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RADIO MEASUREMENTS

SYNTHESIS OF AN INTERFERENCE-RESISTANT SPACE-TIME FILTER FOR HIGH-PRECISION MEASUREMENTS OF NAVIGATION PARAMETERS ACCORDING TO THE SIGNALS OF GLOBAL NAVIGATION SATELLITE SYSTEMS

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We study the problems of serviceability of contemporary high-precision terminals of global navigation satellite systems under the conditions of jamming and spoofing interferences. The application of digital antenna arrays with algorithms of space-time signal processing of the signals can be regarded as a solution of the problem of low interference resistance. We describe well-known, most studied, and currently applied algorithms of space-time signal processing. We also formulate the causes that do not enable one to use well-known algorithms in high-precision terminals of satellite systems of global navigation. We propose an algorithm of space-time signal processing based on a space-time filter of finite length with a theoretically justified requirement of Hermitian symmetry for the matrix impulse response. The proposed algorithm guarantees the absence of distortions of signals under any signal and interference conditions. In this case, the impulse response of the space-time filter is computed according to the criterion of optimal suppression of the interference. The characteristics of the proposed space-time filter and other algorithms of space-time signal processing are investigated. For this purpose, we apply the method of mathematical simulation with random search of numerous parameters of signals and interferences (directions to signals, directions to numerous interferences, and to their reflections, remoteness of the reflectors of interferences, the phases of reflections, and the levels of interferences and reflections, etc.). The results of simulations are presented in the form of the distribution functions of the signal-to-interference ratios at the output of the algorithms of space-time signal processing and the distribution functions of the phase and signal-time biases. The obtained dependences substantiate the absence of the phase and signal-time biases in the space-time filter for any signal and interference conditions with interference multipath. It is shown that the space-time filter guarantees a higher interference resistance than the compensating algorithms of space-time signal processing.

Keywords: *global navigation satellite systems (GNSS), interference resistance, precision, space-time processing, digital antenna array, absence of signal-time and phase biases.*

Introduction. The extensive applications of the terminals of global navigation satellite systems (GNSS) aimed at the solution of various practical and applied problems in the field of transport, power engineering, communications, and agriculture turned GNSS into an element of a critically important infrastructure required for guaranteeing stable economic development of different states and comfortable life of the population. A significant spread of the technologies of telematics of the vehicles based on the GNSS caused active development of the means of individual radio countermeasures to the artificial

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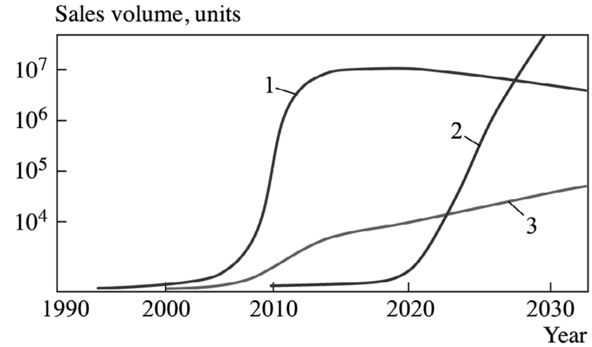

Fig. 1. Life cycles of the evolution of GNSS terminals: *1*) basic service terminals; *2*) augmentation terminals; *3*) noise-immune terminals for all services.

radio-navigation fields of the GNSS. As a result of application of jammers with powers of up to 10 W, just in the regions of high economic activity, the users of GNSS services often encounter the situations in which navigation measurements are impossible due to the presence, in the immediate vicinity, of numerous jammers generating not only jamming but also spoofing interferences [1].

At present, there are four main types of GNSS navigation services used by the customers. The main of these services is provided by the space complex, whereas the other three services (elevated accuracy and reliability, relative navigation, and high precision) are delivered to the customers by using the data of augmentation devices [2]. As a result of the analysis of a typical model of the life cycle of industry [3], in the description of the market of GNSS terminals, we can distinguish three important stages in the evolution of navigation services (Fig. 1). The first stage started together with the deployment of the GLONASS and GPS systems. It was characterized by the introduction of terminals of the main service into the sphere of transport. The phase of most extensive growth of the number of terminals for transport telematics occurred in 2005–2010. At present, this is a mature market with very keen competition for the customers. At the same time, the development of augmentation terminals, which enable customers to determine their actual three-dimensional coordinates with a standard error of at most 0.1 m as a result of differential measurements of current navigation parameters according to the carrier and envelope oscillations of the signals of GNSS space vehicles, was originated seven years later. At present, the market of these terminals can be regarded as stable. Moreover, high-precision terminals are now actively introduced in all sectors of economy and, in addition, the networks of augmentation devices including thousands of stations are functioning worldwide.

Note that the terminals extensively used at present are unable to provide the services required by the customers under the action of jamming and spoofing interferences. Therefore, we can unambiguously state that the market of jamproof terminals of both basic and high-precision services is now in the stage of formation.

