OPTOPHYSICAL MEASUREMENTS

INFLUENCE OF ATMOSPHERIC TRANSMISSION ON THE ACCURACY OF LIDAR MEASUREMENTS OF THE MIE SCATTERING POWER BY AEROSOL PARTICLES

V. E. Privalov,¹ Yu. V. Cherbachi,² and V. G. Shemanin^{2,3} UDC 53.08:535.016

The current issue of improving the lidar measurements accuracy has been considered. For a monostatic aerosol lidar, the degree of influence of atmospheric transmission at the selected laser radiation wave*lengths on the lidar signal error has been estimated. Numerical simulation of the lidar equation for Mie scattering by atmospheric aerosol particles during vertical remote sensing of the atmospheric boundary layer up to 1500 m has been carried out. It has been shown that taking into account the measurement error of the extinction coeffi cient at the selected laser radiation wavelengths leads to limitation of the ranging distances to ensure the specifi ed measurement error of the lidar signal. The results obtained can be applied to the development of new aerosol lidars.*

Keywords: aerosol lidar, equation, Mie scattering, atmospheric transmission, relative error.

Introduction. To measure the parameters of atmospheric aerosol, various laser systems [1, 2] of both aerosol scattering and differential extinction are widely used. These systems allow studying the dependences of scattering and extinction signals of laser radiation on the microphysical parameters of the aerosol at different wavelengths. In [3, 4], the lidar equation for Mie scattering by aerosol particles in the 180° direction has been considered under the assumption that the influence of atmospheric transmission on the error of remote sensing results is insignificant. The use of broadband laser sources in differential absorption and scattering lidars in $[5, 6]$ demonstrated that it is necessary to take into account the influence of the spectral dependence of atmospheric transmission on the remote sensing results.

The objective of this study is to estimate the degree of influence of atmospheric transmission on the lidar signal measurement error at selected wavelengths of laser radiation during lidar remote sensing of atmospheric aerosol. To estimate the relative error of lidar measurements, numerical simulation of the spectral dependence of the atmospheric transmission and the dependence of the Mie scattering power by aerosol particles on the remote sensing distance has been performed.

Aerosol lidar. The optical scheme of a monostatic aerosol lidar is given in [4]. The optical axes of the emitter (laser) and the receiving telescope are directed along axis *Z*. As the emitter, let us consider a solid-state YAG–Nd laser with fundamental, second, and third harmonic wavelengths of 1064, 532, and 355 nm, respectively. Let us characterize the laser emitter of the lidar, similar as in [4, 7], by the power P_0 of laser radiation sent into the atmosphere and the duration of the laser pulse τ_0 ; the lasing line is assumed to be Gaussian with a maximum at the frequency v_0 and a half-width Γ. Then, according to [8], let us represent the laser radiation power in the following form:

¹ Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia; e-mail: vaevpriv@yandex.ru.

² Shukhov Belgorod State Technological University, Novorossiysk, Russia; e-mail: ycherbachi@bk.ru.

³ Novorossiysk Polytechnic Institute, Branch of the Kuban State Technological University, Novorossiysk, Russia;

e-mail: vshemanin@mail.ru.

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TABLE 1. Calculation Parameters of the Lidar Equation

λ , nm	$\alpha(v, z)$, km ⁻¹	$\sigma(\pi, \nu, z), \text{km}^{-1}$	$\epsilon(\lambda)$
1064	0.09	0.0094	0.05
532	0.16	0.0213	0.95
355	0.29	0.0441	0.10

$$
P_1 = P_0 \int_{v_0 - \Gamma}^{v_0 + \Gamma} \Phi(v) \, dv,
$$

where $\Phi(v)$ is the Gaussian function describing the contour of the lasing line:

$$
\Phi(v) = \exp[-(v - v_0)^2/(2\Gamma^2)]/[(2\pi)^{1/2}\Gamma].
$$

Let us denote the area of the telescope receiving aperture as S_0 . Let us specify the optical characteristics of the atmosphere along the remote sensing path *z* by the scattering indicatrix $X(\varphi, \nu, z)$, the scattering coefficient $\sigma(\nu, z)$, and the extinction coefficient $\alpha(v, z)$. The transmission or transparency of the atmosphere at a frequency v of the path section from the lidar to the volume under study $T(v, z)$ according to [9] is

$$
T(v, z) = \exp\left(-\int_{0}^{z} \alpha(v, z') dz'\right),
$$
\n(1)

where z' is the current integration coordinate with the limit z .

