywords: *Lamb waves, initial strains, sandwich plate, wave dispersion, compressible material*

The propagation of flexural Lamb waves in a prestrained sandwich plate made from compressible highly elastic materials is investigated within the scope of a piecewise homogeneous body model by utilizing TLTEWISB. The mechanical relations of layer materials are described by a harmonic-type potential, and numerical results are obtained for the first and second vibration modes. According to the results, the influence of problem parameters and of the initial stretching strain along the layers on the wave propagation speed is examined. The asymptotic values of the speed are considered in the cases of short and long wavelengths, and the influence of the initial strains on these asymptotic (limit) values are also analyzed.

Introduction

The present level of modern engineering and technology requires a more detailed and accurate estimation of the dynamic carrying capacity of structural members, with taking into account their initial distinctive features. One of the features of structural members is the presence of initial (residual) stresses in them. These stresses arise in the members after their manufacture and assembly, in the Earth crust from the action of geostatic and geodynamic forces, in composites, in rocks, etc. Therefore, the studies on wave propagation in bodies with initial stresses are of great significance both for the theory and the actual practice. Up to now, a large number of investigations have been made in this field, and a considerable part of them were performed by utilizing the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies (TLTEWISB). Here, we briefly consider some of them with regard to the subject of the present paper.

The field equations of TLTEWISB were constructed in [1-3]. Moreover, in [3], detailed analyses of the results obtained and of their applications were also carried out. It follows from these references and the review [4] that wave propagation problems for prestressed multilayer cylinders and plates remain practically uninvestigated up to now. Also, the near-surface waves in initially stressed layered half-planes have been examined very poorly. The first attempts in this field were made in [5-8]. In [5], the axisymmetric wave propagation in a prestretched compound cylinder was considered, but in [6, 7] problems on the generalized Rayleigh wave dispersion in a layered half-plane were studied.

---

*Yildiz Technical University, Davutpasa campus No 127, 34210 Topkapi, Istanbul, Turkey. **Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan, AZ-1141 F. Agayev St. 9, Baku, Institute Mathematics and Mechanics NAS of Azerbaijan. ***Azerbaijan University of Architecture and Construction. Russian translation published in Mekhanika Kompozitnykh Materialov, Vol. 44, No. 2, pp. 231-244, March-April, 2008. Original article submitted January 16, 2008.

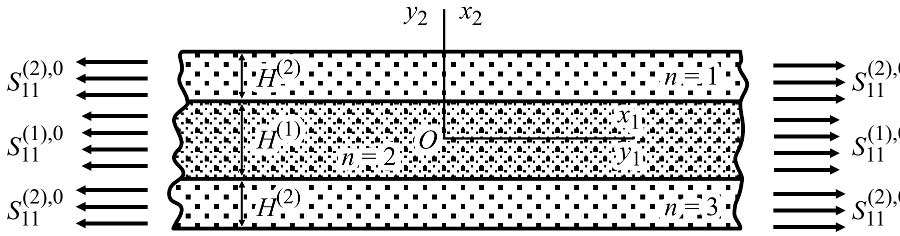

Fig. 1. Geometry of the sandwich plate considered.

In [8], the propagation of Lamb waves in a prestrained sandwich plate made of incompressible high-elastic materials was investigated. However, the character of the investigation and discussions of the numerical results obtained in [8] do not allow one to draw any conclusion about the influence of initial strains on the propagation of Lamb waves in a sandwich plate. Since systematic investigations into the problem are absent, in the present paper, an attempt is made to bridge the gap. It is assumed that the layers of a sandwich plate, which are made of high-elastic compressible materials, are prestrained before their assembling. The mechanical properties of layer materials are described by a potential of harmonic type.

More detailed analyses of recent investigations into the dynamics of initially stressed bodies can be found in [9-17].

1. Formulation of the Problem

We consider a sandwich plate with the structure shown in Fig. 1 and assume that the thickness and materials of the face layers of the plate are the same. We differ three (natural, initial, and perturbed) states of the plate, and relate the points of the plate in the natural (initial) state to the Lagrange coordinates in a Cartesian coordinate system $Ox_1x_2x_3$ $(Oy_1y_2y_3)$. In addition, the midplane of each layer of the plate in the initial state is associated with the corresponding local coordinate system $O_nx_{1n}x_{2n}x_{3n}$ ($O_ny_{1n}y_{2n}y_{3n}$), which is obtained by a parallel transition of the coordinate system $Ox_1x_2x_3$ $(Oy_1y_2y_3)$ along the Ox_2 (Oy_2) axis. The layer materials are high-elastic, and the layers are prestretched by uniformly distributed normal stresses (Fig. 1) before their assembly. As a result, initial strains in the layers, which are homogeneous, are determined by the relations

$$u_i^{(r_n),0} \quad (\; ^{(r_n)}_{i} \quad 1)x_{in}, \quad ^{(r_n)}_{i} \quad \mathrm{const}_{in}, \; r_1 \quad r_3 \quad 2, \; r_2 \quad 1, \quad y_{in} \quad ^{(r_n)}_{i} x_{in} \; ; \quad n, m, i = 1, 2, 3. \tag{1}$$

In Eq. (1) and in what follows, the conventional notation is used, and the quantities related to an nth layer are denoted by the superscript (r_n). The quantities referring to the initial state are labeled by the superscript 0.

