DOI 10.1007/s11018-021-01874-9
Measurement Techniques, Vol. 63, No. 11, February, 2021

FUNDAMENTAL PROBLEMS IN METROLOGY

COSMOLOGICAL DISTANCE SCALE. PART 11. "EXTRAORDINARY" EVIDENCE AND THE "COSMIC JERK" PROBLEM

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UDC 519.245:519.65:52+53:520.12

A statistical test of the "extraordinary" evidence for the "accelerated expansion of the Universe" owing to the "cosmic jerk" over a range of red shifts $z = 0.46 \pm 0.13$ and for $z = 0.763$ based on data from SN Ia supernovae for which photometric distances have been determined is carried out. The transition from "deceleration" to "acceleration" is treated as a "disruption"– a change in the structure and parameters of the model for the cosmological distance scale. It is shown that the data from different sources do not form a compositionally uniform set. "Discrepancies" in the model for the scale are discovered for $z = 0.44–0.48$ in a sample of 10 SN Ia over an interval from $z = 0.30–0.97$ according to data from the High-Z Supernovae Search Team and for red shifts $z = 0.763–0.828$ in a sample of 42 SN Ia over an interval from $z = 0.172–0.83$ according to data from the Supernovae Cosmology Project. The reason for these "discrepancies" may be an unbalanced and random distribution of the SN Ia over the observed range of red shifts for a scale with a clearly distinct non-metric character.

Keywords: *SN Ia supernovae, photometric distance, red shift, Friedman–Robertson–Walker model.*

Introduction. In the section "Extraordinary claims require extraordinary evidence," the Nobel prize lecture of Riess [1] describes a big project to exclude alternative astrophysical hypotheses (absorption by dust or the intrinsic evolution of sources) for the accelerated expansion of the Universe visible in supernovae in a model with hidden mass and dark energy. The year-long program from 2002–2003 to observe distant supernovae on the Hubble space telescope was highly successful. Six Type Ia supernovae at red shifts over 1.25 were discovered and they made it possible to rule out gray dust and evolution and to clearly determine that the Universe was decelerating before it began expanding with acceleration. In physics, a change in the value or sign of deceleration (as a consequence of a change in force) is caused by a sudden *jerk*.

The results from 2002–2003 mentioned in Ref. 1 are listed in Table 1 with the photometric distances D_L from Refs. 2 and 3.

These supernovae belong to the Hubble deep field (HDF) and the Hubble ultradeep field (HUDF). They are found in segments of the sky with a diameter on the order of a few angular minutes in the constellations Ursa Major [$12^h36^m49.4^s$; $+62°12'58''$] and Fornax [$3^h32^m39.0^s$; $-27°47'29.1''$]. Both fields are near galactic poles separated from bright stars of the Milky Way, so it is possible to observe the very dim objects that turn out to be galaxies with large red shifts.

According to Refs. 2 and 4, the purely kinematic interpretation of the SN Ia sample yields evidence with a confidence level of over 99% for a transition from deceleration to acceleration or, analogously, convincing proof of a "cosmic *jerk*." For a *simple* model of the history of the expansion, the transition between the two epochs is bounded by $z = 0.46 \pm 0.13$.

In earlier papers of S. Perlmutter's group it was stated that the transition between deceleration and acceleration took place at $z = 0.73$, when the supernova SN 1997G exploded.

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TABLE 1. Supernovae from the Hubble Deep and Ultradeep Fields [2, 3]

