DOI 10.1007/s11018-017-1220-5
Measurement Techniques, Vol. 60, No. 5, August, 2017

THERMAL MEASUREMENTS

A SYSTEMATIC METHOD OF IMPROVING THE ACCURACY OF AN INFORMATION AND MEASURING SYSTEM FOR DETERMINING THE THERMOPHYSICAL PROPERTIES OF MATERIALS UNDER THE EFFECT OF DESTABILIZING FACTORS

Z. M. Selivanova and T. A. Khoan

UDC 66.067.55

Mathematical models are proposed for evaluating the functional accuracy of an information and measuring system and the measurement channel under the influence of destabilizing factors. A comprehensive method of increasing the accuracy of the system is created that makes it possible, with permissible error, to measure the parameters of the thermophysical properties of materials: the coefficients of thermal conductivity and thermal diffusivity.
Keywords: *accuracy evaluation method, mathematical model, information and measuring system, thermophysical properties of materials, correction algorithm.*

An information and measuring system (IMS) for determining the thermophysical properties (TPP) of materials in actual operational practice functions under the effect of destabilizing factors. Results of experimental studies of IMS and calculations of the relative error of the results of measuring TPP parameters, i.e., coefficients of thermal conductivity and thermal diffusivity (λ, α), show that destabilizing factors increase the error of measurements by 10–30%. Therefore, the creation of an IMS that is resistant to the effect of destabilizing factors is an urgent and important task, but for this purpose it is necessary to develop at first a comprehensive method of increasing the accuracy of such a system.

Based on information obtained experimentally regarding a method of measuring the TPP parameters of materials, input and output signals of the IMS structural components, and permissible values of the error of measurement of coefficients $\Delta\lambda_{\text{perm}}$ and $\Delta\alpha_{\text{perm}}$, it is possible to calculate certain restrictions imposed on the accuracy indicators of the IMS structural components and the parameters of the TPP of materials, i.e.,

$$\Delta y_\lambda(S \setminus H) \le \Delta y_{\lambda\,\text{perm}}; \quad \Delta y_\alpha(S \setminus H) \le \Delta y_{\alpha\,\text{perm}},$$

where $\Delta y_\lambda(S \setminus H)$ and $\Delta y_\alpha(S \setminus H)$ are the errors in determining the thermal conductivity λ and thermal diffusivity α coefficients, respectively.

Errors are provided by a set S of structural components of the measuring system and a set H that includes input signals x_k of structural components of the system, the destabilizing factors D_k affecting the IMS, and the parameters b_k of components of the system.

Figure 1 presents a schematic diagram of an IMS that was developed for determining the TPP of materials. The accuracy of this system is estimated from the error obtained by the results of measuring the coefficients of thermal conductivity

Tambov State Technical University, Tambov, Russia; e-mail: selivanova@mail.jesby.tstu.ru. Translated from Izmerital'naya Tekhnika, No. 5, pp. 44–48, May, 2017. Original article submitted January 25, 2017.

and thermal diffusivity. The information and measuring system consists of the processing, control, and signal correction unit PCSCU, the measurement channel MCh, and the power supply PS [1–3]. The control and signal corrections unit is based on the PIC18F8720 microcontroller MC and includes the signal correction and control unit SCCU, heating control unit HCU and decision device DD, database DB and knowledge base KB, and the program module PM and display D, which displays basic data: the temperatures at specified moments of time and at certain control points of the material being studied MS, the results of calculating coefficients λ and α, and their measurement errors. The measurement channel consists of the measuring probe MP, in which are located the measurement transducer system MTS with temperature sensors (chromel-copel thermocouples) and the heater H; channel selector CS; differential amplifier system DAS; and analog-to-digital converter ADC.

Programs for measuring the parameters of the TPP of materials and algorithms for the functioning of the IMS are included in the program module. Into databases and knowledge bases is placed information on models and algorithms used, mode parameters of system functioning, approximating functions for executing corrections to the parameters of the components of the measurement channel, and the IMS output parameters.