The elastic properties of layer materials are given by the harmonic-type potential

$$\frac{1}{2} \quad s_1^2 \qquad s_2, \tag{2}$$

where

$$s_1 \quad \sqrt{1 \quad 2 \quad _1} \quad \sqrt{1 \quad 2 \quad _2} \quad \sqrt{1 \quad 2 \quad _3} \quad 3,$$
$$s_2 \quad (\sqrt{1 \quad 2 \quad _1} \quad 1)^2 \quad (\sqrt{1 \quad 2 \quad _2} \quad 1)^2 \quad (\sqrt{1 \quad 2 \quad _3} \quad 1)^2. \tag{3}$$

In relation (3), and are material constants, and $_i$ $(i = 1, 2, 3)$ are the principal values of the Green strain tensor. When needed, expressions (2) and (3) are supplied with corresponding indices.

Taking into account the foregoing assumptions, we will investigate the propagation of Lamb waves in a sandwich plate within the scope of a piecewise homogeneous body model, by utilizing TLTEWISB under the plane strain state in the Oy_1y_2 plane, for which $^{(1)}_3 \quad ^{(2)}_3 \quad 1.0$ and $^{(1),0}_3 \quad ^{(2),0}_3 \quad ^{(1)}_3 \quad ^{(2)}_3 = 0$.

The field equations of TLTEWISB for the problem considered are the equation of motion

$$\frac{Q_{ij}^{(r_n)}}{y_{in}} \quad {}^{(r_n)} \frac{{}^{2}u_j^{(r_n)}}{t^{2}}, \tag{4}$$

and the mechanical relations

$$Q_{11}^{(r_n)} \quad {}^{(r_n)}_{1111}\frac{u_1^{(r_n)}}{y_{1n}} \quad {}^{(r_n)}_{1122}\frac{u_2^{(r_n)}}{y_{2n}}, \quad Q_{12}^{(r_n)} \quad {}^{(r_n)}_{1212}\frac{u_1^{(r_n)}}{y_{2n}} \quad {}^{(r_n)}_{1221}\frac{u_2^{(r_n)}}{y_{1n}},$$

$$Q_{21}^{(r_n)} \quad {}^{(r_n)}_{2112}\frac{u_1^{(r_n)}}{y_{2n}} \quad {}^{(r_n)}_{2121}\frac{u_2^{(r_n)}}{y_{1n}}, \quad Q_{22}^{(r_n)} \quad {}^{(r_n)}_{2211}\frac{u_1^{(r_n)}}{y_{1n}} \quad {}^{(r_n)}_{2222}\frac{u_2^{(r_n)}}{y_{2n}}, \tag{5}$$

where

$${}^{(r_n)}_{1111} \quad \frac{{}^{(r_n)}_{1}}{{}^{(r_n)}_{2}}({}^{(r_n)} \quad 2 \, {}^{(r_n)}), \quad {}^{(r_n)}_{2222} \quad \frac{{}^{(r_n)}_{2}}{{}^{(r_n)}_{1}}({}^{(r_n)} \quad 2 \, {}^{(r_n)}), \quad {}^{(r_n)}_{1122} \quad {}^{(r_n)}_{2211} \quad {}^{(r_n)}, \quad {}^{(r_n)}_{1212} \quad {}^{(r_n)}_{2121} \quad \frac{2 \, {}^{(r_n)} \, {}^{(r_n)}_{2}}{{}^{(r_n)}_{1} \quad {}^{(r_n)}_{2}},$$

$${}^{(r_n)}_{1221} \quad {}^{(r_n)}_{2112} \quad \frac{2 \, {}^{(r_n)}({}^{(r_n)}_{2})^{2}}{{}^{(r_n)}_{2} \; {}^{(r_n)}_{1} \quad {}^{(r_n)}_{2})}, \quad r_1 \quad r_3 = 2, \quad r_2 \quad 1. \tag{6}$$

In Eqs. (4) and (5), the following notation is used: $Q_{ij}^{(r_n)}$ are the perturbations of components of the Kirchhoff stress tensor in an *n*th layer, $u_j^{(r_n)}$ are the components of perturbations of the displacement vector, and ${}^{r_n)}$ is the density of the material of the *n*th layer.

Note that, in the theory of large elastic deformation, a symmetric stress tensor $\tilde{S}\,{}^{r_n)}$ is also introduced. The components $S_{ij}^{(r_n)}$ of this tensor are determined through an elastic potential:

$$S_{ij}^{(r_n)} \quad \frac{1}{2}\left(\frac{}{{}^{(r_n)}_{ij}} \quad \frac{}{{}^{(r_n)}_{ji}} \right) {}^{(r_n)},$$

$${}^{(r_n)}_{ij} \quad \frac{1}{2}\left(\frac{u_i^{(r_n)}}{x_{jn}} \quad \frac{u_j^{(r_n)}}{x_{in}} \quad \frac{u_k^{(r_n)}}{x_{in}} \, \frac{u_k^{(r_n)}}{x_{jn}} \right), \tag{7}$$

where ${}^{(r_n)}_{ij}$ are components of the Green strain tensor in the *n*th layer. According to Eqs. (1)-(3) and (7),

$$S_{ii}^{(r_n),0} \quad [\, {}^{(r_n)}({}^{(r_n)}_{1} \quad {}^{(r_n)}_{2} \quad 2) \quad 2 \, {}^{(r_n)}({}^{(r_n)}_{i} \quad 1)]({}^{(r_n)}_{i})^{\;1}, \quad S_{12}^{(r_n)} \quad 0.$$

It follows from the problem formulation that $S_{22}^{(r_n),0} = 0$, from which

$${}^{(r_n)}_{2} \quad [2 \, {}^{(r_n)} \quad {}^{(r_n)}({}^{(r_n)}_{1} \quad 2)]({}^{(r_n)} \quad 2 \, {}^{(r_n)})^{\;1}.$$

The initial strains and stresses in the layers depend on the elongation parameter ${}^{(r_n)}_{1}$, but the perturbations of components of the nonsymmetric Kirchhoff stress tensor $Q_{ij}^{(r_n)}$ are determined by the expressions

$$Q_{ij}^{(r_n)} \quad {}^{j}_{m} \quad \frac{u_i^{(r_n)}}{y_{mn}} \; S_{im}^{(r_n)} \quad S_{im}^{(r_n),0} \; \frac{u_j^{(r_n)}}{y_m}, \tag{8}$$

where $S_{im}^{(r_n)}$ are the perturbations of components of the symmetric stress tensor $\widetilde{S}^{\;r_n)}$.