n	SN	z	D_L, Mpc	Field	n	SN	z	D_L, Mpc	Field
1	1997ff	1.755	11748.97555	HDF	22	HST04Mcg	1.370	11117.31727	HUDF
2	2002dc	0.475	3076.096815	HDF	23	HST05Fer	1.020	6280.583588	HDF
3	2002dd	0.950	6251.726928	HDF	24	HST05Koe	1.230	10814.33951	HDF
4	2003aj	1.307	9954.054174	HUDF	25	HST05Dic	0.638	3784.425847	HDF
5	2002fx	1.400	11376.27286	HUDF	26	HST04Gre	1.140	7726.805851	HUDF
6	2003eq	0.840	5420.008904	HDF	27	HST04Omb	0.975	6950.243176	HUDF
7	2003es	0.954	7244.359601	HDF	28	HST05Red	1.190	5345.643594	HDF
8	2003az	1.265	8472.274141	HDF	29	HST05Lan	1.230	9862.794856	HDF
9	2002kc	0.216	1164.126029	HUDF	30	HST04Tha	0.954	5888.436554	HDF
10	2003eb	0.900	5345.643594	HDF	31	HST04Rak	0.740	4742.419853	HUDF
11	2003XX	0.935	6223.002852	HDF	32	HST05Zwi	0.521	2570.395783	HUDF
12	2002hr	0.526	4130.47502	HUDF	33	HST04Haw	0.490	3221.068791	HDF
13	2003bd	0.670	4345.102242	HDF	34	HST04Kur	0.359	1761.976046	HUDF
14	2002kd	0.735	4246.195639	HUDF	35	HST04Yow	0.460	2792.543841	HDF
15	2003be	0.640	3999.447498	HDF	36	HST04Man	0.854	6194.410751	HDF
16	2003dy	1.340	9638.290236	HDF	37	HST05Spo	0.839	4897.788194	HDF
17	2002ki	1.140	8749.837752	HDF	38	HST04Eag	1.020	8016.780634	HDF
18	2003ak	1.551	10327.61406	HUDF	39	HST05Gab	1.120	8590.135215	HDF
19	2002hp	1.305	7979.946873	HUDF	40	HST05Str	1.010	8994.975815	HDF
20	2002fw	1.300	10280.16298	HUDF	41	HST04Sas	1.390	9549.92586	HDF
21	HST04Pat	0.970	8590.135215	HDF					

In 2004, Visser [5] used an expansion of the Friedman–Robertson–Walker model in a Taylor series with respect to the red shift with a curvature parameter $\Omega_k = 0$ to obtain a third order model for the photometric distance:

$$D_L(z) = \frac{c}{H_0}\left[z + \frac{1}{2}(1 - q_0)z^2 - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^3\right], \tag{1}$$

where c is the fundamental constant of the speed of light, H_0 is the Hubble parameter, q_0 is the deceleration parameter, and j_0 is the jerk parameter.

In 2016, with a new group of coworkers, Riess used the model of Eq. (1) to obtain an new estimate $H_0 = 73.24 \pm 1.24$ km/s/Mpc [6]. It differed significantly from the previous estimate $H_0 = 65.2 \pm 1.3$ km/s/Mpc [4] obtained from the Friedman–Robertson–Walker model with a curvature parameter $\Omega_k = 0$ and from an estimate $H_0 = 67.8 \pm 0.9$ km/s/Mpc based on data from the Planck space probe using the ΛCDM-model [7]. This fact was pointed out by experts from the Carnegie-Chicago Hubble project [8].

Also in 2016, metrological expert evaluation [9] on data from type SN Ia supernovae [4, 10] revealed an anisotropy of the red shift and an asymmetry in its relation to the photometric distance.

In 2017, Freedman referred to discrepancies in model estimates of the Hubble parameter by more than “3 sigma” in a “normal” law, as an “impasse” in cosmology [11], and she saw a way out by bringing the accuracy of the cosmological distance scale to better than 1%. Further measurements [12], however, have only increased the spread in the estimates and the number of people involved in the discussion began to increase.

But, without dramatizing the situation, in terms of the logic of statistical inference according to the standards document R 50.2.004.2000, "State System for Ensuring the Uniformity of Measurements (GSI). Determination of the Characteristics of Mathematical Models of the Relationships Between Physical Quantities in Solving Measurement Problems. Basic Assumptions," the "*jerk* problem" is a standard problem of statistical testing of the hypothesis of the "breakdown" of a model. And the problem of calibrating the cosmological distance scale can be analyzed in terms of the criterion of the minimum of the average modulus of the inadequacy error (AMIE) by the programs "MMK-stat" and "MMK-stat M" [13].

