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# ACCURACY OF DETERMINATION OF THE PARAMETERS OF VIBRATION OF TURBOMACHINE BLADES WITH THE USE OF A NONLINEAR APPROXIMATION OF THE SIGNALS OF PRIMARY TRANSDUCERS

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*We describe a method for the determination of the parameters of vibration of the elements of rotating units of turbomachines. This method is based on the analysis of changes in the shape of the output signal of a primary transducer. The efficiency of the method is demonstrated for broad ranges of operating parameters of the oscillatory processes. We reveal possible problems in the realization of the method and propose some methods for their solution. We also study the accuracy characteristics of the proposed method for various signal-to-noise ratios of the output signal of primary transducer.*

***Keywords:*** *turbine, blades, vibration, discrete-phase method, mathematical model, nonlinear approximation, global optimization.*

As one of important problems of the contemporary machine building, we can mention the problem of guaranteeing high operating reliability and the increase in lifetimes of turbomachines, i.e., gas-turbine engines (GTEs) and steam turbines. The most crucial parts of turbomachines are the blades of compressors and turbines operating under severe conditions of high alternating loads, high temperatures, and erosive and corrosive actions. For the normal operation and prevention of emergencies, it is necessary to control the technical state of blades in the course of operation of turbomachines.

At present, the monitoring of the state of blades is mainly performed on the stopped turbomachines by endoscopic methods, which requires highly qualified technical personnel and is a labor-consuming technological procedure. According to the data presented by the Rolls-Royce Firm, the fraction of endoscopic diagnostics on the aviation GTEs attains 45–70% of the total amount of diagnostic works [1]. Despite the undertaken measures, the operation of GTEs is accompanied by the appearance of emergencies caused by the failures of blades.

There are numerous methods and means used for the diagnostics and monitoring of the strained state of the blades of operating turbomachines [2]. Thus, we can mention the contactless discrete-phase method (DPM) that enables one to determine the individual state of each blade in the blade wheel and can be realized in the systems of monitoring of the strained state of blades [2, 3].

In the classical version of the DPM, it is necessary to measure the intervals between the times of passing of the blades near several impulsive primary transducers (PTs). On the basis of the results of measuring these intervals, we can find the amplitudes and frequencies of vibration of the blades and the displacements of their ends from the initial working position under the action of static and dynamic loads. Then the obtained values of the indicated parameters are interpreted in terms of mechanical stresses and strains. The disadvantages of the classical DPM are connected with the use of several PTs and the accumulation and processing of statistical data. In the process of operation, the changes in the parameters serve as diagnostic signs reflecting the current technical state of the blades.

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For the diagnostics and determination of the parameters of vibration of blades, it is possible to use various impulsive PTs: inductive, eddy-current, capacitive, and optoelectronic. The motion of a blade through the sensitivity zone of the PT leads to the formation of impulsive signals of corresponding shape at the output of the transducer [4–6]. The oscillatory character of motion of the blade and the nonuniformity of its instantaneous velocity lead to changes in the shape of the pulse. In what follows, we describe a procedure of realization of the DPM proposed in [4–6]. According to this procedure, the technical state of the monitored blade is estimated according to the degree of difference between the shapes of pulses of the PT formed by dynamically loaded (vibrating) and unloaded blades. Assume that the PT generates a pulse of bell-like shape caused by the motion of an unloaded blade through the sensitivity zone. It can be described by the following expression for a Gaussian pulse:

$$s_{\mathrm{G}}(t)^* = A_{\mathrm{G}} \exp(-t^2 / 2a_t{}^2),$$

where $A_{\mathrm{G}}$ is the normalized amplitude of the Gaussian signal; $a_t = a_y/(R\omega_{\mathrm{w}})$ is the time parameter of the Gaussian pulse; $a_y$ is the metric parameter of the Gaussian pulse; and $R$ and $\omega_{\mathrm{w}}$ are, respectively, the radius and angular frequency of rotation of the blade wheel.

In the case of vibration of the blade according to the sinusoidal law, the Gaussian output signal, e.g., of an eddy-current PT, takes the form [4–6]

$$s_{\mathrm{G}}(t) = A_{\mathrm{G}} \exp\{-[t + A \sin(\omega_{\mathrm{b}} t + \varphi) / (R\omega_{\mathrm{w}})]^2 / (2a_t^2)\}, \tag{1}$$

where $A$, $\omega_{\mathrm{b}}$, and $\varphi$ are the amplitude, angular frequency, and initial phase of vibration of the blade, respectively.