By employing the linearization procedure, the following relation is obtained from Eq. (7):

$$S_{im}^{(r_n)} \quad {}^{(r_n)}_{im} \; \frac{u^{(r_n)}}{y \;\; n},$$

$$ {}^{(r_n)}_{im} \quad \frac{1}{4} \quad k \quad \frac{u^{(r_n)}}{y_{kn}} \quad \frac{\quad}{{}^{(r_n),0}_{k}} \quad \frac{\quad}{{}^{(r_n),0}_{k}} \quad \frac{\quad}{{}^{(r_n),0}_{im}} \quad \frac{\quad}{{}^{(r_n),0}_{mi}} \quad {}^{(r_n),0}. \tag{9}$$

Substituting relation (9) into Eq. (8) and using expressions (1)-(3), after some mathematical manipulations, we obtain Eqs. (5) and (6).

We assume that the face planes of the plate are force-free:

$$Q_{2i}^{(r_1)}\Big|_{y_{21} \quad h^{(2)} \quad {}^{(2)}_{2}/2} \quad 0, \quad Q_{2i}^{(r_3)}\Big|_{y_{23} \quad h^{(2)} \quad {}^{(2)}_{2}/2} \quad 0. \tag{10}$$

At the same time, between the layers of the plate, the complete contact conditions

$$\begin{matrix} Q_{2i}^{(r_1)} \\ u_i^{(r_1)} \end{matrix}\Bigg|_{y_{21} \quad h^{(2)} \quad {}^{(2)}_{2}/2} \qquad \begin{matrix} Q_{2i}^{(r_2)} \\ u_i^{(r_2)} \end{matrix}\Bigg|_{y_{22} \quad h^{(1)} \quad {}^{(1)}_{2}/2},$$

$$\begin{matrix} Q_{2i}^{(r_2)} \\ u_i^{(r_2)} \end{matrix}\Bigg|_{y_{22} \quad h^{(1)} \quad {}^{(1)}_{2}/2} \qquad \begin{matrix} Q_{2i}^{(r_3)} \\ u_i^{(r_3)} \end{matrix}\Bigg|_{y_{23} \quad h^{(2)} \quad {}^{(2)}_{2}/2} \tag{11}$$

are obeyed.

A more detailed description of deduction of the foregoing basic equations of TLTEWISB is given in the monograph [3]. Note that, in the absence of initial strains in layers of the sandwich plate considered, the foregoing problem transforms to the corresponding one of the classical linear theory of elastodynamics.

2. Solution Procedure and Deriving the Dispersion Equation

Substituting Eq. (5) into Eq. (4), we obtain the equation of motion in displacements

$$ {}^{(r_n)}_{1111} \frac{{}^{2}u_1^{(r_n)}}{y_1^2} \quad {}^{(r_n)}_{2112} \frac{{}^{2}u_1^{(r_n)}}{y_2^2} \quad ({}^{(r_n)}_{1122} \quad {}^{(r_n)}_{2121}) \frac{{}^{2}u_2^{(r_n)}}{y_1 \; y_2} \quad {}^{(r_n)} \frac{{}^{2}u_1^{(r_n)}}{t^2},$$

$$ ({}^{(r_n)}_{1212} \quad {}^{(r_n)}_{2211}) \frac{{}^{2}u_1^{(r_n)}}{y_1 \; y_2} \quad {}^{(r_n)}_{1221} \frac{{}^{2}u_2^{(r_n)}}{y_1^2} \quad {}^{(r_n)}_{2222} \frac{{}^{2}u_2^{(r_n)}}{y_2^2} \quad {}^{(r_n)} \frac{{}^{2}u_2^{(r_n)}}{t^2}. \tag{12}$$

Since $y_{11} \quad y_{12} \quad y_1$, the displacements can be presented as

$$u_i^{(r_n)}(y_1, y_{2n}, t) \quad U_i^{(r_n)}(y_{2n}) \exp i(ky_1 \qquad t). \tag{13}$$

Substituting Eq. (13) into Eq. (12), we obtain the following equations in the unknown functions $U_i^{(r_n)}(y_{2n})$:

$$\begin{aligned} & {}^{(r_n)}_{2112}\frac{d^2U_1^{(r_n)}}{dy_{2n}^2} \quad ik(\ {}^{(r_n)}_{1122} \qquad {}^{(r_n)}_{2121})\frac{dU_2^{(r_n)}}{dy_{2n}} \quad (\ {}^{(r_n)} \ {}^{2} \quad k^2 \ {}^{(r_n)}_{1111})U_1^{(r_n)} \quad 0, \\ & {}^{(r_n)}_{2222}\frac{d^2U_2^{(r_n)}}{dy_{2n}^2} \quad ik(\ {}^{(r_n)}_{1212} \qquad {}^{(r_n)}_{2211})\frac{dU_1^{(r_n)}}{dy_{2n}} \quad (\ {}^{(r_n)} \ {}^{2} \quad k^2 \ {}^{(r_n)}_{1221})U_2^{(r_n)} \quad 0. \end{aligned} \tag{14}$$