We recall that a test for the "discrepancy" in the final results of the Hubble Space Telescope program Key Project for determining the Hubble parameter [14] rejected its estimate of $H_0 = 72.6$ km/s/Mpc independent of the photometric distance $D_L \le 391.5$ Mpc. A reduction in the Hubble parameter to $H_0 = 65.95$ km/s/Mpc with $D_L > 309.5$ Mpc was recognized as more likely [15]. But just this circumstance eliminated the questions regarding the substantial spread in the estimates [7, 8, 11, 12].

Statistical testing of hypotheses. To describe the history of the expansion of the Universe we use a *simple* isotropic model of the scale,

$$D_L(z) = \vartheta_0\theta_0 + \sum_{m=1}^{M} \vartheta_m \theta_m z^m, \tag{2}$$

where $\theta_0, ..., \theta_m, ..., \theta_M$ are the parameters of the model and $\vartheta_0\vartheta_1\vartheta_2...\vartheta_m...\vartheta_M = \vartheta$ is the binary code for its structure; for estimating the parameters by the algorithms of the method of maximum compactness (MMK) in a scheme of crossover observation of the inadequacy error based on the methods of least squares MMKMNK (MMCMNA), least moduli MMKMNM, or median interpolation MMKMEDS (MCODS) in accordance with the standards document R 50.2.004-2000.

We note that a model of maximum complexity $\max\{I, J, M\} \le K$ of the form

$$D_L(l, b, z) = \sum_{i=0}^{I} \sum_{j=0}^{J} \sum_{m=0}^{M} \vartheta_{ijm}\theta_{ijm} l^i b^j z^m \tag{3}$$

was used to detect anisotropy in the red shift in Refs. 9 and 15, where l and b are, respectively, the galactic longitude and latitude of the supernovae for a binary code $\vartheta = \vartheta_{000}\vartheta_{100}\vartheta_{010}...\vartheta_{mij}...\vartheta_{KKK}$ of the structure. The optimum with respect to the criterion of a minimum AMIE turned out to be the MMKMNK model

$$D_L(l, b, z) = 4930.4962z + 2819.7024z^2 + 9.9955969bz - 12.664675lz^2 \tag{4}$$

for an AMIE $\bar{\varepsilon}_\vartheta^{[2]} = 247.42842$ Mpc (here and in the following the computed values of the quantities are given in protocol form, i.e., without rounding), or after reduction to the analytic form (1):

$$D_L(l, b, z) = (c/H_0)[(1 + a_b b)z + (1 - q_0 + a_l l)z^2/2],$$

where $a_b = 2.027311498\cdot10^{-3}b$ and $a_l = -5.137310258\cdot10^{-3}l$ are the anisotropy coefficients, $H_0 = 60.80404234$ km/s/Mpc, and $q_0 = -0.14378664$.

Three samples of N SN Ia with $N = 37$ $\{0.008 \le z \le 0.97\}$ [4], $N = 42$ $\{0.172 \le z \le 0.830\}$ [10], and $N = 41$ $\{0.216 \le \le z \le 1.755\}$ [2, 3] were used to analyze the "cosmic jerk" hypothesis. They are described by a common model (3) of the form

$$D_L(z) = 5581.7251z - 4.1350279lz + 1697.3002z^2$$

with $\bar{\varepsilon}_\vartheta^{[2]} = 519.40485$ Mpc. However, according to the results of hypothesis testing of degeneracy $\mathbf{H}_0$, continuity $\mathbf{H}_{00}$, and compositional uniformity $\mathbf{H}_{000}$ with respect to the criterion of minimal AMIE according to R 50.2.004-2000 for the Friedman–Robertson–Walker model in the approximation of series (2) for $M \le 6$ (Tables 2 and 3, where the boldface entries indicate the AMIE of the best models of form (2) and the underlined boldface, the AMIE of the competing models) do not from a compositionally uniform sample, and for each sample the degeneracy hypothesis is rejected. And the question of combining the data of 1998–1999 and 2003–2004 drops out.

