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DEFORMABILITY OF EXPANDED POLYSTYRENE UNDER SHORT-TERM COMPRESSION

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Keywords*: expanded polystyrene (EPS), compression, design of experiments, elastic modulus, elastic deformation, optimal height of specimens*

The results obtained in an experimental investigation of deformability of expanded polystyrene (EPS) under short-term compression are presented. The density of EPS varied from 13 to 28 kg/m^3. The method of design of experiments was used to determine the elastic modulus and the ultimate strain (corresponding to the end of quasi-linear deformability) under compression stresses operating perpendicularly and parallel to the faces of EPS products. A graphical interpretation of the models is also presented. Based on the experimental data obtained, it was concluded that the expanded polystyrene was homogeneous in mutually perpendicular planes with respect to its deformability in compression.

An important condition for the employment of lightweight structures containing expanded polystyrene (EPS) is the refinement of the methods for their calculation, with deformability parameters of EPS layers accounted for more thoroughly. Compression is the basic form of stress state of an EPS used as a thermoinsulation structural material. In most cases, the efficiency of EPS as a structural material depends not on its strength parameters, but on the deformability. Thus, the EPS serving as a force filler of sandwich panels must primarily possess a certain rigidity; in particular, the filler material must meet particular requirements for the elastic modulus. For experimental determination of the elastic modulus of EPS, tests specimens of various shapes have been utilized: cubes with edges of length $b = 50$ mm [1]; cylinders of diameter 150 mm and height $H = 300$ mm [2]; cylinders of diameter 100 mm and height $H = 200$ mm [3]; rectangular $30 \times 30 \times 100$-mm prisms [4]. In [5], compression tests on rigid-foam specimens with a base size $b = 40$-100 mm and a ratio between the height and base size of 0.5-4.0 were described. It was found that the strength did not depend on their height and cross-sectional area. The ultimate strain ε_B in the quasi-linear region of the dependence $\sigma = f(\varepsilon)$, measured according to the displacement of the crosshead of a testing machine, decreased with increasing ratio between the height and the base size of the specimens, while the elastic modulus increased [5]. According to the data presented in [6], to determine the strength and elastic modulus in compression, specimens of a square or cylindrical form with base sizes $b = 50$ mm (50 mm in diameter) and $H = 50$-150 mm were used for high-density thermoinsulation materials and with $H = 153$ mm (153 mm in diameter) and $b = 25$-153 mm for low-density ones. It is known that there exists a certain correlation between the specimen geometry and experimental results for the mechanical parameters of a material [5]. However, no conclusions for the optimum cross-sectional sizes of test specimens for determining the deformability characteristics of EPS in compression can be found in the literature.

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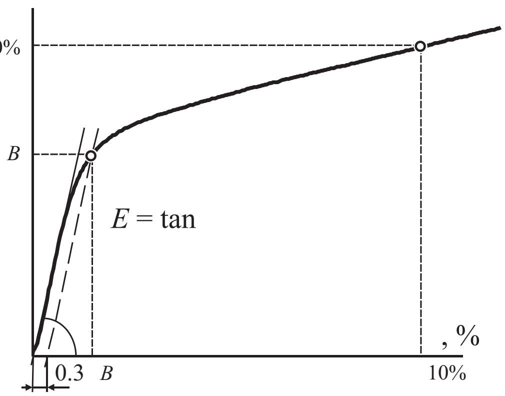

Fig. 1. Determination of the elastic modulus E and the ultimate strain $_B$ of an EPS in compression.

The purpose of the present study is to optimize the specimen height for EPS of various density for determining the elastic modulus E and the ultimate strain $_B$ in the region of quasi-linearity, with the use of total deformation of the specimen measured from the displacement of the crosshead of a testing machine.

The investigations were carried out on EPS plates made not by pressure molding, but by foaming, in a closed volume, a raw material in the form of beads (rigid granules): Styrochem NF-414 (Finland), 1.4-2.5 mm in diameter, for plates of density 13-14 kg/m^3; Styrochem NF-514, 0.9-1.6 mm in diameter, for plates of density 20-21 kg/m^3; Styrochem NF-714, 0.6-0.9 mm, for plates of density 27-28 kg/m^3.

Investigation Method and Mathematical Design of Experiments

The compression tests were performed, according to [7], under normal compression stresses operating perpendicularly (H_N) and in parallel (T) to the surface of the EPS plates, i.e., the operating conditions of foams in lightweight three-layer structures were taken into account. The standard [7] contains recommendations on the geometry of specimens, the test procedure, and the determination of compression strength, but does not indicate the optimum height of specimens for determining the deformability characteristics of a material. We tested square specimens with a base size of $b = 50$ mm (according to [8], accepted for the basic series of experiments) and a height H equal to that of the tested EPS plates —- 50, 100, and 150 mm, as well as specimens with $b = 150$ mm and $H = 150$ mm (additional series of experiments).