After some mathematical manipulations, we have from Eqs. (14)

$$\frac{dU_1^{(r_n)}}{dy_{2n}} \quad i \quad a_n U_2^{(r_n)} \quad b_n \frac{d^2U_2^{(r_n)}}{dy_{2n}^2} \quad , \tag{15}$$

$$\frac{d^4U_2^{(r_n)}}{dy_{2n}^4} \quad a_{1n}\frac{d^2U_2^{(r_n)}}{dy_{2n}^2} \quad b_{1n}U_2^{(r_n)} \quad 0, \tag{16}$$

where

$$a_n \quad \frac{k^2 \ {}^{(r_n)}_{1111}}{k(\ {}^{(r_n)}_{1212} \qquad {}^{(r_n)}_{2211})}, \quad b_n \quad \frac{{}^{(r_n)}_{2222}}{k(\ {}^{(r_n)}_{1212} \qquad {}^{(r_n)}_{2211})}, \quad a_{1n} \quad [b_n(\qquad k^2 \ {}^{(r_n)}_{1111}) \quad a_n \ {}^{(r_n)}_{1111} \quad k(\ {}^{(r_n)}_{1122} \qquad {}^{(r_n)}_{2121} \qquad {}^{(r_n)}_{2112} b_n]^{\ 1}.$$

Let us consider the solution of Eq. (16). Using the representation $U_2^{(r_n)}(y_{2n}) \quad F_1^{(r_n)} \exp(\ _n y_{2n})$, we obtain the equation

$$ {}_n^4 \quad a_{1n} \ {}_n^2 \quad b_{1n} \quad 0$$

for $_n$, wherefrom

$$ {}_n^2 \quad \frac{a_{1n}}{2} \quad \sqrt{D_n}, \quad D_n \quad \frac{a_{1n}^2}{4} \quad b_{1n}.$$

Thus, we have various solutions to Eq. (16) depending on the sign of D_n and ${}_n^2$.

For the case $D_n \quad 0$, the solution is

$$\begin{aligned} U_2^{(r_n)}(y_{2n}) \quad & F_1^{(r_n)}\cosh(\ _n y_{2n})\cos(\ _n y_{2n}) \quad F_2^{(r_n)}\cosh(\ _n y_{2n})\sin(\ _n y_{2n}) \\ & F_1^{(r_n)}\sinh(\ _n y_{2n})\sin(\ _n y_{2n}) \quad F_2^{(r_n)}\sinh(\ _n y_{2n})\cos(\ _n y_{2n}), \\ & {}_n \quad \sqrt{\frac{1}{2}\left(\sqrt{\frac{a_{1n}^2}{4} \quad D_n} \quad \frac{a_{1n}}{2}\right)}, \quad {}_n \quad \sqrt{\frac{1}{2}\left(\sqrt{\frac{a_{1n}^2}{4} \quad D_n} \quad \frac{a_{1n}}{2}\right)}. \end{aligned} \tag{17}$$

In the case D_n 0, the solution of Eq. (16) depends on the sign of the quantities $\ _{1n}^{2}$ $\frac{a_{1n}}{2}$ $\sqrt{D_n}$ and $\ _{2n}^{2}$ $\frac{a_{1n}}{2}$ $\sqrt{D_n}$. These solutions can be classified as follows:

if $\ _{1n}^{2}$ 0 and $\ _{2n}^{2}$ 0,

$$U_2^{(r_n)}(y_{2n}) \quad F_1^{(r_n)}\cosh(\ _{1n}y_{2n}) \quad F_2^{(r_n)}\sinh(\ _{1n}y_{2n}) \quad F_3^{(r_n)}\cosh(\ _{2n}y_{2n}) \quad F_4^{(r_n)}\sinh(\ _{2n}y_{2n}); \tag{18}$$

if $\ _{1n}^{2}$ 0 and $\ _{2n}^{2}$ 0,

$$U_2^{(r_n)}(y_{2n}) \quad F_1^{(r_n)}\cosh(\ _{1n}y_{2n}) \quad F_2^{(r_n)}\sinh(\ _{1n}y_{2n}) \quad F_3^{(r_n)}\cos(\ _{2n}y_{2n}) \quad F_4^{(r_n)}\sin(\ _{2n}y_{2n}); \tag{19}$$

if $\ _{1n}^{2}$ 0 and $\ _{2n}^{2}$ 0,

$$U_2^{(r_n)}(y_{2n}) \quad F_1^{(r_n)}\cos(\ _{1n}y_{2n}) \quad F_2^{(r_n)}\sin(\ _{1n}y_{2n}) \quad F_3^{(r_n)}\cosh(\ _{2n}y_{2n}) \quad F_4^{(r_n)}\sinh(\ _{2n}y_{2n}); \tag{20}$$

if $\ _{1n}^{2}$ 0 and $\ _{2n}^{2}$ 0,

$$U_2^{(r_n)}(y_{2n}) \quad F_1^{(r_n)}\cos(\ _{1n}y_{2n}) \quad F_2^{(r_n)}\sin(\ _{1n}y_{2n}) \quad F_3^{(r_n)}\cos(\ _{2n}y_{2n}) \quad F_4^{(r_n)}\sin(\ _{2n}y_{2n}). \tag{21}$$

