

FUNDAMENTAL PROBLEMS OF METROLOGY

COSMOLOGICAL DISTANCES SCALE. PART 15: COSMIC JERK AND GRAVITATIONAL DIPOLE OF INHOMOGENEITY

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As part of the "Cosmological Distance Scale" series, the paper focuses on the cosmic jerk issue. Drawing on the data for the parametric identification of the Friedmann–Robertson–Walker model as the dependence of photometric distance on Type Ia supernova (SN Ia) redshift used by the High-Z SN Search Team and Supernovae Cosmology Project, it is considered whether the accelerating expansion of the universe can be assumed to be the most plausible hypothesis under the criterion of minimum inadequacy error. The previously detected change points (structural and parameter changes in the systematic component of the model) and rank inversions of SN Ia photometric distances for the systematic component of this model are analyzed. It is shown that these metric disruptions are caused by the isotropy of the Friedmann–Robertson–Walker model. In the anisotropic model of the cosmological distance scale, change points and rank inversions are associated with the gravitational dipole orientation of inhomogeneity in the large-scale structure of the universe. These dipoles represent diametrically opposite "supercluster–giant void" pairs on the celestial sphere. Only the size of the supervoid in the constellation of Eridanus, comparable to that of the observable part of the universe, causes a great imbalance in the gravitational effect of the massive supercluster. This leads to disruptions in the form of change points and rank inversions in the isotropic models of the Friedmann–Robertson–Walker type.

Keywords: redshift, supernovae, SNe Ia, cosmological distance scale, change point, rank inversion, galaxy superclusters, supervoids, gravitational dipole of inhomogeneity.

Introduction. The paper [1] focuses on the accelerating expansion of the universe, specifying several subtle physical effects. The authors of [1] note that "presently, none of these effects reconciles the data with $\Omega_\Lambda = 0$ and $q_0 \geq 0$." Drawing on redshift z data for Type Ia supernovae (SNe Ia) from the Hubble Deep and Ultra-Deep Fields [2, 3] led to a more definite conclusion in [2]: "A purely kinematic interpretation of the SN Ia sample provides evidence at the greater than 99% confidence level for a transition from deceleration to acceleration or, similarly, strong evidence for a cosmic jerk. Using a simple model of the expansion history, the transition between the two epochs is constrained to be at $z = 0.46 \pm 0.13$."

In the same connection, another estimate is given in [4]: "The cross-over between mass-dominated and Λ -dominated energy density occurred at $z \approx 0.37$, for a flat $\Omega_M \approx 0.28$ universe, whereas the cross-over between deceleration and acceleration occurred when $(1+z)^3 \Omega_M / 2 = \Omega_\Lambda$, that is at $z \approx 0.73$. This was approximately when SN 1997G exploded, over 6 billion years ago." In addition, the Nobel lecture [5] clarified: "The very first data (... around a redshift of $z = 0.4...$) appeared to favor a slowing universe with no cosmological constant, but with only 7 supernovae the uncertainties were large. Even one very-well-measured supernova—it had Hubble Space telescope observations — at twice the redshift ... $z = 0.83$... already began to tell a different story. But the evidence really became strong with 42 supernovae ... Now there was a clear bulk of the supernova data indicating a universe that is dominated by a cosmological constant, not ordinary matter."

The Nobel lecture [6] also noted: "Our year-long program from 2002 to 2003 to measure supernovae on Hubble was highly successful. We found six type Ia supernovae at redshifts over 1.25. They allowed us to rule out gray dust and evolution and to clearly determine that the Universe was decelerating before it began accelerating ... In physics, a change in the value or sign of deceleration (which results from a change in force) is called a jerk."

The works [1–4] are distinguished by the logic of statistical inference relying on the normal theory and confidence levels for standard deviations, as well as the default acceptance of the continuity hypothesis with respect to the systematic component of the cosmological distance scale model.