The compression was carried out on an H10KS computerized testing machine (Hounsfield, England) at a loading rate of (0.1 N 25%) mm/min [7], with a force measurement error of 1-11 N. The accuracy of deformation measurements according to the crosshead motion was 0.01 mm.

The tests were carried out at an ambient temperature of 23 ± 2°C and a relative humidity of 50 ± 5%.

In the investigations, the QMAT Professional ver. 3.83 base control programs were used. For each specimen, the conventional compression strength $_{10\%}$ and the shear modulus E were calculated, and the strain diagram was drawn to a (40-60) : 1 scale. Upon processing the stress–strain diagrams, we determined the ultimate strain $_B$. The longitudinal strain was calculated by the formula H/H, where H is the gage length of the specimen H; H is the reduction in the gage length at a fixed load F.

The ultimate strain $_B$, according to the stress–strain diagram, was determined in the following way: through the origin of coordinates, a tangent to the deformation diagram and, in parallel to it, at = 0.3%, a straight line were drawn (Fig. 1). The abscissa of the intersection point of the second line with the deformation diagram determined the ultimate strain $_B$, beyond which a noticeable increase in deformations and a deviation of the dependence from linearity occurred.

TABLE 1. Basic Characteristics of the Design of Experiments

Characteristic	x_1*	x_2**
Basic level	20.6	100
Variation interval	7.0	50
Upper level	27.6	150
Lower level	13.6	50

* Specimen density, kg/m^3.
**Specimen height, mm.

TABLE 2. Matrix of the Total Factorial Experiment of Type 3^2 and Experimental Results

Experiment	x_1	x_2	y_1, MPa		y_2, %		y_3, kPa		y_4, %	
			$\bar{y}_1^{э}$*	$S^2_{y_1}$	$\bar{y}_2^{э*}$	$S^2_{y_2}$	$\bar{y}_3^{э*}$	$S^2_{y_3}$	$\bar{y}_4^{э*}$	$S^2_{y_4}$
1	–1	–1	1.926	0.0416	2.077	0.0177	2.258	0.265	2.237	0.0980
2	0	–1	4.720	0.0496	2.132	0.0161	4.338	0.283	2.307	0.0515
3	+1	–1	7.365	0.1152	2.120	0.0524	6.944	0.273	2.185	0.0320
4	–1	0	2.807	0.0058	1.898	0.0177	2.942	0.244	1.950	0.0202
5	0	0	6.056	0.1076	1.605	0.0093	6.150	0.295	1.627	0.0259
6	+1	0	8.400	0.0508	1.652	0.0090	9.265	0.453	1.640	0.0174
7	–1	+1	3.450	0.0047	1.687	0.0126	3.613	0.205	1.560	0.0240
8	0	+1	6.248	0.1204	1.588	0.0202	6.422	0.257	1.670	0.0303
9	+1	+1	8.643	0.0328	1.630	0.0071	9.667	0.395	1.578	0.0030

*The average values of input parameters from six parallel evaluations are presented.

The elastic modulus in compression was found as the tangent of slope angle of the initial quasi-linear section of the stress–strain diagram by using the formula

$$E - \frac{F}{A}\frac{H}{H} \quad \frac{F}{A}\frac{1}{\ },$$

where F is the increment of the load; A is the cross-sectional area of the working part of the specimen; is the longitudinal strain of the specimen when the load is changed by F (for $_B$).

Such characteristic as the elastic modulus E is necessary in calculations with the use of equations of the mechanics of solid deformable bodies, which are employed to determine the stresses and strains under the action of external loads on EPS as a structural material. Therefore, the optimization of the specimen height for EPS products of various density is important for reliably determining such deformability characteristics as the elastic modulus E and the ultimate strain $_B$ from the total deformation of the specimens. This means that specimen sizes must provide a sufficiently accurate registration of measured deformations.

The system considered includes four output parameters ($E_{(H_N)}$, $E_{(T)}$, $_{B(H_N)}$, and $_{B(T)}$) and two controlled factors (the EPS density and the specimen height H (gage length). Plates of density 13 to 28 kg/m^3 were tested according to a factorial experiment of the type $N \quad n^k$ (where n is the number of levels and k is the number of factors). Altogether, $N \quad 3^2 \quad 9$ tests were performed. For the cut-out specimens of height H = 50, 100, and 150 mm, experimental samples with similar initial mean densities were formed. The tests were carried out in a month and a half after manufacturing the plates. In total, nine series of EPS plates (six specimens for each plate) were tested.