For each of cases (17)-(21), the expressions for $U_1^{(r_n)}(y_{2n})$ are found from Eqs. (15) and the expressions for $Q_{ij}^{(r_n)}$ from Eqs. (5) and (13). Using these expressions, we obtain a linear system of algebraic equation for the unknown constants $F_1^{(r_n)}, F_2^{(r_n)}, F_3^{(r_n)}$, and $F_4^{(r_n)}$ from boundary condition (10) and contact condition (11). For the existence of nontrivial solutions to this system, its determinant must be equal to zero, i.e.,

$$\det\left\| \ _{ij} \right\| \quad 0, \quad i, j \quad 1, 2, \ldots, 12, \tag{22}$$

$$\ _{ij} \quad \ _{ij}(c, kH, \ _1^{(2)}, \ _1^{(1)}, h^{(1)}/h^{(2)}, \ ^{(2)}/\ ^{(1)}, \ ^{(2)}/\ ^{(2)}, \ ^{(1)}/\ ^{(1)}, \ ^{(2)}/\ ^{(1)}),$$

where

$$c \quad /k, \quad H \quad 2H^{(2)} \quad H^{(1)}.$$

Relation (22) is the dispersion equation for the wave propagation problem considered. The explicit expressions for $\ _{ij}$ occupy considerable space and therefore are omitted here. Thus, for every value of the problem parameters $\ _1^{(2)}$, $\ _1^{(1)}$, $H^{(1)}/H^{(2)}$, $\ ^{(2)}/\ ^{(1)}$, $\ ^{(2)}/\ ^{(2)}$, $\ ^{(1)}/\ ^{(1)}$, and $\ ^{(2)}/\ ^{(1)}$, we can construct the dispersion curve $c \quad c(kH)$ from the solution of Eq. (22) and investigate the influence of these parameters on the curve.

3. Numerical Results and Discussions

It is known that, in the case of Lamb waves in a homogeneous plate, the extensional and flexural modes do not depend on each other, and they can be investigated separately. Note that the possibility of such a separation is caused by the geometrical and mechanical symmetry of the problem with respect to the midplane of the plate. Since the problem considered has also this symmetry, we can analyze the flexural and extensional Lamb waves in the sandwich plate individually. The corresponding relations for the modes are

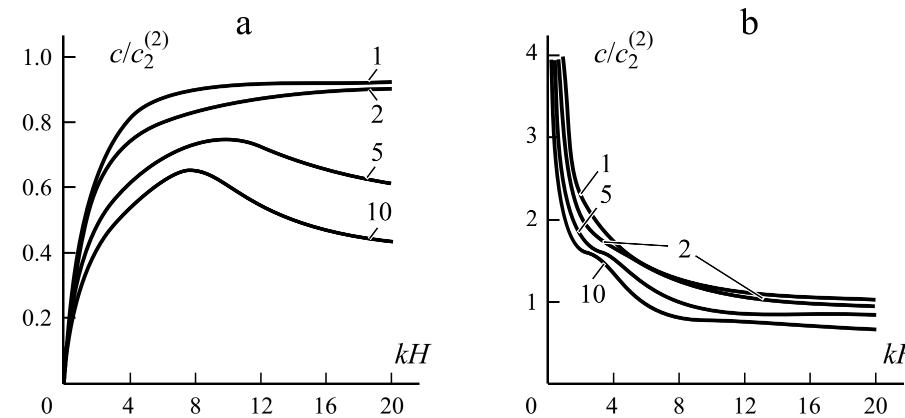

Fig. 2. Dispersion curves for the first (a) and second (b) modes in the absence of initial strains in layers of the plate. Numbers at the curves are the values of $^{(2)}/\ ^{(1)}$. $_1^{(2)}\quad _1^{(1)}\quad 1.0; H\quad 2H^{(2)}\quad H^{(1)}; H^{(2)}\quad H^{(1)}$.

$$u_2^{(1)}(y_{12}, y_{22})\quad (\ 1)^k u_2^{(1)}(y_{12},\quad y_{22}),\qquad u_1^{(1)}(y_{12}, y_{22})\quad (\ 1)^{k\quad 1} u_1^{(1)}(y_{12},\quad y_{22}),$$

$$Q_{21}^{(1)}(y_{12}, y_{22})\quad (\ 1)^k Q_{21}^{(1)}(y_{12},\quad y_{22}),\qquad Q_{12}^{(1)}(y_{12}, y_{22})\quad (\ 1)^k Q_{12}^{(1)}(y_{12},\quad y_{22}),$$

$$Q_{22}^{(1)}(y_{12}, y_{22})\quad (\ 1)^{k\quad 1} Q_{22}^{(1)}(y_{12},\quad y_{22}),\qquad Q_{11}^{(1)}(y_{12}, y_{22})\quad (\ 1)^{k\quad 1} Q_{11}^{(1)}(y_{12},\quad y_{22}).$$

The case $k = 2$ ($k = 1$) pertains to the flexural (extensional) modes. In this paper, we will analyze only the numerical results for the flexural modes. To simplify the considerations, we will assume that $^{(1)}/\ ^{(1)} = \ ^{(2)}/\ ^{(2)} = 1.0$, $H^{(1)}/H^{(2)} = 1.0$, and $^{(2)}/\ ^{(1)} = 1.0$. Moreover, we will consider only the first and second modes, because the character of dispersion curves of the third and subsequent modes are similar to that of the second mode.