In [7], it is shown that for the isotropic model of the cosmological distance scale in the form of the dependence of photometric distance on redshift $D_L(z)$, the cosmic jerk — transition from decelerated to accelerated expansion of the universe — can correspond to a change point — a structural and parameter change in the systematic component of the model. Such scale model disruptions were, in fact, found for the data [1–4] when testing the system of statistical hypotheses according to R 50.2.004-2000;¹ however, the number of change points exceeded that of the specified redshift-based cosmic jerk estimates,² and the estimates did not coincide. The role of rank inversions, i.e., disruptions in the monotonicity of distances to supernovae with respect to the ordered sequence of corresponding redshifts, also remains uncertain.

The paper aims to interpret the data [1–4], which led to the conclusion about the accelerating expansion of the Universe, under the anisotropic model of the cosmological distance scale [10–12] according to the criterion of minimum inadequacy error relying on the logic of statistical inference in R 50.2.004-2000, while taking into account possible reasons for the change points and rank inversions of the isotropic model [1] representing the systematic component of the cosmological distance scale, as well as astronomical discoveries of the last decades [13].

Cosmic Jerk and Change Points. As a factor causing the accelerating expansion of the universe, the cosmic jerk explicitly emerged in the Taylor series expansion of the Friedmann–Robertson–Walker model [14]:

$$D_L(z) = \frac{c}{H_0} \left(z + \frac{1 - q_0}{2} z^2 - \frac{1 - q_0 - 3q_0^2 + j_0}{6} z^3 \right), \quad (1)$$

where c is the speed of light; q_0 is the deceleration parameter; j_0 is the jerk parameter.

A snap parameter can also be introduced; however, in [14] the author points out that: "Since the jerk and snap are rather difficult to measure, being related to the third and fourth terms in the Taylor series expansion of the Hubble law, it becomes clear why direct observational constraints on the cosmological equation of state are so relatively weak, and are likely to remain weak for the foreseeable future."

What is more, it is obvious that model (1) is continuous and does not contain any structural or parameter changes, while the jerk parameter j_0 can only be attributed to the Big Bang. However, 12 years later, model (1) was identified using data on 19 cepheids and 300 supernovae at $z < 0.15$ in [15], which initiated the discussion about the impasse in cosmology (see [16]). A large number of facts had been gathered long before this discussion, so it was already possible to clarify the situation regarding change points in the cosmological distance scale model and the rank inversions of photometric distances to SNe Ia as reference points in the model. Let us present these facts in chronological order.

1967 — attention is drawn to the concentration of quasars with large redshift toward the minimum in the CMB distribution [17].

1974 — the reference book [18, Part 1, Table 31] contains data of 1966–1974 on 383 quasars and radio galaxies.

1977 — the paper [19] links the Doppler effect with the dipole anisotropy of the CMB, showing that relative to it, the Sun moves at a velocity of $350 \text{ km}\cdot\text{s}^{-1}$ to a point ($\alpha \approx 11^{\text{h}}$ and $\delta \approx 4^\circ$) in the constellation of Leo.

¹R 50.2.004-2000. GSI. Characterization of Mathematical Models Representing Dependencies Between Physical Quantities in Solving Measurement Problems. Basic Provisions.

²The search for change points is reminiscent of the 1983 search for the wreckage of the South Korean Boeing 747 airliner conducted in different places by Soviet, South Korean, Japanese, and U.S. ships. At that time, the wreckage of four different planes was found; however, Boeing 747 KAL-007 was not among them [8, 9].

1981 — measurements performed onboard a U-2 aircraft indicate the dipole of CMB anisotropy along the Aquarius–Leo axis [20].

1981–1999 — large-scale galaxy streams moving toward the superclusters of Virgo, Great Attractor, and Hydra were detected (see the review [21, Table 5]).

1997 — an ellipsoid of the local Hubble parameter H_0 (81 ± 3 ; 62 ± 3 ; 48 ± 5) $\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ was identified for the Local Group, with the **major axis** oriented to a point in the constellation of Virgo with equatorial coordinates $\alpha \approx 13^{\text{h}}$ and $\delta \approx -15^\circ$ [22].