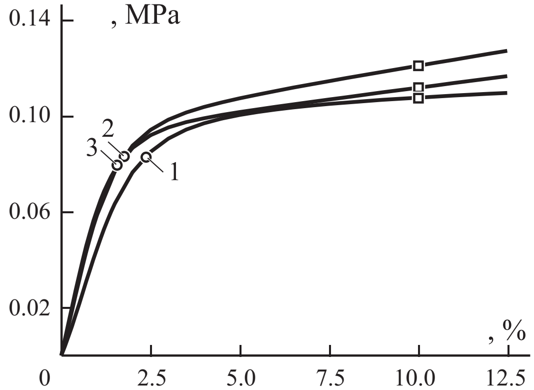

Fig. 2. Compression stress–strain diagrams of EPS specimens of density 20.6 kg/m^3 and thickness 50 (1), 100 (2), and 150 (3) mm. The dots are experimental values: ○ — the strain $_{B(H_N)}$ (on the abscissa axis); □ — the stress (on the ordinate axis) corresponding to a 10% strain of the specimens.

The design of experiments made it possible to vary all factors simultaneously and to obtain quantitative estimates for the basic effects of their interaction. To reduce the multicollinearity of quadratic terms in the two-factor experiment and to simplify the processing of experimental data, the factors were encoded by the formula [9]

$$x_j \quad \frac{\bar{x}_j \quad \bar{x}_{j(i)}}{I_j}, \tag{1}$$

where x_j and $\bar{x}_j$ are the encoded and natural values of a factor, respectively; $\bar{x}_{j(i)}$ is the natural value of the factor of the basic level; I_j is the interval of variation; j is the number of the factor.

The basic level and the interval of variation of the controlled factors are presented in Table 1, and the matrix of design of experiments is given in Table 2.

Upon compression of the specimens cut from the plates and selected according to the matrix of design, the following parameters of EPS were determined:

y_1 and y_3 — the elastic moduli $E_{(H_N)}$ (perpendicularly to the plate plane) and $E_{(T)}$ (in the plate plane), respectively;

y_2 and y_4 — the ultimate strain $_{B(H_N)}$ (perpendicularly to the plate plane) and $_{B(T)}$ (in the plate plane), respectively.

We employed polynomials of the second degree, because, first, there exist well-developed second-order designs and, second, the surfaces of second order can be classified, and hence their extreme points are readily determined. The method of least squares [10] was used to calculate the quadratic models of the relation between the output parameters (y_1, y_2, y_3, and y_4) and the controlled factors (x_1 and x_2). The matrix of design for such a relation is given in Table 3.

In the general form, the quadratic model can be written as

$$y \quad b_0 \quad b_1 x_1 \quad b_2 x_2 \quad b_{12} x_1 x_2 \quad b_{11} x_1^2 \quad b_{22} x_2^2. \tag{2}$$

The significance of the constant coefficients was verified by using the Student t-criterion [11] at a significance level = 5%. The insignificant coefficients were not included in the equation. The adequacy of Eq. (2) was checked by using the Fisher criterion [12, 13].

TABLE 3. Extended Matrix of Design of the Total Factorial Experiment of Type 3^2

Experiment	Matrix of basic functions					
	Design		x_0	x_1^2	x_2^2	x_1x_2
	x_1	x_2				
1	–1	–1	+1	+1	+1	+1
2	0	–1	+1	0	+1	0
3	+1	–1	+1	+1	+1	–1
4	–1	0	+1	+1	0	0
5	0	0	+1	0	0	0
6	+1	0	+1	+1	0	0
7	–1	+1	+1	+1	+1	–1
8	0	+1	+1	0	+1	0
9	+1	+1	+1	+1	+1	+1

TABLE 4. Coefficients of Regression Equations with Respect to Encoded Variables

Regression equations	b_0	b_1	b_2	b_{12}	b_{11}	b_{22}
y_1	5.916	2.704	0.722	-	–0.243	–0.362
y_2	1.672	–0.0433	–0.237	-	0.0690	0.154
y_3	6.119	2.844	1.027	0.342	-	–0.579
y_4	1.739	–0.0573	–0.320	-	-	0.184

Investigation Results

Figure 2 shows the deformation diagrams of EPS under compressive stresses operating perpendicularly to the surface of plates. The shape of the deformation curves is practically quasi-linear up to 60-75% of the stresses $_{10\%}$ corresponding to a 10% strain of the specimens. The limit of quasi-linearity of the dependence $f(\ )$ is the strain $_B$.

Table 4 presents the coefficients of Eq. (2) calculated according to the data in Tables 2 and 3 (the encoded variables), and Table 5 shows results of a statistical analysis. Adequate equations of regression of the second order were obtained, therefore, the effects of interaction between the factors and the quadratic effects (the influence of quadratic terms is also present in the free term b_0, for which a mixed estimate was obtained) found in the experiment become significant. We should note that the experiment was carried out in a local domain in the factor space, and the coefficients of Eq. (2) reflect the effect of factors only in this domain.

The general form of coefficients of the equations shows that not all the factors considered equally affect the output parameters examined (see Table 4). The value of a regression coefficient is the quantitative measure of influence of the corresponding factor on the output parameter upon transition from the zero level to an upper or lower one. The plus (minus) sign indicates that, with increase in the factor, the value of the output parameter increases (decreases).