First, let us consider the case where initial strains in all layers of the plate are absent, i.e., $_1^{(2)} = \ _1^{(1)} = 1.0$. The dispersion curves constructed at various values of $^{(2)}/\ ^{(1)}$ are given in Fig. 2 for the first and second modes, respectively. Note that the dispersion curves constructed at $^{(2)}/\ ^{(1)} = 1.0$ coincide with those following from the linear theory of elastodynamics, which can be found in the monographs [2, 3] and elsewhere. It follows from the graphs in Fig. 2a that, at all the values of $^{(2)}/\ ^{(1)}$ selected, for the first mode, $c/c_2^{(2)}\quad 0$ (where $c_2^{(2)}$ is the speed of the distortion wave in the face layers) as $kH\quad 0$. At the same time, the limit values of $c/c_2^{(2)}$ as $kH\quad 0$ differ from each other at various values of $^{(2)}/\ ^{(1)}$. In the case $^{(2)}/\ ^{(1)} = 1$, this limit is $c_R^{(2)}/c_2^{(2)}$, where $c_R^{(2)}$ is the speed of the Rayleigh wave in the plate material. For $^{(2)}/\ ^{(1)}\quad 1$, this limit is equal to $c_2^{(1)}/c_2^{(2)}$. As $c_2^{(1)}/c_2^{(2)}\quad 1/\sqrt{\ ^{(2)}/\ ^{(1)}}$, the limit values of $c/c_2^{(2)}$ at kH decrease with $^{(2)}/\ ^{(1)}$. This conclusion is supported by the graphs given in Fig. 2a

The dispersion curves depicted in Fig. 2b show that, for the second mode, $c/c_2^{(2)}$ does not tend to a finite limit as $kH\quad 0$. At the same time, the limiting value of $c/c_2^{(2)}$ as kH is equal to min $\{c_R^{(2)}/c_2^{(2)};\ c_1^{(1)}/c_2^{(2)}\}$. For example, at $^{(2)}/\ ^{(1)} = 2$, this limit is equal to $c_R^{(2)}/c_2^{(2)}$, but, in the cases $^{(2)}/\ ^{(1)} = 5$ and 10.0, it is equal to $c_1^{(1)}/c_2^{(2)}$, where $c_2^{(1)}\quad \sqrt{(\ ^{(1)}\quad 2\ ^{(1)})/\ ^{(1)}}$.

Figure 2a shows that the character of behavior of dispersion curves for the first mode, within the foregoing limit values of $c/c_2^{(2)}$, depends on $^{(2)}/\ ^{(1)}$. The ratio $c/c_2^{(2)}$ increases monotonically with kH at $^{(2)}/\ ^{(1)} = 1$ and 2.0, but the relation-

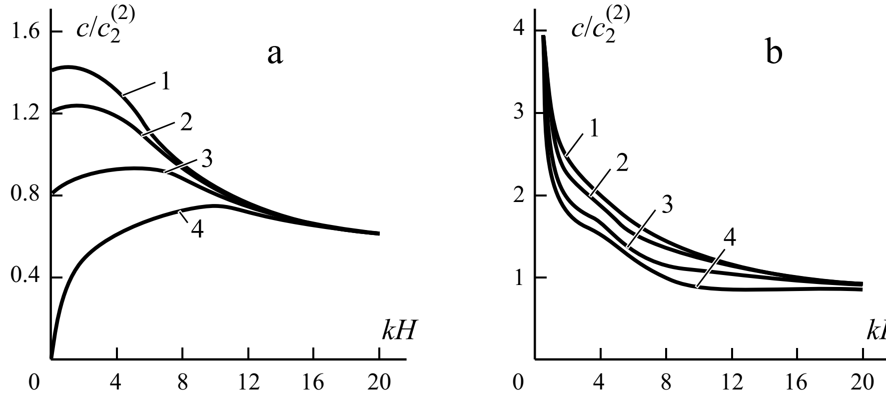

Fig. 3. Influence of initial strains of face layers of the plate on the dispersion curves for the first (a) and second (b) modes. 1 — { $_1^{(2)}$ = 1.7; $_1^{(1)}$ = 1}, 2 — { $_1^{(2)}$ = 1.5; $_1^{(1)}$ = 1}, 3 — { $_1^{(2)}$ = 1.2; $_1^{(1)}$ = 1}, and 4 — { $_1^{(2)}$ = $_1^{(1)}$ = 1}. $H \quad 2H^{(2)} \quad H^{(1)}, H^{(2)} \quad H^{(1)}, \quad ^{(2)}/\ ^{(1)} = 5$.

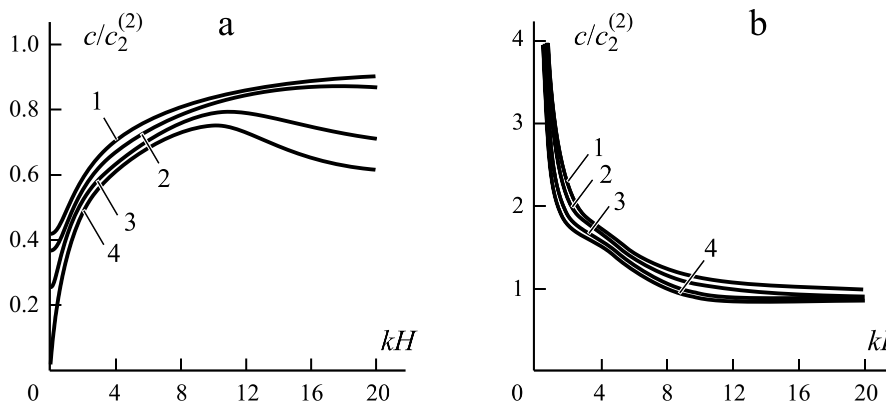

Fig. 4. Influence of initial strains of the midlayer of the plate on the dispersion curves for the first (a) and second (b) modes. 1 — { $_1^{(2)}$ = 1; $_1^{(1)}$ = 1.7}, 2 — { $_1^{(2)}$ = 1; $_1^{(1)}$ = 1.5}, 3 — { $_1^{(2)}$ = 1; $_1^{(1)}$ = 1.2}, and 4 — { $_1^{(2)}$ = $_1^{(1)}$ = 1.0}. $H \quad 2H^{(2)} \quad H^{(1)}, H^{(2)} \quad H^{(1)}, \quad ^{(2)}/\ ^{(1)} = 5$.

ship between $c/c_2^{(2)}$ and *kH* is nonmonotonic at $^{(2)}/\ ^{(1)} = 5$ and 10. Figure 2b shows that, for the second mode, $c/c_2^{(2)}$ decreases with *kH* for all the values of $^{(2)}/\ ^{(1)}$ selected.