At present, for the interference protection of the terminals of basic services guaranteeing the accuracy of determination of coordinates of about 1m, it is customary to use digital antenna arrays, which have been already developed and introduced into the production [4]. From the functional point of view, these arrays are, in fact, antenna interference cancelers (AIC) with algorithms based on multitapped delay lines (MDL). However, the AIC-type digital antenna arrays are unsuitable for application in the composition of promising high-precision GNSS terminals. The main disadvantage of AIC is the distortion of the phase of signals [5] and, hence, the inapplicability of high-precision phase methods of measuring current navigation parameters. One more disadvantage of the contemporary efficient cancelers are the distortions of the signal time caused by the MDL.

The problem of interference protection of high-precision GNSS terminals is an open scientific problem despite the fact that the algorithm of adaptive beam-forming (ABF) that does not distort useful signals due to focusing on these signals is thoroughly described and investigated in the literature [6]. The problem is that this algorithm does not have required level of noise immunity under the conditions of multipath interference due to the absence of MDL.

In the foreign countries, the problem of interference protection of high-precision GNSS receivers is also not solved up to now [7–10]. In the survey [7], one can find the description and experimental investigation of an undistorting algorithm of space-time signal processing (STSP), which combines focusing on signals with suppression of interferences performed

TABLE 1. Characteristics of the Algorithms Aimed at Guaranteeing the Interference Resistance of the GNSS Terminals

Characteristic	Space AIC	ABF	AIC with MDL	STF
Gain in the interference resistance as compared with the existing terminals, dB	20	25	60	60
Phase distortion	Yes	No	Yes	No
Signal time distortion	No	No	Yes	No
Limit measurement error of current navigation parameters, m	1	0.1	10	0.1
Complexity of realization	Simple	Medium	High	Very high

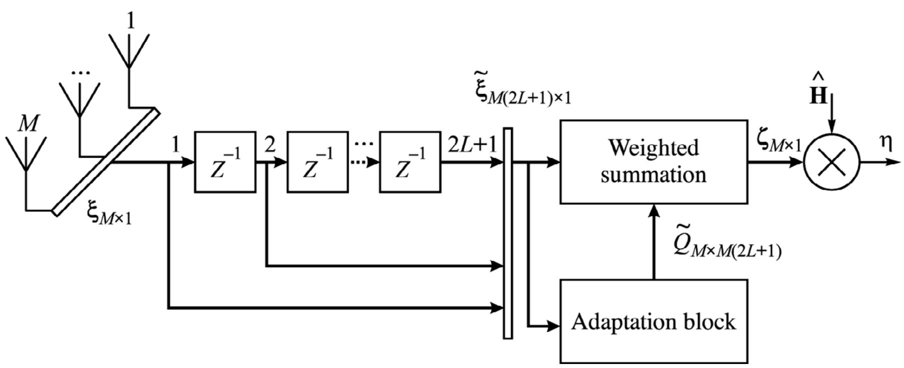

Fig. 2. Structure of the optimal space-time filter with focusing: Z^{-1} is a discrete delay per period of the sampling frequency.

with the use of MDL. However, on the one hand, this algorithm is not optimal, as already indicated. On the other hand, this algorithm has a low computational efficiency because, for each direction of the useful signal, it is necessary to perform independent compensation of the same interferences.

In the present paper, we propose a synthesis of the optimal undistorting space-time filter (STF) on the basis of which it is possible to create digital antenna arrays intended for the solution of the problems of interference protection of promising high-precision GNSS terminals. In Table 1, we present the comparison of the parameters of STF with the other available algorithms for adaptive antenna arrays.

Structure of the optimal processing. In 2016, an optimal undistorting algorithm of STSP was created and published for the multipath conditions [11]. The optimal solution was obtained in the class of filters with infinite impulse response. In fact, this solution cannot be realized in practice. However, in our opinion, the practical significance of the work [11] is connected with the determination of the optimal structure of processing depicted in Fig. 2 and in the generalization of the mathematical models of time filters with an aim to include space processing.

According to the results obtained in [11], in view of the practical restriction of the length of delay lines by an interval $\tau \in [-L, L]$ (the total length of MDL is equal to $2L + 1$, where L is the number of sections of the MDL in one direction), the optimal algorithm of STSP under the conditions of multipath interference is described by the following expressions:

$$\hat{\lambda} = \max_{\lambda}{}^{-1} \operatorname{Re}\left\{\sum_{t=1}^{T} \eta_t s_t^*(\lambda)\right\}; \qquad (1)$$

$$\eta_t = \hat{\mathbf{H}}^* \varsigma_t; \qquad \varsigma_t = \sum_{\tau=-L}^{L} Q_\tau^* \xi_{t-\tau},$$

where λ is the information parameter of a signal $s_t(\lambda)$; η_t is the result of focusing on the signal obtained by estimating the focusing vector $\hat{\mathbf{H}}$; ς_t is the vector output of the space-time interference canceller with matrix impulse response Q_τ; ξ_t is the vector of observations of the outputs of the antenna array, and * denotes the operation of Hermitian conjugation.