The principle of action of an IMS is as follows. The line heater of the measuring probe effects a pulse impact on the studied material. Then the temperature and time characteristics are recorded in the area of the probe and material contact. The obtained data are recorded in the knowledge base of the IMS and used for calculating the specified parameters of the TPP of the materials [4].

In the development of an IMS, destabilizing factors that determine the greatest component of error and influence the accuracy of the system are revealed. The base error is introduced by the measurement channel, specifically the system of measurement transducers, differential amplifiers, and ADC. This error is affected by climatic noise background: ambient temperature T_{env} and humidity W, as well as contact resistance and surface roughness of the studied material [5]. The error of measurements of thermocouples is caused by violation of the temperature mode of a "cold" junction, the ambient temperature T_{env} and random errors associated with special features of manufacturing thermocouples, such as heater thermal capacity, sensitivity, and parasitic thermoelectric force.

It is possible to present the functioning of the IMS under the effect of destabilizing factors in formalized manner by the dependence

$$\mathbf{z} = F(\mathbf{y}, \mathbf{x}, \mathbf{v}) + \mathbf{Q}, \quad \mathbf{y} \in Y, \ \ \mathbf{x} \in X, \ \ \mathbf{v} \in V, \tag{1}$$

where $\mathbf{z}$ is the vector of output parameters of the IMS (parameters of the TPP of materials); $\mathbf{x}$ and $\mathbf{y}$ are vectors of input-controlled and non-driven parameters, respectively, i.e., the strength of the thermal effect on the studied material and signals from outputs of components of the measurement channel; $\mathbf{v}$ is the vector of random signals (T_{env}, W); $\mathbf{Q}$ is the vector of active disturbances when the system is functioning; and X, Y, and V are sets containing the vectors $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$.

The calculation-analysis method is used to assess the accuracy of functioning of the IMS. Based on relationship (1), a mathematical model was developed establishing a dependence between the output parameters of the accuracy of the results of measuring the TPP of materials, and the input parameters of accuracy that introduce error in the results of measuring the TPP:

$$z_n = f(y_{n1}, y_{n2}, ..., y_{nm}), \tag{2}$$

where z_n is the nominal value of the output parameters of the IMS; $y_{n1}, y_{n2}, ..., y_{nm}$ are parameters of the accuracy of components of the IMS measurement channel (measurement transducer system, differential amplifier system, ADC).

The IMS output parameters are measured with error Δz_n, since the parameters of system components introduce errors $\Delta y_{n1}, \Delta y_{n2}, ..., \Delta y_{nm}$:

$$\mathbf{z} = z_n + \Delta z_n = f[(y_{n1} + \Delta y_{n1}), (y_{n2} + \Delta y_{n2}), ..., (y_{nm} + \Delta y_{nm})]. \tag{3}$$

We expand relationship (3) into a Taylor series [6]. We consider only first-order members, since errors y_n and z_n are insignificant in comparison with the values of input and output parameters:

$$\mathbf{z} = f_1(y_{n1}, y_{n2}, ..., y_{nm}) + \sum_{j=1}^{m} (\partial f_1 / \partial y_j) \Delta y_j .$$