We will now analyze the numerical results for the influence of initial strains of layers of the plate on the dispersion curves. Let us consider the case $^{(2)}/\ ^{(1)} = 5$, and first assume that initial strains occur only in the face layers of the plate, i.e., $_1^{(2)} \quad 1$ and $_1^{(1)} \quad 1$. The corresponding dispersion curves are given in Fig. 3 for the first and second modes, respectively. According to mechanical considerations, in the case analyzed, the ratio $c/c_2^{(2)}$ must have the same limit at *kH* for all the values of $_1^{(2)}$ considered, and this limit must be equal to $c_2^{(1)}/c_2^{(2)}$ ($c_1^{(1)}/c_2^{(2)}$) for the first (second) mode. This prediction is supported by the graphs given in Fig. 3. At the same time, Fig. 3a shows that, for $_1^{(2)} \quad 1$, the limiting values of $c/c_2^{(2)}$ at *kH* 0 are finite and differ from zero. The limiting values mentioned increase with $l_1^{(2)}$. Also, the values of $c/c_2^{(2)}$ obtained for each *kH* increase with $l_1^{(2)}$. The foregoing results qualitatively agree with the corresponding results obtained for the homogeneous prestressed plate and analyzed in the monograph [3].

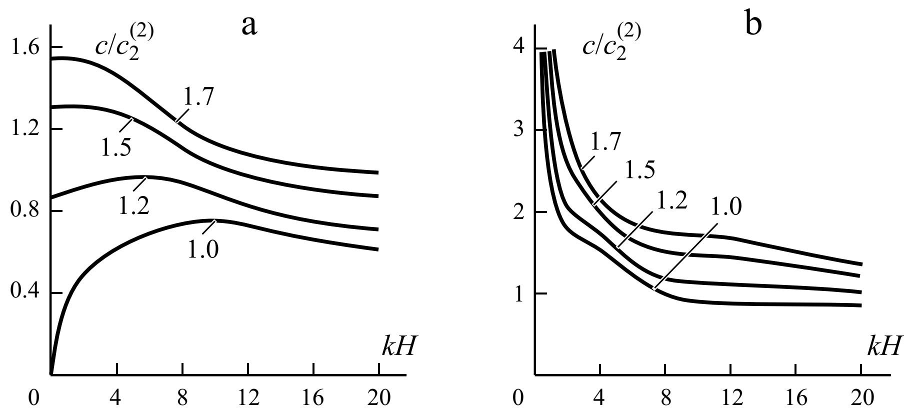

Fig. 5. Dispersion curves for the first (a) and second (b) modes in the case where initial strains occur in all layers of the plate. $_1^{(2)}$ $_1^{(1)}$ (numbers at the curves).H $2H^{(2)}$ $H^{(1)}, H^{(2)}$ $H^{(1)}$, $^{(2)}/$ $^{(1)} = 5$.

Let us consider the case where the initial strains occur only in the midlayer of the plate, i.e., $_1^{(2)}$ 1 and $_1^{(1)}$ 1. The dispersion curves constructed for this case are illustrated in Fig. 4 for the first and second modes, respectively. It follows from these curves that, at kH , the limit of the ratio $c/c_2^{(2)}$ depends on the parameter $_1^{(1)}$. Actually, $c_2^{(1)}$ and $c_1^{(1)}$ depend on the parameter $_1^{(1)}$, i.e., $c_2^{(1)}$ $c_2^{(1)}(\ _1^{(1)})$ and $c_1^{(1)}$ $c_1^{(1)}(\ _1^{(1)})$, and this limit is equal to $c_2^{(1)}(\ _1^{(1)})/c_2^{(2)}$ $(c_1^{(1)}(\ _1^{(1)})/c_2^{(2)})$ for the first (second) mode. Note that the character of the dependences $c_2^{(1)}$ $c_2^{(1)}(\ _1^{(1)})$ and $c_1^{(1)}$ $c_1^{(1)}(\ _1^{(1)})$ was studied in the monograph [3], where it was established that $c_2^{(1)}$ $c_2^{(1)}(\ _1^{(1)})$ and $c_1^{(1)}$ $c_1^{(1)}(\ _1^{(1)})$ both increase with $_1^{(1)}$. Therefore, in the case considered, the limiting values of $c/c_2^{(2)}$ at kH increase with $_1^{(1)}$. Figure 4a shows that, as in the previous case, for the first mode, the limit values of $c/c_2^{(2)}$ at kH 0 differ from zero and increase with $_1^{(1)}$. However, these values are significantly smaller than those in the previous case.

Finally, let us consider the case with $_1^{(2)}$ 1 and $_1^{(1)}$ 1. The dispersion curves obtained are given in Fig. 5 for the first and second modes, respectively. An analysis of these curves shows that, for each fixed $_1^{(1)}$, the limiting value of $c/c_2^{(2)}$ at kH coincides with the corresponding one obtained at $_1^{(2)}$ 1 and $_1^{(1)}$ 1. At the same time, the limiting values of $c/c_2^{(2)}$ at kH 0, as in the previous cases, differ from zero and increase with $_1(\ _1^{(2)}\ \ _1^{(1)})$. Moreover, these values of $c/c_2^{(2)}$ are significantly greater than those in the previous cases.