It is hard to explain why, in the model for the cosmological distance scale, the zero-point parameter $\theta_0 \ne 0$, but it is this property of the optimal MMK-models according to the data of Refs. 2 and 3 for AMIE > 800 Mpc (see Tables 2 and 3) which has become the reason for doubt in the "extraordinary" evidence. But when testing the hypothesis of continuity of the

TABLE 2. Tests of the Hypotheses $\mathbf{H}_0$ and $\mathbf{H}_{000}$ Based on the Data of Refs. 2–4 and 10 for the Algorithm MMKMNK

SN Ia sample	ϑ	$D_L(z) = \theta_0 + \theta_1 z + \theta_2 z^2 + \theta_3 z^3 + \theta_4 z^4 + \theta_5 z^5$						AMIE, Mpc	Combined AMIE, Mpc
		θ_0	θ_1	θ_2	θ_3	θ_4	θ_5		
42 + 37 + 41	100000	3484.839	0	0	0	0	0	4512.452	Isolating the optimal model
42 + 37 + 41	111100	143.26	2064.528	7478.81	–2735.866	0	0	**777.2195**	
42 + 37 + 41	111110	–	–	–	–	–	–	916.7722	
37	111010	–33.39257	5627.184	817.9905	0	1529.156	0	98.76057	Test for uniformity **428.3367499**
42	010000	0	4863.392	0	0	0	0	298.2358	
41	110000	–408.3209	7500.839	0	0	0	0	859.0333	

TABLE 3. Tests of the Hypotheses $\mathbf{H}_0$ and $\mathbf{H}_{000}$ Based on the Data of Refs. 2–4 and 10 for the Algorithm MMKMNM

SN Ia sample	ϑ	$D_L(z) = \theta_0 + \theta_1 z + \theta_2 z^2 + \theta_3 z^3 + \theta_4 z^4 + \theta_5 z^5$						AMIE, Mpc	Combined AMIE, Mpc
		θ_0	θ_1	θ_2	θ_3	θ_4	θ_5		
42 + 37 + 41	100000	3290.644	0	0	0	0	0	4077.102	Isolating the optimal model
42 + 37 + 41	11001	63.84653	4885.396	0	0	1418.503	0	**634.3471**	
42 + 37 + 41	111100	–	–	–	–	–	–	736.4055	
37 + 42	01000	0	4997.367	0	0	0	0	**267.4473**	Test for uniformity **463.7420775**
41	11000	–618.6035	7697.767	0	0	0	0	841.9686	

MMK-models for the data of Refs. 2–4 and 10, we examine the "cosmic jerk" hypothesis as an alternative position characteristic corresponding to the "discrepancy."

The analysis showed that the MMKMEDS-model for the data of Ref. 4 for AMIE = **113.49**13127 Mpc and the MMKMNK-model for the data of Ref. 10 for AMIE = **291.51**11476 Mpc with "discrepancies" at the predicted points are the most likely in terms of the criterion of minimum AMIE. In Table 4 these points are distinguished by bold type with underlining and the model for the data of Refs. 4, 10, 2, and 3 for $z = 0.008$–1.755 is rejected – its AMIE = **497.14**71932 Mpc. Since the models for the position characteristics of the scale with a structure code of 01000 correspond to the Hubble law, while the MMK-models with a structure code of 011100 for a third order model (1) are not among the optimum models, model (1) turns out to be extra.