1998 — divergent anisotropic dipoles in the redshift of radio galaxies and quasars with extrema \bar{z} at right ascensions of $\alpha \approx 1^{\text{h}}$ and $\alpha \approx 13^{\text{h}}$ [24] were found using the MCM-stat M program [23] according to [18, Part 1, Table 31], which improved the result of [17].

2000 — redshift of 167 galaxies and purple shift of 37 galaxies were detected in the Local Group [25].

2005 — an X-ray examination revealed that the mass of the Great Attractor supercluster is only 0.1 of what was originally assumed and the Local Group is, in fact, being pulled toward the most massive supercluster of galaxies in the observable universe, the Shapley Supercluster, which is located behind the Great Attractor [26].

2007 — the so-called CMB Cold spot (or Eridanus Supervoid) was discovered, which is centered at $\alpha \approx 03^{\text{h}}15^{\text{m}}05^{\text{s}}$ and $\delta \approx -19^\circ35'02''$ at $z \approx 1$ [27].

2009 — the work [28] states that the detected coincidence of CMB and redshift anisotropy centers for radio galaxies and quasars seems to be associated with two formations of the large-scale structure of the universe: supercluster of over 2500 galaxies in the constellation of Virgo ($16^\circ \times 10^\circ$ sector); cluster of over 800 galaxies in the constellation of Coma Berenices (a spot measuring 4° in diameter).

2010 — in [29], it is noted that with respect to the CMB, the solar apex is at the border of Leo and Crater in the Dipole⁺ region at $\Delta T > +3$ mK in the region of superclusters Shapley, Virgo, Vela, Coma Berenices, Hydra, Great Attractor, and Leo. The solar antapex corresponds to the Dipole⁻ region at $\Delta T > -3$ mK of the CMB in Aquarius and a supervoid of over 3 Gpc in Eridanus. The dipole test on diurnal harmonic yields a maximum redshift at $\alpha \approx 2.6^{\text{h}}$ for quasars ("Eridanus + Cetus") and at $\alpha \approx 22.1^{\text{h}}$ for radio galaxies (Aquarius), whereas the minimum is observed at $\alpha \approx 14.6^{\text{h}}$ for quasars ("Virgo + Centaurus") and at $\alpha \approx 10.1^{\text{h}}$ (Leo) for radio galaxies [30].

2013 — in [31], it is shown that the dipoles of CMB anisotropy, Hubble constant, and the acceleration parameter coincide with the axis of "South Galactic Pole + Aquarius and Eridanus supervoid system → North Galactic Pole + the largest system of superclusters and quasars Great Attractor — Shapley — Centaurus — Leo — Coma — Virgo," which can be called a dipole of inhomogeneity. Data obtained by the Planck probe confirmed the mean temperature asymmetry on the opposite celestial hemispheres and the existence of a large Cold Spot, whose size (diameter) was much greater than expected — up to 2 billion light years. In the direction of the Northern Galactic Pole in Coma Berenices, the largest structural element of the observable part of the universe was discovered — a supercluster of superclusters. A system of giant voids measuring 11–150 Mpc is found in Eridanus and Aquarius in the direction of the South Galactic Pole [32].

2014 — the paper [10] notes that the North (Coma Berenices) P_N and South (Sculptor) P_S Galactic Poles are near the centers of the transparency windows. In these directions, we can find the largest structural objects in the observable part of the universe — the Great Attractor + Shapley + Centaurus + Leo + Coma + Virgo supercluster system of galaxies, where the Local Group is moving, as well as the Aquarius + Eridanus supervoid system. Also, the Galactic polar axis is aligned with the dipole anisotropy of redshift in the spectra of extragalactic sources, galaxies and quasars, and the red-violet shift dipole in the Local Group. The same axis is linked with the CMB dipole anisotropy; noteworthy is that the Cold Spot is located in Eridanus [10]. According to [25], a red–violet shift dipole was identified in the Local Group that is oriented toward the CMB temperature maximum, the apex of the Sun's motion relative to the CMB, and the dipole of redshift anisotropy [33]. The Laniakea Supercluster was discovered, which measures about 520 million light years in diameter and is two orders of magnitude larger than the mass of the Virgo Supercluster, with its center of gravity in the Great Attractor [34].