In [5, 6, 11, 12], it was repeatedly indicated that the separation of processing into the compensating and focusing components has an advantage connected with the necessity of simultaneous reception of several GNSS signals. The reception of several wanted signals from different directions requires solely the multiplication of focusing blocks, whereas the first (most complicated) unit of multichannel interference compensation remains common for all signals.

The procedure of finding the impulse response Q_τ according to the empirically estimated correlation function K_τ on a finite interval $\tau \in [-L, L]$ was not presented in [11]. This problem is studied in the present paper. It is necessary to find a value of Q_τ for which the interferences are efficiently suppressed, and the estimates of the information parameters of signals do not shift.

Conditions of absence of the phase and signal-time biases. The problem of phase and signal-time biases of wanted GNSS signals in the algorithms of STSP for high-precision terminals has especially high importance. This problem is theoretically independent. It was considered in [12].

In the described structure of optimal processing, the transformation of the wanted signal takes the form:

$$\tilde{s}_t = \sum_{\tau=-L}^{L} [\hat{\mathbf{H}}^* Q_\tau^* \mathbf{H}] s_{t-\tau} = \sum_{\tau=-L}^{L} h_\tau s_{t-\tau}, \tag{2}$$

where h_τ is the impulse response of the "signal" filter.

For the classical filters in the time domain, the requirements of unbiasedness are reduced to the Hermitian symmetry of the impulse response of the filter. In [9], it is shown that, under the condition of precise focusing on the wanted signal $\hat{\mathbf{H}} \equiv \mathbf{H}$, the Hermitian symmetry of the "signal" filter is guaranteed by the Hermitian symmetry of the matrix impulse response

$$Q(\tau) = Q^*(-\tau).$$

We also note that, as a consequence of the presented condition of unbiasedness, we arrive at the necessity of compensation of interferences in the medium (in terms of delay) output of the MDL, which is realized by analyzing a symmetric MDL interval $\tau \in [-L, L]$.

Ordinary Wiener solution of the problem. To find the impulse response Q_τ, it is natural to use the well-known results of the theory of classical Wiener filters (or Wiener–Hopf filters), which are optimal in terms of the criterion of minimum error variance.

It is convenient to represent the families of observations ξ_t and impulse response Q_τ in the block-vector form. A block vector with blocks of dimensions $M \times 1$ and $M \times M$, respectively, has the form

$$\tilde{\xi}_t = [\xi_{t+L} \ \dots \ \xi_{t+1} \ \ \xi_t \ \ \xi_{t-1} \ \dots \ \xi_{t-L}]^{\mathrm{T}} - (2L+1) \times 1;$$

$$\tilde{Q}_t = [Q_{-L} \ \dots \ Q_0 \ \dots \ Q_L]^{\mathrm{T}} - (2L+1) \times 1.$$

Hence, we can write the output of the interference filter (2) in the form

$$\varsigma_t = \tilde{Q}^* \tilde{\xi}_t. \tag{3}$$

The problem of interference suppression is reduced to the determination of the impulse response $\tilde{Q}$ guaranteeing the minimum value of the variance of interference in each channel. In the statement of the problem of synthesis, one may impose the requirement of minimization of the trace of correlation matrix of the vector process ς_t. However, the simplest and most general solution of the problem is given by the principle of orthogonality [13]. According to this principle, the output of the optimal canceling filter must be orthogonal to all inputs except the compensated input. Moreover, it can be described by the expression

$$M\{\varsigma_t \tilde{\xi}_{t-\tau}^*\} = \begin{cases} 0_{M \times M}, & \tau \in [-L, -1] \cup [1, L]; \\ I_{M \times M}, & \tau = 0, \end{cases} \tag{4}$$

In the block-matrix form, this equation can be rewritten as follows:

$$M\{\varsigma_t \tilde{\xi}_t^*\} = (0_{M \times LM} I_{M \times M} 0_{M \times LM}). \tag{5}$$

TABLE 2. Parameters of the Signal-Jamming Scenario No. 4

Parameter	No. of reflection		
	1	2	3
Interference 1 (J/N = 45.2 dB, φ = 272.5°, θ = –16.0°)			
Delay of reflection, m	4.4	1.3	4.4
Level of reflection, dBu	–5.8	–11.5	–5.4
Direction of reflection:			
φ, °	244.2	234.4	134.4
θ, °	–6.4	6.4	–28.0
Interference 2 (J/N = 45.2 dB, φ = 147.1°, θ = 14.5°)			
Delay of reflection, m	13.4	10.6	10.2
Level of reflection, dBu	–7.6	–10.9	–15.5
Direction of reflection:			
φ, °	134.4	356.3	190.7
θ, °	–28.0	0.1	9.2
Interference 3 (J/N = 45.2 dB, φ = 308.7°, θ = –5.7°)			
Delay of reflection, m	4.3	3.4	13.0
Level of reflection, dBu	–13.8	–12.3	–13.7
Direction of reflection:			
φ, °	171.9	147.6	218.5
θ, °	–20.7	–9.6	2.9