The interaction of two factors in the equation for y_3 is significant and means only that the main effect of one factor depends on the fact at what level the other factor is (the existence of an interaction effect between the factors does not mean that they are interdependent). The positive sign of the interaction means that a simultaneous increase, as well as a simultaneous decrease, in the values of two factors increases the output parameter examined, in particular, y_3 (without regard for the linear ef-

TABLE 5. Statistical Analysis of Regression Equations with Respect to Encoded Variables

The quantity calculated	y_1	y_2	y_3	y_4
Number of degrees of freedom				
f_{ad}	4	4	4	5
f_{repr}	45	45	45	45
S^2_{ad}	0.135	0.0530	0.573	0.0739
S^2_{repr}	0.0587	0.0180	0.297	0.0336
F_{cal}	2.30	2.94	1.93	2.20
F_{tab}	2.59	2.98	2.59	2.43
Conclusion on the adequacy of the model	Adequate at the level of significance , %			
	5	2.5	5	5
S_r	0.255	0.093	0.361	0.130

Note. As a measure of scatter of test results about the empirical surface of response, the root-mean-square deviation S_r (the absolute value of the average measure of deviations of test data from the empirical surface of response, constant for all its sections) is accepted:

$$S_r = \sqrt{\frac{\sum_{i=1}^{N}\sum_{u=1}^{m_i}(y_{iu} - \hat{y}_i)^2}{\sum_{i=1}^{N} m_i - l}},$$

where y_{iu} and $\hat{y}_i$ are the ith values of the resultant characteristic — the actual ones and those calculated by Eqs. y_1, y_2, y_3, y_4 (by encoded variables); m is the number of parallel tests at separate points of the design of experiment ($m = 6$); N is the number of runs of the design of experiment ($N = 9$); l is the number of significant coefficients in the regression equation.

fects). At a simultaneous variation of the factors in different directions, for example, $x_1 = +1$ and $x_2 = -1$ or $x_1 = -1$ and $x_2 = +1$, the output parameter y_3 decreases.

The output parameters can be found by using data of the controlled factors x_1 and x_2 from equations for the parameters y_1, y_2, y_3, and y_4 (see Table 4). In investigating the equation for y_1, it is found that, at x_1 varying from 13.6 to 27.6 kg/m^3, the point of extremum is x_2 = 150 mm ("almost stationary area"). The investigation results for y_2 show that the point of extremum y_2, at which the parameter y_2 is minimum with respect to specimens, is the point x_2 = 139 mm. By analyzing the equation for the parameter y_3, it is found that the point of extremum x_2 ("almost stationary area") lies within the range of 130-159 mm when x_1 varies from 13.6 to 27.6 kg/m^3. Analyzing the equation for the parameter y_4, it is found that the point of extremum, at with the parameter y_4 is minimum along the specimen height, is the point x_2 = 144 mm.

After transition from the encoded [see Eq. (1)] to natural variables and averaging of regression coefficients, the equations for the output parameters take the following form:

$$E_{(H_N)} = -7.037 + 0.590 \quad + 0.0434H - 0.00496 \quad ^2 - 0.0001H^2 \tag{3}$$

with a root-mean-square deviation S_r = 0.496 MPa (Nm = 54 points of evaluation);

$$_{B(H_N)} = 3.49 - 0.0642 \quad - 0.0171H + 0.00141 \quad ^2 + 0.0000616H^2 \tag{4}$$

$$(S_r = 0.10\%;\ Nm = 54),$$

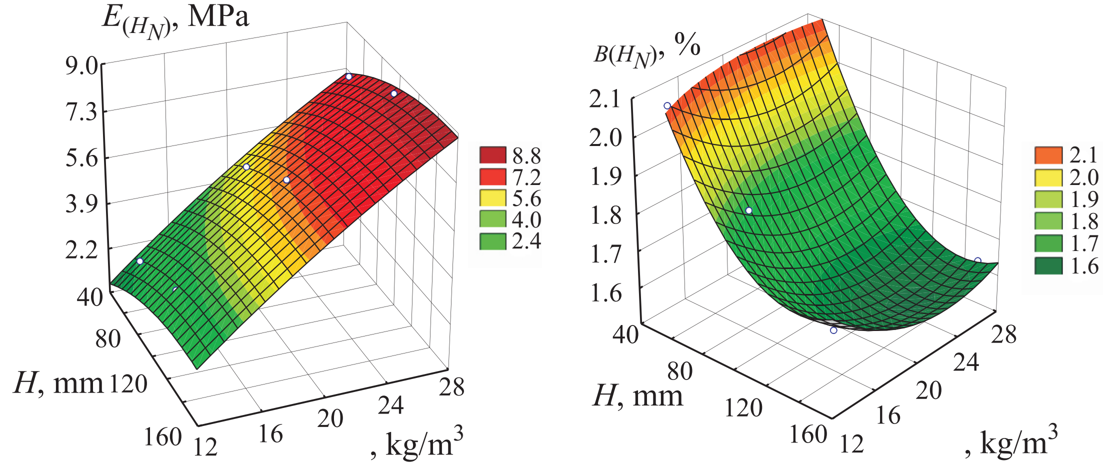

Fig. 3. Second-order surfaces of the output parameters $E_{(H_N)}$ and $_{B(H_N)}$.