The shapes of the output signals of a primary transducer for nonvibrating and vibrating blades were determined in [6]. In order to find the parameters $A$, $\omega_{\mathrm{b}}$, and $\varphi$ of vibration of the blade, it was proposed [4–6] to use the methods of nonlinear approximation. As the input data for approximations, we use the values of the PT signal $s(t_m)$ from an oscillating blade at the times (readings) $t_m$. Through the indicated readings of the signal, it is necessary to draw an approximating function $f_{\mathrm{a}}$ of form (1) guaranteeing the minimum possible deviations of this function from the input readings of the signal. It is customary to use the accuracy criteria of the quality of approximation and the accuracy of approximation to the readings of the input signal realized by the approximating function, such as, e.g., the least-squares method guaranteeing the minimum mean-square error (MSE) of approximation [7].

Thus, the problem of determination of the parameters of the approximating function can be interpreted as the problem of optimization (minimization) of a target function. If several functions satisfy the optimality criterion, then the function for which the MSE of the approximation is minimum is the best function [7]:

$$f = \sum_{m=1}^{M} [s(t_m) - f_{\mathrm{a}}(t_m, \alpha_1, \alpha_2, \ldots, \alpha_n)]^2 = \min,$$

where $f_{\mathrm{a}}(t_m, \alpha_1, \alpha_2, \ldots, \alpha_n)$ is the approximating function that describes the signal, and $M$ is the number of readings of the signal.

In the case of approximation of the shape of a signal by a certain method for different numbers of readings and the absolute values of the input signal, it is expedient to normalize the target function to the power of a signal:

$$F(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{m=1}^{M} [s(t_m) - f_{\mathrm{a}}(t_m, \alpha_1, \alpha_2, \ldots, \alpha_n)]^2 \Bigg/ \sum_{m=1}^{M} s^2(t_m).$$

For the problem of determination of the parameters of vibration of blades, the required parameters are the amplitude, frequency, and initial phase of vibration of the blades. In this case, the target function of optimization can be chosen in the form

$$F(A, \omega_{\mathrm{b}}, \varphi) = \sum_{m=1}^{M} \{s(t_m) - A_{\mathrm{G}} \exp[-(t_m + A \sin(\omega_{\mathrm{b}} t_m + \varphi) / (R\omega_{\mathrm{w}}))^2 / (2a_t^2)]\}^2 \Bigg/ \sum_{m=1}^{M} s^2(t_m). \tag{2}$$

The information signals processed in radioengineering devices suffer the action of internal and external interferences.

In the case of additive action of these interferences, the actual information PT signal is determined by the sum of a Gaussian signal and Gaussian white noise $n(t)$:

$$s_{\mathrm{G+n}}(t) = s_{\mathrm{G}}(t) + n(t).$$

In the case of discretization of a continuous signal $s_{\mathrm{G}}(t)$, we get a sequence of discrete readings $S_{\mathrm{G}}(t_1)$, ..., $S_{\mathrm{G}}(t_m)$ in the form of a discrete signal:

$$S_{\mathrm{G}}(t_m) = \int_{-\infty}^{\infty} s_{\mathrm{G}}(t)\delta(t - t_m)\,dt = A_{\mathrm{G}} \sum_{m=-\infty}^{\infty} \exp\{-[t_m + A\sin(\omega_{\mathrm{b}} t_m + \varphi)/(R\omega_{\mathrm{w}})]^2/(2a_t^2)\}^2 \delta(t - t_m). \tag{3}$$

After discretization, the output PT signal can be represented as the sum of a discrete Gaussian information signal $S_{\mathrm{G}}(t_m)$ and the discrete Gauss additive white noise $n(t_m)$:

$$S_{\mathrm{G+n}}(t_m) = S_{\mathrm{G}}(t_m) + n(t_m). \tag{4}$$

The discrete Gaussian white noise $n(t_m)$ is understood as a sequence of independent Gaussian random variables with zero expectation and Gaussian distribution over the amplitudes [8]. In the analysis of the mathematical model, the dispersion of noise is given by the formula

$$\sigma_{\mathrm{n}}^2 = p/Q, \tag{5}$$

where $p$ is the mean power of the signal, and $Q$ is the signal-to-noise ratio.