Substituting (4) in (5), we arrive at the following expression for the matrix impulse response $\tilde{Q}$, which is optimal according to the criterion of interference variance minimization (in terms of the maximum level of suppression):

$$M\{\tilde{Q}^*\xi_t\xi_t^*\} = \tilde{Q}^* R = (0_{M\times LM} I_{M\times M} 0_{M\times LM}),$$
$$\tilde{Q} = (0_{M\times LM} I_{M\times M} 0_{M\times LM}) R^{-1} = (R^{-1})_{(L)}. \quad (6)$$

Expression (5) shows that the impulse response of STF for the optimal suppression of interferences is equal to the middle block row (Lth row in the collection of $2L + 1$ rows) of the matrix inverse to the correlation matrix $R = M\{\tilde{\xi}_t\tilde{\xi}_t^*\}$.

To analyze the accuracy of evaluation of the information parameters of wanted signals with the use of the STF with impulse response (5), we performed statistical simulation in which the parameters of the signal-jamming scenarios are random variables. The main results of simulations in the form of the distribution functions of the values of phase, delay, and signal-to-noise ratio (SNR) are presented in what follows. We now restrict ourselves to the analysis of the form of characteristics of group delay time for the signal filter (2) and several jamming situations. In the high-precision GNSS receivers, the nonuniformity of group delay time is always a strictly controlled quantity.

In Fig. 3 we present the frequency dependences of the group delay time for four random signal-jamming scenarios whose statistical parameters are presented in what follows. In order to understand the conditions of simulations, in Table 2 we present the parameters of the signal-jamming scenario No. 4: J/N is the ratio of the interference power J to the noise power N, and φ and θ are, respectively, the azimuth and elevation angle of the direction of reception (for the wanted signal, φ =19.9° and θ = 88.8°).

The analysis of Fig. 3 demonstrates that the signal filter (3) has random characteristics of group delay time, which depend on the parameters of the corresponding signal-jamming scenario. At the same time, the bias of the information

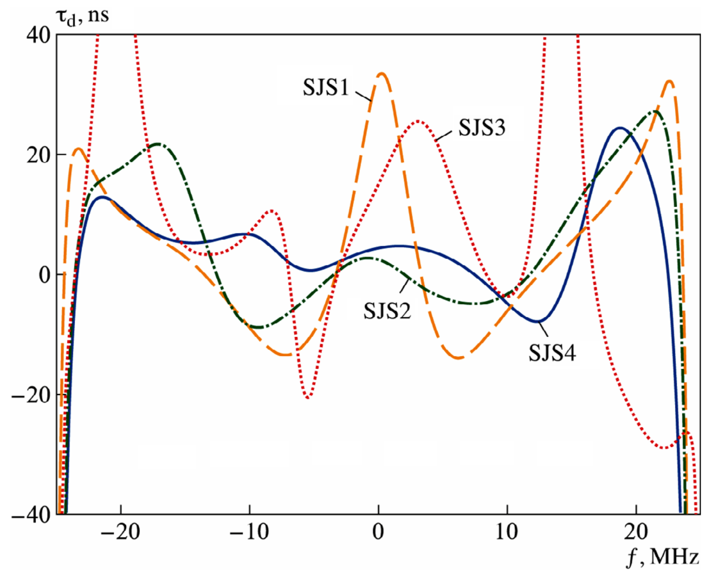

Fig. 3. Frequency dependence of the group delay time τ_d of a space-time filter for four random signal-jamming scenarios (SJS): SJS1, SJS2, SJS3, and SJS4.

parameters of signals may occur not only as a result of changes in the signal-jamming scenarios but also in the case of significant nonuniformity of the group delay time.

The main conclusion that can be made on the basis of this investigation is that the determination of the impulse response of STF for the optimal suppression of interferences performed by using solely the standard criterion of interference variance minimization cannot be applied in the case of high-precision augmentations.

Synthesis of the algorithm of suppression of interferences without biases of the radio-navigation parameters of the signal. We perform synthesis of the STF optimal according to the criterion of maximum suppression of the interferences by restricting ourselves to the class of Hermitian-symmetric filters. The analyzed problem is a problem of Wiener filtering with constraints and can be formulated as follows.

It is necessary to find a Hermitian-symmetric $(2L + 1) \times 1$ block impulse response $\tilde{Q} = \{Q_\tau, \tau = -L:L\}$ whose elements are $M \times M$-matrices minimizing the variance of interference at the output of the $(2L + 1) \times 1$ matrix filter

$$\hat{\tilde{Q}} = \min_{\tilde{Q}}{}^{-1} \operatorname{tr}\{\overline{\varsigma_t \varsigma_t^*}\} = \min_{\tilde{Q}}{}^{-1} \sum_{k=-L}^{L} \sum_{p=-L}^{L} \operatorname{tr}\{Q_k^* \overline{\xi_{t-k} \xi_{t-p}^*} Q_p\}. \tag{7}$$

In order to guarantee the presence of Hermitian symmetry, we represent Q_τ in the form of the sum of its real $X_\tau = \operatorname{Re}\{Q_\tau\}$ and imaginary $Y_\tau = \operatorname{Im}\{Q_\tau\}$ parts specified solely for the positive values of τ within the range $0:L$ (for the taps of the filter with nonnegative numbers). In this case,

$$\left. \begin{aligned} Q_\tau &= X_\tau + jY_\tau; \\ Q_{-\tau} &= X_\tau^{\mathrm{T}} - jY_\tau^{\mathrm{T}}. \end{aligned} \right\} \tag{8}$$

This representation enables us to express $(2L + 1)M^2$ complex parameters specifying $\tilde{Q}$ in terms of $(2L + 1)M^2$ real parameters, i.e., to make the dimensionality of the problem about twice lower.