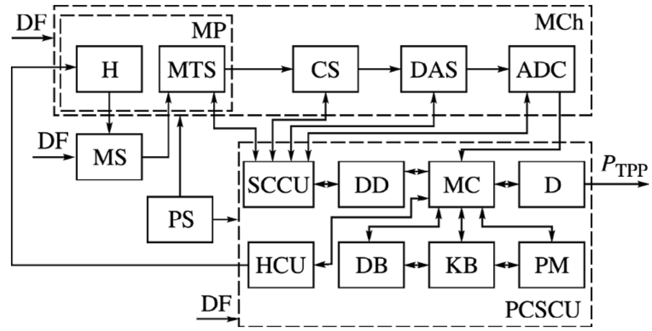

Fig. 1. Structural diagram of information and measuring system for determination of thermophysical properties of materials: DF, destabilizing factors; MP, measuring probe; H, heater; MTS, measurement transducer system; MS, material being studied; PS, power supply; MCh, measurement channel; CS, channel selector; DAS, differential amplifier system; ADC, analog-to-digital converter; SCCU, signal correction and control unit; DD, decision device; MC, microcontroller; D, display; HCU, heating control unit; DB and KB, database and knowledge base, respectively; PM, program module; PCSCU, processing, control, and signal correction unit.

It follows from (3) that the absolute error of results of measurement in the IMS is $\Delta z_{\text{n}} = \mathbf{z} - z_{\text{n}}$. Then

$$\Delta z_{\text{n}} = \sum_{j=1}^{m} (\partial f_1 / \partial y_j) \Delta y_j = \sum_{j=1}^{m} C_j \Delta y_j, \qquad (4)$$

where C_j is a weighting coefficient of the error of the components of the IMS that affect the accuracy of its output parameters.

The relative error, according to which the accuracy of the IMS is assessed, is defined from expression (4):

$$\delta = \Delta z_{\text{n}} / \mathbf{z} = \sum_{j=1}^{m} C_j (\Delta y_j / \mathbf{y}).$$

Based on experimental data of the IMS, analytical dependences were derived for the mathematical model in (2) of output parameters of accuracy z_{n} on the input parameters $\Delta y_{\text{n}m}$. The functioning of the IMS is carried out when ensuring the accuracy of structural components of the measurement channel. For analysis and assessment of its accuracy, it is necessary to perform mathematical modeling, which is an important task when developing any IMS. Such a mathematical model is a description of the transformation, by a linear stationary operator, of measurement information in the measurement channel that arrives from the measurement transducer system via the channel selector for input to the differential amplifier system. At output from this system, data arrives as input to the processing, control, and signal correction unit [7]:

$$z(t) = \int_0^t C(t-\tau) y(\tau) d\tau + V_{\text{df}}(t), \qquad (5)$$

where $z(t)$ is the output signal of the measurement channel; $C(t-\tau)$ is a function of the weighting of components of the measurement channel; $y(\tau)$ is the input signal of the measurement channel; $V_{\text{df}}(t)$ is an additive component of a signal $z(t)$ depending on acting destabilizing factors: temperature T_{env} and ambient humidity W, which is a stationary random ergodic process.

It is necessary, in a verifiable manner and with minimum error, to ensure transfer of the information arriving from the measurement transducer system, the differential amplifier system, and the ADC. As is shown by metrological analysis of the results of measuring the structural components of the measuring complex, the amplifier is the dominant component introducing the greatest error into the measurement results of the IMS. The authors have proposed a functional model of such an amplifier:

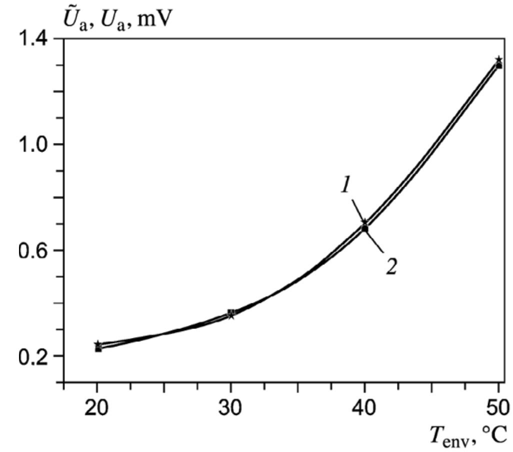

Fig. 2. Experimental (*1*) $U_a = f(T_{env})$ and approximating (*2*) $\tilde{U}_a = f(T_{env})$ dependences of output amplifier voltage measurement results on ambient temperature.