2017 — the conclusions of [24, 30, 31] are confirmed in [35, 36], with the gravitational dipoles of inhomogeneity given the names of Dipole Repeller and Cold Spot Repeller.

2019 — in [37] states that the anisotropy of the local universe is incorrectly interpreted as dark energy.

The first step in analyzing the situation regarding change points and rank inversions of model (1) consisted in checking the data [1, 4] for compositional homogeneity according to R 50.2.004-2000. The check showed that in Taylor series expansions with anisotropy parameters $\theta_k(l, b)$

$$D_L(l, b, z) = \sum_{k=0}^K [1 + \theta(l, b)] z^k, K \leq 3, \quad (2)$$

which includes the isotropic model (1) as a special case, all samples (27 SN Ia at $z = 0.00834\text{--}0.12450$ and 10 SN Ia at $z = 0.30\text{--}0.97$ [1], as well as 42 SN Ia at $z = 0.172\text{--}0.830$ [4]) do not form a compositionally homogeneous population [11]. However, for each sample, the hypothesis of degeneracy was rejected, and the best result, 11.07 Mpc, was obtained using the data in [1]. For the data [1, 4], specifically, due to the unbalanced distribution of supernovae in terms of distance, this representation yielded a non-zero zero point, which is inconsistent with the physical meaning of the distance scale. It was possible to exclude the zero point only in two cases, of which it was chosen to combine the data [1, 4] into a sample of 79 SN Ia at the expense of increasing the average modulus of inadequacy error (AMIE) but at its lower value. According to the criterion of minimum AMIE, the anisotropic MCMLSM model (MCMLSM — maximum compactness and least squares methods) proved to be preferable for the entire set of data [12]:

$$D_L(z, \alpha, \beta) = \frac{c}{H_0} \left[(1 + \alpha b) z + \frac{(1 - q_0)(1 + \beta l)}{2} z^2 \right], \quad (3)$$

where $\alpha = -2.568655129 \cdot 10^{-3}$ and $\beta \approx +2.027311498 \cdot 10^{-3}$ — anisotropy coefficients in galactic coordinates (b, l); $H_0 = 60.80404264 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$; deceleration parameter of $q_0 = -0.14378664$.

In other words, model (1) proved to be redundant, as well as non-metric due to change points and rank inversions of photometric distance estimates. As for model (3), it represents a spatiotemporal trend.

The next step in analyzing the situation involved dividing the causes of change points into structural and parametric, related to the discontinuity of the systematic component for the anisotropic model of the near-distance scale, and spatiotemporal, i.e., arising in isotropic models for redshift-ordered supernovae due to significant differences in their angular coordinates.

Such a measurement problem can be solved in two steps. First, possible change points and rank inversions are established in one-dimensional (i.e., isotropic) $D_L(z)$ models of the form (1). Then, under anisotropic (in this case, three-dimensional) $D_L(z, \alpha, \beta)$ models, supernovae that are at the ends of continuity intervals are considered. According to R 50.2.004-2000, this means the sequential use of the MCM-stat program for the isotropic model (1) and MCM-stat M program for the anisotropic model (3) according to the criterion of minimum AMIE. In this case, due to the lack of compositional homogeneity, which is a generalization of the statistical homogeneity concept to the case of functional dependencies, all data are divided into subsamples [7, Table. 5].