As a technical difficulty that substantially complicates our calculations, we can mention the impossibility of connecting the matrices X_τ^{T} and Y_τ^{T} representing $\tilde{Q}$ for negative τ with the matrices X_τ and Y_τ with the help of a linear transformation in the M-dimensional space despite an obvious relationship between the matrix elements under transposition. At the same time, if we introduce a $M^2 \times 1$-vector $\operatorname{vec}\{X_\tau\}$ formed by the columns of the matrix X_τ, then $\operatorname{vec}\{X_\tau\}$ and the corresponding vector $\operatorname{vec}\{X_\tau^{\mathrm{T}}\}$ are connected via the well-known $M^2 \times M^2$-matrix of permutations T all elements of which are equal to zero except one element in each row, which is equal to one:

$$\operatorname{vec}\{X_\tau^{\mathrm{T}}\} = T \cdot \operatorname{vec}\{X_\tau\}.$$

In a similar way, we can write Y_τ.

By using the indicated procedure of vectorization, we can write

$$\mathrm{vec}\{Q_\tau\} = A_\tau \mathbf{Z}_\tau, \tag{9}$$

where A_τ and $\mathbf{Z}_\tau$ are a matrix and a vector of dimensions $M^2 \times 2M^2$ and $2M^2 \times 1$, respectively, given by the formulas

$$A_\tau = \begin{cases} (I \quad \mathrm{j}I), \ \tau \in [0, L]; \\ (T \quad -\mathrm{j}T), \ \tau \in [-L, -1], \end{cases}$$

$$\mathbf{Z}_\tau = \begin{pmatrix} \mathrm{vec}\{X_\tau\} \\ \mathrm{vec}\{Y_\tau\} \end{pmatrix}, \quad \tau \in [0, L], \quad \mathbf{Z}_{-\tau} = \mathbf{Z}_\tau. \tag{10}$$

Moreover, it follows from the well-known mathematical relations that the elements of the sum in (10) can be represented in the form

$$\mathrm{tr}\{Q_k^* \overline{\xi_{t-k}\xi_{t-p}^*} Q_p\} = \mathrm{vec}\{Q_k^*\}^{\mathrm{T}}(I \otimes R_{p-k})\mathrm{vec}\{Q_p\}, \tag{11}$$

where $I \otimes R_{p-k}$ is the Kronecker product of the matrix, and I is the $M \times M$ identity matrix.

The substitution of (9) in (11) gives the following final representation for the terms in (7):

$$\mathrm{tr}\{Q_k^* \overline{\xi_{t-k}\xi_{t-p}^*} Q_p\} = \mathbf{Z}_k^{\mathrm{T}} A_k^* (I \otimes R_{p-k}) A_p \mathbf{Z}_p = \mathbf{Z}_k^{\mathrm{T}} N_{k,p} \mathbf{Z}_p,$$

where

$$N_{k,p} = A_k^*(I \otimes R_{p-k})A_p \tag{12}$$

is a $2M^2 \times 2M^2$ matrix.

In general, expression (7) takes the form

$$\{\hat{\mathbf{Z}}_\tau, \tau = -L : L\} = \min_{\{\mathbf{Z}_\tau\}}{}^{-1} \left\{ \sum_{k=-L}^{L} \sum_{p=-L}^{L} \mathbf{Z}_k^{\mathrm{T}} N_{k,p} \mathbf{Z}_p \right\}. \tag{13}$$

It is convenient to reduce the double sum in (13) to the canonical vector-matrix form. To do this, it is necessary to represent $\mathbf{Z}_\tau$ in the form of a single $(2L + 1) \times 1$ block vector and $N_{k,p}$ in the form of a $(2L + 1) \times (2L + 1)$ block matrix. In this case, in order to additionally reduce the dimensionality of the problem, we use the mirror dependence of $\mathbf{Z}_\tau$ on τ, i.e., $\mathbf{Z}_\tau = \mathbf{Z}_{-\tau}$.