For the data of Ref. 10 with $\mathbf{0.763} \le z \le 0.828$, there is a "discrepancy" and a transition $D_L(z) = 4948.032z \to D_L = 3533.886$ Mpc. The point at which the "discrepancy" begins, $D_L(0.763) =$ **3854.78**3577 Mpc, actually corresponds to the supernova SN 1997G, but the supernovae 1996ci ($z = 0.828$; $D_L =$ **3801.89**3963 Mpc) and 1997ap ($z = 0.830$; $D_L =$ **3265.87**8322 Mpc) show up at a smaller photometric distance (Fig. 1). Here the red shift of SN 1996ab ($D_{L\max} =$ **4168.69**3835 Mpc) is $z = 0.592$.

There are a number of "discrepancies" for the data of Ref. 4 and one of them, in the red shift interval [0.44, 0.48] fits entirely in the estimate $z = 0.46 \pm 0.13$ [2] for the "cosmic jerk." This corresponds to a transition $D_L(z) = 5800.16z \to D_L(z) = 6244.237z$, or, in reverse time, to a transition $H_0 =$ **48.01**106332 km/s/Mpc $\to H_0 =$ **51.68**692829 km/s/Mpc (Fig. 2). With removal of the inversion the "discrepancy" vanishes and SN 1997ck with $z = 0.97$ becomes the outburst justifying the suspicion of the authors of Ref. 4. The "discrepancies" in the MMK-models for the scale (Fig. 3) separate SN 2003ak ($z = 1.551$; $D_L =$ **10327.61**406 Mpc) and SN 1997ff ($z = 1.755$; $D_L =$ **11748.97**555 Mpc) which are found in opposite directions. In addition, the "piecewise-Hubble" model with code $\vartheta = 01000$ and a parameter $H_0 = \{$**63.28**883891 $\to$ **42.36**803086$\}$ km/s/Mpc (see Table 4) may become the explanation, because as the boundaries of the observed parts of the Universe expanded during the twentieth century, the estimates for the Hubble "constant" decreased by more than an order of magnitude.

TABLE 4. Test of the Hypotheses $\mathbf{H}_0$, $\mathbf{H}_{00}$, and $\mathbf{H}_{000}$ Based on the Data of Refs. 2–4 and 10

Range of compositional nonuniformity with respect to *z* for the algorithm	Interval of continuity in *z*	*N*	ϑ	Model parameters	H_0 km/s/Mpc	AMIE, Mpc	Joint AMIE, Mpc
0.008–1.755 for MMKMNK	0.008–0.830	90	010000	$\theta_1 = 5404.21$	55.47387278	314.9334	**497.1471932**
	0.839–0.840	2	000001	$\theta_5 = 12374.1$	–	491.4998	
	0.854–0.950	4	010000	$\theta_1 = 6591.823$	45.47944597	314.5989	
	0.954–1.010	6	000100	$\theta_3 = 8214.763$	–	627.3931	
	1.020–1.190	6	100000	$\theta_0 = 7501.631$	–	1025.678	
	1.230–1.300	4	010000	$\theta_1 = 7839.505$	38.2412484	719.0664	
	1.305–1.400	6	000100	$\theta_3 = 8268.3$	–	2693.149	
	1.551–1.755	2	010000	$\theta_1 = 6678.834$	44.88694554	59.33789	
0.008–0.970 for MMKMEDS	0.008–**0.079**	24	01000	$\theta_1 = 4736.893$	63.28883891	11.97061	**113.4913127**
	0.088–0.125	3	01000	$\theta_1 = 4967.674$	60.34865774	21.62077	
	0.30–0.44	5	01000	$\theta_1 = 5800.16$	51.68692898	274.2916	
	0.48–0.57	3	01000	$\theta_1 = 6244.237$	48.01106332	68.66854	
	0.62–0.97	2	01000	$\theta_1 = 7075.912$	42.36803086	1134.779	
0.172–0.83 for MMKMNK	0.172–**0.763**	40	010000	$\theta_1 = 4943.809$	60.63997578	279.2859	**291.5111476**
	0.828–0.83	2	100000	$\theta_0 = 3533.886$	–	536.0161	
0.216–1.755 for MMKMEDS	0.216–1.755	41	11000	$\theta_0 = -618.6035$; $\theta_1 = 7897.767$	37.95914187	841.9686	841.9686