$$U_a = f(U_{in}, D, \Delta K, N, R_{in}, R_{out}, R_L, U_L, F_L, I),$$

where U_a is the output voltage of the amplifier; $U_{in} = \{U_{in\,i}, i = 1, ..., n\}$ is the set of measuring input signals; $D = \{D_j, j = 1, ..., l\}$ is the set of acting destabilizing factors (T_{env}, W); ΔK is the instability of the coefficient of amplification; N is the "drift" of the zero of the differential amplifier; R_{in} and R_{out} are the input and output resistance of the differential amplifier; R_L, U_L, and F_L are the instability of the load resistance, supply voltage of the differential amplifier, and output voltage, respectively; and I is signal distortions in the amplifier.

In Fig. 2, the dependences $U_a = f(T_{env})$, derived in measurements with the aid of a developed system of parameters of the TPP of linoleum, are presented as an example of the implementation of model (5). The approximating function $\tilde{U}_a = f(T_{env})$ has the form

$$\tilde{U}_a = -0.0013(T_{env})^2 - 0.0551T_{env} + 0.8264.$$

Approximating dependences of the output amplifier voltage on T_{env} are found experimentally for the following thermal insulating materials: polymethylmethacrylate, mineral wood boards, felt, plastic foam:

$$\tilde{U}_a^{pmm} = f(T_{env}) = -0.0015(T_{env})^2 + 0.1249T_{env} - 1.277; \tag{6}$$

$$\tilde{U}_a^{mwb} = f(T_{env}) = -0.0018(T_{env})^2 + 0.1482T_{env} - 1.640; \tag{7}$$

$$\tilde{U}_a^{f} = f(T_{env}) = -0.0017(T_{env})^2 + 0.142T_{env} - 1.510; \tag{8}$$

$$\tilde{U}_a^{pf} = f(T_{env}) = -0.0024(T_{env})^2 + 0.1961T_{env} - 2.3473. \tag{9}$$

As a result of calculating the amplifier parameters and the experimental studies that were conducted, a mathematical model was created for the process of transforming the measurement information of U_{MTS} by the amplifier. This model is presented as

$$U_a(T) = U_{MTS}K_a \pm \Delta U_{df}, \tag{10}$$

where $K_a = 980$ is the coefficient of amplification of the amplifier; $\Delta U_{df} = \pm 0.61$ is the error of output voltage change of the amplifier under the influence of destabilizing factors.