We combine $\mathbf{Z}_\tau, \tau \in [1, L]$, into a single vector

$$\breve{\mathbf{Z}} = [\mathbf{Z}_1^{\mathrm{T}} \ \ldots \ \mathbf{Z}_L^{\mathrm{T}}]^{\mathrm{T}}, \tag{14}$$

and take into account the mirror dependence by using the reflection matrix

$$[\mathbf{Z}_{-L}^{\mathrm{T}} \ \ldots \ \mathbf{Z}_{-1}^{\mathrm{T}}]^{\mathrm{T}} = E\breve{\mathbf{Z}},$$

where E is an $L \times L$-block reflection matrix with blocks $2M^2 \times 2M^2$ in size. In this matrix, the blocks located in the cross diagonal are equal to the identity matrices and the other elements are null blocks. Then the vector representation $\{\mathbf{Z}_\tau, \tau = -L{:}L\}$ takes the form

$$\{\mathbf{Z}_\tau, \tau = -L : L\} = \begin{bmatrix} E\breve{\mathbf{Z}} \\ \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix} = \begin{bmatrix} 0_{L2M^2 \times 2M^2} & E_{L2M^2 \times L2M^2} \\ I_{2M^2 \times 2M^2} & 0_{2M^2 \times L2M^2} \\ 0_{L2M^2 \times 2M^2} & I_{L2M^2 \times L2M^2} \end{bmatrix} \times \begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix} = B \begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix},$$

and, hence, problem (7) can be represented in the following canonical form:

$$\begin{bmatrix} \hat{\mathbf{Z}}_0 \\ \hat{\breve{\mathbf{Z}}} \end{bmatrix} = \min_{\begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix}}{}^{-1} \left\{ [\mathbf{Z}_0^{\mathrm{T}} \ \breve{\mathbf{Z}}^{\mathrm{T}}](B^{\mathrm{T}} \tilde{N} B) \begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix} \right\}, \tag{15}$$

where $\tilde{N} = \{N_{k,p}, k, p \in [-L{:}L]\}$ is either a $(2L + 1) \times (2L + 1)$-block matrix with $2M^2 \times 2M^2$ blocks or a $(2L + 1)2M^2 \times (2L + 1)2M^2$-dimensional matrix.

The initial quadratic form is a real (not complex) quantity. This enables us to represent (15) in the form

$$\begin{bmatrix} \hat{\mathbf{Z}}_0 \\ \hat{\breve{\mathbf{Z}}} \end{bmatrix} = \min_{\begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix}}{}^{-1} \left\{ [\mathbf{Z}_0^{\mathrm{T}}\ \breve{\mathbf{Z}}^{\mathrm{T}}](B^{\mathrm{T}}\,\mathrm{Re}\{\tilde{N}\}B) \begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix} \right\}. \tag{16}$$

We use the obtained solution (6) to minimize the quadratic form (16):

$$\begin{bmatrix} \mathbf{Z}_0 \\ \breve{\mathbf{Z}} \end{bmatrix} = [B^{\mathrm{T}}\,\mathrm{Re}\{\tilde{N}\}B]^{-1}_{(1)}, \tag{17}$$

i.e., the first column of the inverse matrix, which corresponds, under the proper change of variables, to the middle output of the MDL.

The resulting algorithm aimed at evaluation of the impulse response of undistorting STF follows from (17) if we transform the correlation matrix R into the matrix N (12) and the quantity $\breve{\mathbf{Z}}$ into Q_τ with the help of relations (14), (10), and (8).

Precision and interference-resistance characteristics of the algorithms of STSP. The investigations of stability of the phase and time of signals and the SNR were carried out for the following algorithms of STSP:

– the space-time AIC determined by expressions (1), (2), and (5) under the condition of application of a focusing vector of the form $\hat{\mathbf{H}} = [1\ \ 0\ \ ...\ \ 0]^{\mathrm{T}}$;

– the STF optimal in Wiener's sense and determined by relations (1), (2), and (5) under the condition of exact focusing on the wanted signal $\hat{\mathbf{H}} \equiv \mathbf{H}$;

– the undistorting symmetric STF determined by expressions (1), (2), and (16) under the condition of exact focusing on the wanted signal $\hat{\mathbf{H}} \equiv \mathbf{H}$.

The characteristics of reception of the GNSS signals depend not only on the chosen algorithm of processing but also on the random parameters of signal-jamming scenarios, i.e., on the directions to space vehicles, directions to the sources of interferences, as well as the location and intensity of bright points corresponding to the interference reflectors. In total, there are several tens of parameters of this kind and, hence, the investigation of dependences of the characteristics of reception as functions of these parameters is practically impossible and unreasonable. In this case, the procedure of mathematical simulation accompanied by the search of random signal-jamming scenarios and statistical processing of the results turns into a single practically acceptable method of investigations.

Conditions of simulation. For the purposes of modeling, we use a four-element antenna array with elements located at the corners of a square with the side equal to $\lambda/2$. The elements of the antenna array lie in the horizontal plane. We specify the dependence of the gain factor of antenna elements on the angle of elevation in the form of a cardioid $(1 + \sin\theta)/\sqrt{2}$, which approximately describes the directional diagrams of the commonly used microstrip antennas. Hence, the gain factors are equal to 3 dBi in the zenith direction, –3 dBi in the horizontal direction, and –9 dBi for the angle of elevation equal to –30°.