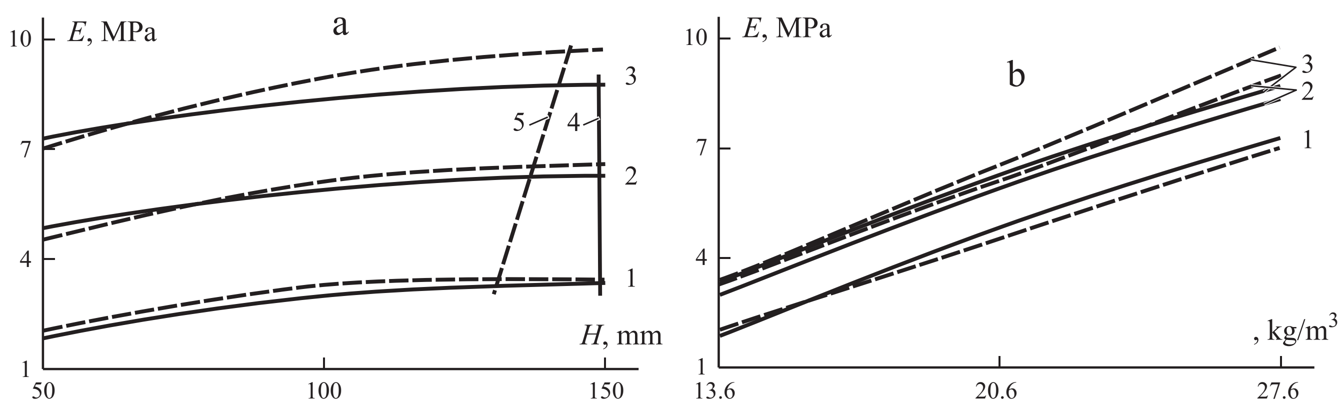

Fig. 4. Graphics of Eqs. (1) and (3): a — at = 13.6 (1), 20.6 (2), and 27.6 (3) kg/m^3; b — H 50 (1), 100 (2), and 150 (3) mm. 4 and 5 — an "almost stationary area". (—) — $E_{(H_N)}$; (- - -) — $E_{(T)}$.

$$E_{(T)} = -4.606 + 0.308 \quad + 0.0467H - 0.000232H^2 + 0.000977 \quad H \tag{5}$$

$$(S_r = 0.405 \text{ MPa}; Nm = 54),$$

$$_{B(T)} = 3.28 - 0.0082 \quad - 0.0211H + 0.0000735H^2 \tag{6}$$

$$(S_r = 0.13\%; Nm = 54).$$

When using these equations, there is no need to transform experimental conditions every time into the encoded variables (in this case, the possibility of interpreting the effect of factors according to the magnitudes and signs of regression coefficients is lost).

The graphic representation of the results of two-factor experiment is the so-called response surface in a three-dimensional space (Fig. 3). The projections of the basic generatrices of response of the output parameters $E_{(H_N)}$, $_{B(H_N)}$ and $E_{(T)}$, $_{B(T)}$ on the coordinate planes are shown in Figs. 4 and 5. For fixed values of the factor x_1 (the density of EPS) at three levels,