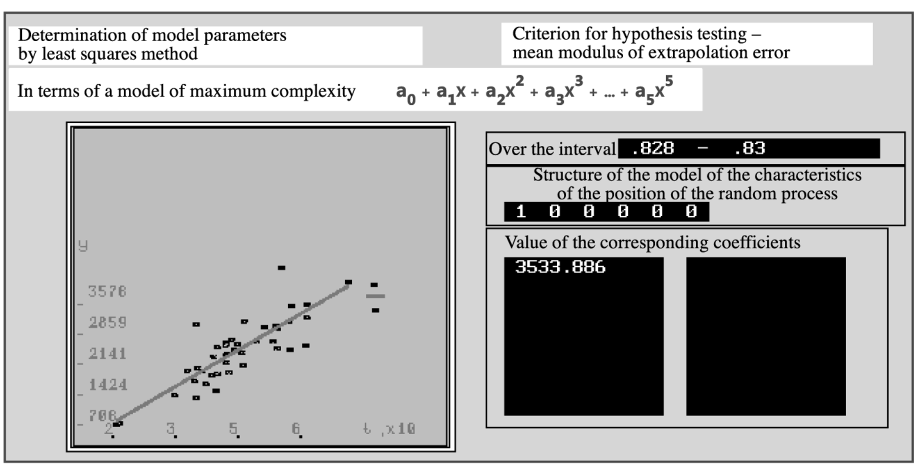


Fig. 1. "Discrepancies" in the MMKMNK (MMCMNA) model of the scale for the data of Ref. 10 with $z = 0.763$–0.828.

Conclusion. The "discrepancies" in the model of the cosmological distance scale for red shifts of $z = 0.44$–0.48 and $0.763 \leq z \leq 0.828$ according to the data of Ref. 10 do not conflict at once with the two hypotheses of a "cosmic jerk" for $z = 0.46 \pm 0.13$ [2] and $z = 0.763$ [4]. However, the very fact of the existence of these "discrepancies" and the inconsistency of these hypotheses has a very much simpler explanation – the cosmological distance scale based on red shift does not have a metric status [9], while the ordered set of red shifts corresponds to a partially ordered set of photometric distances. This failure of the condition of monotonicity for the cosmological distance scale based on red shift also raises doubts about the "cosmic jerk" hypothesis.