5. Conclusions

In the present paper, the propagation of flexural Lamb waves in a prestrained sandwich plate made from compressible high-elastic materials have been investigated within the scope of a piecewise homogeneous body model by utilizing TLTEWISB. It was assumed that the mechanical relations of layer materials could be described by a potential of harmonic type, and particular numerical results were found for the first and second modes.

From the numerical results obtained, the following conclusions can be drawn:

In the case where initial strains are absent in layers of the plate, for the first mode, $c/c_2^{(2)}$ 0 (where $c_2^{(2)}$ is the speed of the distortion wave in the face layers) as kH 0, but the limiting values of $c/c_2^{(2)}$ as kH differ from each other for various values of $^{(2)}/$ $^{(1)}$: in the case $^{(2)}/$ $^{(1)} = 1$, this limit is $c_R^{(2)}/c_2^{(2)}$ $(\ c_R^{(1)}/c_2^{(1)})$, where $c_R^{(2)}(\ c_R^{(1)})$ is the speed of Rayleigh wave in the plate material; however, at $^{(2)}/\ ^{(1)} > 1$, this limit is equal to $c_2^{(1)}/c_2^{(2)}$. As $c_2^{(1)}/c_2^{(2)}\quad 1/\sqrt{\ ^{(2)}/\ ^{(1)}}$, the limiting values of $c/c_2^{(2)}$ decrease with $^{(2)}/\ ^{(1)}$ as kH . For the second mode, the values of $c/c_2^{(2)}$ do not tend to a finite limit with $kH\quad 0$, but the limiting value of $c/c_2^{(2)}$ is equal to $c_R^{(2)}/c_2^{(2)}$; $c_1^{(1)}/c_2^{(2)}\}$ as kH .

The wave propagation speed decreases with $^{(2)}/\ ^{(1)}$.

The initial stretching increases the speed $c/c_2^{(2)}$ of the flexural Lamb wave considered.

For the first mode, as a result of the existence of initial strains, the limiting values of $c/c_2^{(2)}$ as $kH\quad 0$ differ from zero and increase with initial strains.

The limit of $c/c_2^{(2)}$ at kH depends on the parameter $_1^{(1)}$.

REFERENCES

1. M. A. Biot, Mechanics of Incremental Deformations, John Wiley, New York (1965).
2. A. C. Eringen and E. S. Suhubi, Elastodynamics, Finite Motion. Vol. 1, Academic Press, New York–London (1975).
3. A. N. Guz, Elastic Waves in a Body with Initial (Residual) Stresses, "A.S.K." (2004).
4. A. N. Guz', "Elastic waves in bodies with initial (residual) stresses," Int. Appl. Mech., **38**, No. 1, 35-78 (2002).
5. S. D. Akbarov and A. N. Guz, "Axisymmetric longitudinal wave propagation in pre-stressed compound circular cylinders," Int. J. Eng. Sci., **42**, 769-791 (2004).
6. S. D. Akbarov and M. Ozisik, "The influence of the third-order elastic constants on the generalized Rayleigh wave dispersion in a pre-stressed stratified half-plane," Int. J. Eng. Sci. **41**, 2047-2061 (2003).
7. S. D. Akbarov and M. Ozisik, "Dynamic interaction of a pre-stressed nonlinear elastic layer and a half-plane," Int. Appl. Mech., **40**, No. 9, 137-144 (2004).
8. G. A. Rogerson and K. J. Sandiford, "The effect of finite primary deformations on harmonic waves in layered elastic media," Int. J. Solid. Struct., **37**, 2059-2087 (2000).
9. S. D. Akbarov, A. D. Zamanov, and T. R. Suleimanov, "Forced vibration of a pre-stretched two- layer slab on a rigid foundation," Mech. Compos. Mater., **41**, No. 3, 229-240 (2005).
10. S. D. Akbarov, I. Emiroglu, and F. Tasci, "The Lamb's problem for a half-space covered with the pre-stretched layer," Int. J. Mech. Sci., **47**, No. 9, 1326-1349 (2005).
11. S. D. Akbarov, "On the dynamical axisymmetric stress field in a finite pre-stretched bilayered slab resting on a rigid foundation," J. Sound Vibr., **294**, Nos. 1/2, 221-237 (2006).
12. S. D. Akbarov, "The influence of the third order elastic constants on the dynamical stress field in a half-space with a pre-stretched layer," Int. J. Non-Linear Mech., **41**, No. 3, 417-425 (2006).
13. S. D. Akbarov, "Frequency response of the axisymmetrically finite pre-stretched slab from incompressible functional graded material on a rigid foundation," Int. J. Eng. Sci., **44,** Nos. 8/9, 484-500 (2006).
14. S. D. Akbarov and C. Guler, "On the stress field in a half-plane covered by the pre-stretched layer under the action of arbitrary linearly located time-harmonic forces," Appl. Math. Model., **31**, 2375-2390 (2007).
15. S. D. Akbarov, "The axisymmetric Lamb's problem for the finite pre-strained half-space covered with the finite pre-stretched layer," Int. Appl. Mech., **43**, No. 3, 351-360 (2007).
16. A. D. Zamanov and E. R. Aghasiyev, "On some particularities of the dispersion of Lamb's wave propagating in the sandwich plate from compressible material with finite initial strains," Theor. Applied Mech., No. 3, 68-77, Baku (2007).
17. Yu. A. Zhuk and I. A. Guz', "Influence of preliminary stresses on velocities of the plane waves propagation perpendicularly to layers in nanocomposite materials," Int. Appl. Mech., **42**, No. 7, 730-748 (2006).