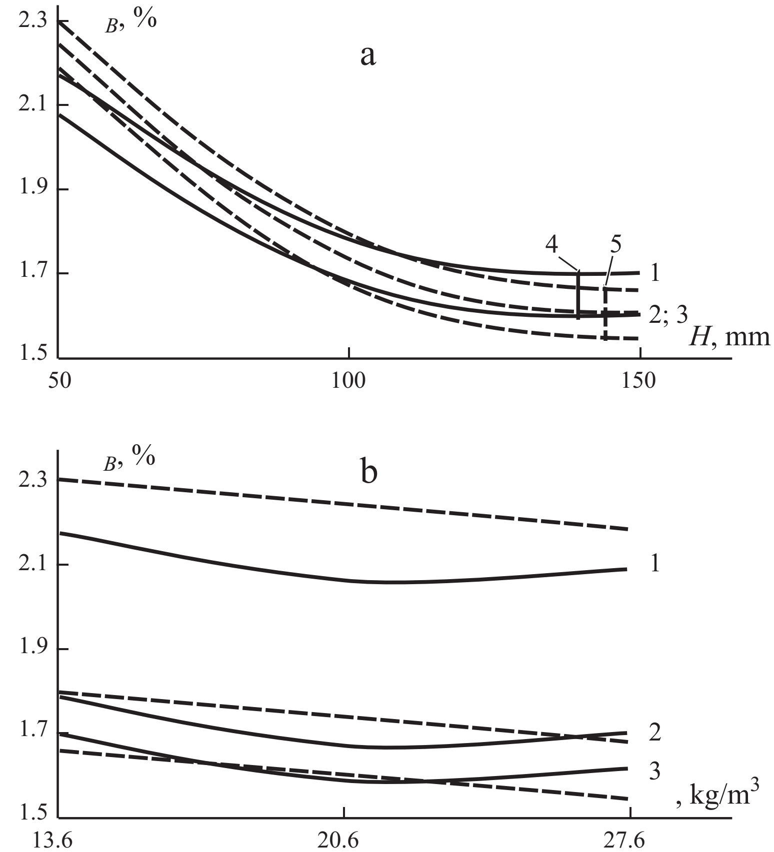

Fig. 5. Graphics of Eqs. (4) and (6): (—) — $_{B(H_N)}$; (- - -) — $_{B(T)}$. Other designations as in Fig. 4.

the figures present the output parameters $E_{(H_N)}$, $E_{(T)}$ and $_{B(H_N)}$, $_{B(T)}$ as functions of the factor x_2 (specimen heights), which confirm the presence of a particular optimum, i.e., of an “almost stationary area”, with respect to the given factor (see the continuous 4 and dashed 5 curves in Figs. 4a and 5a).

Figures 4b and 5b illustrate the output parameters $E_{(H_N)}$, $E_{(T)}$ and $_{B(H_N)}$, $_{B(T)}$ at a fixed value of the factor x_2 at three levels as functions of the factor x_1. With the density of EPS varying on the range 13.6-27.6 kg/m^3, the elastic moduli in compression $E_{(H_N)}$ and $E_{(T)}$ determined on specimens of height H = 150 mm, on the average, are by 6.5% greater than for those of height H = 100 mm (see Fig. 4b). At H = 50 mm, the values of these moduli, on the average, are by 72-30% smaller than at H = 150 mm.

The ultimate strains of EPS $_{B(H_N)}$ and $_{B(T)}$ in compression for the specimens of height H = 150 mm, on the average, are by 0.10% smaller in magnitude than for the 100-mm-high ones. In the case H = 50 mm, the values of $_{B(H_N)}$ and $_{B(T)}$, on the average, are by 0.45-0.56% greater in magnitude than at H = 100 and 150 mm.

The values of the elastic moduli $E_{(H_N)}$ and $E_{(T)}$ and the ultimate strains $_{B(H_N)}$ and $_{B(T)}$ (see Figs. 4 and 5) confirm the assumption that the EPS is practically homogeneous in mutually perpendicular planes with respect to its deformability properties in compression.

To reveal the effect of cross-sectional sizes of specimens on such experimental deformability characteristics as the compressive elastic modulus E and ultimate strain $_B$, by using the total deformation of the specimens (measured according to the motion of the crosshead of a testing machine), the results of compression tests on quadratic specimens with b = 150 mm and H = 150 mm are given additionally in the present study.

TABLE 6. Results of Statistical Data Processing for EPS Compression Tests

Correlated parameters and the number of regression equation (in parentheses)	Specimen size, mm	Number of tests (evaluations)	Values of constant coefficients of Eq. (7)		S_r, MPa	R_E^2	Average values of E (MPa) predicted by empirical equations (10)-(13)	Values of the Student t-criterion		Confidential probability P (1) of the consistency of compared results for the elastic modulus E
			b_0	b_1				(9)	tabulated at a total number of degrees of freedom	
$E_{(H_N)}$ (10)	50 50 150	18	–2..300	0.426	0.148	0.9957	9.458	2.14	2.42	0.98
$E_{(H_N)}$ (11)	150 150 150	27	–2.638	0.415	0.261	0.9883	8.816			
$E_{(T)}$ (12)	50 50 150	18	–2.306	0.429	0.524	0.9606	9.534	1.04	1.99	0.95
$E_{(T)}$ (13)	150 150 150	18	–2.232	0.402	0.372	0.9822	8.863			

*The upper level of density equal to 27.6 kg/m^3 is assumed, at which the maximum deviation of values of the elastic modulus E is observed.
** 1 P is the confidential level (risk), which characterizes the probability that the true value of prediction for E lies outside the field of errors mentioned.

The investigation results on the deformation characteristics E and $_B$ in compression are presented in relation to the density of EPS, because it is the main parameter. For the experimental values of the elastic moduli $E_{(H_N)}$ and $E_{(T)}$, at = 13-31 kg/m^3, linear dependences were assumed, which are simple in calculations and allow one to find, with a sufficient accuracy, their values:

$$\overline{E} \quad b_0 \quad b_1 \quad , \tag{7}$$

where $\overline{E}$ is the average value of the resultant characteristic, and b_0 and b_1 are constants calculated from experimental data by the least-squares method [10].

The degree of connection between two variables in the regression scheme, in the case of a linear dependence between them, is characterized by the correlation coefficient R_E . The coefficient of determination shows what part of variation in the parameter considered depends on the variation in the controlled input factors. In the case of a linear connection, the squared coefficient of determination R_E^2 was used [14].