According to the proposed procedure, the parameters of dynamic displacements of the blade are determined by the methods of nonlinear approximation. In this case, the parameters of the approximating function used to describe the vibrational motion of the blade, i.e., the amplitude, frequency, and initial phase, are determined by processing of readings of the PT signal.

To study the efficiency and noise resistance of the proposed procedure of determination of the parameters of vibration of the blades, we model the additive mixture (4) of a discrete Gauss PT signal and discrete Gaussian noise for various signal-to-noise ratios in the course of numerical experiments carried out on computers. It is reasonable to perform numerical experiments for the signal-to-noise ratios possible in actual radioengineering devices lying within the range $10^4$–$10^8$ (by power). The input discrete Gaussian signal (3) is computed according to the values of amplitude, frequency, and initial phase of the vibration of blades. The discrete Gaussian noise is formed for various signal-to-noise ratios according to (5). The results of numerical experiments give the values of three indicated parameters determined as a result of the solution of the optimization problem.

To estimate the statistical error of evaluation of the required parameters for the additive mixture of signal with noise and any combination of values of the input parameters $A_i$, $\omega_{\mathrm{b}j}$, and $\varphi_k$, it is necessary to perform a series of $N$ tests. In this case, according to the method proposed in [9], we can choose the maximum absolute of the estimation error as a metrological characteristic:

$$\Delta = \max\{|\Delta_n|\}, \quad n = 1, ..., N,$$

where $\Delta_n$ is the absolute error of determination of the parameter.

The number of the tests $N$ depends on the confidence probability $P$: if $P$=0.95, then $N$ =29 independently of the law of distribution of errors.

The detailed description of the procedure of realization of numerical experiments can be found in [6]. We now briefly describe the procedure of realization of the numerical experiments:

1. It is necessary to tabulate three input parameters (amplitude, frequency, and initial phase) in their domains of definition ($A$ = 1–10 mm; $\omega_{\mathrm{b}} = 2\pi(100–150)$ rad/sec; $\varphi = 0–\pi$ rad) in order to get a three-dimensional array of the input parameters $A_i$, $\omega_{\mathrm{b}j}$, and $\varphi_k$.

2. For every combination of input parameters $A_i$, $\omega_{\mathrm{b}j}$, and $\varphi_k$, it is necessary to form the discrete Gaussian signal (3).

3. The experiment is repeated $N$ = 29 times for each combination of the input parameters, i.e.,

- the Gaussian discrete noise (3) and its additive mixture (5) with a discrete Gaussian signal are formed;
- the optimization problem is solved for the target function (2) and the output values of the parameters are obtained;
- the presence of faults is checked according to the Grubbs criterion; if the faults are present, then they are removed, and the optimization problem is additionally solved the required number of times.