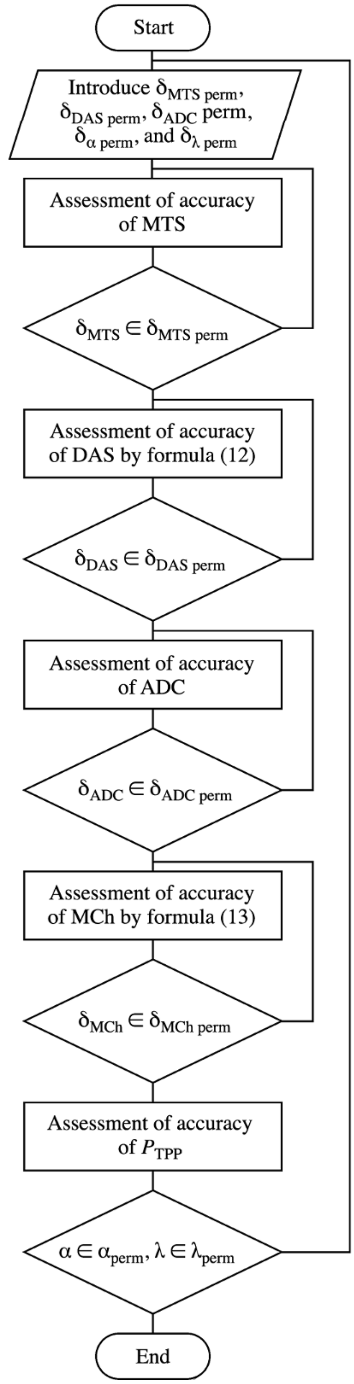

Fig. 3. Block diagram of implementation of a systematic method of increasing accuracy of an information and measuring system for determining parameters of thermophysical properties of materials: $\delta_{\mathrm{MTS\ perm}}$, $\delta_{\mathrm{DAS\ perm}}$, $\delta_{\mathrm{ADC\ perm}}$, $\delta_{\alpha\ \mathrm{perm}}$, and $\delta_{\lambda\ \mathrm{perm}}$ are the admissible relative errors of measurements of system of measurement transducers, systems of differential amplifiers, the analog-to-digital converter, and the coefficients of thermal conductivity and thermal diffusivity, respectively.

In order to implement the proposed models in (5) and (10), a program module was created during design of the measurement channel and correction of its structural components. This made it possible to improve the accuracy of the measurement channel of the IMS.

A systematic method of improving the accuracy of the IMS has been developed for determining the TPP of materials. The method is based on the use of previously-created mathematical models that include models for assessing the accuracy of the structural components of the system and the accuracy of the defined parameters, the coefficients of thermal conductivity and thermal diffusivity. The method consists of the monitoring, correction, and verification of the results of measurements of output signals of the structural components of the measurement channel for adherence to permissible values of the parameters of the TPP according to approximating dependences under the effect of destabilizing factors, derived analytically and on the basis of experimental data. Figure 3 presents a block diagram of the algorithm for implementing the systematic method of improving the accuracy of the IMS for determining the TPP of materials.

In order to confirm the results of implementing the developed overall method of improving the accuracy of IMS for determining the TPP of materials, it is necessary to calculate the error $\delta_{\mathrm{MCh},\Sigma}$ of the measurement channel of the system. This is determined by the error of transformation of the measuring signals by temperature sensors: the thermocouples of the measurement transducer system, the operational amplifiers that are part of the differential amplifier system, and the ADC. Hence, the relative error will be equal to the sum of the errors of the structural components of the measurement channel, i.e.,

$$\delta_{\mathrm{MCh},\Sigma} = \delta_{\mathrm{MTS}} + \delta_{\mathrm{a}} + \delta_{\mathrm{ADC}},$$

where δ_{MTS} is the error of the measurement transducer system; δ_{a} is the amplifier error; and δ_{ADC} is the ADC error.

Thermal transducer error depends on many factors. These include: sensitivity error occurring because of a deviation of the characteristics of the thermal transducers from nominal static characteristics (NSC); heterogeneity of the thermal transducers; parasitic thermal EMF; and temperature change of cold junctions in thermal transducers. The effect of the specified factors on the temperature measurement error of the thermal transducers is determined based on experimental data using Hromel–Kopelev thermocouples as temperature sensors at the stage of development of the IMS. Sensitivity of thermal transducers in the temperature range 0–100°C, in which thermophysical measurements are conducted, is determined by the formula [8]

$$L = \Delta E/\Delta T = (E_2 - E_1)/(T_2 - T_1) = 6.842/100 = 0.068 \text{ mV/°C},$$

where E_2 and E_1 are the values of parasitic thermal EMF at 100 and 0°C, respectively; and T_2 and T_1 are temperatures 100 and 0°C, respectively.

The maximum relative error of the sensitivity of thermal transducers ($\delta_{\max}$) due to deviation of their actual characteristics from the nominal static characteristics (NSC), i.e., nonlinearity of thermal transducers in the temperature range 0–100°C, is estimated by the maximum relative error by the ratio

$$\delta_{\max} = [(E_{\mathrm{NSC\,i}} - E_{\mathrm{NSC\,a}})/(E_2 - E_1)]\cdot 100,$$

where $E_{\mathrm{NSC\,i}}$ and $E_{\mathrm{NSC\,a}}$ are ideal and actual NSC, respectively.