As a measure of scatter of observation results about the empirical regression line, the root-mean-square deviation S_r (the absolute value of the average measure of deviations of test data from the regression line, which is constant for all its sections) is assumed:

$$S_r \quad \sqrt{\frac{\sum_{i\ 1}^{i\ n} (E_i \quad \overline{E}_i)^2}{n \quad m}}, \tag{8}$$

where E_i and $\overline{E}_i$ are the factual and calculated [by Eq. (7)] ith values of a resultant characteristic; n is the number of experiments; m is the number of constant parameters evaluated in the empirical equation ($m = 2$ for the linear equation).

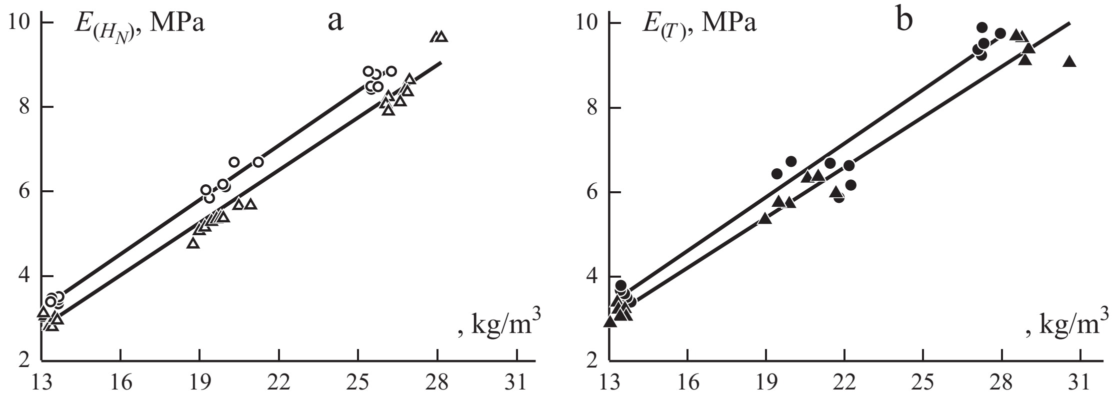

Fig. 6. Regression lines of the compressive elastic moduli $E_{(H_N)}$ (a) and $E_{(T)}$ (b) for the EPS of thickness 150 mm vs. the density . Dots —- experimental values according to test data for the specimens at $b = 50$ (○, ●) and 150 mm (△, ▲).

The value of t for two predictions with average point values $\overline{E}_{(1)}$ and $\overline{E}_{(2)}$, determined by regression equations (7), and with root-mean-square deviations $S_{r(1)}$ and $S_{r(2)}$ was calculated by the formula [15]

$$t \quad \frac{\left|\overline{E}_{(1)} \quad \overline{E}_{(2)}\right|}{\sqrt{S^2_{r(1)} \quad S^2_{r(2)}}}. \tag{9}$$

Then, using the table of the Student t-criterion at the total number of degrees of freedom $f \quad f_1 \quad f_2 \quad (n_1 \quad m_1) \quad (n_2 \quad m_2)$, [where n_1 and n_2 are the numbers of E_i values in the density interval of EPS 13-31 kg/m^3 in calculating $S_{r(1)}$ and $S_{r(2)}$; m_1 and m_2 are the numbers of parameters in the regression equations of form (7)], we determined the probability $P \quad 1$ (the level of reliability) that the compared prediction results for $E_{(1)}$ and $E_{(2)}$ are consistent. The consistency condition $t \quad t_{\text{tab}}$ (where t_{tab} is the tabulated value of the Student t-criterion [11]) is fulfilled.

The results of statistical processing of experimental data for the elastic moduli $E_{(H_N)}$ and $E_{(T)}$ of expanded polystyrene are presented in Table 6 [the constants b_0 and b_1 of Eq. (7), calculated from experimental data (Fig. 6); the root-mean-square deviation S_r; the coefficient of determination R^2_E; the elastic moduli $\overline{E}_{(H_N)}$ and $\overline{E}_{(T)}$ for the upper level of EPS density, equal to 27.6 kg/m^3 (see Table 1); the values of the Student t-criterion, both calculated by Eq. (9) and taken from the table; the level of reliability $P \quad 1$ of consistency of the compared prediction results for the elastic moduli]. It is seen that, for the empirical dependences $\overline{E}_{(H_N)} \quad f(\;)$ and $\overline{E}_{(T)} \quad f(\;)$ obtained, the coefficients of determination in Eqs. (10)-(13) vary from 0.961 to 0.996 and considerably exceed the threshold values (lower boundaries) of R^2_E for the corresponding values of n (the number of tests) [16]. Thus, the regression equations presented can be used to predict the elastic modulus of EPS in compression under short-term loading.