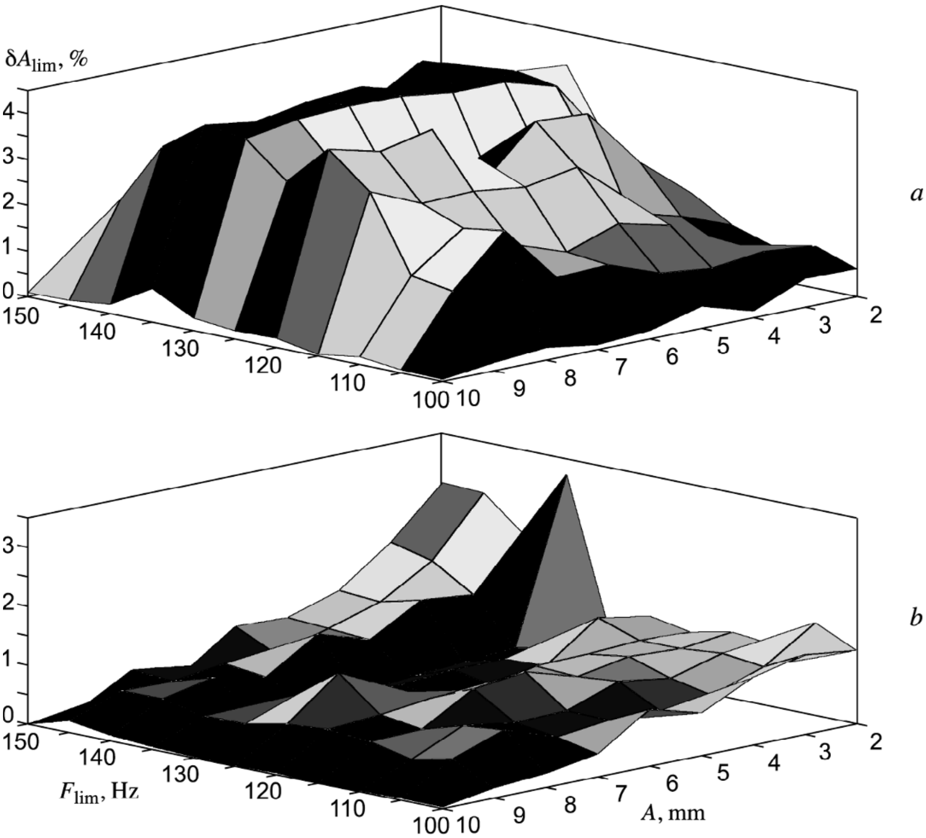


Fig. 1. Maximum reduced errors of determination of the amplitude (*a*) and frequency (*b*) of vibration of turbine blades.

4. The maximum moduli of the errors of estimation are determined for each of the analyzed three parameters:

$$\Delta_A(i,j,k) = \max\{|\Delta_{An}|\}; \quad \Delta_\omega(i,j,k) = \max\{|\Delta_{\omega n}|\}; \quad \Delta_\varphi(i,j,k) = \max\{|\Delta_{\varphi n}|\},$$

where $\Delta_{An}$, $\Delta_{\omega n}$, and $\Delta_{\varphi n}$ are the absolute errors of determination of the corresponding parameter in the $n$th experiment, and $n = 1, \ldots, N$ is the number of the experiment.

As a result of experiments, we establish the dependences of the maximum values of the moduli of errors on the input values of $A_i$, $\omega_{bj}$, and $\varphi_k$ for each analyzed parameter.

In the engineering analysis of vibration characteristics of the blades, the most significant parameters are the amplitude and frequency of vibration of the blade [2, 3, 10], whereas in most cases, it is not necessary to determine the initial phase of vibration. Hence, we computed the errors of $\Delta_{An}$, $\Delta_{\omega n}$ for several fixed values of the initial phase $\varphi_k = \varphi_1, \ldots, \varphi_{k\max}$ of vibration of the blade and chose the maximum errors for subsequent consideration:

$$\Delta_A(i,j) = \max\{|\Delta_A(i,j,k)|\}; \quad \Delta_\omega(i,j) = \max\{|\Delta_\omega(i,j,k)|\}, \qquad k = 1, \ldots, k_{\max}.$$

For the general comparative analysis, it is reasonable to normalize the obtained results to the limiting values of the amplitude $A_{\lim}$ =10 mm and frequency $F_{\lim}$ = 150 Hz by getting the maximum values of the reduced errors: $\delta A_{\lim.\max} = \Delta_A(i,j)/A_{\lim}$ and $\delta F_{\lim.\max}\ \Delta_\omega(i,j)/F_{\lim}$. In Figs. 1*a* and *b*, we present the plots of the indicated errors for the signal-to-noise ratio $Q = 10^{10}$ (100 dB).

**Existing problems and procedures of their solution.** The indicated method of finding the parameters of vibration of the blades is connected with the necessity of solution of certain encountered problems. From the geometric viewpoint, the values of the target function (2) can be interpreted as the heights of the surface topography in the space of the input parameters $A_i$, $\omega_{bj}$, and $\varphi_k$. The nonlinear dependence of the target function on the parameters and the ambiguity of these dependences lead to significant irregularities of the topography of the target function.

In the solution of the optimization problem for relation (2), it was established that there exist some values of the parameters $A_i$, $\omega_{bj}$, and $\varphi_k$ optimal in a sense of minimum of the target function but not optimal from the expert viewpoint, i.e., these parameters give erroneous results.

The optimal solution is defined as a solution minimizing or maximizing the criterion of quality of the optimization model (optimality criterion) under given conditions and restrictions of the analyzed model and corresponding to the global extremum. The suboptimal solution corresponds to a local extremum of the optimality criterion. The analysis of these suboptimal solutions shows that the causes of their appearance are the irregularity of the topography of target function in the space of input parameters and the significant dependence of the obtained solution on the initial approximation, i.e., on the choice of the initial point.