Let us consider that the exponent $\alpha(v, z)$ in Eq. (1) is measured with a relative error $E = 10\%$. Let us calculate two transmission curves under the assumption that in one case the error at each subsequent step in the remote sensing distance with the number *i* in comparison with the previous step $(i - 1)$ increases as

$$
\alpha_i(\mathbf{v}, z) = \alpha_{i-1}(\mathbf{v}, z) + \alpha_{i-1}(\mathbf{v}, z)E
$$
 (2)

and in the other case – at each subsequent step it decreases as

$$
\alpha_i(\mathbf{v}, z) = \alpha_{i-1}(\mathbf{v}, z) - \alpha_{i-1}(\mathbf{v}, z)E. \tag{3}
$$

For comparison, let us also calculate the third curve for a homogeneous atmosphere with a constant exponent $\alpha(v, z)$ in Eq. (1). All the parameters needed for calculations at the selected laser radiation wavelengths of 1064, 532, and 355 nm were taken from [3, 7, 9, 10] and summarized in Table 1, where λ is the wavelength, and $\xi(\lambda)$ is the relative spectral sensitivity of the photodetector.

Results of the lidar equation simulation and discussion. The results of numerical simulation of the lidar equation of the aerosol lidar variant considered above are shown in Fig. 1. As an example, two families of three curves for wavelengths of 355 and 532 nm are presented. For each family or for each wavelength, the dependences of the atmospheric transmission on the remote sensing distance have been plotted: curves *3*, *4* are calculated according to Eq. (1); curves *1*, *2* – according to Eq. (1) taking into account the increase in the extinction coefficient according to Eq. (2); curves $5, 6$ – according to Eq. (1) taking into account the decrease in the extinction coefficient according to Eq. (3) . The obtained dependences of the atmospheric transmission on the remote sensing distance are in good agreement with the data of [11, 12].

 The relative error of measuring the atmospheric transmission in the range of remote sensing distances up to 1.5 km, determined according to Fig. 1, is shown in Fig. 2. The relative error exceeds 10% when sensing at laser radiation wavelengths of 355, 532, and 1064 nm at sensing distances of more than 1000, 1200, and 1350 m, respectively.

 Let us consider the data in Fig. 1 in the numerical simulation of the lidar equation for Mie back scattering for a lasing line with a finite half-width Γ [4]:

$$
P(v, z) = \frac{c\tau_0 S_0 P_0 G(z)}{2z^2} \int_{v_0 - \Gamma}^{v_0 + \Gamma} \Phi(v) T^2(v, z) \sigma(\pi, v, z) A(v) dv,
$$
\n(4)

Fig. 1. Calculated dependences of the atmospheric transmittance *T* on the height *z* for a homogeneous atmosphere and laser radiation wavelengths of 355 nm (curves *1*, *3*, *5*) and 532 nm (curves 2, 4, 6) when calculating the extinction coefficient $\alpha(z)$ according to Eq. (3) (curves *1*, *2*) and Eq. (2) (curves *5*, *6*), as well as for $\alpha(z) = \text{const}$ (curves 3, 4).

Fig. 3. Logarithmic dependences of the lidar signal power $P(v, z)$ on the ranging distance *z* calculated according to Eq. (7) for a homogeneous atmosphere and laser radiation wavelengths of 355 nm (curves *4*, *5*, *9*), 532 nm (curves *1*, *2*, *3*), and 1064 nm (curves 6, 7, 8) when calculating the extinction coefficient $\alpha(z)$ according to Eq. (3) (curves *1*, *5*, *6*) and Eq. (2) (curves *3*, *8*, *9*), as well as for $\alpha(z) = \text{const}$ (curves 2, 4, 7).

Fig. 2. Dependence of the relative measurement error of atmospheric transmission on the remote sensing distance in the range up to 1.5 km for laser radiation wavelengths 355 (*1*), 532 (*2*), and 1064 (*3*) nm, calculated according to Fig. 1.

Fig. 4. Dependences of the relative error of measuring the Mie back scattered radiation power *E* in relative units on the remote sensing distance *z* for the selected laser radiation wavelengths of 355 nm (*1*), 532 nm (*2*), 1064 nm (*3*) calculated according to Eq. (9).

where *c* is the light velocity; $G(z)$ is the geometric function of the lidar or the geometric factor [3, 9];

$$
A(v) = A_0 \exp[-(v - v_0)^2 / (2\Gamma_a^2)]
$$

is the spectral transmittance coefficient of the lidar receiving system $[4, 7]$ or its instrumental function, which is also approximated by a Gaussian curve with amplitude A_0 and half-width Γ_a .

The back scattering coefficient $\sigma(\pi, \nu, z)$ [3] can be expressed from the relation for the scattering indicatrix in [4]:

$$
\sigma(\pi, \nu, z) = X(\pi, \nu, z)\sigma(\nu, z)/(4\pi). \tag{5}
$$

The amplitude A_0 in the calculations is taken equal to the relative spectral sensitivity of the lidar photodetector (see Table 1). As a lidar photodetector, a highly sensitive line of a charge-coupled photodevice TCD1304DG(M/X) (Toshiba, Japan, 3500 pixels) of an FSD-8 microspectrometer (TsVO IOF RAN, Russia) was used. Let us assume that, at the wavelengths of the remote sensing radiation, absorption by atmospheric gas molecules is negligibly low compared to aerosol scattering [7]. Then, the extinction coefficient inside the laser line slightly depends on the frequency v in the studied range 2Γ and α(ν, *z*) = α(*z*). Similarly, σ(π, ν, *z*) ≈ σ(*z*). The remaining lidar parameters are: peak power of the laser radiation pulse $P_0 = 1$ MW; laser pulse duration of 10 ns; receiving telescope area $S_0 = 0.12$ m²; one measurement duration of 1 μs; the Mie back scattering coefficients [3, 7, 10] and the average value of the Mie scattering coefficients are presented in Table 1. Next, let us rewrite Eq. (4) as