In the calculation, the maximum value of the difference is considered to be $E_{\mathrm{NSC\,i}} - E_{\mathrm{NSC\,a}}$. Then, $\delta_{\max} = [0.081/(6.842-$ $-0)]\cdot 100 = 1.19\%$.

Thermoheterogeneity of the thermocouple influences the change in NSC when the thermal transducer during thermophysical measurements is in the region of a temperature gradient that results in the occurrence of parasitic thermal EMF. The error was estimated during heating of the thermal transducer in the range of 0–100°C. The relative error of measurement of the thermal transducer, taking into account thermoheterogeneity, was calculated using the formula

$$\delta_{\mathrm{nd}} = [(E_i - E_{\mathrm{n}})/E_{\mathrm{n}}]\cdot 100,$$

where E_i and E_{n} are the current and nominal values of thermal EMF.

The maximum relative error of measurements was $\delta_{\mathrm{nd\,max}} = [(1.953 - 1.947)/1.947]\cdot 100 = 0.31\%$, and the total overall relative thermal transducer error was 1.5%. In order to eliminate the effect of the temperature of cold junctions of the thermal transducer on the results of measurements, differential thermocouples were used in the measuring probe for the IMS.

The differential amplifier error is determined by the voltage change of a shift of the operational amplifier, i.e., $U_{shift} = 25$ μV. In the summation of the derived errors, the absolute additive zero offset error of the operational amplifier is normalized by the nominal value of the maximum input signal of the measurement transducer system, and is 2 mV:

$$\delta_{shift} = U_{shift}/U_{in\ max} = (25 \cdot 10^{-6})/(2 \cdot 10^{-3}) = \pm 0.0125\%. \tag{11}$$

The zero offset error of the operational amplifier is distributed normally. The relative error of the operational amplifier because of the effect of T_{env} is determined from expressions (6–9):

$$\delta_{a\ max} = (\tilde{U}_2/U_1) \cdot 100 = [(1.657 - 1.647)/1.647] \cdot 100 = 0.61\%, \tag{12}$$

where U_1 and $\tilde{U}_2$ are the voltage at output of the operational amplifier under normal conditions of use and under the effect of ambient temperature, respectively.

The ADC error depends on T_{env} and on the instability of the power supply U_{PS}, affecting the change of reference voltage ($U_{ref} = 2.5$ V). This is also a multiplicative error. The error of quantization of the ADC is equal to one half unit of the lower order. A temperature change results in an initial zero offset of the ADC, an additive error. It has been established as a result of calculations that the random error of the ADC $\delta_r = 0.1\%$. The systematic error (quantization error) $\delta_q = 0.062\%$.

The IMS measurement channel error is calculated according to the metrological characteristics of its components [9]. The confidential interval was calculated with the maximum permissible lower $\delta_{L\ MCh}$ and upper $\delta_{U\ MCh}$ bounds. The measurement channel error is located in this interval with specified probability $P = 0.95$. The assumption has been adopted that errors of components of the measurement channel are random variables with the normal law of distribution of density.

The mean square deviation (SD) of a random error of the measurement channel $\sigma[\delta_r]$ is determined by geometrical summation of the SD of random errors of the components of this channel by formula [9]

$$\sigma[\delta s] = \left[\sum_{i=1}^{s} \sigma^2[\delta s_i] \right]^{1/2},$$

where s is the number of structural components that are part of the IMS measurement system; and δs_i is the SD of a random error of the ith channel component.

The mean square deviation of a random component of the error of the measurement channel of the system is defined as follows [10]:

$$\sigma[\delta s] = \sqrt{(\delta_{tt}^2 + \delta_a^2 + \delta_{ADC}^2)/q}, \tag{13}$$

where $\delta_{tt} = 1.5\%$ is the relative error of the thermoelectric transducer; $\delta_a = 0.61\%$ is the relative error of the differential amplifier, calculated in formulas (11), (12); $\delta_{ADC} = \pm 0.4\%$ is the relative error of the ADC; $q = 3$ is the number of differential amplifiers, for which changes of the metrological characteristics of components of the measurement channel are normalized.