To find the global minimum of the target function, we propose to use the MultiStart method of the Global Optimization package from the MATLAB software for mathematical modeling. The MultiStart method enables one to find multiple local minima of the target function and its global minimum. In this case, we can use the efficient fmincon local solver supporting the use of gradients of the target function and the restrictions in the form of equalities and inequalities. The MultiStart method generates a set of starting points, transfers these points to the local solver, starts the operation of the solver, creates (when the operation of operation is terminated) a vector containing the obtained local minima, and marks the best of these minima as the global minimum.

The application of the MultiStart method significantly decreases the sensitivity of the proposed procedure of determination of the parameters of vibration of the blades to the appearance of suboptimal solutions but does not exclude them completely. In order to find the optimal solution in the family of suboptimal solutions both from the viewpoint of minimum of the target function and from the expert viewpoint, it is necessary to take into account a certain additional condition. This condition can be formulated on the basis of

1) the conditions of verification of the possibility of realization of the mechanical vibrating system (by using the boundary conditions in the form of inequalities);

2) the available vibration characteristics of the blade obtained, e.g., in the course of testing of the blades on a vibration bench; and

3) and the analysis of time intervals between the pulses of PT from a specific blade after one or several rotations of the blade wheel.

In the last case, the additional condition can be represented in the form of the condition of comparison of the approximated signal with certain parameters (obtained as a result of approximation) of vibration of the blade for the first rotation of the blade wheel with the PT signal obtained in the next rotation of the wheel. The equation of the Gaussian PT signal from the investigated blade after one rotation of the blade wheel takes the form

$$s_{GT} = s_G(t-T) = A_G \exp\{-[t - T + A\sin(\omega_b t + \varphi)/(R\omega_w)]^2/(2a_t^2)\},$$

where $T$ is the period of rotation of a blade wheel.

Then the additional condition in the form of a restriction (equality) for the determination of the optimal solution can be written as follows:

$$F_T(A, \omega_b, \varphi) = \sum_{m=1}^{M} [s_{G2} - A_G \exp\{-[t_m - T + A\sin(\omega_b t_m + \varphi)/(R\omega_w)]^2/(2a_t^2)\}]^2 \Bigg/ \sum_{m=1}^{M} s_{G2}^2 . \tag{6}$$

The graphical representations of the maximal reduced errors of determination of the amplitude and frequency of vibration (see Fig. 1) are obtained as a result of the solution of the optimization problem with regard for restriction (equality) (6). The results of investigation of the dependence of the indicated maximal errors on the signal-to-noise ratio of the input PT signal are presented in Fig. 2.

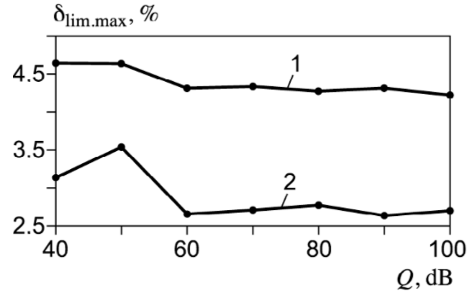


Fig. 2. Maximal reduced errors of determination of the amplitude (*1*) and frequency (*2*) of turbine blades versus the signal-to-noise ratio.

**Conclusions.** The parameters of vibration of the blade end, i.e., the amplitude, frequency, and initial phase, can be directly found according to the results of analysis of the changes in the shape of the information PT signals with the use of the proposed method based on the nonlinear approximation of PT signals. The results of our computer simulations reveal the efficiency of the proposed procedure in broad ranges of the operating values of the amplitude, frequency, and initial phase of vibration of the blades. We also present the graphical representations of the maximum reduced error of determination of the required parameters.

The application of the MultiStart method from the MATLAB software used for mathematical modeling enables us to exclude the suboptimal solutions caused by the nonoptimal choice of the initial approximation of the algorithm and characterized by elevated errors of the parameters of vibration of the blade. The subsequent decrease in this error should be realized with the help of the additional physical conditions formulated in the form of conditions-equalities and conditions-inequalities. Thus, the application of an additional condition formulated on the basis of the equality of the required parameters of pulses from the monitored blade with a delay by one rotation of the blade wheel enables us to get an additional severalfold decrease in this error.

It is established that the computational modeling of the dependence of the maximal error of evaluation of the parameters of vibration of the blade can be used within the range of actual signal-to-noise ratios observed in the radioengineering circuits.

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