In [4], a sample of 42 supernovae at $z = 0.172\text{--}0.830$ was examined. In this sample, the MCM-stat program detected a change point between SN 1997G ($z = 0.763$ and $D_L = 3854$ Mpc; $\alpha = 4^{\text{h}}58^{\text{m}}$, $\delta = -3^{\circ}16'$, Eridanus) and SN 1996cl ($z = 0.828$ and $D_L = 3801$ Mpc; $\delta = -3^{\circ}37'$, "Sextans–Leo") which simultaneously constitutes a rank inversion in the isotropic model. This corresponds to the prediction in [4] at $z \approx 0.73$, with SN 1997R ($z = 0.657$ and $D_L = 3388$ Mpc; $\alpha = 10^{\text{h}}57^{\text{m}}$, $\delta = -3^{\circ}55'$, Leo) as the closest supernova having a lower redshift to SN 1997G. In the considered sample, SN 1997ap ($z = 0.83$ and $D_L = 3266$ Mpc; $\alpha = 13^{\text{h}}47^{\text{m}}$, $\delta = +2^{\circ}24'$, Virgo) comes last. This is also consistent with the gravitational dipole axis of inhomogeneity "Eridanus → Leo" [30], with SN 1997G located in the diametrically opposite direction to SN 1997ap, 1996cl, and SN 1997R. Also, of these three, the "well-measured" SN 1997ap at $z = 0.83$ constitutes a rank inversion.

In [1], 37 supernovae were used at $z = 0.008\text{--}0.970$. In this data sample, the MCM-stat M program revealed several continuity intervals: [0.008–0.079]; [0.088–0.125]; [0.30–0.44]; [0.48–0.57]; [0.62–0.97]. Similarly, in [4], two continuity intervals were identified for 42 SN Ia: [0.172–0.763] and [0.828–0.83]. The boundaries of these intervals correspond to the supernovae summarized in the Table 1.

It is not necessary to comment on the coincidence of change points and rank inversions on the gravitational dipole axis of inhomogeneity "Eridanus → Virgo–Leo," which is formed by giant voids and massive superclusters of galaxies.

TABLE 1. SN Ia Parameters

SN Ia, constellation	z	D_L , Mpc	α	δ
1995D, Sextans–Leo	0.008	40	9 ^h 41 ^m	+5° 08′
1992bp, Eridanus	0.079	339	3 ^h 36 ^m	−18° 21′
1992br, Eridanus	0.088	439	1 ^h 46 ^m	−56° 06′
1996ab, Boötes	0.125	634	15 ^h 21 ^m	+27° 56′
1996J, Sextans–Leo	0.300	1888	10 ^h 11 ^m	−12° 37′
1997ce, Hercules	0.440	2455	17 ^h 08 ^m	+44° 01′
1995K, Sextans–Leo	0.480	3090	10 ^h 51 ^m	−9° 15′
1996I, Virgo	0.570	3565	12 ^h 01 ^m	−0° 16′
1996H, Virgo	0.620	3945	12 ^h 29 ^m	−0° 06′
1997ck, Hercules	0.970	7556	16 ^h 53 ^m	+35° 03′
1997I, Eridanus	0.172	762	04 ^h 59 ^m	−3° 09′
1997G, Eridanus	0.763	3855	04 ^h 58 ^m	−3° 16′
1996cl, Leo	0.828	3802	10 ^h 57 ^m	−3° 37′
1997ap, Virgo	0.830	3266	13 ^h 47 ^m	+2° 24′

Conclusion. The most remarkable aspect of this analysis proved to be that seemingly formal statistical tests for the inadequate mathematical treatment of data as per R 50.2.004-2000, on the basis of which a conclusion was drawn about the accelerating expansion of the universe, led to a physically meaningful result. An alternative to the accelerating expansion of the universe could be the hypothesis about the gravitational dipole of inhomogeneity. This conclusion emerged from a string of unexpected results and coincidences [33, 38], to which cosmologists have recently turned their attention.

Conflict of interest. The authors declare no conflict of interest.

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