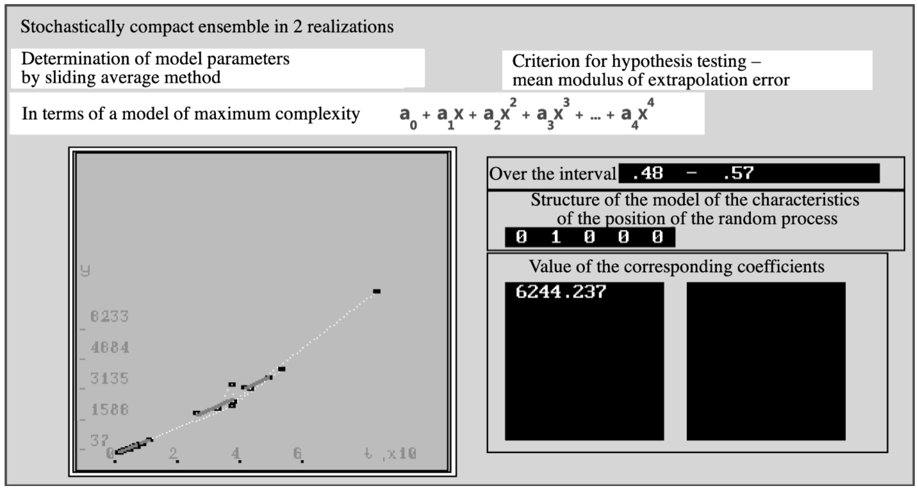
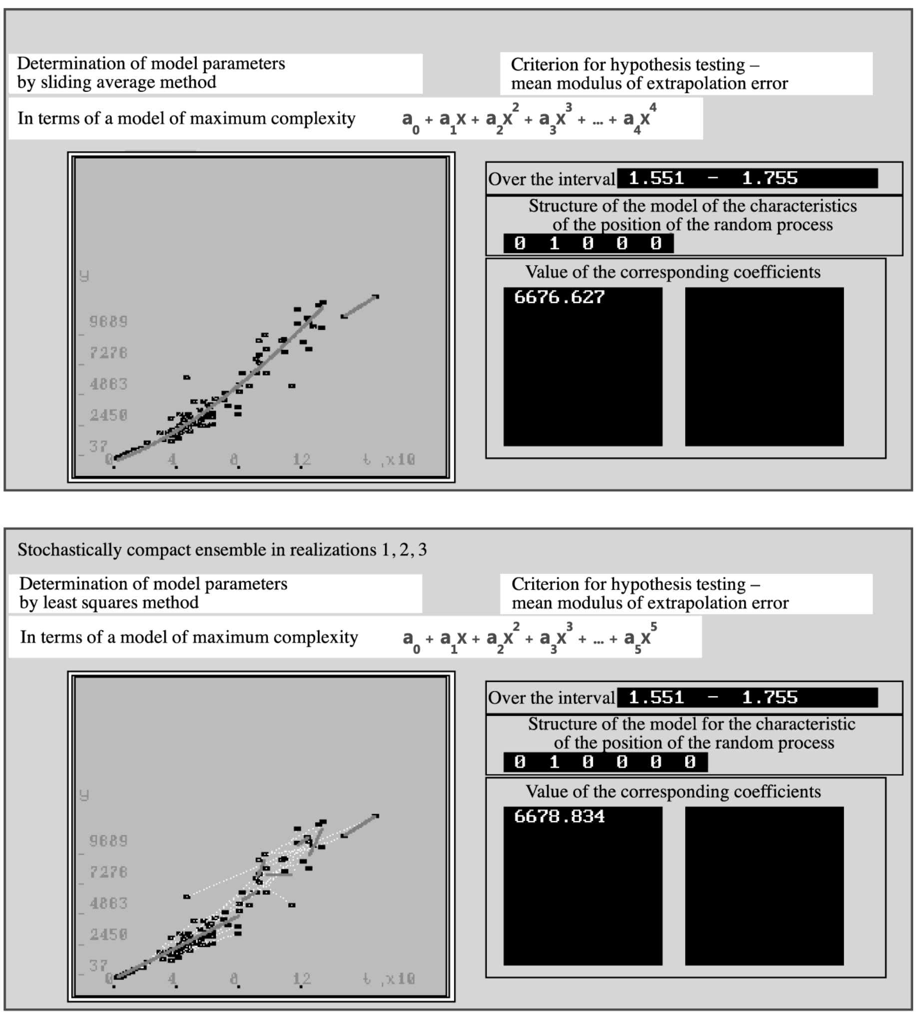


Fig. 2. “Discrepancies” in the MMKMEDS (MCODS) model for the data of Ref. 4 with $z = 0.44–0.48$.



a

b

Fig. 3. The program MMK-stat: MMKMEDS (MCODS) (*a*) and MMKMNK (MCMSC) (*b*) of the model for the scale.

In addition, the anisotropy of the red shift and the solution of the calibration problem by the method of weighted least squares without using a scheme of crossover observations of the inadequacy error, which is more effective in these problems than the more complicated algorithms of confluent analysis, contributes to the perception of this situation.

Nevertheless, the "discrepancies" in the model of the cosmological distance scale based on red shift have reduced the statistically significant differences in the estimates of the Hubble parameter that led to an "impasse" [6–8, 11, 12] which most likely may not even exist.

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