As a result of calculation according to formula (13), the SD of the total error of the IMS measurement channel of the TPP of materials was

$$\sigma[\delta s] = \sqrt{[(1.5)^2 + (0.61)^2 + (0.4)^2]/3} = 0.963\%;$$

$$\sigma[\delta_r] = \pm 0.963\%.$$

The maximum permissible error of the system's measurement channel is calculated according to the formula [9]

$$\delta_{MCh,L(U)} = \pm K_n \sigma[\delta_r].$$

If there are more than four measurements, the distribution of total error reduces to normal ($K_n = 1.96$):

$$\delta_{MCh,L(U)} = \pm(1.96 \cdot 0.963) = \pm 1.89\%.$$

Hence, the value of the lower (upper) bound of the confidence interval in which the IMS measurement channel error is located with probability $P = 0.95$ is

$$\delta_{\text{MCh,L(U)}} = \pm 1.89\%.$$

Conclusion. The mathematical model for assessing the accuracy of functioning of the IMS for the TPP of materials was developed in the form of an analytical dependence of the output parameters on the input, taking into account the effect of external factors. A mathematical model of the IMS measurement channel was created, making it possible to analyze the transformation of information in this channel with the effect of destabilizing factors and reduce channel error to 1.89%.

The proposed systematic method of increasing the accuracy of functioning of an IMS as a result of the monitoring, correction, and verification of the output parameters of the measurement channel's structural components for adherence to permissible values and the results of measuring the TPP parameters of materials under the influence of destabilizing factors on the derived approximating dependences makes it possible to ensure an error of no more than 2–3% in determining the coefficients of thermal conductivity and thermal diffusivity of materials. The results obtained in the IMS research that was conducted are recommended to be used during the development and operation of the IMS of the TPP of materials in actual conditions with the influence of destabilizing factors.

REFERENCES

1. Z. M. Selivanova and A. A. Samokhvalov, "Intelligent information and measuring system for determining thermophysical properties of materials and products," *Izmer. Tekhn.*, No. 9, 38–42, (2012).
2. Z. M. Selivanova and T. A. Khoan, "Increasing the accuracy of data-measuring systems for the nondestructive testing of the thermal properties of solids," *Izmer. Tekhn.*, No. 9, 45–48 (2015).
3. K. S. Stasenko and Z. M. Selivanova, "A methodology and an intelligent information and measuring system for enhancing accuracy of quality control of thermal conductivity of thermal insulating materials," *Vestn. Tambov. Gos. Tekh. Univ.*, **21**, No. 1, 42–49 (2015).
4. Yu. L. Muromtsev and Z. M. Selivanova, Patent 2301996 RF, "Method of nondestructive monitoring of thermophysical properties of materials and products," *Izobret. Polezn. Modeli*, No. 18 (2007).
5. D. Yu. Muromtsev, S. V. Artemova, and A. N. Gribkov, "Prediction and compensation of perturbations in optimal control systems," *Vestn. Tambov. Gos. Tekh. Univ.*, **9**, No. 4, 632–637 (2003).
6. D. A. Kabanov and Yu.Ya. Korotkov, *Statistical Testing in Analysis of the Accuracy of Electronic Equipment: Textbook*, GPI im. Zhdanova, Nizhnii Novgorod (1984).
7. MI 222-81, *Methodology for Calculating Metrological Characteristics of Measurement Channels of Information and Measuring Systems According to Metrological Characteristics of Components*.
8. GOST P 8.585-2001, *GSI. Thermocouples. Rating Static Characteristics of Conversion*.
9. RD 153-34.0-11.201-97, *Methodology of Determining Generalized Metrological Characteristics of the IMS and ACSTP Measurement Channels According to Metrological Characteristics of Modular Measuring Instruments*.