The experimental values of the compressive elastic moduli of EPS shown in Fig. 1 and the results of their statistical processing given in Table 6 indicate that the confidence interval of the moduli for the square EPS specimens with sides of 50 and 150 mm, at $H = 150$ mm, is $P = 0.98$ for $E_{(H_N)}$ and $P = 0.95$ for $E_{(T)}$. In this case, the deviation of average values for $\overline{E}_{(H_N)}$ and $\overline{E}_{(T)}$ in the interval of EPS densities 13.6-27.6 kg/m^3 does not exceed 7%.

The results of statistical processing of experimental values of the ultimate strains $_{B(H_N)}$ and $_{B(T)}$ of EPS in compression on the interval of EPS densities 13-29 kg/m^3 are shown in Table 7. It should be noted that, at a coefficient of variability of EPS density varying within 27-31%, the variability of ultimate strains does not exceed 4% (see Table 7). This makes it possi-

TABLE 7. Statistical Data Processing for the Ultimate Strains $_{B(H_N)}$ and $_{B(T)}$

Specimen size, mm	Number of tests (evaluations)	Parameter	Variation interval of the tests		Average value	Root-mean-square deviation	Coefficient of variation*, %
			from	to			
50 50 150	17	, kg/m^3	13.3	26.3	19.7	5.30	26.9
		$_{B(H_N)}$, %	1.57	1.73	1.65	0.0523	3.2
150 150 150	25	, kg/m^3	13.1	28.2	19.4	5.45	28.1
		$_{B(H_N)}$, %	1.78	1.95	1.86	0.0529	2.8
50 50 150	17	, kg/m^3	13.4	27.9	21.1	5.72	26.9
		$_{B(T)}$, %	1.54	1.76	1.61	0.0615	3.8
150 150 150	17	, kg/m^3	13.0	29.2	20.4	6.40	31.4
		$_{B(T)}$, %	1.72	1.91	1.81	0.0569	3.1

*The root-mean-square deviation in fractions of the arithmetic mean, expressed in percentage.

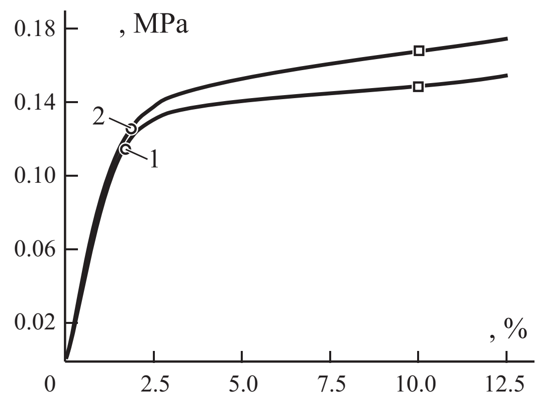

Fig. 7. Compression stress-strain diagrams of specimens of thickness 150 mm and density 27 kg/m^3. 1 — b = 50 and 2 — b = 150 mm. Dots are experimental values: ○ — the strain $_{B(H_N)}$ (on the abscissa axis); □ — the stress (on the ordinate axis) corresponding to a 10% strain of the specimens.

ble to compare the average values of ultimate strains in compression of specimens with sides of 50 and 150 mm (at H = 150 mm) on the interval of EPS densities from 13 to 29 kg/m^3. By using the Student t-criterion, it can be found that the discrepancy between the average values of $\bar{}_{B(H_N)}$ and $\bar{}_{B(T)}$ is insignificant with the reliability P = 0.98 for compressed specimens with b = 50 mm and P = 0.998 for those with b = 150 mm. This allows us to refine the estimate of mathematical expectation for the ultimate strain $_B$ and the standard deviation with respect to the pooled samples of specimens tested in compression perpendicularly and in parallel to the surface of polystyrene plates.

The values of the ultimate strains $\bar{}_{B(H_N)}$ and $\bar{}_{B(T)}$ of EPS obtained in compression of specimens with a side of 50 mm, in their magnitude, on the average, are by 0.2% smaller than in the case of specimens with a side of 150 mm (see Table 7). In our opinion, the influence of the areas with a disturbed stress state near the end faces of a specimen on its total deformation decreases with increasing cross-sectional area, and therefore the larger specimens will have higher ultimate strains $_B$ (Fig. 7).

Thus, in the present study, by using the method of design of experiments, mathematical models for optimizing the height of square (b = 50 mm) specimens were constructed to determine the elastic modulus and the ultimate strain (in the region of quasi-linear deformation) under compressive stresses operating perpendicularly and in parallel to the surface of EPS products. A graphic interpretation of these models is presented: the level lines of elastic modulus and ultimate strain in relation to the height of test specimens of EPS with densities from 13.6 to 27.6 kg/m^3. The consistency of experimental results for the elastic modulus obtained on specimens with sides of 50 and 150 mm at a height of 150 mm is also shown. The difference between the average values of ultimate strains is insignificant (random).

A comparison of empirical values of the elastic moduli and ultimate strains of EPS in mutually perpendicular planes confirms the homogeneity of the material with respect to its deformation characteristics in compression.

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