The directions to interferences and repeated reflections are regarded as random with uniform distribution inside the following sector of elevation angles: (–30°)–(+15)°. Signals are received from the upper hemisphere within the range 10–90°.

The maximum number of interferences was set equal to 3, i.e., to the limit of serviceability of four-element antenna arrays. The interferences have a rectangular spectrum 50 MHz in width. They have identical powers and their total power at the point of reception is higher than the level of internal noise by 50 dB.

Each source of interferences generates three random repeated reflections. The reflections have random phases, random delays relative to the direct beam of interference (path difference) within the range 0–15 m, and random levels within the range from –5 to –20 dBi.

Thus, for $L = 4$, the MDL has nine taps. The distance between the taps of delay lines is equal to the discretization interval. The discretization frequency was set equal to 50 MHz. For the complex representation of the processes, this guarantees a 50-MHz band for analysis.

The evaluation of the phase and signal time is performed for the wanted signal with correlation function of the GLONASS signal with open access. In the band of analysis, the signal corresponds to a frequency equal to zero.

The determination of the space-time correlation matrix *R* is performed on the basis of geometrically specified parameters of the signal-jamming scenarios with multipath and of the antenna array configuration by the method proposed by Kharisov and Pavlov [V. N. Kharisov and V. S. Pavlov, Advantages of the Space-Time Antenna Jammer Compensator of GNSS under the Conditions of Interference Multipath, www.gk-nap.ru/images/stories/Foto/14-19.09.2015/14.pdf].

The results of statistical analyses of the biases of the estimates of signal time (pseudorange) and phase are presented in Fig. 4 in the form of a family of distribution functions of the pseudorange offsets and phase biases plotted for random signal-jamming scenarios with fixed numbers of interferences.

The main result of simulations is that the distribution functions of the errors of pseudorange and phase biases in the symmetric STF for any signal-jamming scenario have the form of step functions, which means that the distortions are absolutely absent. At the same time, both the AIC and the STF optimal in Wiener's sense shift the estimates of the pseudorange and exhibit a regular growth of distortions in the case of complication of the analyzed signal-jamming scenario (increase in the number of interferences).

It is necessary to draw attention to the fact that the bias of the phase estimate in the STF optimal in Wiener's sense is insignificant, which is explained by the exact focusing on the GNSS signals. However, it is impossible to get any practical benefits of this effect for the improvement of the accuracy of determination of coordinates due to the large biases of the signal time.

In Fig. 5 we present the distribution functions of SNR. In the plots, a SNR equal to 0 dB corresponds to the reception of a navigation signal on one antenna element without interferences. The critical SNR is determined by the inherent interference-resistance margin of the GNSS receiver (sensitivity margin of the receiver). Thus, an interference-resistance margin of 10–15 dB most typical of the contemporary GNSS receivers corresponds to a range from –15 to –10 dB in the distribution curves of SNR.

First, it is worth noting that curves plotted for the Wiener-optimal and symmetric STF practically coincide in Fig. 5. This means that the requirements to the symmetry of STF have almost no negative influence on the interference resistance.

In the absence of interferences (see Fig. 5), the distribution is determined by the amplitude directional diagram of elements of the antenna array specified earlier. The range of values of the SNR for AIC is –4.6 to 0 dB. Moreover, for both STF, the corresponding range is 1.4–6 dB, i.e., becomes higher as a result of focusing by 6 dB.

As the number of interferences increases, the range of possible values of SNR sharply increases due to the following effects:

– deeply negative values of the SNR are obtained if interference cannot be completely suppressed and a false zero of the directional diagram is formed in the direction of the wanted signal;

– positive values of the SNR are formed in the case of complete suppression of the interferences and successful focusing of the directional diagram.

The analysis of the curves also demonstrates that, in the absence of interferences or in the case where their number is small, STF has an advantage over AIC due to focusing. However, for the critical number of interferences, their characteristics are practically identical.

Conclusions. The results of our investigations made it possible to propose a new algorithm of STSP aimed at the realization in digital antenna arrays of high-precision GNSS terminals. This algorithm guarantees the absence of phase and signal-time biases in all signal-jamming scenarios, including the most complex scenarios with multipath interference. From the viewpoint of interference resistance, the proposed algorithm is, in fact, not inferior to the classical algorithms of Wiener filtering, which are regarded as optimal according to the criterion of the best possible interference suppression.

The unbiasedness of phase and signal time guarantees the preservation of the accuracy of determination of the absolute and relative coordinates of the customers in the case of application of the GLONASS augmentation services.