$$
P(v, z) = \frac{CG(z)}{z^2} \sigma(z) T^2(v, z) \int_{v_0 - \Gamma}^{v_0 + \Gamma} A_0 \exp[-(v - v_0)^2 / (2\Gamma_a^2) - (v - v_0)^2 / (2\Gamma^2)] dv,
$$
 (6)

where $C = (c\tau_0/2)S_0P_0$.

Let us transform the last two exponents in Eq. (6) using the error function erf (y) [11]:

$$
P(\mathbf{v}, z) = \frac{CG(z)}{z^2} \sigma(z) T^2(\mathbf{v}, z) \frac{2A_0 \Gamma_a}{\sqrt{\pi(\Gamma^2 + \Gamma_a^2)}} \operatorname{erf}\left(\frac{1}{\Gamma} \sqrt{\frac{\Gamma_a^2 + \Gamma^2}{2}}\right).
$$
 (7)

Let us assume that $G(z) = 1$ for the experimental situation under consideration and analyse the dependence of the numerical simulation results of Eq. (7) for values of the half-width ratio Γ*a*/Γ from 1 to 10, which is presented in Table 2.

 According to Table 2, with an increase in the ratio Γ*a*/Γ from 1 to 10, the Mie scattering signal power increases to a maximum value in accordance with [12, 13]. Moreover, the value $\Gamma_a/\Gamma = 10$ shows that the half-width of the instrumental function is an order of magnitude higher than the half-width of the lasing line, which is acceptable for industrial lasers and microspectrometers as part of a lidar [2, 3, 7].

Let us consider the solution of the lidar equation (7) for all selected wavelengths, the calculated values of the atmospheric transmission, and the ranging distances up to 1.5 km. The results of these calculations are shown in Fig. 3.

 Comparison of curves in Figs. 1, 3 shows that the calculated dependencies in Fig. 3 are similar to the dependencies in Fig. 1, but the differences in them for the atmosphere transmission and the Mie scattering signal power at the selected wavelengths are less pronounced. A sharp increase in the Mie scattering signal power for a wavelength of 532 nm is associated with the entry of laser radiation of this wavelength into the area of maximum photodetector sensitivity (see Table 1).

Calculation of the lidar measurements error. Let us estimate the errors of solving Eq. (7) assuming that the main contribution is made by the errors in measuring the atmospheric transmission and the back scattering coefficient. To calculate the error, let us determine the differential of Eq. (7):

$$
dP(v,z) = \frac{CG(z)}{z^2} [\sigma(z)2T(v,z)dT(v,z) + d\sigma(z)T^2(v,z)] \frac{2A_0\Gamma_a}{\sqrt{\pi(\Gamma^2 + \Gamma_a^2)}} \text{erf}\left(\frac{1}{\Gamma}\sqrt{\frac{\Gamma_a^2 + \Gamma^2}{2}}\right).
$$
 (8)

Replacing the differentials with finite increments and dividing Eq. (8) by Eq. (7) , an expression for the relative measurement error is obtained:

$$
E = \frac{\Delta P(\mathbf{v}, z)}{P(\mathbf{v}, z)} = \sqrt{4 \left(\frac{\Delta T(\mathbf{v}, z)}{T(\mathbf{v}, z)} \right)^2 + \left(\frac{\Delta \sigma(z)}{\sigma(z)} \right)^2}.
$$
\n(9)

 The results of calculations according to Eq. (9) for the selected laser radiation wavelengths and remote sensing distances up to 1500 m are shown in Fig. 4 and differ from the data in Fig. 2. A relative error of measuring the lidar signal of 25% was obtained when sensing at laser radiation wavelengths of 355, 532, and 1064 nm at remote sensing distances of more than 750, 1000, and 1200 m, respectively.

Conclusion. The results of computer simulation have shown that the relative error in the lidar measurement of the back scattered Mie radiation power is due to errors in measuring the atmospheric transmission and back scattering coefficient, which corresponds to the results of [14]. For the half-width ratio $\Gamma_a/\Gamma = 10$ for the case of single scattering and a homogeneous atmosphere, the value of the back scattered Mie radiation power is maximum and is determined only by the spectral dependence of the atmospheric layer transmission to the remote sensing area at a distance *z* for equal laser peak power at different wavelengths.

 The spectral dependence of the atmospheric transmission at the selected laser radiation wavelengths is taken into account in the form of a relative error of the exponent $\alpha(v, z)$ in Eq. (1) of 10%. It led to a relative error of measuring the lidar signal of 25% when sensing at laser radiation wavelengths of 355, 532, and 1064 nm at distances higher than 750, 1000, and 1200 m, respectively.

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