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A COMPARATIVE ANALYSIS OF THE DISCHARGE COEFFICIENTS OF VARIABLE PRESSURE-DROP FLOWMETERS

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Possible additional errors in measurements of flow rate and quantity of gas with the use of the discharge coefficient of standard orifices are considered.
Keywords: *orifice, flow rate, pressure drop.*

The variable pressure-drop method is widely used for measurements of flow rate and quantity of liquids and gases. Hydraulic resistances that create a contraction in a section that produces a pressure drop as a function of the flow rate is usually used as the flow rate transducer. A standard orifice in the form of a flat disk with concentric round hole is the most commonly used contraction. An important parameter used in this method is the discharge coefficient C, which characterizes the ratio of the effective value of the flow rate of a substance passing through a contraction to the corresponding value calculated from a theoretical model of the flow rate of the substance [1]:

$$C = 4Q_m\sqrt{1-\beta^4}\Big/\left(\varepsilon\pi d^2\sqrt{2\Delta p\rho}\right),$$

where Q_m is the mass flow rate of the substance; β, relative diameter of orifice, $\beta = D/d$; D, diameter of the measurement pipe; d, diameter of hole of contraction; ε, correction factor for expansion (for incompressible substances $\varepsilon = 1$); Δp, pressure drop; and ρ, density of substance under working conditions.

The required precision is not realized in theoretical determinations of the initial discharge coefficients of a contraction [2], hence the coefficients are found experimentally and represented in the form of empirical interpolation equations. A comparative analysis of the discharge coefficients obtained from the empirical equations along with a comparison of these coefficients with the results of recent experimental studies [3] will be proposed in order to estimate the precision of flowmeters.

Empirical Equations of Discharge Coefficients. Different researchers defined the discharge coefficient by the formula $\alpha = CE$, where $E = 1/(1-\beta^4)^{1/2}$ for different values of β, but did not take into account the dependence of C on the Reynolds number Re, which is responsible for an additional error in the measurement of flow rate [2]. The introduction of corrections decreased the total error only down to 0.3%.

The results of the experimental data were processed by Stolz [4] based on the Reynolds number and interpolated in the following equation [4]:

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 0.0029\beta^{2.5}(10^6/\mathrm{Re})^{0.75} + 0.09L_1\beta^4/(1-\beta^4) - 0.0337L_2\beta^3, \quad (1)$$

where L_1 and L_2 depend on the method used to sample the pressure at the orifice. The error of the discharge coefficient calculated from (1) is as follows: $\delta_C = 0.6$ with $\beta \le 0.6$ and $\delta_C = \beta$ with $\beta > 0.6$.

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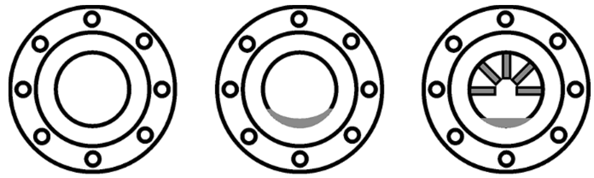

a *b* *c*

Fig. 1. Test models of contractions: *a*) undisturbed orifice; *b*) PTB model; *c*) SP model.

Studies designed to update the discharge coefficient (for example, [5]) then began. In 1998, Reader-Harris and Gallagher proposed the following improved solution for the discharge coefficient (RHG equation), which was used in [6] and then in [7]:

$$C = 0.5961 + 0.0261\beta^2 - 0.216\beta^8 + 0.000521(10^6\beta/\mathrm{Re})^{0.7} +$$

$$+ (0.0188 + 0.0063A)\beta^{3.5}(10^6/\mathrm{Re})^{0.3} + \left(0.043 + 0.08\mathrm{e}^{-10L_1} - 0.123\mathrm{e}^{-7L_2}\right)\times$$

$$\times (1 - 0.11A)\beta^4/(1-\beta^4) - 0.031\left(M_2 - 0.8M_2^{1.1}\right)\beta^{1.3} + M_3. \qquad (2)$$

Here $A = (19000\beta/\mathrm{Re})^{0.8}$; $M_2 = 2L_2/(1-\beta)$;

$$M_3 = \begin{cases} 0, & D \geq 0.07112 \\ 0.011(0.75-\beta)(2.8 - D/0.0254), & D < 0.07112. \end{cases}$$

The errors of the discharge coefficients calculated with the use of (2) are as follows: $\delta_C = \pm(0.7 - \beta)$ with $0.1 \leq \beta \leq 0.2$; $\delta_C = \pm 0.5$ with $0.2 \leq \beta \leq 0.6$; and $\delta_C = \pm(1.667\beta - 0.5)$ with $0.6 < \beta \leq 0.75$.

The precision with which the coefficient C is determined with the use of (2) is higher than that found with the use of (1), and a comparative analysis of these equations is given in [1]. Let us compare these results with recent experimental data obtained at the national metrological institutes of Germany (Physikalisch-Technische Bundesanstalt, PTB) and Sweden (Sveriges Tekniska Forskningsinstitut, SP).