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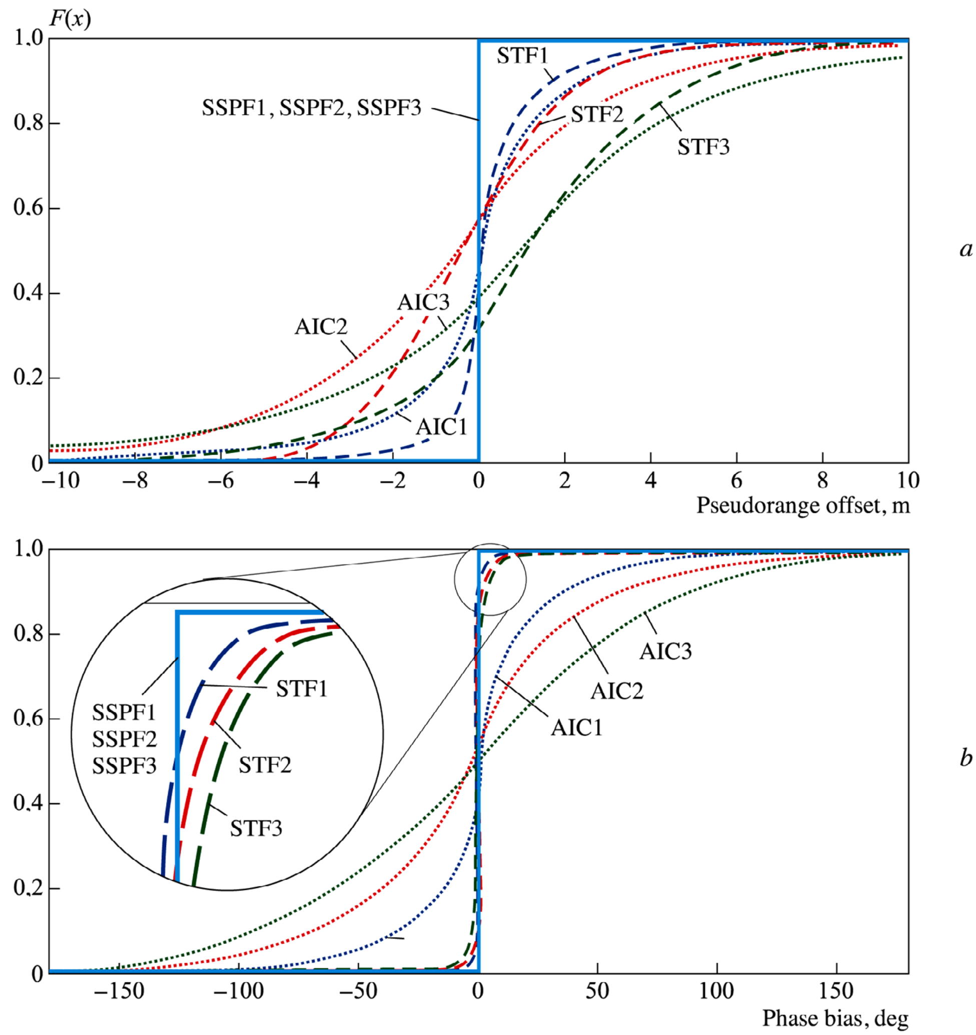

Fig. 4. Distribution function $F(x)$ of the pseudorange offsets (a) and phase biases (*b*): STF1, SPF2, and SPF3 are space-time filters; SSPF1, SSPF2, and SSPF3 are symmetric space-time filters; AIC1, AIC2, and AIC3 are antenna interference cancellers; the digits denote the numbers of the corresponding interferences.

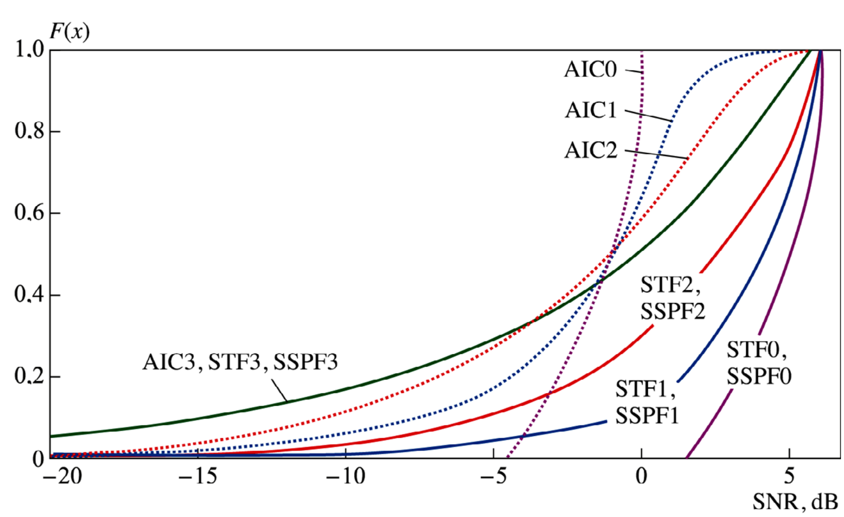

Fig. 5. Distribution functions $F(x)$ of the signal-to-noise ratio: STF0, STF1, STF2, and SPF3 are space-time filters; SSTF0, SSTF1, SSTF2, and SSTF3 are symmetric space-time filters; AIC0, AIC1, AIC2, and AIC3 are antenna interference cancellers; the digits denote the number of affecting interferences.

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