Comparative Analysis of Discharge Coefficients. Experiments related to the influence of the temperature and profile of a flow on the coefficient C are described in [3]. An orifice with asymmetric opening in which a small part of the flow section is reduced by an arched insert is used in the model proposed by PTB. In the model proposed by SP, one half of the orifice of the flow section is reduced by a small chord, while five equidistant rectangular inserts are situated in the other half, thus significantly disturbing the flow by comparison with the PTB model. The models are represented in Fig. 1.

All the experiments demonstrated that temperatures in the range 20–85°C do not have any effect on the coefficient C. The dependences $C(\mathrm{Re})$ for orifices with $\beta = 0.5$ calculated on the basis of (1) and (2) as well as the results of experimental studies on an undisturbed orifice are presented in Fig. 2, whence it follows that the results of verifying experiments for an undisturbed orifice significantly exceed the values obtained from the empirical equations (1) and (2). In [3], it is reported that the coefficient C calculated on the basis of (2) coincides with the experimental values for the contraction model represented in Fig. 1*b*, in the range of Reynolds numbers $\mathrm{Re} = (1\text{–}8)\cdot 10^5$. Does this mean that the use of standard orifices as primary flow transducers introduces a systematic error in measurements of the flow rate of a gas when C is calculated on the basis of (2)?

Let us compare the values of C obtained for an undisturbed flow with values calculated with the use of (1) and (2). For this purpose, we will use the relative difference equation

$$\delta = (1 - C_{(1),(2)}/C_{\mathrm{e}})\cdot 100,$$

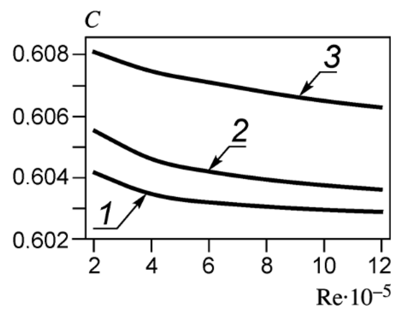

Fig. 2. Discharge coefficient C as a function of the Reynolds number Re for $\beta = 0.5$: *1*, *2*) calculations with the use of (1) and (2), respectively; *3*) results of an experiment with an undisturbed orifice.

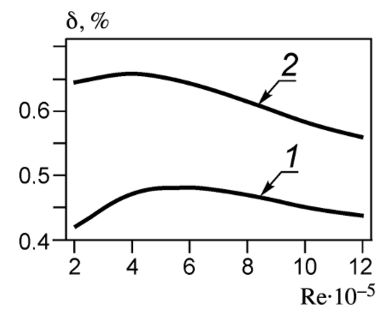

Fig. 3. Relative difference as a function of the Reynolds number Re for Eqs. (1) and (2) – curves *1* and *2*, respectively.

where $C_{(1),(2)}$ are the discharge coefficients calculated on the basis of (1) and (2), and C_e is the coefficient obtained in the course of experiments on the model *a* (cf. Fig. 1).

The dependences $\delta(\mathrm{Re})$ for an orifice with $\beta = 0.5$ are shown in Fig. 3. It follows from Fig. 3 that Eq. (2) yields an average deviation equal to 0.45% relative to the experimental data for an undisturbed contraction, while Eq. (1) yields a deviation of 0.62% on average. On the other hand, the following alternative equation for C, which does not depend on the Reynolds number Re and is a function of the ratio of the pressure drop Δp at the orifice to the absolute pressure p before the contraction, is proposed in [6]:

$$C = a_1 + a_2\beta^{3.75} + a_3\beta^4 + a_4(\Delta p/p)^{1.25} + a_5(\Delta p/p)^{2.25}, \quad (3)$$

where $a_1 = 0.59865$, $a_2 = 0.81891$, $a_3 = -0.86143$, $a_4 = 0.25169$, $a_5 = -2.2216$.

The main advantage of Eq. (3) is that the coefficient C calculated from this equation is not a function of the Reynolds number Re and depends on the measurable parameters of the flow, whereas the Reynolds number Re must be constantly refined in the course of repeated iterations with the use of formulas (1) and (2).

Thus, a comparative analysis of the results of a calculation of the coefficient C with the use of Eqs. (1) and (2) has been carried out using recent experimental data. From the analysis, it is clear that the use of Eq. (2), which was presented in [5], yields a deviation of 0.45% relative to the refined experimental data for a relative diameter $\beta = 0.5$. If this is so, the error (uncertainty) of the discharge coefficient is one of the components of the error (uncertainty) of measurements of the flow rate and is equal to 0.45%, disregarding the other components of the measurement error. What this means is that studies focused on improving the value of C for a contraction of variable pressure-drop flowmeters must be continued. It is possible that a correction factor must be introduced into the technique used to carry out the measurements in order to increase the precision of Eq. (2) or efforts must be focused on obtaining a dependence of the type of (3) that would be simpler to interpret and more easily implemented in the computer components of flowmeters.

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